

SHELLS 1970—HISTORY AND OUTLOOK

By ANTON TEDESKO

The history of the development of shells in the United States is reviewed. Current trends and practices are examined. Observations and recommendations are made with respect to the future.

Fewer industrial shell structures are built today than in former years, when the few engineers who designed shells and knew how to build them were active in their promotion. Today's shell designers are too often far removed from the construction and are not sure of construction costs. The author's observations are applicable also to other fields of building construction, as more and more contractors become merely brokers and, lacking extensive concrete experience, prefer solutions which keep their field forces small, and as clients with a limited budget tend to purchase prefabricated structures which can be bought from fixed price lists. The author, therefore, earnestly hopes for greater collaboration, as in the past, between designer and constructor.

Outstanding and monumental structures will be designed and built, and computers, excellent tools in the hands of experienced engineers, will be used. Computers can show us what new shapes are possible, but they are no substitute for engineering judgement. Rationally designed, imaginative, and economical shell structures will result if the abilities of the new generation of talented, esthetics-minded designers are blended with the effective use of computer technology and the experience and know-how of the older generation.

Keywords: history; reinforced concrete; shells (structural forms); structural engineering.

INTRODUCTION

□ These opening remarks, made at an ACI Symposium on Concrete Thin Shells, are a look back and some reflections on the history of the development of shells in the United States, a look at current trends and practices, and a look forward to what can be expected if present trends continue.

Having been deeply involved in shell design and construction in this country for more than 30 years, my comments are based on personal experiences and my desire to share these experiences with the profession.

The history of shells in this country can be broken down into several distinct periods. Shell theory and standards were of European origin. Names such as Zeiss-Dywidag, Dischinger, Finsterwalder, and Torroja come to mind.

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Period 1 First came the introduction and the years of pioneering in the nineteen thirties. (Shells in this country were never copies of designs used abroad.) Almost all the design work, the promotion, and the supervision of construction in this country was done by a single company of engineers (and I am proud to have belonged to that group),* assisted by the promotional staff of the Portland Cement Association. Most architects of those days considered a shell as something to be hidden, just as steel trusses were usually concealed. Shells were not seen as a fresh concept of space enclosure.

As an example, many years ago I designed a flat and what I considered quite an elegant dome for the McAlister Auditorium at Tulane University. This dome is supported at 8 points, but when it was finished one could not see the structural system. A shell was not considered fit to be seen; it had to be covered by an architectural veneer. There are heavy walls, a ceiling underneath the dome, and above, a patterned roof in colored tile.

The architects of those days chose shells only when they could be built for less money than the conventional steel structure, and especially when the owner could be persuaded that in using a shell there was some advantage from the standpoint of fire safety, although insurance companies at that time were too conservative to look into the question of lowering insurance rates.

Economical use of shell materials and efficient use of labor did not mix. Labor-material cost relationships in America were different from those in Europe. Low cost labor, which makes possible economical construction of thin shells in Mexico, is simply not available in New York. As a result, American shells are different from their foreign counterparts, with the designs emphasizing labor savings at the expense of additional material. Ingenious contractors contributed new ideas and new job applications, as the design of forming and the handling of labor were of much greater importance here than they were in Europe. These

* Roberts and Schaefer Company, Engineers, founded in 1903 as an Illinois corporation, performed more than 3000 engineering assignments during the first half of the century. Beginning in 1954 the company went through several corporate mergers and became a subsidiary of Thompson-Starrett and other publicly-owned corporations. After 1968, only the Chicago branch, active in the design and construction of plants for the mining industry, continued under the Roberts & Schaefer name as a division of Elgin National Industries.

improvements in construction technique made possible the success of American shells, which were the product of resourceful designers who visualized the construction job as it would be built while the structure was still in the design stage.

Many shells were built during World War II, including industrial plants, military installations, and airplane hangars. They required less strategic materials, did not use much steel. Structural efficiency and performance were the important requirements during these years, with the architect playing a minor role. There was no emphasis on appearance, but the structures built turned out to be clean and simple, based on function and not on architectural dreams (Fig. 1-1).¹ Corbetta* built a shell structure, 180 by 1560 ft in 34 calendar days (still a record), his price being less than the price of temporary wood construction.

The shell boom continued after the war. At this point a second firm of engineers† entered the field and by the end of this first period shells were the domain of two recognized engineering firms who at times associated themselves with local engineers (Fig. 1-2).

By now modern architects had been installed as heads of all prestige schools of architecture in the United States, giving rise to a new generation of architects. This had its influence on the shape of shells in Period 2.

Period 2 This period was initiated with the publication in 1952 of the ASCE Manual 31 on Shell Design,²‡ which ended the technical domination by the two firms and, with the help of the Portland Cement Association, enabled more engineers to design cylindrical shells. Other firms of engineers entered the field and the purity of the shell form became more generally appreciated. The Portland Cement Association's promotional support shifted from the two pioneering firms to small, local engineers; this brought on some unexpected economic side effects which will be mentioned later. Some of the new men in the field erroneously assumed that the design of shells would cost no more than the design of any other type of construction.

In this Period 2, there should be mentioned a variety of shells built for the Space Program and our national defense: underground installations, such as missile launching facilities, launching control centers, often designed for very substantial loads as may be produced by misfiring space vehicles or nuclear explosions. Many thousands of cubic yards of concrete were used in these shells (Fig. 1-3, 1-4, 1-5, and 1-6).^{3,4,5,6}

Period 3 The third, current stage was initiated by the 1964 report of the shell committee.⁷ By this time, many engineers and architects were at work, often competing with each other. The results were a great variety of new ideas and designs and better looking shells in the hands of creative architects (Fig. 1-7).⁸ Unfortunately, there were a number of poor shells and of failures when men with insufficient knowledge did the designing. As in the medical field, there were many good practitioners around, and some who are not so good.

*Corbetta Construction Company, New York.

†Ammann & Whitney, Engineers, New York

‡Primarily based on the work of Alfred L. Parme.

ECONOMICS OF DESIGN

Using the medical field for another comparison; the firm of shell designers of the first period (no longer operating in this field of engineering) might be compared with the Mayo Clinic. Doing their own testing, sponsoring university research, keeping labor cost records, they maintained a competent staff of shell experts and construction advisors. They helped interested contractors plan and estimate a shell job, showed films of the construction of shells (taken at their own expense), and planned jobs in cooperation with ingenious contractors. Engineering fees were ample, with money allowed for extensive engineering-design services during the construction phase. These consultants could afford to maintain a competent organization. The result was fully-engineered shell projects constructed on a planned manufacturing cycle, which were successful in competing with other types of framing on large projects. Millions of square feet of shells built for industrial use were the result. These engineers in a sense paid back to the P.C.A. the debt for its past promotional efforts by opening up the market for shells. Many large jobs which had been designed for wood or steel were converted into concrete. This firm, of course, spent thousands of dollars on the design of structures which for various reasons did not reach the construction stage or did not go ahead as shells. Some may remember that in New York, many years ago, I was part of a team designing a shell roof, column-free over spans of 540 by 580 ft, for a convention hall to be built in the New Jersey meadows; Walter Ahlschlager was the architect. More than 5000 pre-war dollars were invested, but due to political changes in New Jersey the job never went ahead and the engineers were never paid. During this period fees had to be higher than conventional fees. Engineers furnished more than just the minimal design drawings. This helped the contractor, brought down his costs, ultimately benefiting the client.

Engineering fees have decreased during the last 20 years (Periods 2 and 3) as the field became more competitive. Costs to design a shell are higher than to design a comparable conventional structure. The completeness of the designs suffered in this squeeze. The pioneering companies could no longer afford to keep their all-around experienced staffs and to make available the construction advice of earlier days. Films which previously were shown during pre-bid conferences to create a fuller understanding of problems and costs were not shown anymore. Fewer competent contractors were induced to bid. Contractors who did not have shell experience came in with high prices and few engineers knew whether their cost estimates would be met during actual bidding; as a matter of fact, hardly any shell designs were made from then on for competitive bidding with other types of construction.

Today there are fewer shells for big industrial installations than there were 20 years ago. Shells are being designed by engineers who are inexperienced in the field and although the number of shell designers increased under this policy, the total area covered by shell structures built must have decreased.

It is ironic that in earlier days the design fee could be paid out of savings over other forms of construction made by the owner. Assuming for the purpose of this comparison that all costs remain relatively constant, the owner now pays more for a shell job, and the engineer gets less, due to greater competition. Of

course, labor prices have not remained constant; great advances have been made in prestressing techniques; these are other reasons that the number of shell jobs has not attained the promise of the late fifties and early sixties at which time there was great enthusiasm for shells and their future possibilities.

CURRENT TRENDS

More and more of the general contractors who turn in low bids on public jobs are merely brokers who do not have extensive concrete experience. Where these men have influence on decisions, the decisions are in favor of solutions requiring fewer men on the job. In this age of demonstrations, stoppages, and interruptions of all kinds, a cast-in-place shell is more vulnerable to unforeseen events and to pressure tactics than is a pre-cast job. If concrete finishers on a shell structure walk off at 4 p.m. because they do not want to work overtime, the final product is affected. If all workers are on the job for an early start in concreting a large shell on a hot summer day, and if the concrete supplier sends a fleet of trucks delivering concrete which is not of the type specified for hot-weather concreting, a loss in time and dollars will result. The contractor who handles merely precast parts of concrete, avoids this problem.

With the mass market for shells fading, there is a tendency to go to plant-produced structures, made of precast, prestressed elements for which firm prices can be obtained. This type of construction requires fewer men, is easier to design and takes some of the guesswork out of estimating.

In America, shells for single-story industrial buildings are not utilized unless aesthetic considerations play a dominant role, or unless an enlightened owner assembles a competent team, pays a competent engineer a sufficient fee to enable him to do a complete planning job, inclusive of model testing if deemed advisable (design of centering, etc.), and a competent contractor can be interested in the construction. Shells particularly benefit by joint planning of design and construction.

(Thinking of competent engineer-contractor teams, names like Candela and Arthur Anderson come to mind; unfortunately, there are not many such teams in existence. Abroad, Finsterwalder, Nervi, and Esquillan have great teams.)

It is the separation of design and construction which has worked against shells in our economy. Unfortunately, because of our economic system, too many engineers are far removed from the construction, exert no control, and feel their job is done when their drawings are finished; they do not get paid for supervision of construction and their attitude if the contractor gets into trouble is: that's his own business. Such engineers are co-responsible for ending up with delays, a patched job, and with shell projects which do not come up to their full potential.

Those designing shells are usually unable to tell a client accurately what the job will cost, as they are dependent on whether a contractor with or without previous shell experience happens to bid the job.

On moderately-sized shells which are part of a larger contract, it does not make much difference whether these shells cost more or less, the shell cost usually is hidden. On larger jobs I would recommend that we follow the original concept of Period 1, as much as possible, that is, maximum engineer-contractor collaboration; if you want progress, this is one way of getting it, although it is not the American practice at this time. The ultimate goal is to get a building, not a design! I also recommend making use of precast parts in cast-in-place shell structures. Such jobs in which the contractor had appreciable influence on the design have recently been built abroad and are well suited to American conditions. The larger the job the greater the possibility that an on-the-job plant producing monolithic shells will do the job better and more economically than a plant manufacturing prefabricated parts which have to be shipped to the site and then erected.

Let us learn from the past that there is no substitute for competence. It pays to design a job right in the first place and to avoid correcting deficiencies later, after things have gone wrong. The costs of corrective measures usually are a multiple of the costs of items not taken care of at the beginning. The average practitioner in the medical field calls in a specialist; why can not the average engineer do the same? In this day of specialization no one has to be ashamed of not knowing everything. Would anyone encourage a medical practitioner to perform major surgery in a field in which he lacks experience? Not likely! So let us drop the idea that every engineer is competent to design shells.

In the interest of better shells and of making the designer aware of the construction aspects, I would rather consult with an engineer at the beginning of a significant job as to what he should do with his shells, than to be called in by the contractor later, and placed in the position of having to protect the public, help the contractor, and bail out the engineer.

THE FUTURE

What does the future hold?

There is a field for shell roofs of moderate size for churches and other eye-catching buildings where a shell adds an attractive or unusual element and gives an air of elegance. For long spans where the saving in weight is an important factor, shells must be seriously considered.

What constitutes a pleasing shell structure?

Flat surfaces are easier to form than most curved surfaces; some people thereby justify their preference for folded plates. Some types of shells were used until they became boring architecturally; then the pendulum swung back as architects tried to build something different. Too often an architectural designer thinks he has to incorporate something new, even though the new is not necessarily good. I have seen streamlined shapes with good aerodynamic properties which might be more appropriate for flying saucers. I, personally, do not believe that a "free form shape" is the solution, even though a computer might be able to calculate the stresses. Nature builds simply and with economy and I believe that our shells would be more pleasing to the eye if they followed some geometrical law, in the

manner of snow flakes or a rainbow. Countless combinations of cylinders, cones, and simple doubly-curved shapes have not been utilized and would produce logical and attractive solutions. It is an exception if complicated or far-fetched solutions turn out to be good. Good designers can produce excellent effects with simple shapes. Consider the outstanding shells of men like Finsterwalder, Nervi, Torroja, and Candela. These men refined in a subtle way simple shapes; the refinements had structural meaning and resulted in elegant solutions.

Questions of cost and re-use of forms may have influence on the solution to be adopted. In many cases, however, the architect and the engineer are picked to do the job as an artist is commissioned to do a painting. Economy in the execution and the final price of the work are seldom criteria in the award.

