

Prediction of FRP Debonding Using the Global-Energy-Balance Approach

Chris J. Burgoyne¹, Mithila Achintha², Garfield X. Guan³

¹ *Dept of Engineering, University of Cambridge, Cambridge, UK, cjb@eng.cam.ac.uk*

² *Dept of Eng. Science, University of Oxford, Oxford, UK, mithila.achintha@eng.ox.ac.uk*

³ *Dept of Engineering, University of Cambridge, Cambridge, UK, xgg20@cam.ac.uk*

SYNOPSIS:

A major research program was carried out to analyze the mechanism of FRP debonding from concrete beams using global-energy-balance approach (GEBA). The key findings are that the fracture process zone is small so there is no R-curve to consider, failure is dominated by Mode I behavior, and the theory agrees well with tests. The analyses developed in the study provide an essential tool that will enable fracture mechanics to be used to determine the load at which FRP plates will debond from concrete beams. This obviates the need for finite element (FE) analyses in situations where reliable details of the interface geometry and crack-tip stress fields are not attainable for an accurate analysis. This paper presents an overview of the GEBA analyses that is described in detail elsewhere, and explains the slightly unconventional assumptions made in the analyses, such as the revised moment-curvature model, the location of an effective centroid, the separate consideration of the FRP and the RC beam for the purposes of the analysis, the use of Mode I fracture energies and the absence of an R-curve in the fracture mechanics analysis.

Keywords: Branson's model, external strengthening, fracture mechanics, FRP, interface debonding, mixed-mode crack

Chris J. Burgoyne is Head of the Structures Group at the University of Cambridge. He has carried out extensive research into the use of advanced fibers for use in a variety of Civil, Structural and Marine applications. He has published extensively and was a founder member of ACI 440.

Mithila Achintha is a post-doctoral research assistant in the Department of Engineering Science, University of Oxford, UK. He received his BSc(Eng) from the University of Moratuwa, Sri Lanka and PhD from University of Cambridge, UK in 2003 and 2010 respectively. His research interests include the application of fiber-reinforced polymers in civil engineering and fracture mechanics of concrete.

Garfield X. Guan is a PhD student at the University of Cambridge. He received his B.Eng. degree in Civil Engineering from the University of Hong Kong. He is currently studying the application of concrete fracture mechanics in RC structure retrofitting, and he has wider research interests in RC structures and seismic engineering.

INTRODUCTION

A comprehensive study of the debonding of FRP plates from concrete beams using the global-energy-balance approach (GEBA) has been undertaken; many of the concepts used have been described in elsewhere^{1,2,3,4}. This paper summarizes the overall logic but space does not permit a detailed discussion of the individual elements.

A high stress may cause a crack to form near the interface between the concrete and the FRP, but that crack will only propagate if more energy is thereby released than it takes to form the new fracture surfaces. It is thus a fracture mechanics problem, not a stress-analysis problem. The Global Energy Balance Approach to the study of debonding of FRP plates from concrete beams is a very simple concept, but requires understanding of some complex mechanics. When a crack forms, the beam loses some of its stiffness so the load does more work; most of which is stored by an increase in strain energy within the beam. So to calculate the energy that is released these two quantities have to be calculated with reasonable accuracy.

This paper addresses a number of issues that are important in the analysis of debonding of FRP plates from concrete beams. The evaluation of energy states in cracked concrete beams using the stress-strain (σ - ϵ) behavior over the whole beam is very complex, so in the present model a simpler integration of moment-curvature (M - κ) is used, but even determining the curvature is complex. Branson's model⁵ was conceived only for beams with conventional steel reinforcement, and only up to the point where the steel yields. When external FRP is added there is an additional layer of reinforcement, with different bond characteristics so it is incorrect to incorporate the FRP as a second layer of steel reinforcement in the Branson's model. The model also has to be applicable after the steel has yielded. The present paper shows how these issues have been addressed.

The debonding analyses rely on knowing the fracture energy of concrete (G_C), a parameter that is easy to define but less easy to determine, and one that is rarely assessed in experimental studies, even if it is the most important concrete parameter when studying debonding. Unlike glass, the fracture process zone (FPZ) in concrete is large, typically over 300 mm (1 ft) long, and may have a width of several times the aggregate size⁶. Conventional fracture analysis would require modelling the whole of this zone, but a debonding fracture occurs in a narrow zone of the concrete cover that has FRP on one side and the steel rebar on the other (Fig. 1a). Propagation of a short crack is unlikely to allow the FPZ to develop fully. The present GEBA analyses rely on the fact that fracture energy is not affected by the length of the debonding crack; this is discussed.

Premature FRP debonding hampers efficient use of externally bonded FRP plates in flexural strengthening of reinforced concrete (RC) beams, and uncertainty about the governing mechanisms means that there is no reliable theory that can be applied by designers. The earlier work of the present authors has shown that the area near the plate end, and zones where widening of flexural cracks causes interface flaws, are those most susceptible to the initiation of debonding (Fig. 1b); the two modes are referred to as "plate-end" (PE) and "intermediate-crack-induced" (IC) debonding respectively³. PE debonding initiates from the vicinity of the plate end and propagates towards the mid-span of the beam, whereas IC debonding initiates at a high-moment zone and propagates towards a low-moment zone (Fig. 1b). It has also been shown that the present GEBA model can be used to analyze debonding of steel plates provided that the plates remain within elastic limits.

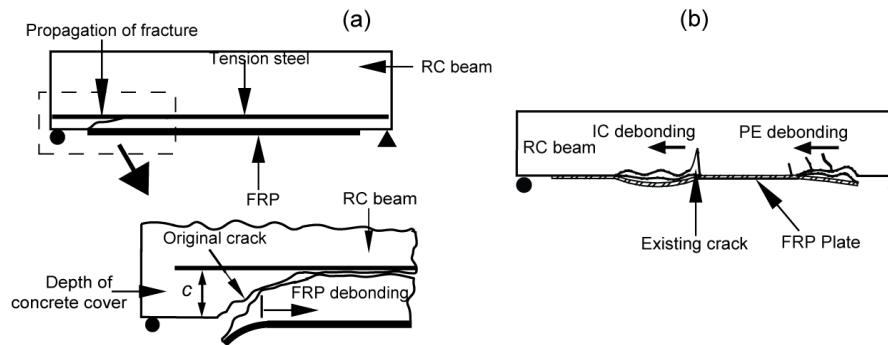


Figure 1. – (a) Debonding propagates in the concrete just above the interface
(b) PE and IC debonding

Manufacturers have now developed adhesives that are sufficiently tough that, if used correctly, debonding usually takes place in the concrete beam just above the interface (Fig. 1a). It has been observed that FRP debonding initiates from the propagation of a dominant crack in the vicinity of the interface, and hence, fracture-mechanics-based analyses have often been used in the literature to determine failure loads. However these analyses, (for example Günes⁷) were often based on the pioneering theories of Hutchinson and Suo⁸, which were intended for the analysis of interface debonding in thin-layered elastic materials. Because of the long FPZ associated with fractures in concrete, a reliable solution for FRP debonding cannot be obtained from the theories of linear-elastic-fracture-mechanics (LEFM). Furthermore, the non-linear FE models, such as the J-integral method, that would be needed to simulate the debonding of FRPs, require far more detail of the interface properties than will ever be available, even to the analyst of laboratory experiments, and certainly not to designers. Most analyses reported in the literature have only been calibrated against individual researchers' own, usually limited, test data, and none of these analyses has received a wide acceptance.

There is a need for a more physically-based fracture mechanics model that represents energy balance requirements, rather than an unreliable analysis of crack-tip stress field. The model has to be based on governing parameters that can be reliably determined and should be able to analyze all modes of debonding in beams with a wide range of dimensions. The earlier work of the present authors has shown that the energy states in beams can be determined to an accuracy good enough for models of RC beams, and the incorporation of these energy estimates in the GEBA-based debonding model correlated well with test data reported in the literature³. The work obviates the need for unreliable FE analyses which have often been used in the studies reported in the literature.

The "current state" of a system will be at a position of minimum total potential energy. The GEBA model determines that debonding will occur if the energy available for a potential small extension of an existing interface crack exceeds the energy needed to form the new fracture surfaces formed during this crack propagation. If the energy release rate (G_R) associated with an existing crack exceeds the fracture energy of concrete (G_C) the crack will propagate. How the initial crack developed up to the current state is immaterial and it is sufficient to assume that flaws of the relevant size are *likely* to exist in critical locations. The model can be used to determine the shortest crack that triggers failure at a given load and also the failure load of a beam with an existing crack of known length³.

However, determinations of both G_R and G_C , either theoretically or experimentally, are complex even in a research context; the present study has developed appropriate methods to calculate both the parameters to an accuracy that is reliable enough to be used in the analysis of FRP debonding. The G_R associated with a given crack can be determined by considering the energy changes that take place in the system during a potential unit extension of the crack. However, this analysis is not trivial; integration of $M-\kappa$ relationships determined on the assumptions that the section is uncracked (i.e. concrete in the tension zone is fully effective) significantly underestimates the energy state of the beam, while if the beam is assumed to be fully-cracked (i.e. no tensile contribution from concrete) the energy state is significantly overestimated. Various tension-stiffening models exist, although these were primarily developed to calculate the deflections of RC beams. Branson's model⁵ indirectly incorporates the effects of tension-

stiffening into the stiffness of cracked RC beam sections by defining an effective stiffness in the deflection analysis of conventional RC beams (i.e. with steel reinforcement only) and the model has been widely verified in the literature. In the present work the model has been modified to take account of several factors so that it can be used for the energy analysis of strengthened beams².

Fracture propagates in the concrete substrate, so it is necessary to know G_C of the concretes from which the beams are made. In different modes of fracture (i.e. opening; shear; or a combination of both), different stress–displacement fields will develop in the vicinity of the crack, so it is necessary to determine G_C corresponds to the correct fracture mode. The present study has shown that the interface of a strengthened beam, which is primarily carrying shear, actually fails in tension. The relative vertical displacements between the two crack faces of the original shear crack and the difference in the curvatures between the RC beam and the FRP at the plate end introduce significant peeling (tensile) stresses in the vicinity of the interface flaw that causes PE debonding (Fig. 2a). During IC debonding, the force in the FRP (F_p) acts with an eccentricity with respect to the tip of the interface crack and induces significant tension at the crack tip (Fig. 2b). The fracture propagates locally by opening (i.e. as a Mode I crack)⁴. The work has also shown that the results of shear–lap experiments, which have often been reported in the literature in studies of FRP debonding, only provide an estimate for the shear mode fracture energy which is not relevant for the debonding analyses. The incorporation of the Mode I fracture energy of concrete (G_{CI}) in the debonding analyses provided results that match with test data reported in the literature³. The present paper explains why particular values for G_{CI} have been chosen in that study.