Outstanding and monumental shell structures will be built. Prestige architects will be involved, some of them without a particularly well-developed sense of the practical elements of construction. Shell structures in the past were built with the construction cost occasionally going much higher than anticipated. A high price can be justified in our complex economy when added attractiveness is achieved, a cost which in a sense represents a form of advertising. TWA did not "lose" by erecting a costly and beautiful terminal at J. F. Kennedy Airport. The terminal is a piece of sculpture and should not really be called a shell. Political aspects have influenced decisions in favor of expensive solutions which an engineer would not have recommended. The history of cost increases due to naiveté and inexperience is not limited to this country. Very well-known structures are examples of cost multiplication. The Sidney Opera House is an illustration of what may happen when shape comes first and design comes later. When funds are not limited, there need be no limit to new shape, the scope, and size of our shell structures.

There is no generation gap when it comes to shells and shell designers. There is no gap between the good designers of earlier days and good engineers of today. Progress in shells is in the hands of forward-looking men, the young and the more mature, those who make optimum use of good practice, old and new, using the powerful weapons of today: the computers. Some of our younger men know more of shell analysis and computer solutions; the older men may know more of practical problems and the cost of construction. In the interest of progress this great talent and experience which are available must be blended.

Having been involved in many shells, I know that none of the great classic shells of the past would have become a better shell by the use of any one of the computer solutions available today. Computers cannot replace engineering judgment and are no substitute for good design. Computers can show us what new shapes are possible; they opened up new opportunities and are a wonderful tool in the hands of experienced engineers for speeding up calculations and replacing the tedious work of former days. They make it easier to cope with complicated shapes.

Computer solutions should not be misused for mathematical exercise. The purpose of calculations is not to keep us in trim intellectually. Their purpose is "to build"!

CONCLUSIONS

This brief look at the history of the development of shells, and the examination of current trends in the United States, lead me to the conclusion that while in present design practice there are some misguided applications of shell structures and lost opportunities for using shells, there is also strong hope for the future. A new generation of shell designers has grown up in the United States. They add the gift of imaginative, artistic sensibility, and greater computer technology to the practical knowledge, understanding, and intuition of the older generation. These younger men are more structural sculptors than mass-producers of shells, more partners in the architectural team than engineer-workers producing, as in former days, an inexpensive structural system to carry the architect's loads. Shell design has been advanced as a result of this development.

Rationally-designed shell structures will have the greatest promise whenever there is cross-fertilization between the minds of the designer and the constructor.

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Fig. 1-1—Ellipsoidal shell domes for trickling filters, Hibbing, Minn., 150 ft span (author's photograph)



Fig. 1-2—Shell units for aircraft hangar of US Air Force, clear span of 340 ft was longest at that time, stiffening arches restrained by heavy buttresses follow catenary (photographed by author) (similar hangars for American Airlines of 260 ft span, engineered by Ammann and Whitney, followed the same year)

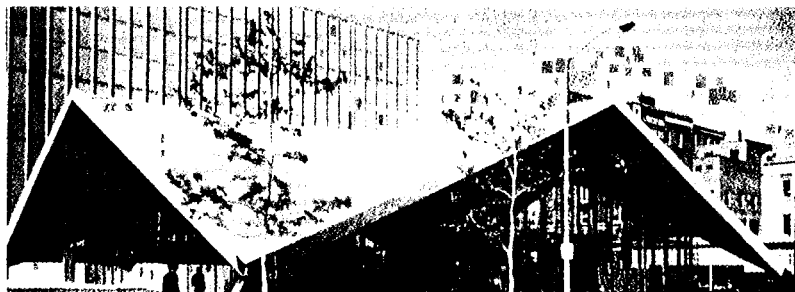


Fig. 1-3—Shell at Denver was longest span hyperbolic paraboloid (author's project)



Fig. 1-4—Cast-in-place shells for US Government installation proved competitive with precast concrete structure (author's project)

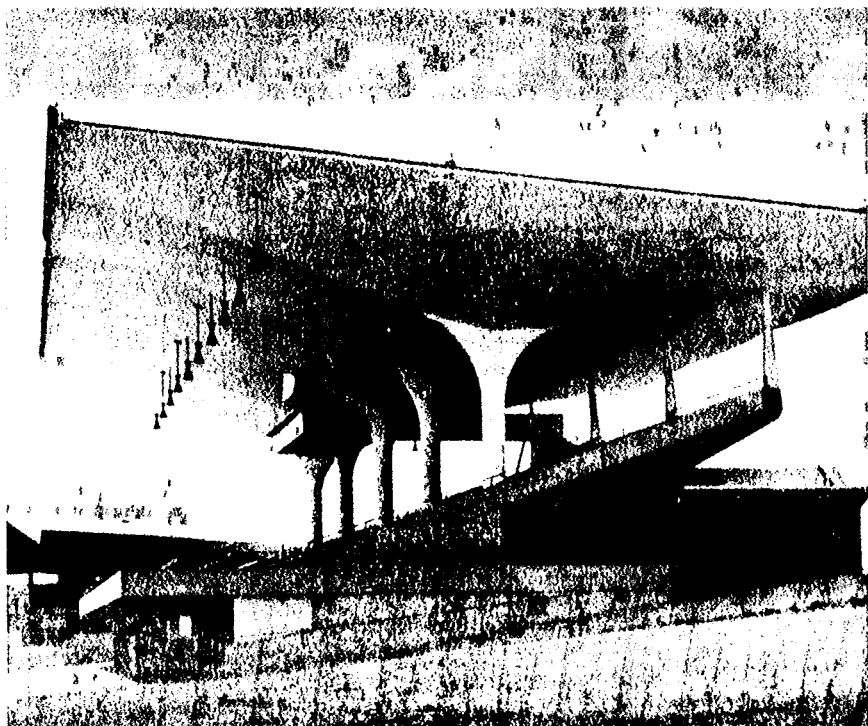


Fig. 1-5—Grandstand at Columbus, Ohio (R. M. Gensert Associates, Engineers)

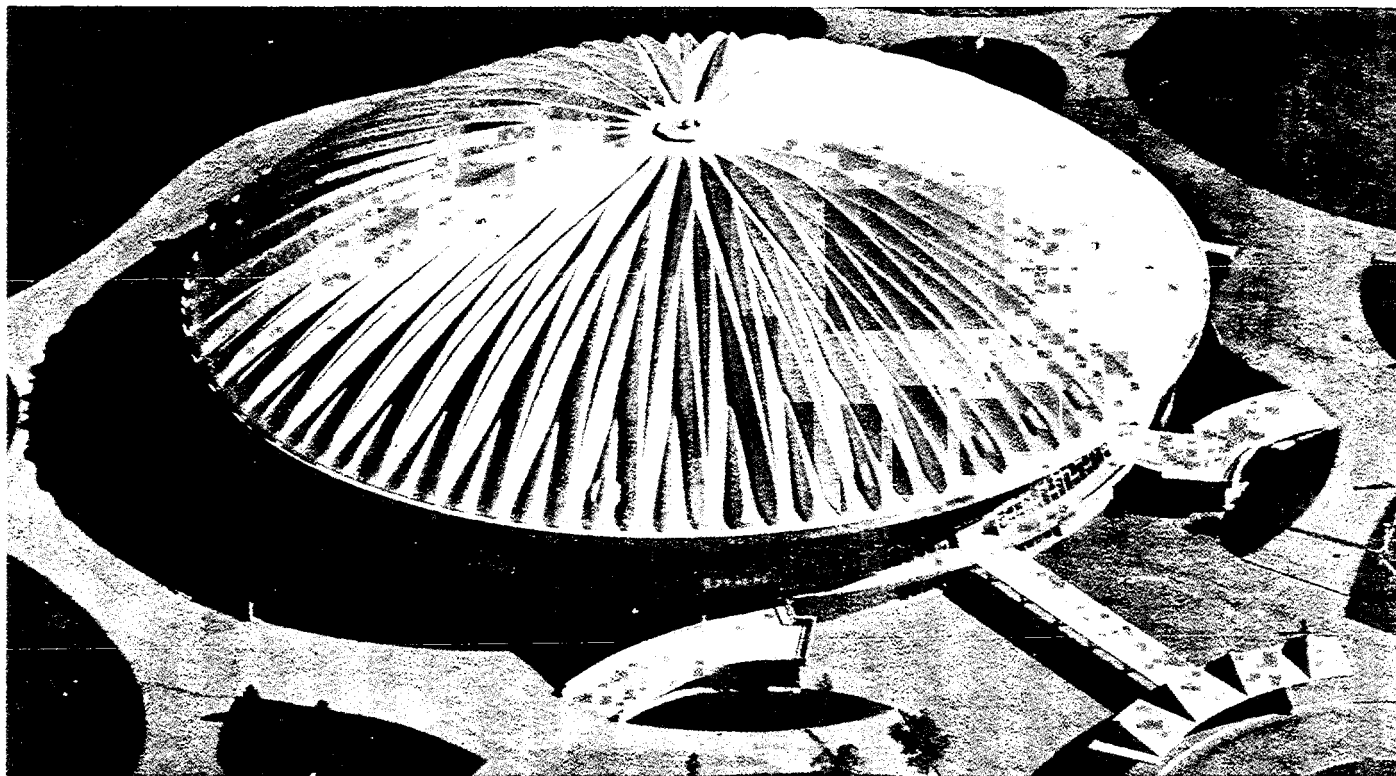


Fig. 1-6—Pleated shell for assembly hall, University of Illinois (Harrison and Abramovitz, Architects;
Ammann and Whitney, Engineers)

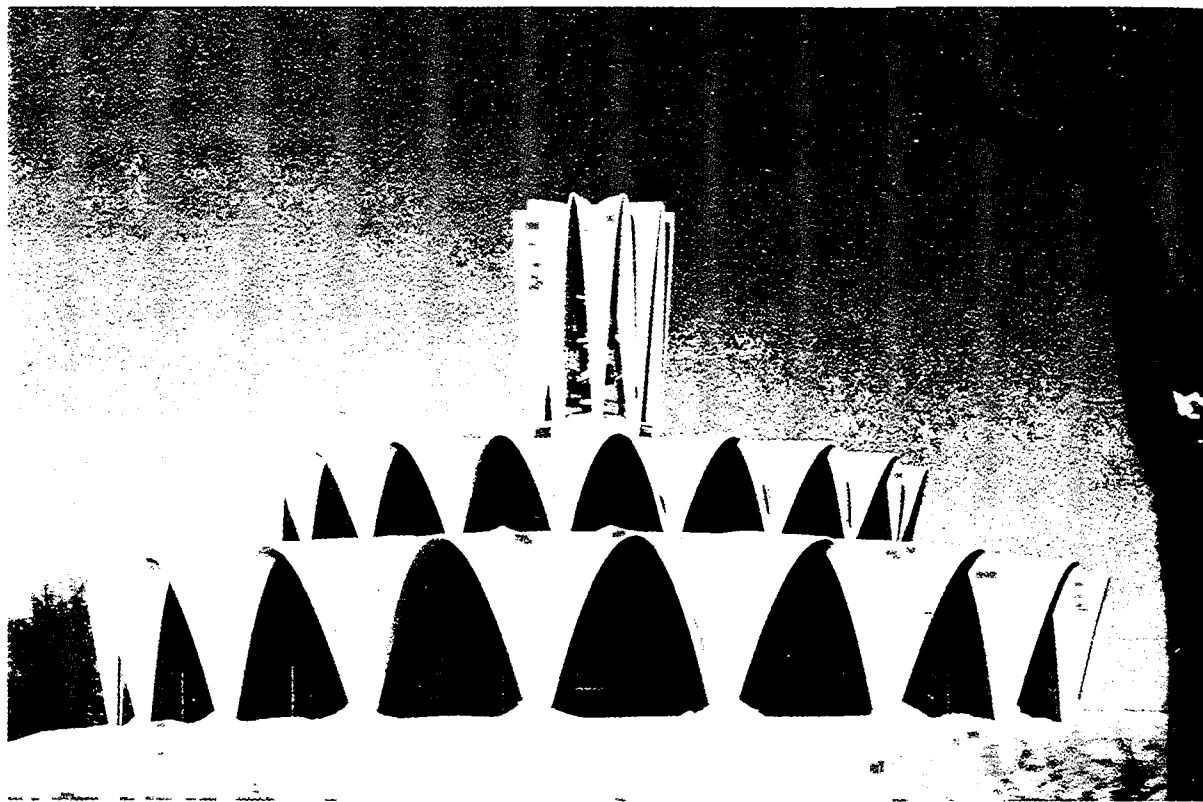


Fig. 1-7—Priory Chapel, St. Louis, Mo. (Hellmuth, Obata and Kassabaum, Architects; Paul Weidlinger, Engineer)



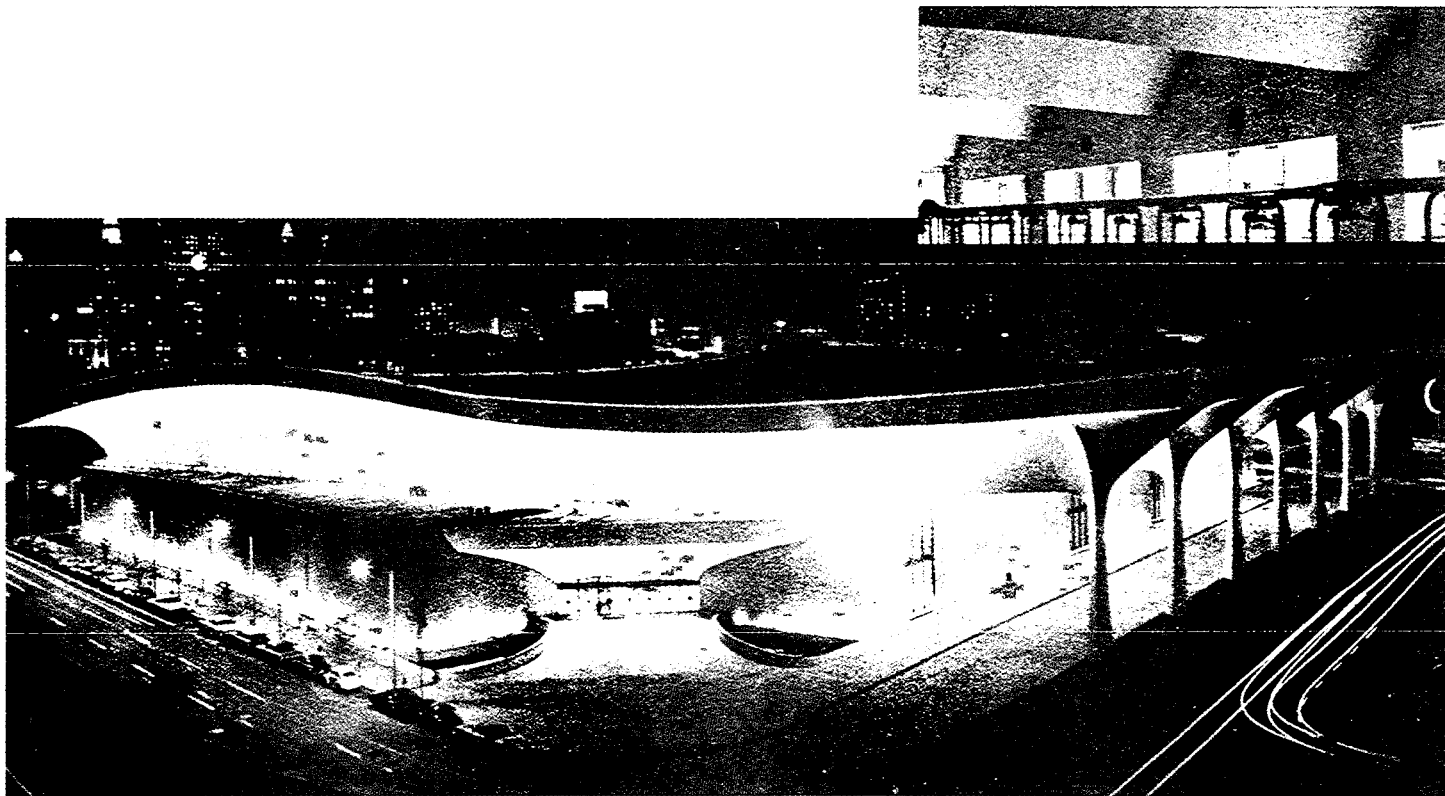


Fig. 1-8—First small translational barrel shells were built in Chicago (designed by the author), thirty years later Skilling-Helle-Christiansen-Robertson designed these eye catching translational shells of vastly greater magnitude

CONCEPTUAL DESIGN OF SHELLS

By STEFAN J. MEDWADOWSKI

A review of the state-of-the-art of conceptual design of concrete thin shells is presented. An examination of shell forms and of the factors which shape their development is followed by a critical review of design goals and of the methods for achieving these goals. The nature of human response to a work of architecture is considered, and some conclusions are drawn regarding its importance to the conceptual design process.

Keywords: architecture; history; masonry; planning; reinforced concrete; shells (structural forms); structural design; structural engineering.

INTRODUCTION

□ Thin shells are unique among structural systems for the interest they inspire. Architects like the dramatic space-creating possibilities inherent in their forms, while engineers are impressed by the efficiency with which material is utilized. The fascination with shells is shared by critics. As a form, and as an expression of the age, Huxtable considers concrete shells as "...undeniably today's most significant architectural adventure,"¹ Giedion sees in them "...the vault of our period in the great public buildings to come," and observes that the "...egg-shell-thin slabs which can be bent like cardboard have both the buoyancy we like and the permanence we like,"² while Von Eckardt comments: "...here is at least a promise ... that the new forms might also have new social meaning."³

By definition,⁴ a shell is a structure which, in the large, has the form of a surface or a combination of surfaces in space. The thickness of a shell is small compared to its other dimensions. From this geometry shells derive their most important characteristic: the ability to carry loads to supports primarily through the action of in-plane stresses. This permits a very efficient utilization of material and allows shells to cover large areas without intermediate supports. It is seen that, in consequence of this definition, folded plate structures also belong to the category of shells although, inherently, they are subject to appreciably larger bending stresses, and hence utilize material less efficiently. Also, since the nature of the in-plane stresses is not further specified, shells include membranes and tension structures. Finally, skeletal systems which form a surface in space also meet the definition of this paragraph and hence are classified as shell structures. Further justification for this is afforded by the observation, already noted by such early pioneers in the field as Bauersfeld,^{5,6} and Torroja,⁷ and since confirmed by other researchers,^{8,9} that the structural action of such systems is the same as the action of a shell, in that loads are transferred primarily through in-plane stresses. Historically, some of the earliest shells were partially, at least,

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skeletal in form. Thus the inclusion of skeletal surface-type structures among shells appears both reasonable and proper.

All projects, and hence also the projects which involve shell structures, are realized in stages. These may be divided into conceptual design, detailed design (which includes stress analysis), and construction. These three broad stages are obviously interrelated and closely influence one another. Thus, as an example, the choice of the method of construction may be influenced by such design decisions as the type and quality of concrete finishes. Conversely, the knowledge of the method of construction, should it be predetermined at the time conceptual design is being prepared, will influence the choice of form. The form will be also influenced strongly, often decisively, by the type of the structural system selected for the project; this is particularly true of shells in view of their near identity of architecture and structure.

Conceptual design is then the first stage of the process of realization of a project; it is the stage during which the major decisions are made regarding the form and the structural action of the building. In the case of shells, because of the strong identification of the structural and architectural form,¹⁰ the decisions made in the course of conceptual design almost completely determine the degree of success achieved in fulfilling the functional requirements of a shell project; here the word "functional" must be interpreted in the broadest possible terms.

This paper is concerned with the conceptual design of concrete shells; an examination of shell forms and of the factors which shape their development is followed by a critical review of design goals and the methods for achieving these goals. On the assumption that beauty is one of the aims of design, the extent to which a completed project is beautiful, and the interrelation between structural form and beauty, will form an important part of this review.

FACTORS AFFECTING FORM OF SHELLS

Works of architecture, unlike other works of art, have a utilitarian background.¹¹ The designer of a shell does not operate in a vacuum: his project, after design, must be built and must function. Hence many factors affect the selection of form of a shell. These factors may be listed as follows:

Tradition: by this we mean shell forms which exist in nature as well as shell forms previously used on other projects, perhaps even in the distant past. The projects need not necessarily involve shells—they might simply have had an impact on the shell designer.

Construction technology: information on the material; availability of construction equipment, power, and skilled labor; cost of construction of various systems.

Structural mechanics: the extent to which a given shell form has been studied, and the availability of methods and design aids for the performance of the necessary calculations; today it means as well the availability of computers to perform these calculations.

Considerations of utility: the extent to which a given shell form is suitable for the specific project; thus a hyperboloid of revolution suggests itself as the appropriate shape for a cooling tower.

Theories of design: they are the means through which the aims and methodology of design are systematized, permitting a rational approach to such design problems as shape selection.

Shells, as their very name implies, occur in nature in a multiplicity of forms. Egg shells, shells of nuts, plants and leaves of various type, snails, many insect forms, all represent nature's attempt at efficient use of material, possible because of the relatively few factors affecting the growth of a given plant or given species. The natural forms of shells have been known to man from time immemorial and must have influenced his design choices, even if not consciously.

Shell-like structures have been built by man since antiquity, when the arcuated forms of Roman architecture were extended into three dimensions. The material was masonry and concrete, probably developed from the pisé system of construction through the addition of the "wonderful dusky-red pozzolana" natural cement.¹² The forms used by the Romans were a hemispherical dome, as in the 142.5 ft diameter roof of the Pantheon completed in the middle of the second century, or a semicircular barrel vault, such as used in the roofs of some of the *termae*. While the Roman examples of domes and vaults do not constitute thin shells in the modern sense, they certainly represent a tremendous engineering achievement when considered in the context of the day. In fact, the span of the Pantheon had not been exceeded until after the industrial revolution brought with it the application of iron and steel to construction. Architecturally, the effect on the modern visitor is one of awe and admiration, "by reason of the severe simplicity and unity of the design."¹³

Domes and vaults were commonly used during the Byzantine period. The best and the most famous example is the church of St. Sophia built in Istanbul in the first half of the sixth century and later converted into a mosque, with the resulting addition of minarets. The roof consists of a series of domes, the largest a hemisphere 107 ft in diameter, supported vertically by circular arches, and laterally, in part, by a pair of ancillary semi-domes augmented by four additional excedra shells. According to Fletcher, "the detailed effect, with the great hemicycles and the smaller excedrae, is one of extreme intricacy, in spite of the

simplicity of the general scheme" (cf. Reference 13, p. 283). This apparent intricacy, combined with the delicate structure of the base of the main dome may be responsible for the extraordinary lightness and the sense of space of the interior, of which Zevi spoke as "space in the process of expansion,"¹⁴ thus contrasting its dynamic nature with the more static qualities of the Pantheon. This apparent intricacy, as may be of interest to note, has led also to some speculation on the role of the ancillary shells and exedrae¹⁵; however, the structural action appears to be as described above. Another example is afforded by the domed roof of the church of San Vitale in Ravenna, noted for its use of pottery as the material of the shell. A timber roof was placed above the masonry dome, protecting it from elements but also completely hiding the form from the outside.

The use of concrete was almost forgotten in the middle ages, but the tradition of masonry vaults continued. Thus, as an example, the narthex of Notre Dame in Paris, built in the twelfth century, is framed of 4 in. thick masonry and spans 40 ft. The form itself changed from the Roman (and Romanesque) semicircular to the pointed arch, possibly due to the Arab influence. The shell itself is articulated through the use of stone ribbing placed at the intersection of the pointed cylindrical vaults, thus forming voussoir arches which transferred the load to the columns and to the buttresses. In time the pattern of the ribs became gossamer-like in the richness of its lines; always it remained a direct expression of the method of construction.

The early Renaissance saw an important contribution to the development of form: the octagonal dome of the Florence cathedral, designed by Brunelleschi and completed in 1434. Called "a miracle of design" (cf. Reference 13, p. 677), the dome is formed from an intersection of eight cylinders, with the base diameter of 138.5 ft and a tall lantern at the top. Eight primary and sixteen secondary arches support the dome; hoop tension is resisted by circumferential wood planks tied at the joints by iron clamps, apparently the first application of hoops to domes, later used in the dome of St. Peter's in Rome and St. Paul's in London. Reputedly the dome was erected without centering. The baroque dome of St. Peter's, designed by Michelangelo and built in 1590, is slightly smaller in diameter than the Duomo in Florence, but exceeds it in all other characteristics but one: its beauty. The triad which makes up the structure of the dome of St. Paul's in London, built to Wren's designs in 1710, added an important new form to the vocabulary of shells: a cone of revolution. The middle of the three, it is not visible either from the outside or from the inside of the church.

The shells of the past share a common characteristic: the methods of construction remained essentially static through the ages. Further progress could not be achieved without basic improvement in the technology of construction, in the quality of construction materials, and in the understanding of the laws of mechanics. Nevertheless, even the brief review of the preceding paragraphs will give us an idea of the scope of achievements of the designers and builders of the past. It also gives us a sense of perspective, a new point of view in architecture, a scale by which to measure our own efforts, and a sense of humility in the face of the accomplishments of the past.

The advent of the industrial revolution, the rapid growth of population, the vastly increased needs of housing, goods and transportation, all provided the

impetus for the necessary development. First, following the rediscovery of cement and its application to the traditional *pisé** to obtain concrete, a new material was created by the addition of steel: reinforced concrete. Unlike the Roman concrete, or the masonry of the past, reinforced concrete was capable of transferring tensions, and was moldable into any desired shape. The application of prestressing in the twentieth century brought additional freedom to the designer. Second, the technology of construction has grown at an unprecedented rate following the discovery of an effective process for the manufacture of iron and steel; this technology was later easily adapted to the realization of reinforced concrete projects. Lastly, in the second half of the nineteenth century structural mechanics reached maturity.¹⁷ First practical theory of continuous beams was devised prior to 1855 by Clapeyron; the theory of arches was worked out by Bresse by 1859; Kirchhoff's re-examination of the Lagrangian plate theory dates to 1849, while the first theory of shells was proposed by Aron in a remarkably complete form in 1874. Today, a century later, the understanding of the laws of mechanics as applied to problems of shells encountered in building construction is nearly complete, and the development of computers (which the author believes to be the central development of the 20th century) makes analysis of stress of any shell problem not only possible but practical.

The developments in the construction technology as related to steel structures and early reinforced concrete structures were of particular importance in the development of shell forms. Almost invariably, a designer is very strongly affected by the example of other, already built projects. Without the bridges of Eiffel there might not be the bridges of Maillart, or his cement hall; without the work of Cotancin, without the Gallery of Machines, the Breslau Centennial Hall or the lamella structures there might not be the many magnificent projects of Nervi. In addition, reinforced concrete as a material has brought with it new possibilities of structure and form. Its monolithicity made a cantilever simple to construct and a continuous beam a necessity as well as a convenience. Its plasticity made possible the fluid surface-like forms of flat plates, as early devised by Hennebique,† as prophecised by Garnier¹⁸ and as used by Turner¹⁹ and Maillart.²⁰ The extension to three dimensions and shells, as used in the traditional manner in the nineteenth century, and on a more rational basis in the twentieth, logically followed.