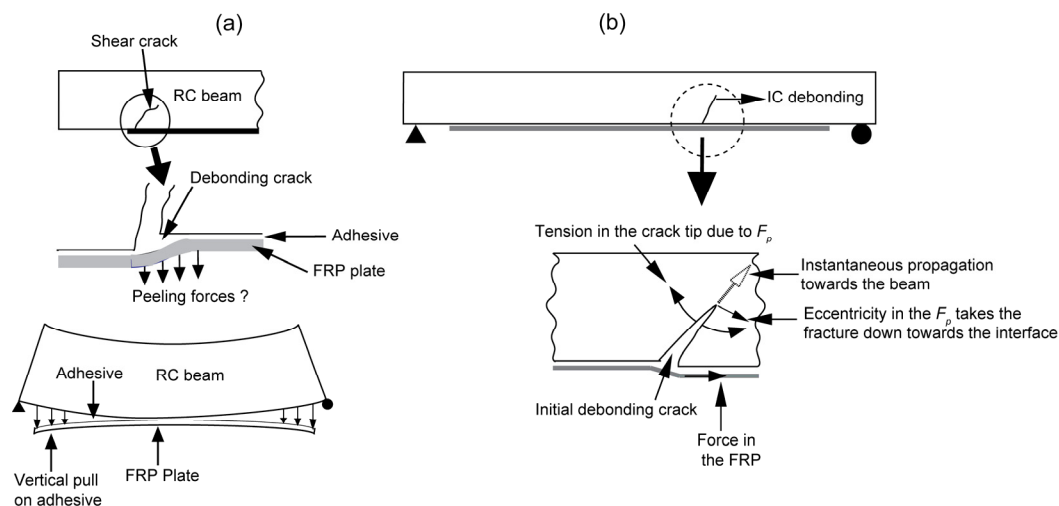


Figure 2. – Tensile stresses of significant magnitudes are developed in the crack tip
(a) PE debonding (b) IC debonding

MECHANISM OF FRP DEBONDING

PE debonding forms when an interface crack forms due to the widening of a shear crack in the vicinity of the plate end, whereas IC debonding is triggered by an interface crack formed due to the widening of a critical flexural crack in the high moment zone (Fig. 1b). Shear and peeling stress concentrations develop due to geometric restraints and also due to the relative vertical movements of the faces of the critical shear/flexural crack, which triggers further propagation of the already-formed interface crack, causing separation of FRP from the concrete beam⁴.

The whole concrete cover of the beam usually separates during PE debonding, whereas a concrete layer of only a few millimeters thick separates during IC debonding⁹. This observation has led some researchers to analyze PE debonding as a shear failure of the RC beam and IC debonding as an interface failure⁹. However, both modes are essentially fractures in the concrete substrate and the difference in the fracture path is due to the effect of the difference in the magnitudes of force in the FRP (F_p) in the corresponding locations. The principal interfacial stress

in the vicinity of the debonding crack will be at about 45° to the interface, and thus it is expected that the crack will move into the beam. PE-debonding cracks usually move up to the level of tension steel bars and the final failure occurs at this level. However, during IC debonding, the large force F_p that acts eccentrically to the crack tip takes the fracture back down towards the interface, and debonding propagates in the concrete just a few millimeters above the interface (Fig. 2). The present GEBA model has been used to analyse the both modes of FRP debonding³.

Experimental studies reported in the literature suggest that PE debonding is the likely failure mode of most strengthened beams⁹, and hence, a number of studies investigated the effectiveness of the use of plate-end anchoring systems (e.g. FRP jackets¹⁰) and the use of long FRP plates right up to the beam end¹¹ as methods to resist PE debonding. Although the methods improved the strength and the ductility of beams, the fact that PE debonding still takes place may be due to the stress concentrations developed due to the anchoring devices. The use of long FRPs usually eliminated PE debonding, but this can lead to premature IC debonding¹¹.

ANALYSIS OF FRP DEBONDING USING GEBA METHOD

Fracture mechanics investigates the possible propagations of existing cracks and it better simulates the mechanism of interface debonding than any other method. Even though numerous cracks are inevitably present in the interface between the FRP and the concrete, most are either not long enough, or not weak enough to trigger failure. Only the propagation of a dominant crack triggers failure and it is this that is analyzed by the GEBA method. Analytical methods that compare interfacial stress concentrations with the interface strength might not be able to distinguish the critical crack that triggers debonding from other minor flaws. Conventional fracture mechanics analyses that determine G_R at the crack tip, such as the J integral method, cannot be performed because the microstructure in the vicinity of the interface is unknown. Nonlinear finite element packages use special types of element to model crack tips and the FPZ, such as collapsing elements to model stress singularity at a crack tip and special “spring” type elements that can incorporate the effects of the cohesive forces in the FPZ. However, because of the heterogeneous nature of concrete the details can never be known in sufficient detail, even for laboratory specimens, let alone when designing a new structure.

As an alternative, the present GEBA model predicts FRP debonding by comparing two governing parameters (G_R and G_{CI}), both of which can be determined to an appropriate accuracy. An essential first stage of the calculation of G_R associated with an existing crack, is the determination of the energy state of the beam at a given applied load, derived using an appropriate $M-\kappa$ model that is discussed below.

Moment–curvature analysis of strengthened beams

Although the analysis of a RC beam section with the assumptions that the section was uncracked or fully-cracked is straightforward, an accurate $M-\kappa$ analysis of a partially-cracked section whilst incorporating the effects of tension-stiffening of cracked concrete is not trivial. Branson’s model (Eq. 1) incorporates the effects into the section’s stiffness indirectly by defining an effective stiffness (I_{eff}) as an interpolation between those of the uncracked (I_{un}) and fully-cracked (I_{fc}) sections, primarily with a view to being able to predict the deflections of beams (I_{un} and I_{fc} can be determined to an acceptable accuracy from an elastic and a cracked-elastic analysis respectively). The interpolation coefficient (K in Eq. 1) takes account of the current cracking level of the section and is defined as the ratio between the moment that causes the first flexural crack in the section (M_{cr}) and the current applied moment (M_{app}). The model has been widely validated against experimental results of deflections of RC beams, and also, with appropriate modifications, the method has been used in the analyses of prestressed concrete beams^{11,12}.

$$I_{eff} = K I_{un} + (1 - K) I_{fc} \quad \text{where} \quad K = \left(M_{cr} / M_{app} \right)^4 \quad (1)$$

I_{eff} in Eq. (1) is the effective second moment of area of the equivalent transformed concrete section of modulus E_c , so curvature of the section (κ) can be determined as:

$$\kappa = M_{app} / (E_c I) \quad (2)$$

A modified Branson’s model for strengthened beams

In a strengthened beam, the M - κ relationships of uncracked/fully-cracked sections can be determined as those of a conventional beam whilst taking account of the effect of the force in the FRP (F_p). The objective is to use the Branson's concept to determine stiffness and hence the curvature in a partially-cracked section and is discussed below. However, the original Branson's model will need modifying here because it is not correct to consider FRP as a second layer of internal steel reinforcement and it is also necessary to consider the strain state when the FRP is partially debonded. The earlier work of the present authors² has shown that, if the effect of F_p was incorporated as an external prestressing force on the RC beam alone, the analyses can be simplified conceptually since it allows analyzing the RC beam portion as a conventional beam (Fig. 3). However, this complicates the analysis since now it requires analyzing the RC section for a combined action of a compressive force and moment, both acting at the section's centroid. At any given location along the beam, the moment due to the applied load is generally known (M_{app}). This acts on the combined beam section (i.e. RC beam section + FRP plate). The portion of this applied moment which is resisted by the RC section alone (M_{eff}) can be determined if the location of the centroid is known. The energy in the RC beam can be then determined as $\int_L M_{eff} \kappa dx + \int_L F_p \varepsilon_0 dx$ (κ – curvature, ε_0 – strain in the beam section at the location of centroid and L – beam span). The determination of the energy in the FRP is trivial since FRP is assumed to be linear elastic. The separation of the energies due to moment and force in this way is only valid if the actions are calculated relative to the section's centroid, the determination of which is discussed below.

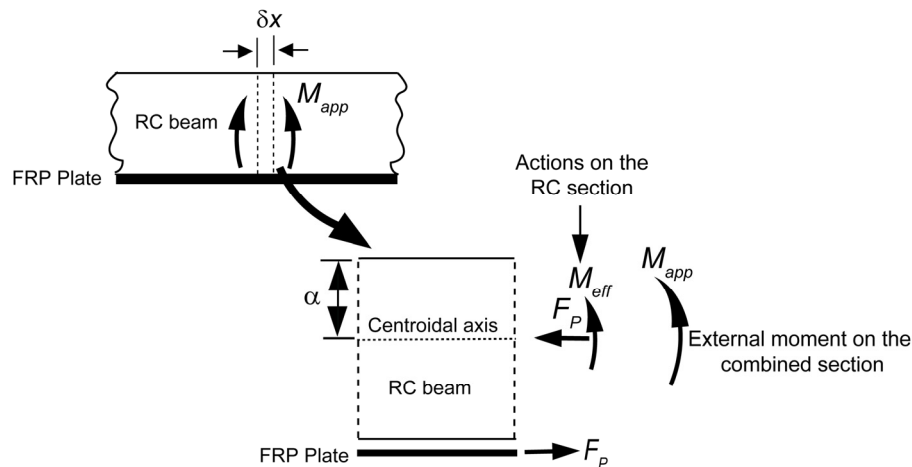


Figure 3. – Taking the effect of FRP as an external prestressing force requires analyzing the RC section for the combined action of M_{eff} and F_p

Complexities over conventional Branson analysis

The original Branson's model only applies to RC beams subject to pure bending, which can be regarded as a simple couple, so there is no need to define a particular reference axis. This will no longer be true in the analysis of strengthened beams because the RC section has to be analyzed under M_{eff} and F_p (Fig. 3). The modification of Branson's model to take account of the effects of the axial force is discussed below.

Force in the FRP

Branson's original model is only concerned with stiffness and is not used to determine the strains in the beam that are assumed to be adequate because separate checks (either permissible stress or section strength) would be performed in association. However, in a strengthened beam, if the FRP is bonded to the beam section then the strain in the FRP is locally compatible with that in the extreme tension fiber of the RC section, and if the FRP is partly debonded over a zone in the beam span, then the extension of the FRP over the unbonded region is compatible with that of the extreme tension fiber of the RC beam over the same zone². The new model, therefore, requires the satisfaction of a compatibility condition between the FRP and the concrete, which means strong assumptions need to be made about the strains, and hence stresses, in the beam section; these have to be determined from the effective stiffness. As a result, F_p at a given location in the beam span cannot be known *a priori*, so it is treated as a variable and determined numerically using a least-squares method². Once an accurate value for F_p is known, all other parameters may be evaluated.

Location of equivalent centroid

It is impossible to find an axis in a cracked RC beam section that satisfies the requirements of centroid in a linear elastic analysis (i.e. $F_p = \int_A \sigma dA$ and $M_{eff} = \int_A \sigma y dA$ where A is the cross sectional area and y is the distance from the centroid; strain energy given by $\int_L M_{eff} \kappa dx + \int_L F_p \varepsilon_0 dx$). The concept was thus developed of an “equivalent centroid” (α) of strengthened RC beam sections that will allow the separation of M_{eff} and F_p approximately, and hence, to determine the energy state in the usual way to an accuracy good enough to be used in the debonding analyses². For uncracked and fully-cracked sections the σ - ε distributions are reliably known from respective section analyses, and hence, the relevant equivalent centroids can be determined by considering the equivalent transformed sections (α_{un} and α_{fc} respectively), whilst taking account of the secant modules of non-linear materials. The centroidal location of a partially-cracked section (α_{eff}) will then be interpolated between respective α_{un} and α_{fc} using Branson’s concept; this analysis is presented elsewhere². It should also be noted that the materials are non-linear, and the secant modulus varies with stress, and hence, the location of α changes with the applied load. Thus, there is no fixed centroid that is a section property.

Modified interpolation coefficient

When the amount of cracking of a RC section increases, the tension-stiffening effects eventually become ineffective. In Branson’s model, however, the stiffness in the section becomes asymptotic to the fully-cracked state but never reaches it. That model was intended to represent sections at working loads and well below yield of the steel bars. However, the fully-cracked state will be reached in strengthened beams because that is why they needed strengthening in the first place. In the present model, it was assumed that a beam section will be fully-cracked at the moment that causes first yielding of tension steel (M_y); and the interpolation coefficient (K_p) is determined by considering the magnitudes of M_{cr} , M_y and M_{app} ². This modification does not cause significant changes to the predictions from the original model for conventional RC beams but avoids a discontinuity in stiffness when the steel yields (an example is shown in Fig. 4).

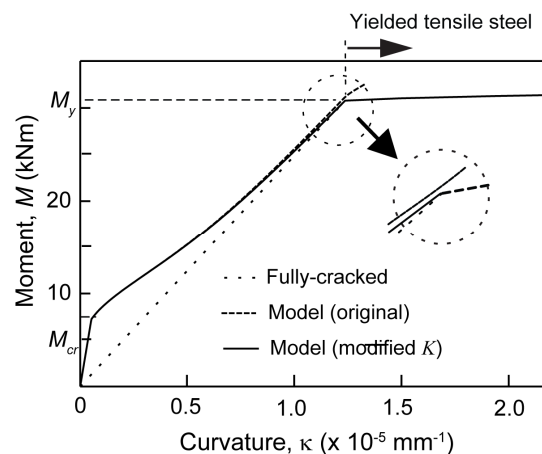


Figure 4. – For a conventional RC beam, the model predictions with the assumption of the fully-cracked state at M_y do not cause significant changes to the results at moments below M_y
(Curvature - 1.10^{-5} mm = $2.5.10^{-4}$ ins; Moment - 10 kNm = 7376 ft-lbs)

As a further complication, because of the presence of the axial load, the effective moment acting on the RC section alone depends on the choice of the axis about which it acts. The obvious choice would be the centroid, but as shown above this is not at a fixed location, either along the beam or as the loading increases. To avoid complications, it was decided to use a fixed axis about which to calculate the effective moment used to determine the interpolation

factor K_p ; the mid-depth axis of the beam was chosen, the corresponding moments are M_{cr-m} , M_{y-m} , and M_{eff-m} respectively (Eq. 3).

$$K_p = (M_{cr-m}/M_{eff-m})^4 \left[1 - \left\{ (M_{app-m} - M_{cr-m}) / (M_{y-m} - M_{eff-m}) \right\}^4 \right] \quad (3)$$

Equivalent elastic stiffness

As discussed previously, M_{eff} and κ of uncracked and fully-cracked sections can be directly determined from respective section analyses. The objective is to use Branson's concept to determine effective stiffness and hence κ of partially-cracked sections. Since the present model is to be applied to sections where the nonlinearity of material behavior needs to be taken into account, the cracked-elastic analysis used in the Branson's model is not applicable. Since the Young's modulus of concrete is no longer fixed, there is no value in defining an equivalent second moment of area. Instead, an equivalent elastic stiffness (B) is defined in place of the product of E_c and I used in the original model. The values of B for uncracked and fully-cracked sections (B_{un} and B_{fc} respectively) can be determined from the direct section analyses². For a partially-cracked section, the corresponding B_{un} and B_{fc} are first calculated, and that of the actual section (B_{eff}) is then interpolated (Eq. 4). The location of α_{eff} and hence M_{eff} of the actual section is known so combining M_{eff} with B_{eff} the κ of the section can be determined (Eq. 5). Flowcharts of the complete process of determining M_{eff} and κ of sections with bonded or partially debonded FRPs are shown in Fig. 5 (a) and (b) respectively.

$$B_{eff} = K_p B_{uc} + (1 - K_p) B_{fc} \quad \text{where } K_p \text{ is from Eq. (3)} \quad (4)$$

$$\kappa = M_{eff} / B_{eff} \quad (5)$$

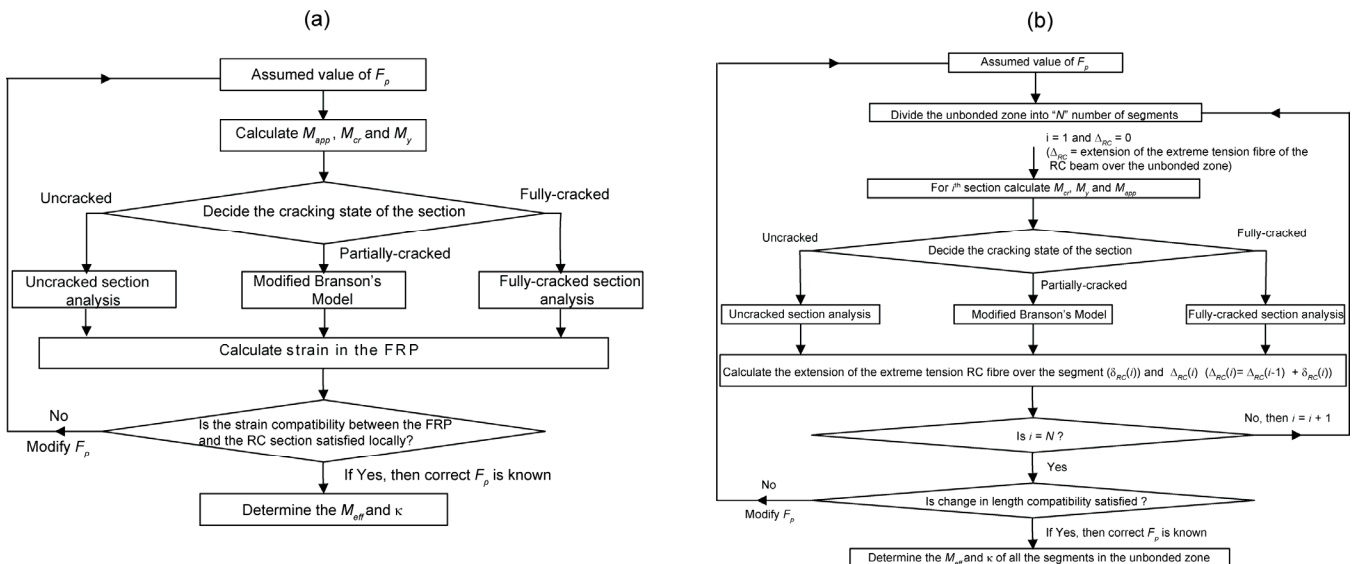


Figure 5. – Step-by-step procedure to calculate M_{eff} and κ in sections with (a) fully-bonded (b) unbonded FRP

VALIDATION OF THE PRESENT M - κ MODEL

The M - κ model was applied to several sets of beam tests reported in the literature and the model was found to be accurate enough for a model of RC beams². All the beams analyzed in the study were tested as simply-supported beams and a large database of specimens, including a variety of material/geometric properties, was investigated. The axial force in the beam can be either externally applied or exists due to unbalanced stress resultants acting on the RC section as in the case of strengthened beams. Comparisons with the test data and the present model were made under both of these categories. The model was also used to determine strain and deflection profiles of strengthened beams. A single example for each of M - κ and strain profile comparison in strengthened beams are shown in Fig. 6.

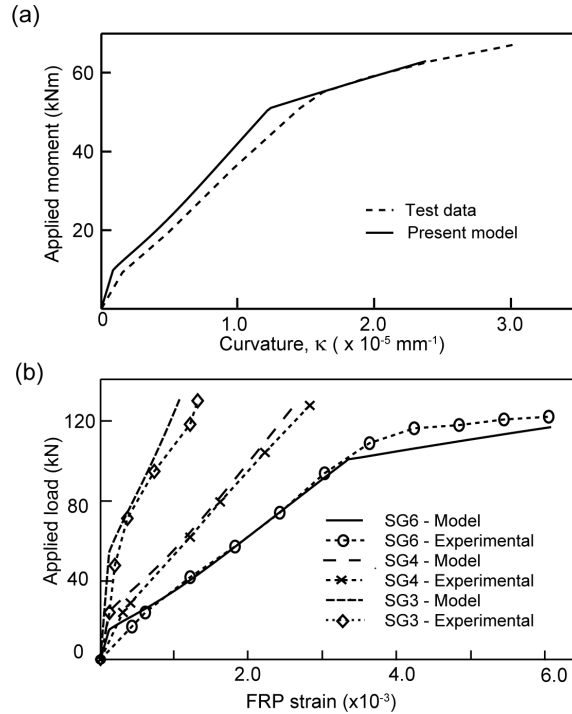


Figure 6. – (a) M – κ comparison for Beam A3.1¹³ (b) FRP strain comparisons along the span of Beam CB4-2S¹⁴
(Curvature - $1.10^{-5} \text{ mm} = 2.5.10^{-4} \text{ ins}$; Force – 100 kN = 22500 lbs; Moment - 10 kNm = 7376 ft-lbs)

Fig. 6a shows M – κ comparisons for a strengthened beam (Beam A3.1) tested by Spadea et al. (1998)¹³. It shows that the model can successfully predict behavior for all uncracked, partially-cracked, and fully-cracked regimes. The predicted M_{cr} and M_y are slightly higher than those actually observed, which may be attributed to the overestimation of the concrete tensile strength or the yield strength of steel. The small variations in the stiffness predictions may be attributed to a slight overestimation of the material stiffness. Comparisons with the measured strains in the FRP at three different span locations in Beam CB4-2S tested in four-point bending by Alagusundaramoorthy et al.¹⁴ are shown in Fig. 6b. Locations SG6, SG4 and SG3 quoted in the figure correspond to positions in the constant moment zone, center region of one of the shear spans and at distance one quarter of the shear span from the beam support, respectively. Good correlations can be observed in all cases. This shows that, not only does the present model correctly predict the curvatures, but it also correctly predicts the neutral axes, from which it can be assumed that the strain profiles will be correct.

DETERMINATION OF G_R ASSOCIATED WITH EXTENSION OF AN INTERFACE CRACK

In the GEBA model the G_R associated with a small potential extension of a given interface crack is required to compare with G_{CI} to decide whether the crack will propagate. The objective is to use the present M – κ model to determine G_R at a given applied load. When the crack extends, the beam loses some of its stiffness, so work is done by the external loads. The curvature increases in the beam, storing some of this extra work as strain energy, but some is available to cause the crack to propagate. Thus, according to the global energy balance of the system, the G_R is the rate of change of the system's total potential energy, Π_{sys} (i.e. the sum of the potential energies in the applied loads (W_{load}) and the work done on the beam (W_{beam})) with respect to the crack length (a) (Eq. 6).

$$G_R = -1/b_p (\partial \Pi / \partial a) = -1/b_p (\partial W_{beam} / \partial a + \partial W_{ext} / \partial a) \quad (b_p = \text{width of the FRP}) \quad (6)$$

In order to focus on the basic mechanics, analysis of a simply-supported beam is assumed in the present discussion. The procedure would need modifying if a statically indeterminate beam was to be analyzed when the distribution of moments caused by applied loads would change as the beam's stiffness changes during crack extension. When a RC beam bends, a part of the energy put into the beam by the loads is dissipated in cracking and

steel yielding. The rest is stored as the beam's strain energy, shown schematically in Fig. 7. It is not correct to determine G_R simply as the rate of change of strain energy, as is usually done in a LEFM analysis where it is assumed that the total work done on the beam by the loads is stored as strain energy. A number of existing FRP debonding analyses, however, are based on this incorrect theory⁶. It should be pointed out that this analysis applies only to statically determinate beams, so when debonding takes place at one location, the moment distribution, and hence the cracking state, elsewhere do not change. If the beam were statically indeterminate, changing the stiffness in one location could change the moment distribution, and a more complex set of energy changes would need to be considered.