The purpose which a building is to serve might often decide the form of the shell. The example of the use of a hyperboloid of revolution for cooling towers was already mentioned. Similarly, grain silos are one of the earliest uses of shells; in fact, the early pioneer of *pisé* and concrete, Cointereaux, proposed the use of concrete in the construction of silos in the first decades of the nineteenth century (cf. Reference 16, p. 24). Cylindrical containers, though not entirely in the form in which they appear today, were used as water containing structures in antiquity.

The last of the influences on our list to which a designer is subjected is perhaps the most important: theories of design. Whether an architect's design selections

*A fascinating account of this method of construction has been given by Collins.¹⁶

†For a building at the Paris Exposition of 1900; cf. Reference 18, p. 29.

are deliberate, or whether they are rooted in the subconscious, the goals of design, and its methodology, affect his choice very strongly. Before undertaking a more detailed examination of these matters we shall consider the development of forms currently used in shell construction.

DEVELOPMENT OF FORM

The oldest form of a shell is probably that of a cylindrical barrel vault which developed from the arcuated forms of Roman architecture. As used by the Romans, it was an elongated semicircular vault supported along the springing. Two such vaults at right angles to one another create an intersecting vault, used very commonly by the Romans and their heirs. In the hands of medieval masons two changes of form took place: the cross-sectional arch became pointed, and the method of construction introduced voussoir arch ribs. At first, these were placed at the intersection of the two surfaces, later more ribs were added in the area between these intersections to reduce the span of the thin masonry. With time the patterns of the ribs became very intricate, no longer reflecting the need to limit stresses, but rather the exuberant spirit of the designer-builder. Where the space to be covered was an elongated rectangle, but the support of the vault was not continuous, the ribbed framing of the intersecting vault was also used, bringing the forces to the columns; lateral thrusts were taken sometimes by ties, more commonly by buttresses. The problem of the end of the elongated vault was handled by providing a half-vault at right angles, as done by Romans.

In modern days cylindrical shells were first used in order to achieve economy of forming. The shape of the cylindrical vault remained essentially the same, but some new variations have been added. First, the profile is now not only circular but parabolic, elliptic, or catenary, reflecting the desire for more dramatic forms, and a deeper understanding of structural mechanics. In the case of "short" shells, the profile might be a combination of a series of circular shapes; also longitudinal dilatations were often introduced to facilitate stress calculations. Second, cylindrical barrel vaults are used for north light buildings, with the cross section gradually evolving to reflect the optimum stress distribution from a circle to catenary to S-shape;²¹ the efficient use of the system was made possible only by the introduction of prestressing. It should be noted that short cylindrical shells have been used for north light roofs as well, as in the highly imaginative roof of a factory in Switzerland (cf. Reference 6, p. 62-63). Third, the surface of the shell itself is sometimes obtained by combining flat segments: a folded plate structure, particularly suitable for cylindrical shells because of the zero curvature of the generatrix. This type of structure was applied with success to north light roofs, using a series of "Z" shapes. Finally, Roman-like end form of a cylindrical vault occurs occasionally in modern architecture, some of the works of Nervi being notable examples.

Spherical dome represents one of the first applications of shells. We have already noted its use in the roof of the Pantheon in Rome, the domes of the Hagia Sophia in Istanbul, St. Peter's Basilica in Rome, and the interior shell of the dome of St. Paul's in London. Initially the shape was hemispherical, due to the

requirements of observed behavior. The first modern shell, designed and built with the benefit of the knowledge of structural mechanics, was the roof of an experimental planetarium in Iena, completed in 1923.⁵ This was still hemispherical, corresponding to the functional shape of the project. Soon a shallow dome was built, with the central angle substantially less than 180 deg. If the dome was circular in plan, and if the shell was to be supported continuously along the base perimeter, the spherical shape was typically used. The introduction of point supports, and the requirement that the shell admit light above the base line led to modifications. A number of forms were developed, of which Torroja's Market Hall in Algeciras is a notable modern example. As in the case of vaults, folded plate domes have been used, such as the domed roof of the University of Illinois arena, apparently as a result of the decision to use prefabrication as the construction method. Finally, skeletal domes* (such as the geodesic structures of Buckminster Fuller) have gained wide acceptance during the last few years. The U.S. Pavilion at the Montreal Expo in 1967 is a spectacular example of a Fuller dome executed in metal. Much of its appeal lies probably in the fact that it encompasses an angle substantially more than 180 deg. We note the inevitable: the geometry of the concrete version of the geodesic dome (cf. Reference 6, p. 122) was anticipated by Leonardo da Vinci.²²

Domed roof on a polygonal base constitutes a separate class of shell forms. The octagonal Brunelleschi dome in Florence was already mentioned. Similarly, intersections of cylindrical surfaces were used for domes on a square base and on a hexagonal base, as in the very large domes of the Market Hall in Leipzig. In the case when light is to be admitted between point supports, the shape is basically altered. The Kresge auditorium roof is a spherical triangle bounded by great circles and supported at the three points of intersection. An intersection of three parabolic cylinders was used by Esquillan in creating the form of the largest shell structure built so far; the roof of the giant Exhibition Hall near Paris, completed in 1958, where the distance between the points of support is over 700 ft.²³

A domed roof on a square or rectangular plan occupies a special place in the development of form. The simplest type was described in the previous paragraph. Again, if light is to be admitted above the support line, the form has to be modified. Two possible solutions exist: Yamasaki used the Roman intersecting vault as the basic unit, separating it from the neighboring units by skylights as in the roof of the airport in St. Louis;²⁴ he later used it also to roof the warehouse of a plant in California,²⁵ where all the structural elements were precast. Yamasaki's solution was later extended so that the dome was formed by intersection of two toruses, thus obtaining better structural behavior and a change in appearance. Similarly, the curvature of the intersecting vaults could be made reverse, retaining the benefits of double curvature but permitting the edges of the shell to be thin and, again, changing substantially the appearance of the structure. This type of dome could then be readily extended to create roofs with many points of support in plan, as in the famous restaurant by Candela, later also used by others. A second type of solution is afforded by a continuous surface, such as a segment of a sphere or an elliptic paraboloid supported either only at the corners or along

*For many other applications of skeletal systems see Reference 9.

the sides. This shell form was considered by the early pioneers of shells in Germany; it came of age with the Brynmawr factory roofs of Jenkins.²⁶ Minimal surface bubble shells of Isler are another example of this type of solution.²⁷

Many of the domes are surfaces of revolution. This class of shells contains a number of other forms, primarily of importance in the already mentioned functional applications such as grain silos, container structures and cooling towers. Commonly used shells are a sphere, a paraboloid of revolution, a cone, and a torus. Applications to other than functional structures should be noted: the planetarium in St. Louis is an example.

Much of the developmental work in the early twenties was directed toward reducing the cost of construction of shells, primarily the cost of formwork. This led Freyssinet to the use of conoidal surfaces for the north light roofs of the railway workshops at Montrouge built in 1925, and later in a similar project at Bagneux, filling the interior space with light and adding an important new form to the architecture of shells.

The now famous hyperbolic paraboloid, previously used as an essentially architectural rather than structural element by Gaudi, was developed at approximately the same time. Aimond proposed a series of shapes possible within the general class of shells of negative curvature, and developed the membrane theory of such shells.²⁸ He was followed by Lafaille,²⁹ and as early as 1934 Baroni built his first hyperbolic paraboloid roofs for a foundry in Milan. Some additional theoretical work was done in the early forties by Hruban, who applied the form to a few projects in Czechoslovakia, but the promise of the form had not been fully realized until Candela unveiled its true potential in a series of projects.³⁰ Working in the tradition of such masters as Freyssinet, Torroja, and Nervi, and acting often as a designer and builder at the same time, Candela demonstrated not only the basic economy of the hyperbolic paraboloid, but also its dramatic architectural possibilities. Furthermore, in the simplest possible way—by the example of realization of a number of striking projects—Candela removed some of the stress analysis mystique which had been associated with shell construction from the early days of its reappearance in the twenties. Candela's influence has been greatest, perhaps, in the United States, where many architects appreciated the possibilities of the form, and utilized it in a great many projects. These range from utilitarian roofs of warehouses, built for their economy, to the dramatic mood-creating shapes of some of the churches.

Draped forms were already mentioned in connection with the inverted domed roofs, and also with some north-light cylindrical shells. However, the first modern tension structure, the roof of the arena in Raleigh, North Carolina, built to Nowicki's design in 1952 had the form of a hyperbolic paraboloid. This type of construction quickly caught the imagination of architects and engineers.³¹ In the United States spans have reached the dimensions up to 400 ft.³² Some truly ingenious structures and forms have been built in Japan in connection with the 1964 Olympic Games in Tokyo (although these are in metal).³³ A comprehensive study of the structural possibilities was made by Hottinger.³⁴ Current applications are primarily in the field of temporary structures, such as the mammoth tents of Otto.³⁵ A very promising path of development was suggested by Caminos, et al.,³⁶ but it appears that it had not been followed sufficiently far to

result in realized projects. The author believes that the wedding of concrete technology and the tension structure principle has not been yet fully achieved.

We have covered above those shell forms which can be classified as belonging to some specific geometry or class of surfaces, whether they were devised as a result of search for economy, structural efficiency, or beauty of form. We now need to consider the shells of not readily classifiable form which were developed as plastic shapes, much as a sculptor might create his work. Clearly, this can be achieved only by listing some examples. Apart from the work of Gaudi, which occurred before modern shell construction and cannot be classified as such, one of the first free form shells was probably Torroja's proposal for the roof of a clubhouse in Venezuela. Some of the most famous include the TWA terminal at the Kennedy airport in New York, and the roof of the Sidney Opera House. The use of forms such as these is normally reserved by architects to projects of unusual importance, since the freedom of form provides an opportunity for creation of space and mood not normally attainable within the restrictions of a more utilitarian project, or one with narrower budgetary limits.

Two European engineers must be given particular recognition in any review, however short, of the development of shell form. We have already noted in a number of instances the contribution of Torroja, whose productive years span the period from the twenties through the fifties, and Nervi, who is still active in design. Eduardo Torroja's projects include the dome of the market in Algeciras (1933), the Zarzuela Hippodrome Grandstand (1935), Fronton Recoletos in Madrid (1935); and the workshops of what is now the Torroja Institute in Castillares near Madrid (1948); one should add to this list the wonderful brick water tank at the Zarzuela development. Pier Luigi Nervi's first important project was the stadium in Florence (1928), where the plastic forms of the stairs show the promise of things to come. Nervi designed the hangars at Orvieto (1936), which is a poured in place skeletal cylindrical shell vault in the lamella tradition; later (1939), the design was modified so that the lamella structure was precast, with six supports at the perimeter. Essentially the same method of prefabrication was later used by Nervi in a number of projects and became almost his trademark. Examples include the Exhibition Hall in Turin (1949), with the strongly one-directional central hall and a half-dome at one end, the Exhibition Hall C at Turin (1950), Beach Casino at Ostia (1950), and the sport arenas built for the Rome Olympics in 1960. Torroja and Nervi both had an immense influence on the conceptual design of shells and the development of form, both through their own work and through their publications.^{7,37,38}

It may be of interest to compare shell construction in the United States to that in Europe and elsewhere. It is markedly different. As often pointed out, the economics of construction differ from European: the labor/material cost ratio is often cited as the primary reason why the refined methods of construction, including shells, are not economical in this country. In a recent contribution,³⁹ Parme convincingly argues that the whole trend in the construction in the United States is toward placing it on the basis of a manufacturing industry, with much of the structure fabricated in plants, shipped to the field and rapidly erected, thus reducing the very high costs of field labor. At this time it does not appear that shells, and in particular the very individualistic pace setting and widely

imitated shells designed by the top architects of the country, can be built on this basis. The author believes that the high labor/material cost ratio is not the predominant factor; rather, he believes the central reason lies in the very structure of the construction industry in the United States. The French system may be considered representative of the European approach. There, design-build contracts are quite common, and often the architects and their consultants limit their services to what might almost be called advanced preliminary design, in which a number of design decisions such as the precise nature of the structural system are left open. The contractors are then invited to submit proposals for the building, including not only construction, but also the design of the structural system itself. The already mentioned Exhibition Hall in Paris is a case in point. Thus the contractor is encouraged not only to use economical methods of construction, but to contribute his ingenuity to the invention of new structural systems. Often competitions for important public buildings are arranged along these lines, with contractor and architect together forming a team which proposes a solution, and places a cost value on this solution. Nervi's Palazzo del Lavoro in Turino is but one of the many projects which was built as a result of winning such a competition. Thus the organization of the construction industry in Europe encourages invention of new structural systems. The situation in the United States is different in that important buildings are designed by architects, who are often men of strong convictions, and who are only rarely disposed to accept the advice of their consultants, let alone that of contractors, should indeed such advice be available at the time major design decisions are made.

With rare exceptions, architects and engineers do not necessarily have the incentive to invent new systems because of the almost inevitable premium added to the cost of an untried idea, and because of the associated higher personal risk. Contractors, on their part, direct their ingenuity wholly toward finding new, more economical ways of building a given structure specified in a set of contract documents. The incentive for contributing toward the invention of a new structural system is thus lacking for both. As a consequence of this situation, the United States contribution in the field of traditional shell construction has been primarily not in the invention of new shapes or systems, but rather in the invention of construction methods aimed at the reduction of costs. In the case of shells this means reduction of the cost of forming. Thus we see shells poured on prepared soil, which may later be excavated from under the shell to create an enclosed arena, reusable forms, pneumatic forms and, above all, precasting.^{4,40} Because of the availability of heavy construction equipment, precasting in the United States tends in the direction of very large elements restricted in size by the requirements of transportation rather than by weight. At the same time it makes possible better control of concrete finishes through factory construction. In this it differs from the precasting methods of Europe, where traditionally the size of the prefabricated elements was limited by the capacity of the construction equipment.