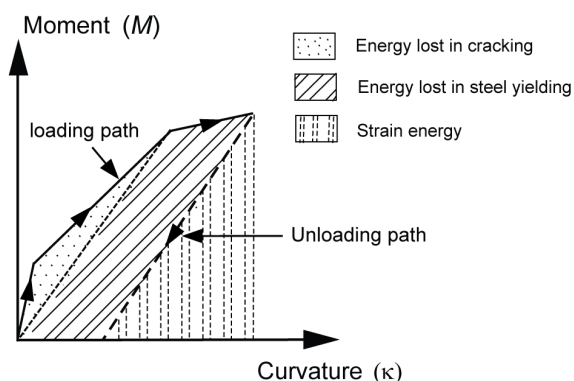


Figure 7. – Only a part of the total work done is stored as beam's strain energy

The method used to calculate G_R

G_R is determined as the change in system's total potential energy ($\Delta\Pi$) per unit area of new interface crack (G_R has the units N/mm). Due to the crack extension, the beam softens, but not uniformly. Over most of the length of the beam the curvature and hence the strain energy remain unchanged; it is only the portion of the beam near the crack tip where significant changes of curvature occur¹.

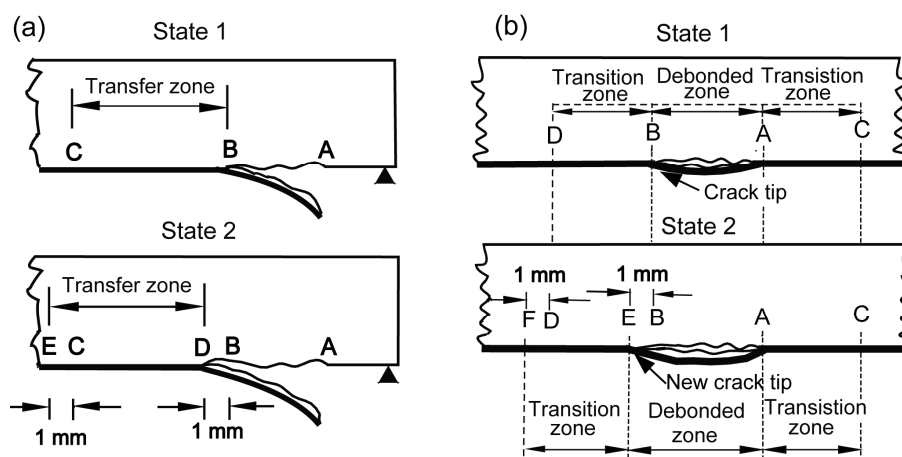


Figure 8. – Energy released zones: before (State 1) and after (State 2) small crack extension
(a) PE debonding (b) IC debonding extension

G_R associated with PE debonding is determined by considering the energy and κ changes take place in beam segments within the plate-end transfer zone (BE in Fig. 8a); for IC debonding κ changes take place in the unbonded zone and the two transition zones (CF in Fig. 8b). Outside these zones, the FRP is fully-bonded to the concrete, so if the load does not change during the debonding increment, the moment and hence the curvatures remain constant.

It is important to know the length of the transition zone. A simplified model, based on the more rigorous interfacial stress analysis of Täljsten¹⁵, was used to determine the distribution of F_p in the plate-end stress transfer

zone and also that in the transition zones associated with an existing IC debonding crack¹. The results show that the length of each of this stress-transfer zone is about 30 times the thickness of the FRP for most FRP/adhesive/concrete combinations; this value is used in all subsequent analyses.

Calculation of G_R

The objective is to determine the $\Delta\Pi$ (i.e. the sum of the additional work done on the beam (ΔW_{beam}) and the change in the potential energy of externally applied loads (ΔW_{ext}) due to the assumed small crack extension) by using the present $M-\kappa$ model. Only the changes in the energy state and κ in the critical zone (Fig. 8) are considered. The critical zone is first divided into segments 1 mm long and the additional work done (δW_{beam}) and the change in curvature ($\delta\kappa$) in each segment after the assumed crack extension are calculated as shown below. The δW_{beam} in each segment is then summed to obtain ΔW_{beam} . By numerically integrating $\delta\kappa$ of individual beam segments, the change in deflection profile and hence ΔW_{ext} can be calculated.

Calculation of ΔW_{beam} — Due to the crack extension, both the effective moment (M_{eff}) and the axial force (F_p) in RC segments alter and there will also be a change in the strain energy in the FRP. According to the present $M-\kappa$ model, the δW_{beam} in a beam segment consists of three components: additional work done in the RC section due to the change in M_{eff} (δW_M), additional work done in the RC section due to change in F_p (δW_F) and change in strain energy in the FRP (δW_{FRP}). Fig. 9 shows the changes in all these three action–deformation relationships in a beam segment, with the assumption that all have increased during the transformation. All M , κ , F_p and ϵ_0 of the segment before and after the crack extension (i.e. States 1 and 2 respectively) can be calculated from the $M-\kappa$ model, and hence, δW_{beam} and ΔW_{beam} can be determined:

$$\delta W_{beam} = \delta W_M + \delta W_F + \delta W_{FRP} \quad \text{and} \quad \Delta W_{beam} = \sum_{critical\ zone} \delta W_{beam} \quad (6)$$

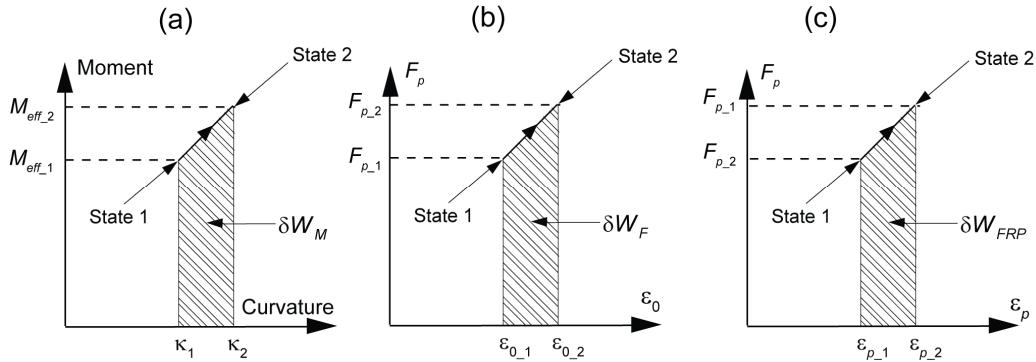


Figure 9. – Changes in the actions of a beam segment due to the crack extension
 (a) moment (b) axial force acting in the concrete beam section and (c) in the FRP plate

Calculation of ΔW_{ext} — By numerically integrating $\delta\kappa$ in the beam sections within the critical zone, the change in beam's deflection profile (Δ_{dis}) and hence the δW_{ext} can be determined (Fig. 10).

$$\Delta W_{ext} = \sum_{for\ all\ loads} P \Delta_{dis} \quad \text{where } P \text{ is the applied load} \quad (7)$$

Finally G_R can be calculated:

$$G_R = -1/b_p (\Delta W_{beam} + \Delta W_{ext}) \quad (\text{typically, } \Delta W_{ext} < 0) \quad (8)$$

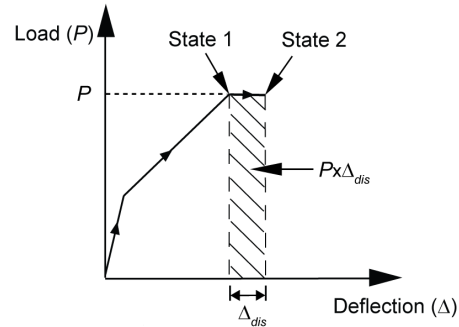


Figure 10. – Change in potential energy of the applied loads

Validation of G_R calculated from the $M-\kappa$ model against that from direct $\sigma-\epsilon$ analyses

Since accurate $\sigma-\epsilon$ relationships cannot be developed to model tension-stiffening effects, it is impossible to verify the present $M-\kappa$ model generally, but if a case is considered in which the debonding takes place either in a region which is completely uncracked or in a region which is fully-cracked, the “exact” G_R value can be determined from direct $\sigma-\epsilon$ integration, so the error in the results calculated from $M-\kappa$ integration can be investigated. Fig. 11 shows the G_R values calculated from $M-\kappa$ and $\sigma-\epsilon$ integrations respectively for an interface crack that initiated IC debonding in a typical beam, with all the sections in both unbonded and the transition zones are in the state of fully-cracked. G_R here can thus also be calculated from the integration of $\sigma-\epsilon$ relationships. The figure shows that G_R calculated from the $M-\kappa$ model agrees well with that from $\sigma-\epsilon$ analysis with about 5% error, which is well below the variance of about $\pm 10\%$ usually associated with experimentally determined G_{CI} ¹⁶. Analysis of other beams shows similar results.

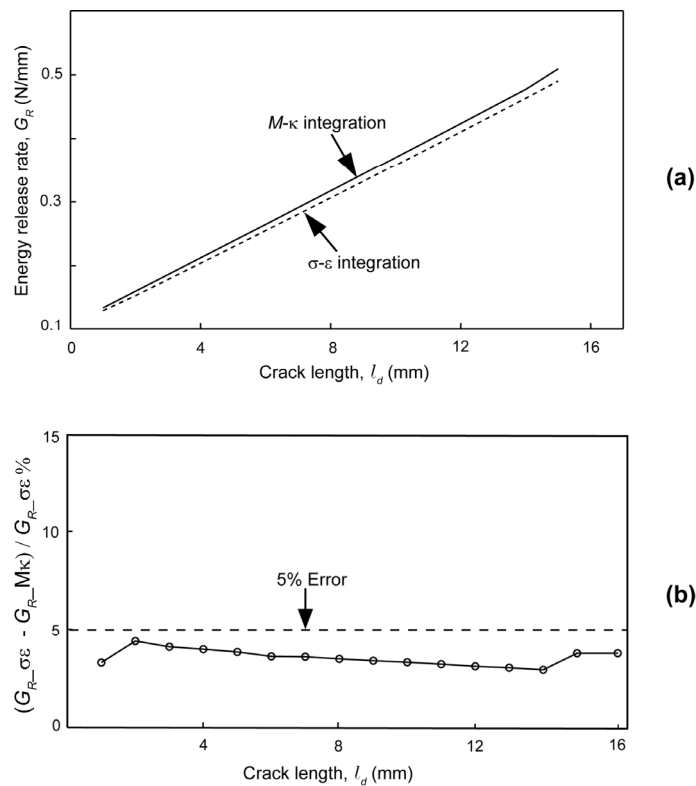


Figure 11. – (a) G_R estimation from $\sigma-\epsilon$ and $M-\kappa$ analyses
 (b) percentage error in G_R calculated from $M-\kappa$ integration
 (Energy Release Rate – 0.1 N/mm = 0.57 lbs/in; Length – 12 mm = 0.47 ins)

Effect of tortuous crack paths

Due to the material heterogeneities and the complex crack-tip stress fields, an interface crack does not propagate on a unique plane, so the G_R calculated as above represents the G_R per unit area of crack projected onto the horizontal plane. Nevertheless, G_R calculated this way can still be compared with G_{CI} , which is either experimentally determined or based on approximate theoretical models, both of which inevitably include accounts for tortuous crack paths and also for the development of new microcracks⁴. The present GEBA model only requires a reasonable estimate of G_{CI} , which is usually available, whereas a FE-based analysis would require accurate details of crack paths and the development of microcracks all of which are difficult to know with any certainty.

INTERFACE FRACTURE ENERGY

Debonding fracture usually takes place in the concrete substrate, and the G_C in the vicinity of the interface depends on the mode of fracture. The present study has shown that FRP debonding can be regarded as a Mode I fracture in concrete, despite the interface being loaded primarily in shear⁴.

The mechanism of fracture of concrete during FRP debonding

The geometric restraints, and the relative vertical movements of the faces of the pre-existing critical shear/flexural crack, cause significant shear and peeling stress concentrations in the vicinity of already-formed interface micro cracks, which then propagate as debonding cracks⁴ by opening (i.e. Mode I fracture) in the direction perpendicular to that of the maximum principal tensile stress (MPTS). Since the principal stress would be at about 45° to the interface, the crack would move into the beam. Usually, in PE debonding, the original interface crack moves up to the level of tension steel bars and the final failure occurs at this level (Fig. 2). However, during IC debonding, the F_p which carries a high tension acts eccentrically to the tip of the crack, taking the fracture back down towards the interface, and hence, the fracture propagates close to the interface along a path approximately parallel to it. Based on the orientation of the fracture path, a number of studies reported in the literature wrongly modeled IC debonding as a shear fracture of concrete⁹. The high F_p causes a significant tension in the crack tip causing opening of the crack tip – i.e. debonding locally propagates by Mode I. This can be further confirmed by the fact that, during IC debonding, the FRP remains attached to the RC beam at the other end of the original interface crack, and hence, the relative sliding between the two crack faces could not take place, and any movement of the crack faces must be normal to the crack tip.

FRACTURE ENERGY OF CONCRETE AGAINST FRP DEBONDING

The GEBA model relies on knowing the G_C of concrete, which is rarely assessed in experimental studies, even though FRP debonding is clearly a fracture event.

A combination of normal and shear stress concentrations will be present in the vicinity of an existing interface crack. It might be supposed therefore that a mixed-mode fracture energy would be relevant. However, because of the relatively high shear fracture resistance of concrete, and also because crack plane separations with a magnitude similar to the size of the aggregate are required to activate shear fracture mechanisms, it is a reasonable approximation to assume that G_C for mixed-mode fractures is dominated by the Mode I fracture energy (G_{CI}), which is much less than the G_{CII} ⁴. This assumption has been widely validated in the literature in relating various other mixed-mode fracture problems¹⁷, and is also assumed in the present GEBA model. Details of experimental and theoretical investigations of G_{CI} of concretes are presented elsewhere⁴; the methods by which the G_{CI} values of the beams quoted in that study were determined and the values that results in are briefly discussed below.

Determination of G_{CI}

The stress vs. crack-separation relationships in the FPZ, which is required to determine G_{CI} , depend on many microstructural features such as size, shape, surface texture and location of the aggregate pieces, and also on the distribution of voids in the mix; no accepted direct method is quoted in the literature for this analysis. Although, a reliable estimate for G_{CI} of a given concrete can be determined from experiments, the experimental investigations are often associated with practical and conceptual difficulties. Alternatively, simplified models, which are usually based on more readily known properties of concrete such as compressive strength (f'_c), size and type of the aggregate may be used to estimate G_{CI} to an accuracy good enough for debonding analyses; in the present work,

three widely quoted approximate simplified models and an empirical model reported in the literature were used to estimate G_{CI} of the concretes used in the beams⁴.

Estimation of G_{CI} using simplified models

Approximate simplified models determine G_{CI} as the work required to open the tip of a traction-free crack to a critical value where there are no stress transfers across the crack tip. The stress vs. crack-opening (σ_I-w_I) relationship at the crack tip is referred to as the tension softening response of concrete and is usually approximated by a bi-linear or a polynomial form. In the models quoted in the literature, the governing parameters of the σ_I-w_I curve are represented in terms of tensile strength of concrete (f_t), and the type and size of the largest aggregate. The area under the tension-softening curve is adjusted to match reliable estimates of G_{CI} obtained from more accurate analyses or from experiments. The models can then be used to estimate G_{CI} of other concretes.

G_{CI} values used in the comparisons with test data of FRP debonding

The GEBA-based FRP debonding analysis was applied to several sets of beam tests reported in the literature³. G_{CI} values of the concretes used were not measured, so it is necessary to decide on values that can be used in the debonding analysis. The f'_c of the concretes used in the beams being tested were in the range 30–55 N/mm²; (4. - 8. ksi) crushed aggregates of 20 and 10 mm and 10 mm (0.8 and 0.4 ins) rounded aggregates were used in the mixes (the complete database can be found elsewhere⁴). The G_{CI} of the beams under consideration, were calculated from concrete data quoted by the experimenters, according to the bi-linear tension-softening models of Guinea et al.¹⁸, and Gustafsson and Hillerborg²⁰, and the polynomial model of Reinhardt¹⁹; and also using the empirical model of Bazant and Becq-Giraudon²¹. The results show that for a given concrete, the predictions from these models are very similar. Also, since f'_c of the concretes did not vary significantly, the variations in G_{CI} of the beams mainly depend on the aggregate type and size⁴. Based on the model predictions, G_{CI} of mixes with crushed aggregates of 20 and 10 mm and 10 mm (0.8 and 0.4 ins) rounded aggregates were assumed to be 0.15, 0.10 and 0.07 N/mm (0.85, 0.57 and 0.40 lbs/in) respectively. These values agree with experimentally obtained G_{CI} values quoted in the literature for concretes with similar properties¹⁶. The experimentally determined G_{CI} values were often associated with a scatter about 10%, so results for a $\pm 10\%$ variation in G_{CI} was considered in the present debonding analyses.

Effect of the length of short cracks on G_{CI}

The earlier work of the present authors has shown that interface cracks of 10–30 mm (0.4 – 1.2 in) long cause PE debonding, whereas critical cracks with lengths less than 5 mm (0.2 in) trigger IC debonding³. The final issue in the discussion of G_{CI} is whether the FPZ, which is typically over 300 mm long, will be fully developed during the propagations of these short cracks. Although reliable analyses exist for some non-linear materials (e.g. metals, FRPs) to study the fracture parameters associated with short cracks (i.e. R-curve analysis), a similar analysis of concrete is not trivial and no reliable models are quoted in the literature⁴. As a further complication, fracture occurs in a narrow zone of concrete between the FRP and the internal steel bars in the beam (Fig. 1a); the strain limit imposed by the steel bars almost certainly means that the FPZ cannot develop fully. However, measured values of G_{CI} of concrete are usually obtained from small test specimens, where the FPZ could not fully develop. In addition, the present GEBA model considers the change in energy state for a short extension of the debonding crack. Any FPZ present in State 1 will be mirrored by a similar FPZ in State 2. Thus, it is assumed here that the same amount of energy is locked up in the two FPZs so that G_{CI} is unaffected by the crack length. The use of G_{CI} , determined in this way, in the debonding analyses gives results that match the test data reported in the literature. A detailed study of a "typical crack", including the R-curve behavior, is the subject of a new study.

RESULTS OF FRP DEBONDING ANALYSES

The predictions of the method have been compared with experiments reported in the literature. A number of test beams, including a variety of material/geometric properties, and covering beams that failed in all possible modes of debonding, was investigated. Comparisons made in this study show that the present model predicts results that match the test data³. A single example for each of PE and IC debonding are presented below.

Example: PE debonding

The critical shear crack that develops in the vicinity of the plate end usually propagates at about 45° to the interface, up to the level of tension-steel bars (Fig. 12). The peeled part of the plate carries no force, so the effective plate end location (L_{0_e}) is now placed slightly away from the actual plate end location (L_0). How far the initial shear

crack develops prior to the attainment of the critical debonding state determines the location of L_{0e} , which in turn governs the associated G_R . The variation between L_0 and predicted L_{0e} is expressed in terms of the depth of the cover (c): results are normally fitted between $L_0 - 2c$ and $L_0 + 2c$. The best possible agreement between the model predictions and the experimental results would be for failure at $P = P_f$ (observed failure load); $G_R = G_{CI}$ (including a range with $\pm 10\%$ variation in G_{CI}) to occur at the observed L_{0e} . Fig. 13 shows the variation in G_R vs. L_{0e} for a pair of beams (F9 and F10) selected from the study of Fanning and Kelly²². The G_{CI} of the concrete mix with 20 mm (0.8 in) crushed aggregate was assumed to be 0.15 N/mm (0.85 lbs/in). The figure shows that taking L_{0e} to be 10 mm (0.4 in) higher than the actual L_0 (i.e. $L_0 < L_{0e} < L_0 + c$), the P_f predicted from the model compares well with the observed P_f . The figure also shows that, at P_f , any L_{0e} shorter than the L_0 could not cause PE debonding, since an interface crack of a positive magnitude is required to trigger failure. Fig. 13 also shows that loads 10% higher/lower than P_f are too strong/too weak respectively to cause failure within the range $L_0 < L_{0e} < L_0 + c$. Thus, the model predictions match with the observed P_f and failure mode; this confirms that, although shear stress is present in the interface between the FRP and the concrete, at the instant of debonding the tip propagates as a Mode I tensile crack.

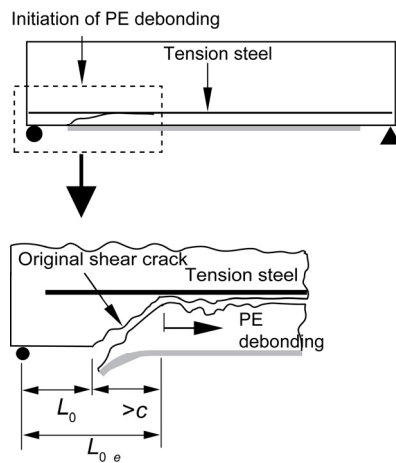


Figure 12 – Location of the effective plate end (L_{0e})

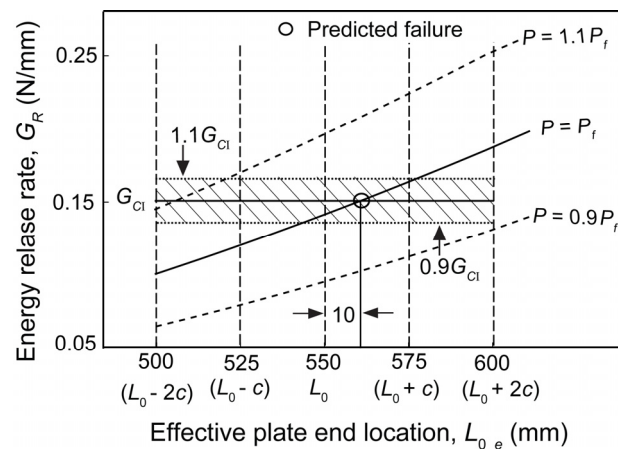


Figure 13 – G_R vs. L_{0e} plots for beam set F9 and F10²¹²²

Example: IC debonding

Analyses have shown that, in four-point bend beams, interface cracks formed at about a half beam depth (h) away from the loading point (x_c) cause IC debonding (Fig. 14)³. Analysis of the failure observed in a set of beams (Group 1) reported in the study of Ross et al.¹¹ is discussed here. Possible propagations of interface cracks that are assumed to initiate at x_c and, further distances $\frac{1}{2}h$ and h towards the nearest beam end were investigated. A concrete with 10 mm crushed aggregate was used in the beams, so G_{CI} was assumed to be 0.10 N/mm (0.57 lbs/in). The solid line in Fig. 15a shows the variation in G_R vs. l_d (crack length) for an interface crack that initiates at x_c , at the observed P_f . The figure shows that l_d of 2 mm (0.1 in) would cause debonding here; experiments reported in the literature observed that widening of a flexural crack in the high moment zone forms interface cracks of this magnitude²³. The dashed lines in Fig. 15a show that, if the debonding initiated at distances $\frac{1}{2}h$ and h away towards the nearest beam end, much longer cracks of lengths 3.5 and 6 mm (0.14 and 0.23 in) respectively, would be required to cause debonding at the observed P_f ; so these are less likely to occur. The conditions needed for debonding to initiate at x_c at 90% of P_f is also investigated in Fig. 15b, and the figure shows that l_d about twice that required at P_f would be required here to cause failure; so again is less likely.