CURRENT STATUS OF SHELL DESIGN

Today shells are being built in a variety of forms. Shells of revolution, segmental domes, barrel vaults, cones, elliptic and hyperbolic paraboloids, conoids, toroids, all of these shapes and their combinations have been used, acting structurally as traditional shells, as skeletal shells or, as tension structures. Applications are to a variety of structures. Some shells are purely functional, such as grain silos, containers, and hyperbolic cooling towers. In other cases shells are used to cover large areas, as in roofs of stadiums. In a number of cases shells serve to fulfill function other than just enclosing space and providing shelter; examples range from churches, which are designed to create an evocative mood, to bowling alleys or automobile showrooms, where an attention attracting shape of a shell may be used for advertising purposes. Finally, a shell may be used by the designer to provide a plastic accent to an otherwise plain project, whether as a canopy over the entrance to an office building, roof penthouses, or as a small restaurant which serves as the foil to a large hotel building in the tropics. Whatever their form and structural action, with the development of the computer and in the present state of construction technology it can be truly said that any shell structure is possible. Critics' view of the significance of shells was already noted in the first paragraph. It would thus appear that after the long initial period of trial and error shells have reached maturity and are destined for even greater achievements.

The promise of shells anticipated by Giedion has not been fulfilled as yet, and in spite of their glamor the field of shell design seems to be sterile today when compared to what it was a few decades ago. A number of observers share this view.* Von Eckart wonders why shell "... hasn't aroused the engineers themselves to the more daring, more exciting, more beautiful structures they are surely capable of" (cf. Reference 3, p. 169). It can be seen also by considering again the early shells; few of the more recent structures are as impressive in the context of their architecture as Freyssinet's shops at Bagneux, or Torroja's market hall at Algeciras. It appears that the progress in construction techniques and methods of stress analysis was not matched by progress in the conceptual design of shells. Architects and engineers both became involved in designing projects for new span records, shapes, and structural systems for their novelty, all the while mindful of the architectural magazines waiting in the background to proclaim the current masterpiece. Shell structures are, on the whole, expensive to construct, the exception being such utilitarian structures as the previously mentioned silos and cooling towers. Hence the justification for shell projects lies in the form, with its promise of beauty.

Two facts emerge. First, conceptual design stage is the phase primarily affecting the degree of success achieved by the project as a work of architecture. Second, the judgment (such as the judgments expressed in the preceding paragraph), as to whether or not a given project is successful, is subjective. In this vital respect the conceptual design differs from the other two phases of realization of a project. The goal of the detailed design phase is to produce a set of contract

*A recent report on the 1969 meeting of the International Association for Shell Structures is entitled "Do Shells Have a Future?" Its author's conclusion: the prospects are bleak.⁴¹

documents from which a project can be successfully executed, while the aim of the construction phase is to build the building in accordance with the plans and specifications, with the added provision of economy. In either case the degree of success achieved can be directly measured once the structure is completed and, even before that, at the time of bidding. In contrast, the fundamental goals of conceptual design are not specified with equal clarity, nor are there simple standards available to measure the degree of success attained in reaching these goals. This, in spite of the fact that, for millenia, man has been laboriously considering theories of design.

HISTORICAL THEORIES OF DESIGN

Some two thousand years ago an obscure Roman soldier-architect, Vitruvius, wrote in retirement a book which, in addition to building materials and methods, considered also the aims of architecture. The manuscript was rediscovered during the Renaissance and strongly influenced such architects as Alberti and Palladio. Through them it reached a wider European audience. Thus Vitruvius' stated goals, paraphrased in the time-honored English Renaissance formulation,⁴² came to our own time: "Commodity, Firmness, and Delight." The importance of these three requirements is usually not questioned. What varies is the weight attached to any of the three by a given architect, to a given project, in a given time.

To the Greek builders of the temples, or the medieval creators of the gothic cathedrals, beauty of a building was synonymous with the beauty of the structure, perhaps due in part to the state of technology of the time. Thus in each case the structure is shown, though the structure itself is different. The Greek temple was of post and beam construction, massive stone and masonry evoking a feeling of dignity and strength, of mastery over the forces of nature through successful support of loads. The mood of the Roman buildings is essentially the same, though a new arcuated form might be used. The structure of the gothic cathedral represents a departure. The masonry is no longer large and massive; on the contrary, the proportions become very thin, material is reduced almost to an absolute minimum, often requiring introduction of additional strength-increasing measures such as metal ties and clamps. The structure itself formed the integral ornament of the building, though the impression of strength no longer depended on large inert mass, but rather on the perception of lines of the columns and ribbing of the vaults. Thus the difference in the mood between the classical beauty of the Greek temple, relying on unifying principles and perfection of design and execution operating on a restricted set of variables, and the romanticist beauty of the gothic cathedral where the freedom of creation led to more emotional, organic, though possibly no less disciplined design. Even though unconsciously, in both the Greek temple and the gothic cathedral the structure was the principal means for achieving beauty.

This approach was radically altered in the Renaissance. A number of factors contributed: the structure of the society, the need for rapid construction of many buildings, the opportunity for adaptation of existing structures. Perhaps

the strongest was the high regard in which artists were held by the cultivated classes, the only class of people with the means to sponsor construction of important buildings. If a painter, or a sculptor, can create a work of great beauty in his own field, surely he can be trusted with designing a building. Thus designers considered structure subordinate to form, “. . . they used structure decoratively, they concealed structure, they contradicted it or even ignored it” (cf. Reference 10, p. 160). (After all, a sculptor is accustomed to employing concealed armature within his creations.) This attitude, for better or for worse, created buildings with wall paneling which did not correspond to, but imitated, columns, to fake windows and doors inserted in the elevation for the sake of an aesthetic appeal. Vitruvian “Delight” was still the goal, but its interpretation changed.

This attitude led to the creation of rococo and, eventually, to a reaction: strict functionalism. It first appeared in Italy in the eighteenth century. Lodoli proposed that “only that shall show in a building that has a definite function and derives from the strictest necessity” (as quoted in Reference 10, p. 69-70), a doctrine that soon proved impractical in application. This did not stop its proponents. More and more architects professed adherence in principle, while readily breaking the rule in their own work. The body of the doctrine grew, with important modifications aimed at accommodation between theory and the practical requirements, finally reaching maturity in the works of the French architect of the nineteenth century, Viollet-Le-Duc.⁴³ To functionalism he added the thesis that taste and artistic sensitivity must have a role in the design process, but that their influence must be exerted through a rational process of thought rather than through the subconscious.* The teachings of Viollet-Le-Duc occurred at the time when new materials, iron (later steel) and reinforced concrete, were revolutionizing the scope and the pace of construction, and when the need for some generally applicable rules of design was felt particularly keenly. This was also the time of the positivist teachings of Comte and the time of the industrial and scientific revolution. Followers were easily found. Through the admiration of nature and natural philosophy the modified functionalism of Viollet-Le-Duc developed into what is now known as the engineer’s aesthetic and eventually, constructivism.

The doctrine of the engineer’s aesthetic can be summarized as follows: if the laws of nature are applied to the materials involved in construction, the resulting structure, and hence the building, will be inevitably beautiful. The structural beauty again becomes identified with the architectural beauty. This theory of design, we should add, was developed not by egocentric engineers, but by architects and theoreticians of architecture, as a result of their dissatisfaction with the prevailing works of architects and their admiration for the works of the engineers of the era. The critics drew further conclusions. Noting a number of extremely impressive and beautiful projects designed by engineers, and assuming that, almost by definition, engineers are not in the slightest interested in beauty, the critics inevitably concluded that the person of the designer is not important to the design process (insofar as the conscious desire to create delight is concerned),

*For a recent admiring comment on Viollet-Le-Duc see a paper by Billington.⁴⁴

that only the fact that the laws of nature are rationally followed matters. Thus human element becomes separated from the design effort. (This attitude is still implied in such important critical works as Giedion's.²) The approach to design through nature, and the admiration for engineer's aesthetic, led eventually to the constructivism of Mies with his total belief in, and commitment to, technology. Like the Italian functionalists of the eighteenth century Mies, too, found it hard to reconcile doctrine with life, to mold his designs to the principle. The engineers' aesthetic, and constructivism, contain within them one particularly difficult problem. The process of design, which consists of applying laws of nature, must be in essence an optimization process. The question arises: what constitutes efficiency? It is variously interpreted as the minimum amount of material, minimum cost, minimum total energy expended. The answer varies with time, place and the designer of a given project. The multiplicity of solutions which may all be, on the face of it, correct, is a point of weakness in the doctrine. Constructivism continues a very strong force in architecture today. The Kresge Auditorium and the Oakland Coliseum are but two of the more successful and impressive examples, surely representing an important contribution to the architecture of our time. We could offer criticism of these buildings on their own terms, that is on the basis of the extent to which they represent efficient and rational use of materials and structures. Instead we observe that something less easily definable appears to be lacking in these projects, perhaps some sterility of emotion, possibly due to some failure in the design process. Buildings must be not only designed for humans, they must give also an impression of being designed for humans. As pointed out by Schlaepfer (cf. Reference 10, p. 140), structural efficiency is not synonymous with beauty. Le Corbusier recognizes this by noting the intervention of taste in the engineer's design process.⁴⁵

Just as the Renaissance preoccupation with form led first to the excesses of rococo and later, in reaction, to the strict functionalism of Lodoli, the constructivism of present day, through its sterile strict adherence to the professed mastery of technology led eventually to a reaction. In the words of Scott "... 'theory'—the attempt to decide architectural right and wrong on purely intellectual grounds—is precisely one of the roots of our mischief." And: "What we feel as 'beauty' in architecture is not a matter for logical demonstration."⁴⁶ Scott accepts the three Vitruvian precepts as valid, but believes that delight is independent of the others. He defines the architecture of Humanism as that which "... by Mass, Space and Line responds to human physical delight, and by Coherence answers to our thought" (cf. Reference 46, p. 178). The human sources of architecture founded in realism are recognized by many designers.⁴⁷ However, in many cases the reaction to the doctrinaire technological axioms of constructivism led to preoccupation with the personality of the designer. Creighton calls this "The New Sensualism," and illustrates his thesis with a number of examples.⁴⁸ The human response of the user of the project often remains neglected.

HUMAN RESPONSE TO ARCHITECTURE

Since all projects are designed and built for human use, and if the Vitruvian principles of commodity, firmness, and delight are valid, then delight, human delight, is an inseparable pre-condition of architecture. Thus when speaking of functionalism, delight must be inevitably included within the definition. It is a relatively simple matter to achieve commodity and firmness, since both involve a finite number of variables and are susceptible to a systematic, more or less scientific mode of attack. Not so delight; there is no systematic procedure which would ensure its presence as part of the human response to architecture. To arrive at a choice from the infinitely many alternatives available (which the author believes to be the essence of art, as contrasted to science) involves a creative process which, so far, is not understood, or at least not fully understood. At the same time this phase of design goals has received less than its share of attention from engineers, if not architects. In the following we shall make an attempt to understand better the nature of delight. We note that delight has three sources: moral, sensual, and intellectual. No weight should be attached to the order in which these sources are listed; certainly all three are closely related. For it is through the senses that we acquire the perceptions which form the basis for our moral and intellectual judgments.

The moral source of delight is itself twofold. Firstly, the morality of the project as a whole, in the sense referred to by Giedion when he speaks of the desire “. . . to reconcile methods of feeling with methods of thinking (cf. Reference 2, p. 25).^{*} The architects themselves are deeply aware of the moral foundation of their work, in the sense that it is founded in a real world, functions in a society, and fulfills its needs.^{49,50} Secondly, the morality of the project as executed, often spoken of as honesty of design, being true to materials, or being true to some principles. The first question which arises in this connection is: should the structure of a project be exposed, or may it remain hidden? While sensual and intellectual arguments in favor of exposing the structural system are many, from the point of view of moral delight the designer is free to expose it fully, in part, or not at all. After all, Vitruvian precepts speak of delight, but do not say how it is to be achieved. In the case of concrete shells, in which structural form is almost invariably coincident with architectural form, this problem is of somewhat less importance. Whatever the structure, however, it should be used in a manner consistent with its intrinsic properties, and should be built of materials which match its characteristics. Conversely, the structural system selected for a given project should reflect the properties of the materials used in its construction. As an example, an observer's moral sense of fitness of things would be offended by the use of aluminum as the counterweight material in a bascule bridge.

^{*}At an even more fundamental level, morality might involve the question whether it is possible at all to design a project such as a prison (a concentration camp?) and as the end result to produce a work of art. This question has been (and continues to be) much debated in relation to literature and fine arts, but not with regards to architecture.

An architectural project is experienced directly through the senses. The primary sense is that of vision. Oster, a biophysicist and a painter, calls the eye-brain complex the "mother of art." Through the sense of vision we distinguish line, shape, form, light and shadow, color. Sight plays also an important contributory role in the sense of space, sense of scale, sense of texture, sense of mass. The actual mechanics of vision, and in particular the manner in which the imprint from the retina is transmitted to and interpreted by the brain are not completely understood.⁵¹ It is known that the role of the brain in the interpretation is all important, to the point where the brain may create its own images, apparently based on past experience, occasionally only vaguely or not at all connected with the external stimuli.⁵² Similar observations hold for response to color, which is part of the sense of sight, and for response to sound. In all cases the brain responds to a stimulus, and the response may be heightened, essentially by attracting and holding attention of the person who experiences an event, provided a sufficient time is given for the response to develop. The action of senses may be heightened through a number of possible devices, one of these being contrast. An example is afforded by entering a large space through a low and narrow passage; this will make the space look even larger. Similarly, a sense of scale of an object is provided if another object already within a viewer's experience is placed next to it. Thus placing a horizontal line at the height of a man along the walls will accentuate the height of a hall, a device often used in architecture. So far we noted that the responses evoked by sensual stimuli may be heightened. The real point lies in the observation that some stimuli are associated with delight; thus certain lines, shapes, colors, etc., evoke a feeling of pleasure to a degree greater than others.⁵³ The knowledge gained in this connection by Gestalt psychologists is as yet insufficient to explain fully why this is so.^{51*} However, it appears that some visual stimuli cause essentially the same response in all individuals, independent of the individuals' past experience. Stimulation of the senses can be used also to create a state of emotional tension and release. Again, by contrasting the two, the degree of delight associated with it may be heightened. To a designer, what is important is the fact that certain forms, lines, colors, etc., are delightful, and that through the device of contrast, with the associated added stimulus, the delight may be heightened. Similarly, the delight experienced as a result of some emotional tension can be heightened by contrasting it with release. The reason why this is so is of secondary importance in this connection, just as it is of secondary importance to a man who has just eaten chocolate to know why the taste of sweet is universally liked. The essence of genius in a designer lies in his ability to teach us that they do. And in all cases he will use the device of contrast, the ability to create tension and release in turn, much as a writer of genius uses the same devices in his writing, and a master composer in his music.

Intellectual delight derives, by definition, from the rationality of man. The ability to translate the impressions received through the senses into a coherent pattern brings with it a feeling of pleasure. Thus the degree of delight evoked by a work of architecture will vary with the viewer's sophistication. A layman is capable of perceiving immediately the manner in which a suspension bridge

*Refer also to the already cited work of Arnheim.¹¹

carries loads. On the other hand, a trussed space framework which might delight a skilled stress analyst could appear as a hopelessly confused collection of aimlessly arranged elements to a sidewalk superintendent. As in the case of sensual delight, the effect may be heightened by contrasting simplicity with sophistication, by letting the observer go through a process of discovery of the structure, by using systems which appear complicated at first sight, only to prove simple and logical on more detailed inspection. In effect, an intellectual game between the designer and the observer might take place, with the structure intentionally yielding its secrets, step by step, with the elements of mystery and surprise playing an essential role, always bearing in mind that a coherent pattern must emerge, that logic remains supremely in evidence, whether the logic of intellect or emotion.

One other source of intellectual delight must be mentioned: simplicity. Ironically, the idea which the word is intended to convey is very subtle, and other terms, such as economy of expression, or elegance, are often used to describe it. Simplicity manifests itself at two levels. An event, an idea, or a work of man may be simple, *per se*; in addition, the manner of manifestation of the event, the means of communication of the idea, or the method and means used in the creation of the work of man may also be to a varying degree simple. In science, a law of parsimony “demands that when several hypotheses fit the facts, the simplest should be chosen” (cf. Reference 11, p. 46). Davies distinguishes two kinds of scientific simplicity: that derived from a restriction in the number of independent variables, and the subjective simplicity of, say, number 2 as contrasted to number 1.99.⁵⁴ The desire to reduce all experience to simplest possible terms, no matter how intricate or subtle the experience itself might be, seems to have been part of man’s heritage from time immemorial, causing him to search for the simplest possible natural laws which would explain, and hence govern (perhaps to make safer?) the unknown world. Thus the metaphysical attempts of Maupertuis and his Principle of Least Action, thus the researches of mathematicians and physicists into the domain of variational principles. This emotional intellectual need for simplicity, deeply rooted in the nature of man, translates itself from the field of natural philosophy into the field of art. Goethe: “Simplicity has genius, power, and magic in it.”* Willa Cather: “Art should simplify. That, indeed, is very nearly the whole of the higher artistic process. . . .” Economy of expression is one of the goals of the artist, and those periods of the past during which simplicity was lacking are somehow considered superficial, perhaps on the theory that a truly great idea must be capable of being expressed simply and concisely, whether in the field of thought or as a work of art. Plato in philosophy, Hemingway in literature, Brancusi in sculpture are but three examples. In architecture, the idea of structural efficiency as one of the fundamental rules of design, misguided or not, is an example of an attempt at a single, simple natural law which would guide all designers. Certainly, whatever the professed approach to design, the means of expression are always simple in the hands of a master. And our delight at the presence of simplicity is enhanced by contrasting

*For this quotation the author is indebted to an architect friend, Don Knorr, who has it prominently exhibited in his office.

it subtly with planned disorder, perhaps introduced at a different scale of the project. Thus a building as a whole, in the large, may be made simple by the introduction of a modular grid, imposing serenity and elegance, while the detailing of the parts, in the small, might be exuberantly, defiantly rich. Conversely, a simple basic building block, say a prism, might be used in a large scale project to create a complex whole. Thus delight results not only from simplicity, but possibly from just the appearance of simplicity; this may, perhaps, partially explain the success of constructivists, and the admiration for their work.

Is there a hierarchy of importance of sources of delight? Can one of the three—moral, sensual, intellectual—be said to be predominant? The writer believes not; all three are a necessary condition for creation of delight as part of response to a work of architecture. According to Aristotle, “Noblest art appeals equally to intellect as to feeling.”⁵⁵ Further, each of the three heightens the delight due to the others. Finally, in experiencing a work of art, the delight is a feeling, a state of mind, and not often is it possible to distinguish its sources, so closely intertwined are they in the observer’s subconscious.⁵⁶ To express a structure is honest, it appeals to our senses through the (presumably) pleasant forms, but it also causes intellectual delight by letting us unravel the mystery of transfer of loads, while increasing our sense of security and well being and providing reassurance regarding the strength and stability of the building. Thus the same architectural feature results in multiplicity of delight. In any event, as far as delight is concerned, the structure of a building is important only to the extent it contributes toward the realization of the basic architectural idea of a project. To argue otherwise, to endow the structure with importance in excess of this is to stand in the face of many examples to the contrary, buildings where the designer of genius successfully chose to subordinate structural expression in favor of the requirements of human needs.

DESIGN OF CONCRETE SHELLS

An attempt will now be made to translate the observations regarding the sources of delight to the field of conceptual design of thin concrete shells. As in almost no other architectural work, the structural form of a shell is essentially synonymous with the architectural form, the structural system strongly identified with the building itself. The architectural and the structural beauty fuse into one, and whatever the success in ensuring delight as part of the human response to the structure will largely carry over to the larger field of experiencing the project as a work of architecture. That delight can be inevitably achieved is by no means certain and the degree of it will certainly vary. Shells are sensitive in this regard, perhaps more so than other structural systems. As an example, it is not easy to design a suspension bridge which would not be in a large measure beautiful. Not so with shells; these are fragile and easily hurt, and in the hands of a doctrinaire constructivist or a drama seeker, however well-meaning, a shell may not only lack in delight, but may create a feeling of regret at the opportunity irretrievably lost.

The predominant feature of concrete is its plastic moldability. Alone among the current major building materials, concrete can be molded into any desired

monolithic shape. Thus, in addition to the linear forms (which themselves, due to the monolithicity, added the cantilever and the rigid frame to the structural vocabulary), surface-like forms are also valid structural concrete systems, creating a whole new approach to architectural design. These characteristics are strikingly captured in Max Bill's sculpture "Endless Ribbon," executed in 1936. Their use in architecture is evident in the works of Perret, of Freyssinet, and most of all, perhaps, in the bridges of Maillart. The freedom with which concrete can be used, as linear or surface forms, flat or curved, gives it unique versatility. It also creates additional problems for the designer. Consider structural steel: its very technology almost dictates the manner in which it is used, and the architectural expression of its buildings. The richness of form, possible with concrete, makes the selection of shape, and the structural system more difficult, while enlarging both the challenge and the opportunity.

Among concrete systems, shells belong to the category of surface structures; however their visual characteristics may vary. Schlaepfer believes that a typical linear structure manifests itself in a "... sharp rhythmic succession of structure and spatial intervals, enhanced by the play of light or the reflection in the glass," while in the case of shells their surface, plain or ribbed, causes "... a reduction of light and spatial interpenetration, but an increase in the energy and the flowing movement of interior space" (cf. Reference 10, p. 135). However, Torroja (in the Fronton Recoletos) and Nervi (in the Turin Exhibition Hall, as well as other projects using prefabricated ribbed system of construction), have both shown that the element of light not only need not be lost, but may be even enhanced. Further enrichment is possible with the increased use of skeletal-type shells. With regards to the structural action of shells, the dominant characteristic is their ability to carry loads predominantly through in-plane stresses rather than through bending. As previously noted, this is due to their geometry in the large, which is that of a surface curved in space. This principle should be emphasized in the selection of form, preference being given to those forms which minimize bending. This would apply also to folded plate structures, although these are inherently subject to larger bending stresses. From the structural point of view, shells of double curvature are preferred to shells of single curvature, although the application of prestressing may reduce or even eliminate bending. Reduction in the magnitude of bending stresses in a shell can result also from local variation of the shape as proposed originally by Dischinger.⁵⁷ The supports of thin shell structures may also be arranged to emphasize the in-place nature of internal stresses, as in the hangars of Nervi, or in his Palazzetto di Sport, or in Candela's church piers. Another feature of shells is their thinness, due to the efficient utilization of material. In many cases, such as Candela's restaurant at Xochimilco, the shape of the shell is so arranged that large edge beams are not required, and a thin boundary line delights the viewer with the simplicity of form and sophistication of design. In other cases the thinness of the shell can be successfully accented through devices such as Torroja's use of extension of shape beyond the line of support (cf. Reference 7, page 140). Conversely, when rather large edge beams are required and similar devices are not used (as in the Kresge Auditorium), the results tend to be disappointing. Openings through the surface of a shell can also be used to indicate the essential thinness of the structure. Perhaps the most effective, certainly the method

which offers the widest choice to the designer, is the visual lightening of the shell by transforming the surface into a network of ribs. Much of the strength of the shell is concentrated in these ribs; accordingly, they should be placed along the lines of stresses, much in the tradition of the gothic cathedral. Many of the projects of Nervi are a brilliant example of this approach to design which is particularly suitable in the case of precast shells, but which could also be used for shells poured in place. Again, the possibility of placing prestressing tendons within thickened portions of the shell, thus creating a pattern of ribs placed along the precisely most efficient lines, offers exciting possibilities for architectural and structural expression, so far attained only in the case of linear structures.⁵⁸ Equally valid is the variation in the strength of a shell through increase in thickness or, most commonly, through an equivalent variation in the amount of reinforcing.

The choice of the method of construction influences strongly, sometimes decisively, conceptual design and the choice of form. Prefabrication led Nervi to the use of delicate ribbing to which many of his projects almost entirely owe their mood. The form of the domed roof of the stadium at the University of Illinois was devised largely to accommodate erection stresses; again, the dome was prefabricated. In general, design and construction influence each other, and the pattern is cyclic. First a new structural form is devised, requiring new methods of construction. These methods are in turn revised, perhaps radically altered, to achieve economy. The new construction techniques give rise to new structural systems, and the cycle is repeated. As an example, this pattern is discernible in the development of tension structures. The cyclic nature of progress in shell design and construction was discussed by the author in a different context.⁵⁹ But not only the overall shape and the structural expression are affected by the method of construction of a shell. The finish of the concrete, with its multitude of possible surface treatments available, the scoring (or the absence of scoring) of construction joints, the manner in which the joints between precast elements are treated, these all affect the final architectural expression of a building. It appears that in this case as in others, technology outstripped progress in design. However, the problem of finishes is the same in shells as it is in all other concrete structures; in any event it has an effect on form only if it enforces the use of prefabrication. The designer should specify finishing procedures appropriate to the construction method to be used. Thus stricter standards of performance may be expected from precast construction, and if the shell is to be poured in place a realistic appraisal of current field practice would lead to an expectation of a fairly crude surface appearance, which might well be incorporated into the design.

The remarks of the preceding chapter are intended as a contribution toward clearer perception of goals of conceptual design and methods for their achievement. Of necessity, their interpretation varies with each individual. For if anything is clear at this time, it is that the problem is infinitely complex. In the present state of knowledge even the relatively simple problems of technology have not yet been resolved. For example, assume that a given shell is to be ribbed; assume further that the designer wishes to place the ribs along the lines of principal stresses. The location of these lines is known only in some few cases

of shells of simple geometry, supported in an easily ascertainable manner, and loaded by well defined loads. A general case of this nature is simply not tractable at this time. This leaves the designer in the position of subscribing to some rules which offer no immediate guidelines in the decision making process. The much more complex problems of human response, intellectual as well as emotional, have not been yet fully formulated, even, let alone resolved. Even the mechanics of human response are not understood. No wonder, then, that solutions to a given problem are many. Are, then, all of them equally valid? Only a subjective answer is possible, but in all cases Vitruvius' precepts should be the guide in arriving at these solutions.

SUMMARY AND CONCLUSIONS

This paper presents a review of the state of art of conceptual design of concrete thin shells, and offers a contribution toward a clearer perception of its goals and its methodology. Such review, if successful, would be useful both as a part of a wider effort aimed at an examination of the current state of design and construction of shells, and because of the importance of conceptual design in the process of realization of a project. For it is during the conceptual design stage that the basic geometry and shape of the shell are determined, and the structural action established. Since in the case of thin shells the form of a shell is also almost invariably the architectural form, structural and architectural beauty become one, and the decisions made in the course of conceptual design are fundamental to the success of the project as a work of architecture.