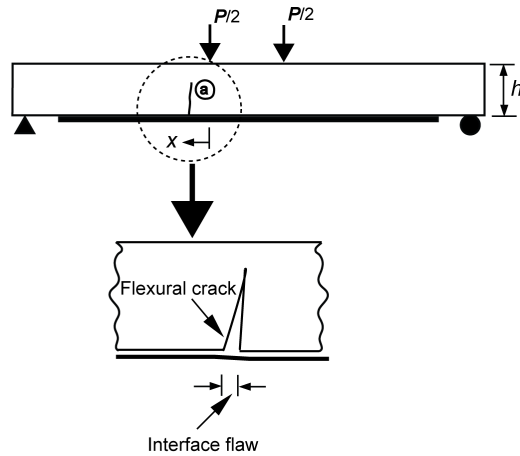


Figure 14 – Initiation of IC debonding by widening of flexural cracks

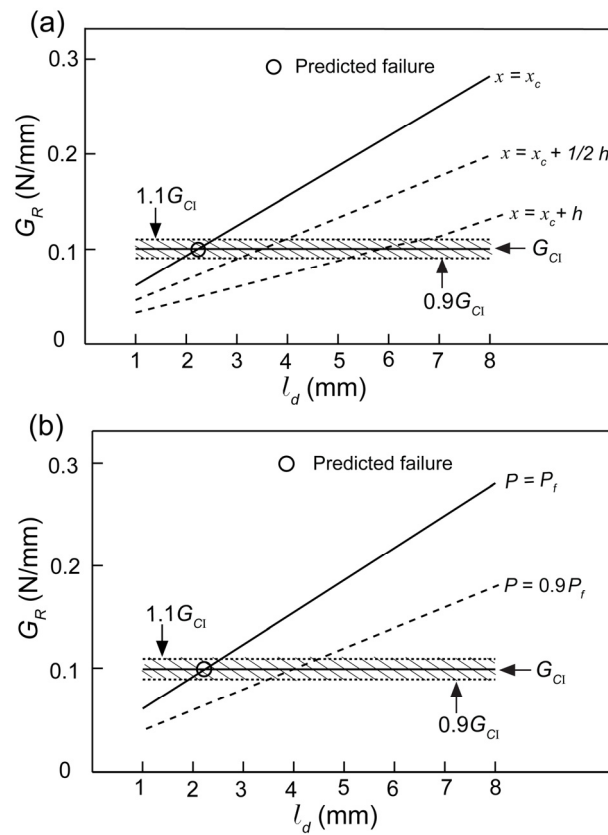


Figure 15 – G_R vs. l_d plot for Group 1 beam¹¹ (a) for fractures starting at different locations
(b) 90% of the failure load
(Energy Release Rate – 0.1 N/mm = 0.57 lbs/in; Length – 5 mm = 0.2 ins)

CONCLUSIONS

The study has shown that FRP debonding can be studied by means of a global-energy-balance based fracture mechanics model, which obviates the need for a FE analysis of dubious validity. It was necessary to produce a modified form of Branson's model to calculate the energy release during a potential small extension of an existing interface crack. Fracture propagates in the concrete just above the interface and it has been shown that debonding is a Mode I fracture as an average; comparisons with test data validate the assumption.

The results from a FE model significantly depend on the assumption of the crack path and geometry of the initial crack. The present model is also approximate but the results are not heavily influenced by the assumption. The present analysis also differs from conventional fracture mechanics because it is wrong to model the complete FPZ (which is over 300 mm long). In a conventional fracture approach the effect of this long FPZ and the R-curve behavior have to be modeled. It is assumed here that the fracture FPZ does not change with the crack length and hence there is no R-Curve behavior.

Despite exact geometries of interface cracks being unknowable, it has been shown that sensible assumptions on likely cracks give comparable predictions with test data. The basis of this work could be effectively used in applications such as the debonding analysis of continuous beams and pre-cracked beams, where external strengthening may be most useful.

Although, model can be used to explain why a particular test beam fails, a comprehensive understanding of the likely sizes and locations of interface cracks is required prior to design. There is thus a significant difference between analyzing the mechanism of failure of a laboratory test and designing a beam for use in the real world. The model can however form the basis of a parametric study that could identify the parameters that are most important in controlling debonding, identify how sensitive the failure load is to changes in those parameters, and provide guidance about approximate values of the parameters that should be used when designing beams. Results of this study will be published in due course.

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Fracture approaches to debonding in FRP systems for double sided flexural upgrade

Oded Rabinovitch¹

¹ *Faculty of Civil and Environmental Engineering, Technion – Israel Institute of Technology, Haifa, Israel,
cvoded@tx.technion.ac.il*

SYNOPSIS:

In this paper, the problem of debonding in flexural members strengthened with FRP layers bonded on their tensed and compressed faces is investigated using the fracture mechanics theory. This problem is particularly relevant to double sided FRP applications for the strengthening of masonry or reinforced concrete walls to resist cyclic or dynamic loading. The paper adopts an analytical methodology and compares between two fracture mechanics based approaches for the assessment of the initiation, evolution, and stability of the debonding process. The first approach uses the nonlinear fracture concept of the cohesive interface. The second approach adopts the classical fracture mechanics concept of the energy release rate. In both models, the effect of geometrical nonlinearity and buckling of the compressed layer and its role as the driving force for the debonding process are considered. The two approaches are compared and emphasis is placed on the stability of the debonding process and the post-debonding behavior. These aspects are illustrated through a numerical study that focuses on a masonry specimen strengthened with double-sided FRP systems and subjected to flexure. Conclusions on the behavior of the unique structural system, its stability, and its handling using the fracture mechanics approaches close the paper.

Keywords: Fracture mechanics, Energy release rate, Cohesive interface, Debonding failure, Flexural elements, Masonry walls, Buckling, Geometrical nonlinearity, Stability

Oded Rabinovitch is an associate professor at the Faculty of Civil and Environmental Engineering at the Technion – Israel Institute of Technology.

INTRODUCTION

The evolution of brittle and unstable debonding processes is one of the critical aspects of strengthened concrete, steel, and masonry elements strengthened with externally bonded layers of fiber reinforced plastic (FRP). The debonding processes and the unstable propagation of an interfacial crack between the bonded layers or few millimeters inside the substrate can dictate the global failure of the strengthened structure. These observations are relevant to the common case where the externally bonded layer is subjected to tension and the stress concentrations near irregular points (edges, cracks, masonry joints) are the driving force of the debonding process. However, the exposure of the FRP system to axial compression and the potential evolution of a buckling driven debonding process make the impact of the interfacial instability even more acute, see Fig. 1. The strengthening of concrete and masonry walls to resist cyclic or dynamic out-of-plane loads, the strengthening of such walls to resist cyclic shear due to sign reversing lateral loads, and the upgrade of columns are few cases where at least one externally bonded FRP layer is subjected to compression. The severity of the debonding mechanism in these cases, and particularly in the case of FRP strengthened masonry walls (Fig. 1) sets forth a critical analytical and practical challenge.

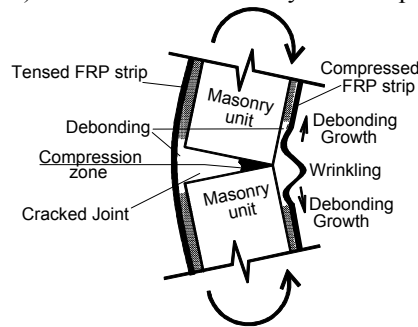


Figure 1—Local buckling driven debonding in a masonry wall strengthened with FRP to resist out-of-plane loads

The nature of the debonding process and the nature of the associated instabilities designate two main possible approaches for its analytical and numerical handling. The first approach is based on the classical concepts of Fracture Mechanics and uses Griffith's energy criterion for the propagation of the cracked area. For example, Yang et al¹ adopted the finite element and the virtual crack extension methods for the assessment of the energy release rate (ERR). Au and Buyukozturk² developed analytical expressions that are based on direct differentiation of the total potential energy. Greco et al³ used a layered shear deformation model and the virtual crack closure method for the assessment of the ERR. Bruno et al⁴ addressed the edge debonding problem through evaluation of the ERR using interfacial displacement jumps. Achinta and Burgoyne⁵ adopted the modified Branson concept for the analysis of the plated beam and the energy criterion for the crack initiation and propagation. In Achinta and Burgoyne⁶, the results are experimentally validated. Rabinovitch and Frostig⁷ adopted high order solutions and the J-integral formulation⁸. A simplified assessment of the ERR through numerical differentiation of the total potential energy is discussed in Rabinovitch⁹. This approach was also adopted for the debonding analysis of RC slabs strengthened with circular FRP sheets (Rabinovitch and Frostig¹⁰) and for the assessment of the ERR in FRP plated beams under loading conditions that yield compression in the bonded reinforcement (Rabinovitch and Frostig¹¹). The analyses in Rabinovitch and Frostig⁷ and Rabinovitch^{9,10,11} are, however, limited to the assessment of the ERR for a given crack length or a given load level. The evolution of the debonding process was not addressed.

The second approach for the handling of the debonding process uses the cohesive interface methodology (Xu and Needleman¹²). Here, the debonding interface is modeled through an interfacial surface in which the tractions across the interface are nonlinearly related to the crack opening and sliding (displacement jumps). When the cohesive interface separates, the magnitude of the tractions at first increases, reaches a maximum, and then approaches zero with increasing separation. In the field of FRP strengthening of existing structures, Taljsten¹³ used a cohesive interface concept for the modeling of the debonding of a composite strip bonded to a concrete prism. Cohesive interface analyses of debonding in beams strengthened with FRP plates were discussed in Wu and Yin¹⁴, Yuan et

al¹⁵, Niu and Wu¹⁶, Lu et al¹⁷, Wang^{18,19}, and Teng et al²⁰. Rabinovitch²¹ applied the cohesive zone concept to the analysis of beams strengthened with composite materials. Qiao and Chen²² presented a detailed experimental and numerical characterization of the CFRP-Adhesive-Concrete tri-layer system. This work highlights the softening nature of the interfacial effect. The studies surveyed above highlight the strength of the cohesive interface approach as a tool for the systematic handling of the debonding process. However, like many models that use the ERR approach, they focus on the case where the FRP layer is subjected to tension and where the main trigger and driving force for the debonding mechanism are the interfacial stress concentrations near irregular points. The case where the debonding layer is subjected to the compression makes the handling of the problem even more challenging.

Rabinovitch²³ presented a nonlinear fracture model that applies a cohesive interface approach for the analysis of buckling driven debonding in structural systems that combine stiff blocks, compliant and brittle joints, and adhesively bonded layers of FRP. The study focused on FRP strengthened masonry walls under axial compression. In cases the FRP strengthened masonry construction is mainly subjected to bending, the level of axial compression in the FRP layer may significantly increase and the deteriorated bonding conditions near the joints may trigger a localized wrinkling of the thin layer. This effect triggers a buckling driven process that combines the geometrical nonlinearity with the interfacial one.