This review led the author to the conclusion, shared by others, that the field of construction of shells is not as dynamic and vigorous as it was in the three decades following the first Iena shell, in spite of the tremendous progress made in the technology of construction and the methods of analysis. With rare exceptions, current applications seem sterile, with the size of span often the principal criterion of the success of design. The author believes that a partial reason for this may be that the progress in conceptual design, and in the understanding of its aims and techniques, has not matched either the progress in the technology of construction, or the progress in stress analysis.

The goals of design as stated by Vitruvius are *Commodity, Firmness, and Delight*; these seem as applicable today as they were 2000 years ago. While challenging, the problems of fulfilling the first two are relatively simple, and systematic procedures exist for their attainment. The difficulty lies in attaining delight since it forms a part of human response to architecture, and may vary from person to person. Thus the interpretation of delight changes with time and place. In addition, an agreement on the nature of delight does not necessarily carry with it an agreement as to the ways of attaining it. Whatever the difficulties in this regard, shells, as all works of man, are built for humans to use and take delight in; they should not only be built for, but give the appearance of being built for humans. The criterion of success of a project becomes then: does it enrich the life of man? Whenever the answer is no, as it often appears to be in current architecture, the reason may lie precisely in the neglect of human values, of human response, by the designer.

The methodology of design, the manner in which a stated design goal is to be achieved, is not at this time at the point where a set of rules could be stated which, if followed, would inevitably lead to the creation of a work of art. The author believes, and hopes, that this time will never come. The world of total efficiency and total delight would be very hard to live in, even if it were possible. And it is not possible, because the term "total delight" contains within it a contradiction: like its component elements, delight is enhanced by contrast; it does not exist on its own, apart from the rest of the world. This allows man the hope that some day a new pioneer of genius will show how, by ignoring rules, new heights can be scaled.

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ECONOMIC PROPORTIONING OF CAST-IN-PLACE CONCRETE THIN SHELLS

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Material and construction costs of various types of thin shell structures are presented. Form cost factors are described for shell surface types and for possible boundary member configurations. Cost of forms, based on the number of reuses, the slope of the surface, and the radius of curvature is evaluated. Scaffolding and concrete placing costs are described. Shell geometry, slab and boundary member dimensions, optimum depth-span ratios for various spans, relative material costs, and economic span ranges are indicated for each shell type. All optimum span ranges and depth-span ratios are based on the total cost per unit area, not on the values of stress.

Keywords: concrete construction; construction costs; costs; curvature; cylindrical shells; depth/span ratios; domes (structural forms); economics; edge beams; falsework; formwork (construction); hyperbolic parabolic shells; optimization; reinforced concrete; shells (structural forms); spans; structural design.

INTRODUCTION

□ The construction of cast in place thin shell concrete structures involves many operations, all of which affect the final cost. Quantities of materials may have less of an influence on the cost of structure, whereas the methods and techniques of installing materials may be more significant. Consideration of these cost factors should prompt one to design toward optimum solutions with construction as a prime factor, rather than stress values. With this in mind, we have attempted to evaluate certain shell types to find their optimum proportions.

All of the shell structures in this study were analyzed for primary and secondary stresses. Methods of construction and their resulting costs were determined by the Ernest F. Donley's Sons, General Contractors, Cleveland, Ohio and Campbell-Ellsworth, Inc. General Contractors, Pittsburgh, Pennsylvania. Stress analysis, quantities, and cost data were processed by a Hewlett Packard 2116 digital computer.

There are certain basic considerations that should be established, i.e., all shell structures include the shell slab, the boundary members, stiffening ribs (if required), and traverses and ties. Construction costs include forms for shell slabs, bottom forms for boundary members, fixed and supported side forms for boundary members, stiffening ribs, and end traverses, and top forms for surfaces inclined more than 45 deg with the horizontal.

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Reinforcing steel, concrete material costs and installation costs are also included in the study. Scaffolding costs represent the total amount of scaffolding to support the shell from its lowest springline. Vertical columns or piers that support the shell below its lowest springline are not included; however, tension ties are included since some shell types automatically furnish this important aspect of structural stability.

COST FACTORS

We might begin the evaluation by observing the difference in form costs of four common types of shell curvature. This comparison can be made on the basis of cost factors rather than actual monies. (Fig. 3-1)

A ruled surface of single curvature does not require variations in the bracing of stringers nor does it require tapered form fillers, as does a doubly curved surface. This simplicity allows a form cost factor of 60.

Double curvature surfaces that are negative but contain a family of straight lines are often referred to as ruled. The ease of forming with straight lines, whether level or inclined allows for a form cost factor of 70.

A double curvature that is positive cannot be formed with straight lines. However, if it is a uniformly rotational surface, it has the advantage of large duplication of form sections. This curvature type has been assigned a cost factor of 90.

The most expensive surface to form is that of negative double curvature, which cannot be generated with straight lines. Although this curvature type is translational; supports, stringers, and forms must be cut to shape. Thus we assign it a form cost factor of 100.

Boundary members and stiffening ribs may be upset or downset with respect to the shell slab. Forms for these varying conditions may be fixed or supported or floating as the case may be. (Fig. 3-2).

Type I edge beam or ridge beam is the least expensive to form. The two side forms are supported from the shell form.

The side forms for Type II beams are supported from the beam bottom form. The break between slab form and beam bottom form accounts for the increase in the form cost factor.

Side forms for Type III require a major change in the staging level of the shores as well as a complete break in the shell slab form.

Type IV beams have all of the additional form operations required in Types II and III. Thus they have the highest form cost factor.

Fig. 3-3 indicates the cost of forms based on number of re-uses. This cost decreases by approximately 5 percent per re-use up to a 20 percent decrease; re-setting and repairs remains fairly constant.

Variations in form costs relative to slope of shell surface is a function of the stabilization of stringers supporting the plywood forms and the top form required on slopes greater than 45 deg. The upward jump in the curve reflects the increase in cost due to top forming. (Fig. 3-4).

As previously stated, form costs will vary depending upon type of curvature. The degree of curvature will also affect these costs as is shown in Fig. 3-5. In this instance, the warping and fitting of forms becomes more difficult as the radius of curvature becomes smaller.

Scaffolding supporting the forms is another major part of thin shell costs, and it varies with type and number of re-uses. (Fig. 3-6).

The placing and finishing of concrete will vary with the slope of the shell surface. This variation is shown in Fig. 3-7, and indicates an appreciable increase when top forms are required.

Fig. 3-8 shows a savings in skilled and unskilled labor costs relative to the workmen's familiarity with an operation based on the number of times it occurs.

SCOPE AND CRITERIA OF DESIGN

Considering all cost factors as outlined, we attempted to establish comparisons and proportions of the following types of shell structures:*

Structures

- Spherical Domes
- Sectional Domes—Square Plans
- Sectional Domes—Rectangular Plans
- Long Cylindrical Shells

*See the design charts and cost comparisons following the figures at the end of Paper 28-3.

Short Shells—Non Rigid Edge Beams
 Short Shells—Rigid Edge Beams
 Saddle Shells—Level
 Saddle Shells—Tipped
 Hyperbolic Paraboloid Domes—Square Plans
 Hyperbolic Paraboloid Domes—Rectangular Plans
 Hyperbolic Paraboloid Umbrellas—Square Plans
 Hyperbolic Paraboloid Umbrellas—Rectangular Plans
 Hyperbolic Paraboloid Inverted Umbrellas—Square Plans
 Hyperbolic Paraboloid Inverted Umbrellas—Rectangular Plans

Design Criteria

Live Load = 30 psf on horizontal projection
 Self Weight = Shell slab and ribs
 Roofing = 5 psf
 Miscellaneous = 5 psf
 Concrete Weight = 150 lb per cu ft
 Concrete Strength = 4000 psi in 28 days
 Reinforcing Steel = Grade 40 in Shell Slab
 Grade 60 in Boundary Members

Each shell type and plan proportion is indicated by curves showing the optimum economical depth/span ratios as spans vary. It should be noted that the depth/span ratio is not constant as is normally assumed when only stresses are considered.

Depth/span ratio charts are preceded by tables indicating the dimensions of shell slab, boundary member, and stiffening ribs, as chords, spans, and heights vary. These tables are presented as a guide in planning the preliminary proportions of a shell structure.

A third curve for each shell type illustrates the breakdown in costs for forms, scaffolding, concrete, reinforcing, and installations, as spans and shell proportions vary.

Finally, all shells, their costs, and spans are combined to show their relative efficiency. Shell costs are based on no re-use of forms, as well as a minimum of five form re-uses.

ECONOMICS OF DIFFERENT SHELL FORMS

Spherical Domes

All domes were designed for a uniformly varying thickness from the crown t_c to the boundary t_b .

The shell slab was designed for secondary stresses at the boundary due to strain compatibility and restraint from the ring beam. The ring beam was designed for the same secondary effects; however, it was assumed to be supported at frequent intervals, thus minimizing the bending effects from beam action.

The optimum depth/span ratio increases as the span increases. Shaded areas on either side of the line indicates the range in which variations would have no effect on the total cost of shell structure.

Relative construction costs are based on optimum designs. It should be noted that high form costs are most significant for the smaller spans. Increasing the spans requires more concrete, reinforcing, and scaffolding, thus decreasing the percentage of form costs when compared with the total shell.

Sectional Domes

Sectional domes were designed for a uniformly varying thickness from the crown t_c to the corner supports t_b . Shell thickness at the high point of the edge beam is t_c . The shell slab was designed for secondary stresses at the boundary due to strain compatibility at the corner supports, and the restraint from the edge beam at its high point.

The edge beam was designed as an arch, loaded by tangential shears from the shell slab, and its self weight.

Unlike the other shell types in this study, the optimum depth/span ratio curve has a much wider band. As h_x/h_y changes from 0.2 to 0.6, the curves for optimum h/L becomes lower on the graph and less inclined with the horizontal axis until $h_x/h_y = 1.0$ where the curvature is reversed.

Relative construction costs for forms, scaffolding, concrete, and reinforcing varied with the spans; however, there was little difference between rectangular and square plans, and upper and lower limits of h/L .

Short Cylindrical Shells

Short shells were designed for a constant thickness t_c except near their springlines where transverse bending takes place at the edge member, and is a function of the edge member stiffness. This region of bending stress extends from the springline toward the crown for a distance designated as 'c' in the tables. The 'c' distance has been arbitrarily adjusted to create a transition curve from shell to edge member.

Secondary stresses in the shell slab occur in the region near the edge beam and the crown of the stiffening rib.

Although the tie is shown above the floor, it could just as easily have been placed below it.

The shell slab is extended beyond the stiffening rib 1/10 of the bay dimension. This is not a design requirement, merely a design possibility.

Stiffening ribs for short shells with rigid and non rigid edge members were designed to carry their self weight only. Their depths are critical at the crown of the shell, but not necessarily so at the springline. Accordingly, two possibilities are shown—the constant depth for shells with non rigid edge members, and a variable depth for shells with rigid edge members.

Optimum depth/span ratios vary with the span of the shell. The curves for different bay dimensions are similar, but show a definite separation.

Scaffolding and forms represent the greatest influence on cost. The span L_1 where both are equal appears to change with the bay dimension L_2 .

Although concrete and reinforcing costs for short shells with non rigid edge beams, decrease as spans increase, the rate of decrease varies with the bay dimension L_2 . Concrete and reinforcing costs for short shells with rigid edge beams vary in a much more complicated manner.

Long Cylindrical Shells

Long cylindrical shells were designed for a constant thickness t . Secondary stresses occurred as transverse bending moments at the edge beam, and longitudinal restraining moments at the end frame.

Depth/span ratios are a function of the span L as well as the chord dimension c . The largest variation in optimum h/L ratios occurs for a c/L of $1/3$.

Hyperbolic Paraboloids

All hyperbolic paraboloids were designed for a nearly constant thickness of shell slab. Secondary stresses in shell slabs occurs where the curvature approaches a flat horizontal plane, and where high axial stresses in those boundary members at column supports introduces comparable strains in the shell slab.

Secondary stresses in boundary members occurs where these members are subjected to bending stresses from beam action.

Depth/span ratio curves for all hyper shells reflects costs and not necessarily efficiency of the structure. This is a result of the minimum concrete and reinforcing quantities in the shell slab as compared to the boundary members.

SUMMARY AND CONCLUSIONS

As spans increase, we might consider hyperbolic paraboloid umbrellas, (inverted or not), hyperbolic paraboloid domes, spherical cones, and short shells with rigid edge beams as the most economical shells to use in that order. Architectural requirements may not be compatible with the shell form or the shell support requirements, therefore shell forms appearing to be more costly may have a valid position in the economical order of thin shell concrete structures.

It should be pointed out that the shell forms appearing to be more costly, can be decreased in cost since all of these forms are capable of multi-use in formwork and scaffolding. This is not necessarily true of the hyperbolic paraboloids and spherical domes.

Optimum values are not always attainable or even practical for a particular structure. Variations from these dimensions and costs may be entirely justified; however, it is necessary to evaluate the efficiency of a structure based on stress values as well as construction costs.

There are many other shell proportions and shell forms that should be included. Prestressing of boundary members for large span shells can produce considerable savings in the boundary members and the shell slabs. The prestressing of shell slabs may offer still further savings.

Other factors not reflected in this study might include the effect of lightweight concrete materials, wind pressures, temperature effects, floor to springline dimensions, eccentricities between ties, springline of shell structure, and cable supported or air supported forms.

APPENDIX

List of Symbols

B, b	Width of boundary member of rib
D, d	Depth of boundary member or rib
c	Portion of shell slab contributing to edge beam stiffness (short shell) chord dimension (long shell)
h	Height (depth) of shell
h_1, h_2	Greater and smaller height, respectively (hyperbolic paraboloid)
h_x, h_y	Smaller and greater height, respectively (sectional dome)
h/L	Depth/span ratio
L	Shell span
L_1, L_2	Greater and smaller shell span, respectively
t	Shell thickness
t_c, t_b	Shell thickness at crown and at base, respectively

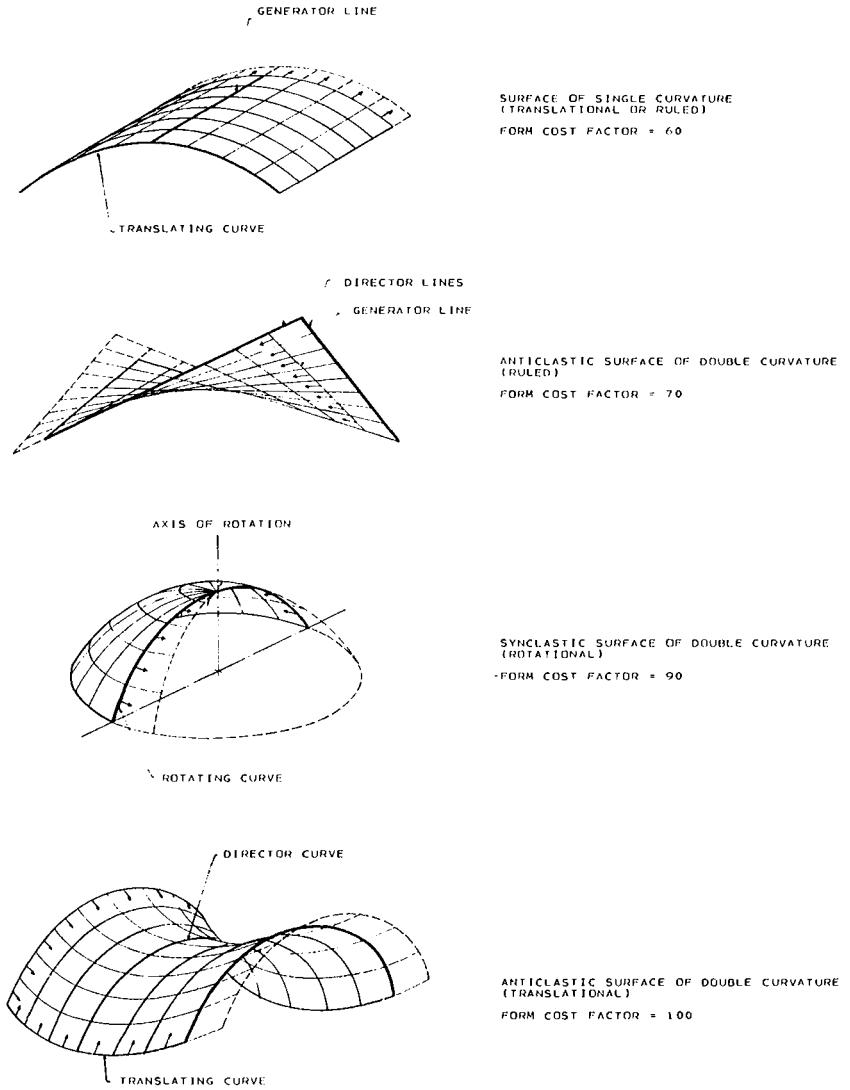


Fig. 3-1—Form cost factors for various types of surfaces

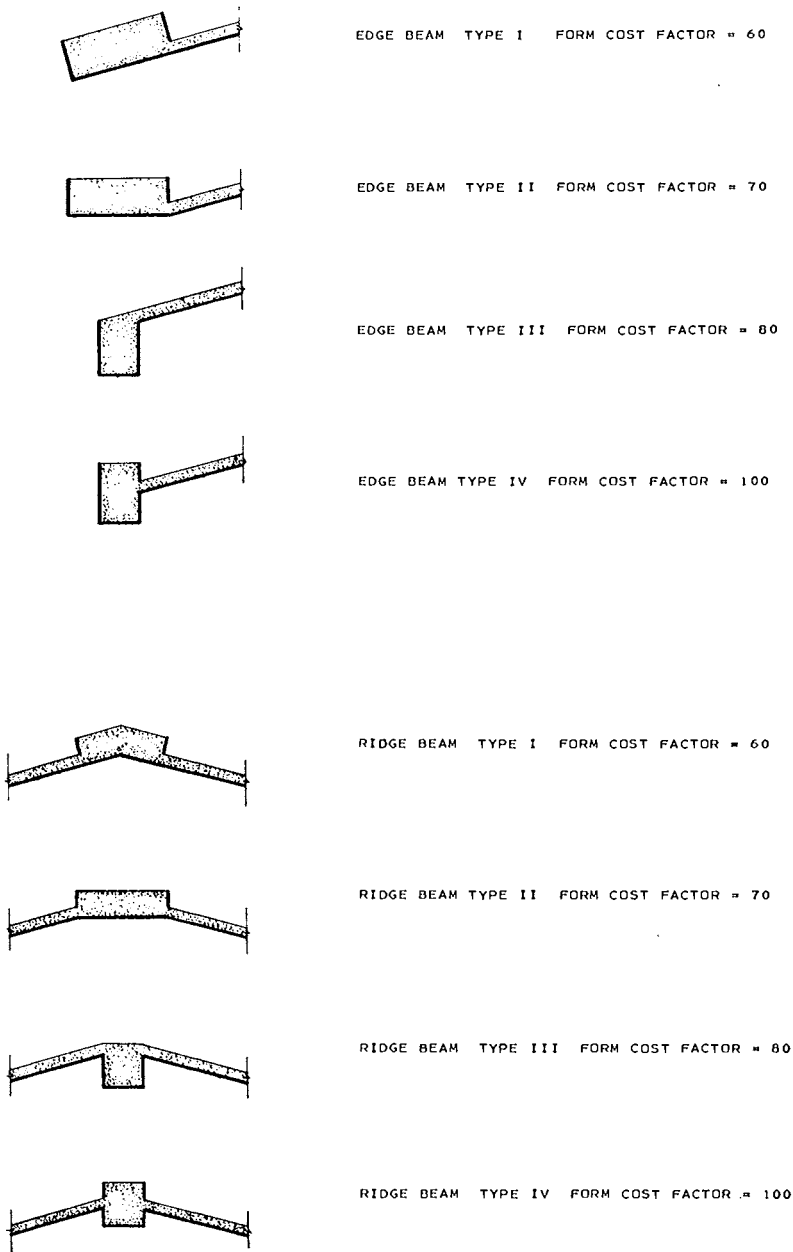


Fig. 3-2—Form cost factors for various types of boundary members

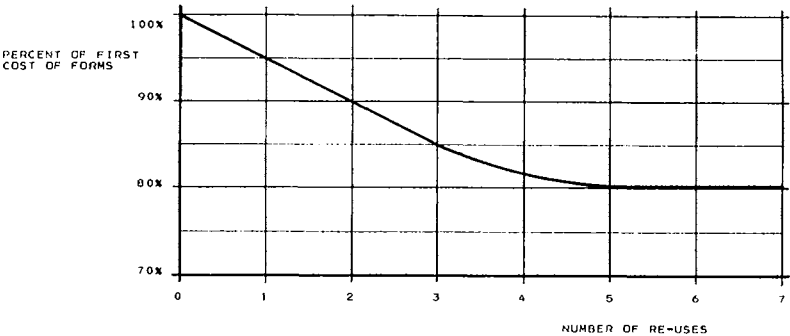


Fig. 3-3—Cost of forms based on number of reuses

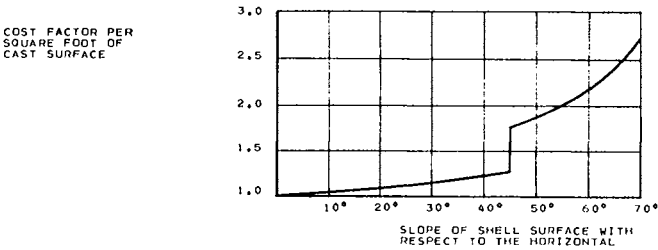


Fig. 3-4—Form costs based on slope of shell surface

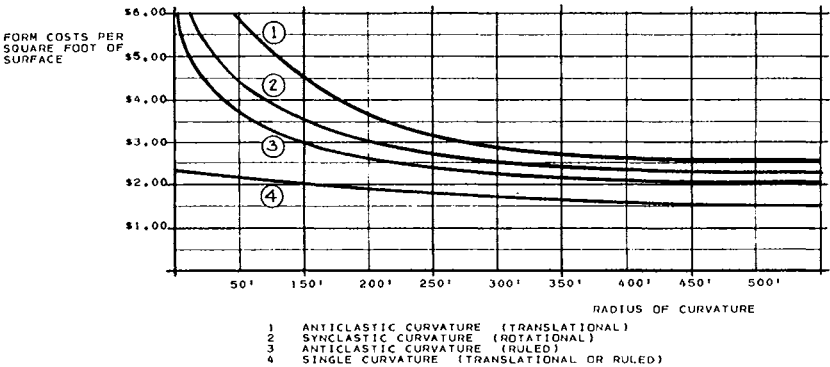


Fig. 3-5—Form costs based on radius of curvature of shell slab

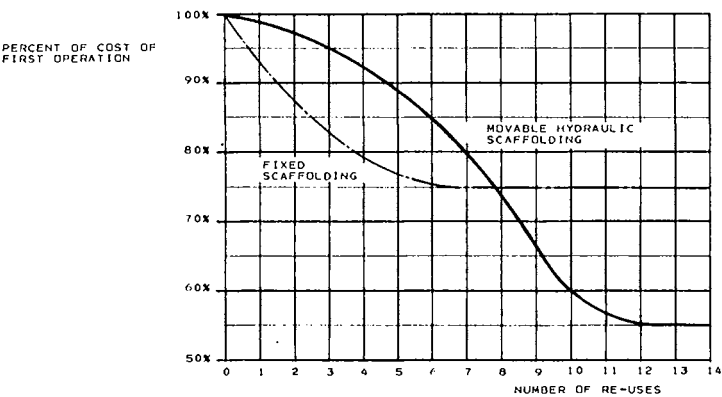


Fig. 3-6—Scaffolding costs based on number of reuses

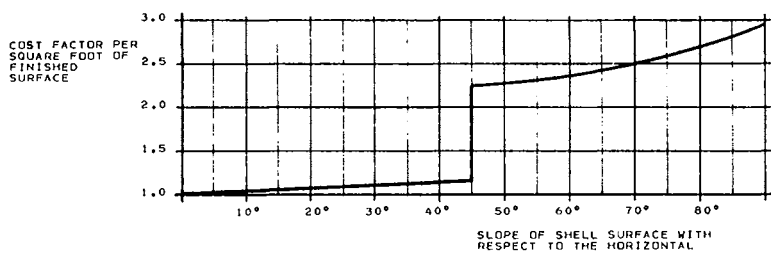


Fig. 3-7—Concrete placement and finishing costs based on shell slab slope

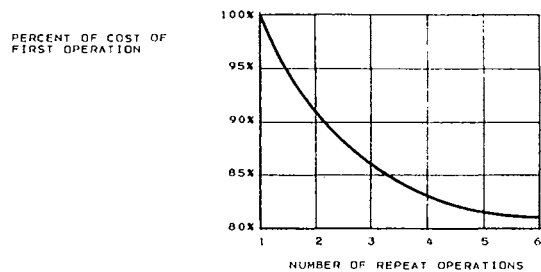


Fig. 3-8—Labor costs for shell structures based on familiarity factor of workmen

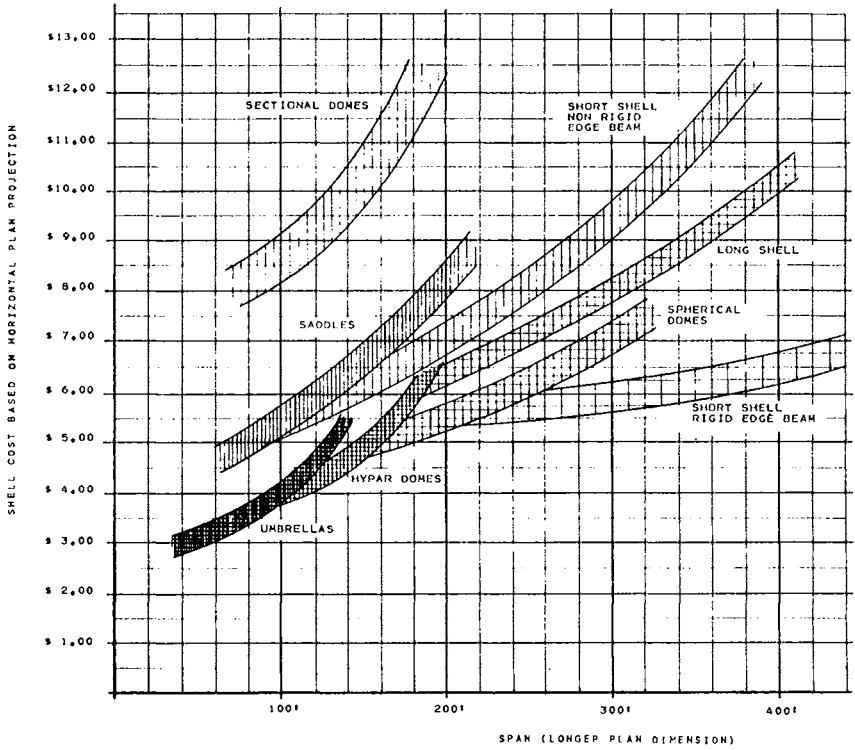


Fig. 3-9—Economic span range for various types of shells based on no reuse of forms and scaffolding