Although the cohesive interface is a powerful tool for the assessment of the debonding mechanism in general and the buckling driven debonding mechanism in particular, it is based on heavy nonlinear computational algorithms (mostly due to the softening effects) and requires significant computational efforts and time. The ERR approach is usually "lighter" in terms of computational load and yet it allows for the assessment of the debonding initiation and, by means of the crack length control approach, the simulation of the fracture process^{24,25}. For example, simulations of the crack propagation process and a discussion of the associated snap through and snap back instabilities appears in Carpinteri et al^{26,27}. Rabinovitch²⁸ compared between geometrically (and physically) linear simulations of the debonding process by means of the ERR approach and simulations that are based on a cohesive interface model. However, this comparative analysis is limited to the geometrically linear range and it does not take into account the buckling and wrinkling phenomena as a driving force for the crack propagation instability. Comparison between fracture and cohesive interface analyses of curved FRP strengthened elements and for shear-governed plate end debonding in plated beams appear in De Lorenzis et al²⁹ and De Lorenzis and Zavaris^{30,31}.

The general agreement observed in Carpinteri et al^{26,27}, Rabinovitch²⁸, De-Lorenzis et al²⁹ and De Lorenzis and Zavaris^{30,31} between the ERR and the cohesive interface characterization of the interfacial instability designate the ERR analysis as a potential alternative to the computationally heavy cohesive interface one. However, the ability of the ERR based model to describe the debonding process in cases where the interfacial instability is coupled with a geometrical one and where the debonding process is driven by buckling and wrinkling of the external layer still have to be addressed. This challenge is faced here.

The objectives of this paper are to gain insight into the buckling driven debonding process in flexural elements strengthened with FRP on their tensed and compressed faces and to comparatively examine the capabilities of the cohesive interface and the ERR approaches to describe the debonding process and the associated instabilities. To achieve these goals, two fracture mechanics based models for the debonding analysis are presented and compared. The first model uses a cohesive interface approach. The model follows the formulation developed in Rabinovitch²³ for the buckling driven debonding analysis in axially loaded FRP strengthened masonry walls and applies the concept to the geometrically nonlinear bending problem studied here. The second model applies the ERR concept of the classical fracture mechanics theory to the geometrically nonlinear problem. This formulation follows Rabinovitch and Frostig¹¹ but extends the assessment of the ERR presented there to the characterization of the entire debonding process. This is achieved by means of the crack length control approach (Carpinteri^{24,25}, Carpinteri et al^{26,27}, Rabinovitch²⁹). The comparison between the two models aims to throw light on the nature of the physical problem and to examine the ability of the two models to quantify the buckling driven debonding process.

The paper is organized as follows. The cohesive interface model for the buckling driven debonding process is briefly presented first. This section is followed by a brief presentation of the ERR model for the geometrically nonlinear problem. Then, the crack-length control algorithm for the assessment of the debonding process path and the associated snap-back or snap-through instabilities is presented. In the numerical study, the results obtained using the two approaches are compared. The paper closes with conclusions on the physical nature of the debonding problem and on the various aspects of its assessment.

COHESIVE INTERFACE MODEL

The formulation of the nonlinear cohesive interface model is presented in detail in Rabinovitch²³. For clarity and completeness, the important aspects are briefly presented next. The formulation assumes that the behavior of the double-sided strengthened element is unidirectional in the sense that it transfers load to supporting systems located at two opposite edges only. This type of assumption is suitable for beams, one-way slabs, and one-way monolithic or masonry walls. The latter case of FRP strengthened masonry walls is probably the most common application of double-sided FRP strengthening systems. Therefore, the discussion in this paper focuses on this application. The topology of the FRP strengthened masonry wall is further simplified and, in accordance with the assumption of unidirectional action, it is assumed that the stress and deformation fields are uniform through the width of the element. This allows using the periodicity of the masonry construction in the width direction and referring to a characteristic strip of the strengthened wall. (More details on the sub-structuring approach appear in Rabinovitch²³).

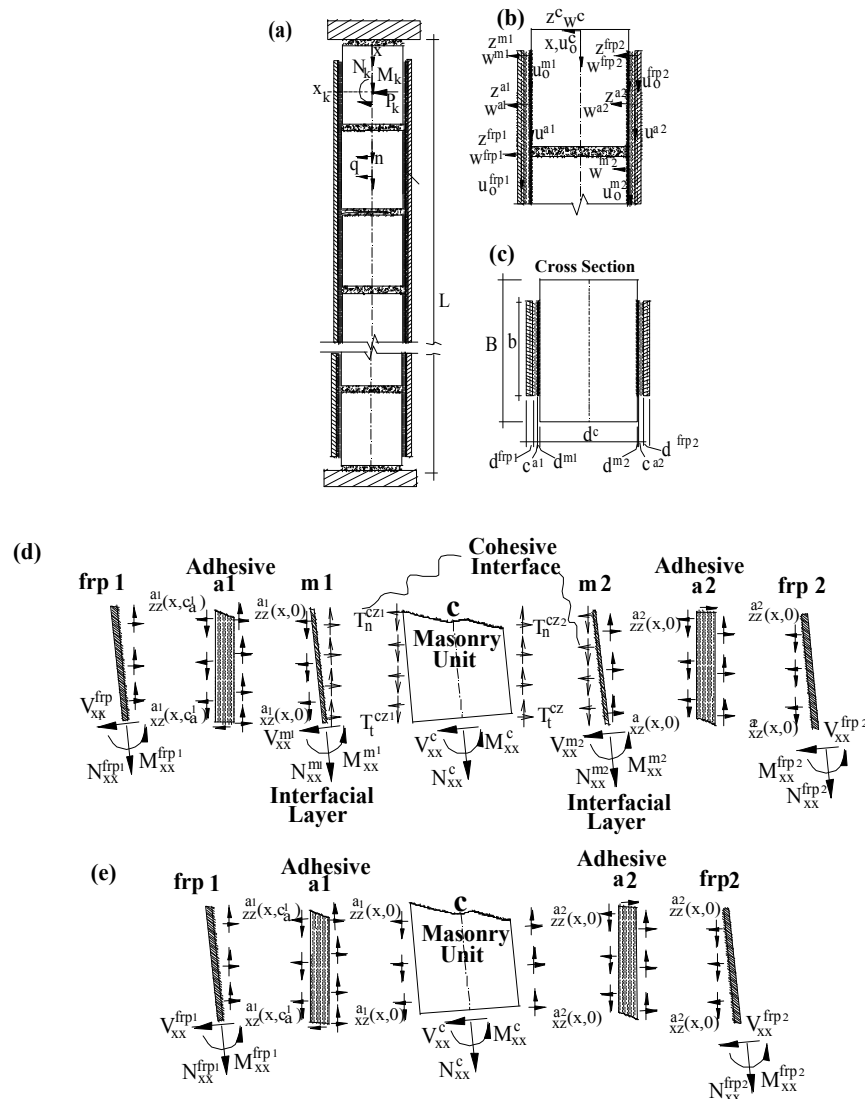


Figure 2– Notation and sign conventions: (a) overall geometry (not to scale); (b) displacements and coordinate systems; (c) cross section; (d) Cohesive interface model: stresses, interfacial tractions, and stress resultants; (e) Energy release rate model: stresses, and stress resultants;

The notation and the sign conventions appear in Fig. 2. The modeling uses the nine layers that are illustrated in Fig. 2d. The middle and the outer layers are the masonry and the FRP layers, respectively. These layers are designated

with the superscripts "*c*" "*frp1*" and "*frp2*", respectively. Each layer is independently modeled as a Bernoulli-Euler beam. It is assumed that in these layers, the displacements may be relatively large but the rotations are moderate and the strains are small. The masonry wall itself is comprised of masonry units and mortar joints. It is assumed that the material nonlinearity associated with the masonry construction is concentrated in the mortar joint. Under bending, the masonry joints are subjected to cracking and evolution of a limited compressive region (balanced by tension in the tensed FRP strip). The concentrated compression may yield a nonlinear material response at the compressed mortar. However, for simplicity, and in order to focus on the fracture mechanics handling of the interfacial nonlinearity but yet to approximate the locale-scale conditions near the cracked joint, the effect of cracking in the joints is introduced by means of eccentric hinges. The adhesive layers are designated with the superscripts "*a1*" and "*a2*". The kinematic relations for each adhesive layer adopt a 2D elasticity modeling and assume that the strains are small. In addition, it is assumed that the in-plane rigidity of the adhesive, which is much smaller than the ones of the FRP layer or the masonry wall, is neglected.

In most cases, debonding occurs at, or very close to, the adhesive-masonry interface. Therefore, following Rabinovitch²¹, each interface is modeled using two components. The first component represents the thin layer of substrate material that remains attached to the adhesive after failure. These layers are designated with the superscript "*m1*", "*m2*" and modeled as Bernoulli-Euler beams with intermediate class of deformations. The second component is the cohesive interface in which the tangential (shear) and normal (peeling) tractions are nonlinear functions of the slip (tangential displacement jump) and the separation (normal displacement jump) across the interface. The cohesive interfaces are designated with the superscript "*cz1*" and "*cz2*". Akin to the assumptions that the strains are small compared with unity and that the re-orientation of the stresses at the interfaces of the adhesive layer is not significant, it is also assumed that the rotations of the cohesive interface are sufficiently small to neglect their re-orientation during deformation (for more details also see the discussion in Rabinovitch²³).

Kinematic assumptions and equilibrium equations

The kinematic relations for the masonry wall (*c*), the FRP layers (*frp1*, *frp2*), and the interfacial layers (*m1*, *m2*) are:

$$w^i(x, z^i) = w^i(x) \quad i=c, frp1, frp2, m1, m2 \quad (1)$$

$$u^i(x, z^i) = u_o^i(x) - z^i w_{,x}^i(x) \quad i=c, frp1, frp2, m1, m2 \quad (2)$$

$$\varepsilon_{xx}^i(x, z_i) = u_{o,x}^i(x) + \frac{1}{2} (w_{,x}^i(x))^2 - z^i w_{,xx}^i(x) \quad i=c, frp1, frp2, m1, m2 \quad (3)$$

where $w^i(x)$ and $u_o^i(x)$ are the out-of-plane and axial displacements, respectively, of the reference line of each component; z^i ($i=c, frp1, frp2, m1, m2$) is measured from the reference line of each component, and the comma stands for a derivative. The reference lines are independently located at the mid-surface of each component.

The kinematic relations for the adhesive are:

$$\varepsilon_{zz}^{aj}(x, z^{aj}) = w_{,z}^{aj}(x, z^{aj}) \quad i=1, 2 \quad (4)$$

$$\gamma_{xz}^{aj}(x, z^a) = u_{,z}^{aj}(x, z^{aj}) + w_{,x}^{aj}(x, z^{aj}) \quad i=1, 2 \quad (5)$$

where ε_{zz}^{aj} ($j=1, 2$) are the out-of-plane normal strains in the adhesive layers, γ_{xz}^{aj} ($j=1, 2$) are the shear angles in the adhesive layers, w^{a1} , w^{a2} and u^{a1} , u^{a2} are the out-of-plane and in-plane displacements, respectively, of the adhesive layer and z^{a1} , z^{a2} are the out-of-plane coordinates of the adhesive layer.

The displacement jumps across the cohesive interface are given by:

$$\Delta_n^{cz1} = w^c(x) - w^{m1}(x) \quad (6)$$

$$\Delta_n^{cz2} = w^{m2}(x) - w^c(x) \quad (7)$$

$$\Delta_t^{cz1} = u_o^c(x) + \frac{d^c}{2} w_{,x}^c(x) - u_o^{m1}(x) + \frac{d^{m1}}{2} w_{,x}^{m1}(x) \quad (8)$$

$$\Delta_t^{cz2} = u_o^{m2}(x) + \frac{d^{m2}}{2} w_{,x}^{m2}(x) - u_o^c(x) + \frac{d^c}{2} w_{,x}^c(x) \quad (9)$$

where Δ_n^{czj} and Δ_t^{czj} ($j=1,2$) are the normal (separation) and tangential (slip) displacement jumps, respectively, d^c , d^{m1} , and d^{m2} are the thicknesses of the masonry wall and the interfacial masonry layers, respectively.

Equilibrium equations

The equilibrium equations are:

$$N_{xx,x}^c - bT_t^{cz1}(x) + bT_t^{cz2}(x) = -n^c \quad (10)$$

$$N_{xx,x}^{m1} + bT_t^{cz1}(x) - b\tau_{xz}^{a1}(x, z^{a1} = c^{a1}) = -n^{m1} \quad (11)$$

$$N_{xx,x}^{m2} - bT_t^{cz2}(x) + b\tau_{xz}^{a2}(x, z^{a2} = 0) = -n^{m2} \quad (12)$$

$$N_{xx,x}^{frp1} + b\tau_{xz}^{a1}(x, z^{a1} = 0) = -n^{frp1} \quad (13)$$

$$N_{xx,x}^{frp2} - b\tau_{xz}^{a2}(x, z^{a2} = c^{a2}) = -n^{frp2} \quad (14)$$

$$M_{xx,xx}^c + (N_{xx}^c w_{,x}^c)_{,x} + \frac{bd^c}{2} (T_{t,x}^{cz1}(x) + T_{t,x}^{cz2}(x)) - bT_n^{cz1}(x) + bT_n^{cz2}(x) = -q^c \quad (15)$$

$$M_{xx,xx}^{m1} + (N_{xx}^{m1} w_{,x}^{m1})_{,x} + \frac{bd^{m1}}{2} T_{t,x}^{cz1}(x) + \frac{bd^{m1}}{2} \tau_{xz,x}^{a1}(x, z^{a1} = c^{a1}) + bT_n^{cz1}(x) - b\sigma_{zz}^{a1}(x, z^{a1} = c^{a1}) = -q^{m1} \quad (16)$$

$$M_{xx,xx}^{m2} + (N_{xx}^{m2} w_{,x}^{m2})_{,x} + \frac{bd^{m2}}{2} T_{t,x}^{cz2}(x) + \frac{bd^{m2}}{2} \tau_{xz,x}^{a2}(x, z^{a2} = 0) - bT_n^{cz2}(x) + b\sigma_{zz}^{a2}(x, z^{a2} = 0) = -q^{m2} \quad (17)$$

$$M_{xx,xx}^{frp1} + (N_{xx}^{frp1} w_{,x}^{frp1})_{,x} + \frac{bd^{frp1}}{2} \tau_{xz,x}^{a1}(x, z^{a1} = 0) + b\sigma_{zz}^{a1}(x, z^{a1} = c^{a1}) = -q^{frp1} \quad (18)$$

$$M_{xx,xx}^{frp2} + (N_{xx}^{frp2} w_{,x}^{frp2})_{,x} + \frac{bd^{frp2}}{2} \tau_{xz,x}^{a2}(x, z^{a2} = c^{a2}) - b\sigma_{zz}^{a2}(x, z^{a2} = c^{a2}) = -q^{frp2} \quad (19)$$

$$\tau_{xz,x}^{a1}(x, z^{a1}) + \sigma_{zz,z}^{a1}(x, z^{a1}) = 0 \quad (20)$$

$$\tau_{xz,x}^{a2}(x, z^{a2}) + \sigma_{zz,z}^{a2}(x, z^{a2}) = 0 \quad (21)$$

$$\tau_{xz,z}^{a1}(x, z^a) = 0 \quad (22)$$

$$\tau_{xz,z}^{a2}(x, z^a) = 0 \quad (23)$$

where N_{xx}^i , and M_{xx}^i are the in-plane and moment stress resultants, respectively, in the masonry wall ($i=c$), the interfacial layers ($i=m1, m2$), and the FRP layers ($i=frp1, frp2$); T_n^{czj} and T_t^{czj} ($j=1,2$) are the normal and tangential tractions across the cohesive interfaces, respectively; $\tau_{xz}^{aj}(x, z^{aj})$ and $\sigma_{zz}^{aj}(x, z^{aj})$ ($j=1,2$) are the shear and out-of-plane normal stresses and strains in the adhesive layers; q^i and n^i are the distributed loads; d^{frp1} and d^{frp2} are the thicknesses of the FRP layers, c^{a1} and c^{a2} are the thicknesses of the adhesive layers, and b is the width of the cohesive interfaces, the adhesive layers, and the FRP layers.

Equations (10) and (15) stand for the axial and out-of-plane equilibrium conditions, respectively, in the masonry units. Eqs. (11-12) and (16-17) stand for the axial and out-of-plane equilibrium conditions, respectively, in the interfacial layers ($m1$ and $m2$). Eqs. (13-14) and (18-19) represent similar equilibrium conditions in the two FRP layers ($frp1$ and $frp2$). Equations (20-21) stand for the out-of-plane equilibrium in the adhesive layers and equations (22-23) stand for the axial equilibrium in the adhesive layers.

For brevity, the boundary and continuity conditions are not explicitly outlined. However, it is noted that in the present work, the flexural cracks, which are usually restricted to the mortar joints of the masonry substrate, are modeled using an eccentric hinge. This condition is introduced through the continuity conditions at the masonry wall as follows:

$$u_o^{c(-)} - h_{hinge}^{(-)} w_{,x}^{c(-)} = u_o^{c(+)} - h_{hinge}^{(+)} w_{,x}^{c(+)} \quad (24)$$

$$(-N_{xx}^{c(-)} + \bar{N}_k^{c(-)}) - (N_{xx}^{c(+)} + \bar{N}_k^{c(+)}) = 0 \quad (25)$$

$$M_{xx}^{c(-)} - (N_{xx}^{c(-)} - \bar{N}_k^{c(-)})h_{hinge}^{(-)} + \bar{M}_k^{c(-)} = 0 \quad (26)$$

$$M_{xx}^{c(+)} - (N_{xx}^{c(+)} + \bar{N}_k^{c(+)})h_{hinge}^{(+)} + \bar{M}_k^{c(+)} = 0 \quad (27)$$

where h_{hinge} is the z^c coordinate of the eccentric hinge, $\bar{M}_k^{c(-)}$, $\bar{M}_k^{c(+)}$ are concentrated bending moments exerted at the left and right sides (in the sense of the positive direction of the x-axis) of the crack, respectively, and $\bar{N}_k^{c(-)}$, $\bar{N}_k^{c(+)}$ are axial loads exerted at the left and right sides of the crack, respectively.

Constitutive relations and tractions across the cohesive interface

The constitutive relations for the masonry wall assume that the blocks are elastic and resilient and that the material nonlinearity is concentrated in the joints. Under these assumptions, the constitutive laws for the masonry units read:

$$N_{xx}^c = EA^c \left(u_{o,x}^c + \frac{1}{2} (w_{,x}^c)^2 \right) \quad (28)$$

$$M_{xx}^c = -EI^c w_{,xx}^c \quad (29)$$

where EA^c and EI^c are the axial and flexural rigidities of the masonry unit. The constitutive relations for the interfacial masonry layers ($m1$, $m2$) are:

$$N_{xx}^{mj} = EA^{mj} \left(u_{o,x}^{mj} + \frac{1}{2} (w_{,x}^{mj})^2 \right) \quad (j=1,2) \quad (30)$$

$$M_{xx}^{mj} = -EI^{mj} w_{,xx}^{mj} \quad (j=1,2) \quad (31)$$

where EA^{mj} and EI^{mj} ($j=1,2$) the stiffnesses of the interfacial layer. The constitutive relations for the FRP layers ($frp1$, $frp2$) read:

$$N_{xx}^{frpj} = A_{11}^{frpj} \left(u_{o,x}^{frpj} + \frac{1}{2} (w_{,x}^{frpj})^2 \right) \quad (j=1,2) \quad (32)$$

$$M_{xx}^{frpj} = -D_{11}^{frpj} w_{,xx}^{frpj} \quad (j=1,2) \quad (33)$$

where A_{11}^{frpj} and D_{11}^{frpj} are the extensional and flexural rigidities of the j 'th FRP layer, multiplied by its width. The constitutive relations of the adhesive material are:

$$\sigma_{zz}^{aj} = E^a \varepsilon_{zz}^{aj} \quad (34)$$

$$\tau_{xz}^{aj} = G^a \gamma_{xz}^{aj} \quad (35)$$

where G^a and E^a are the shear and the elastic moduli of the adhesive.

The tractions across the cohesive interface are nonlinear functions of the displacement jumps. These relations are phenomenological in nature. This opens the way to a broad range of traction-displacement jump laws. In the present work, the cohesive law of Volokh and Needleman³² is used:

$$T_n^{ci} = \frac{\phi_n \Delta_n^{ci}}{\delta_n^2} \cdot \exp \left(-\frac{\Delta_n^{ci}}{\delta_n} - \frac{(\Delta_t^{ci})^2}{\delta_n^2} \right) \quad (36)$$

$$T_t^{ci} = 2 \frac{\phi_n \Delta_t^{ci}}{\delta_n^2} \cdot \left(1 + \frac{\Delta_n^{ci}}{\delta_n} \right) \cdot \exp \left(-\frac{\Delta_n^{ci}}{\delta_n} - \frac{(\Delta_t^{ci})^2}{\delta_n^2} \right) \quad (37)$$

where ϕ_n is the work of separation per unit area and δ_n is the characteristic length parameter of the cohesive interface. While other forms of the cohesive interface laws also apply, this form is preferred here due to its simplicity, the use of continuous functions (which somewhat simplifies its implementation) and the consideration of the coupling between the shear and the normal responses. The latter feature incorporates the reduction in the interfacial shear strength due to normal separation and the reduction of the interfacial tensile strength due to sliding. Yet, in the context of the choice of the cohesive interface law, it should also be noted that the results of the analysis may be affected by the specific shape and the specific parameters of the traction-separation laws. This aspect, which has been demonstrated, for example, by Volokh³³ and Alfano³⁴ for rigid block separation and pure mode fracture simulations, respectively, ties the predictions of the cohesive interface analysis to the choice of specific parameters.

Adhesive layer – Stress and Displacement Fields

The 2D stress and displacement fields in the j'th (j=1,2) adhesive layer follow Rabinovitch and Frostig³⁵ and take the following form:

$$\tau_{xz}^{a1}(x, z^{a1}) = \tau_{xz}^{a1}(x) = \tau^{a1} \quad (38)$$

$$\sigma_{zz}^{a1}(x, z^{a1}) = \frac{w^{m1} - w^{fip1}}{c^{a1}} E^a + \tau_{,x}^{a1} \cdot \left(\frac{c^{a1}}{2} - z^{a1} \right) \quad (39)$$

$$w^a(x, z^a) = \frac{-\tau_{,x}^{a1} z^{a1} \cdot (z^{a1} - c^{a1})}{2E^a} - \frac{(w^{fip1} - w^{m1}) \cdot z^{a1}}{c^{a1}} + w^{fip1} \quad (40)$$

$$u^a(x, z^{a1}) = \frac{\tau^{a1}}{G^a} z^{a1} - \left(\frac{\tau_{,xx}^{a1}}{E^a} \right) \left(\frac{c^{a1} (z^{a1})^2}{4} - \frac{(z^{a1})^3}{6} \right) - \frac{(z^{a1})^2}{2c^{a1}} (w_{,x}^{m1} - w_{,x}^{fip1}) - w_{,x}^{fip1} z^{a1} + \left(u_o^{fip1} - \frac{d^{fip1}}{2} w_{,x}^{fip1} \right) \quad (41)$$

$$\tau_{xz}^{a2}(x, z^{a2}) = \tau_{xz}^{a2}(x) = \tau^{a2} \quad (42)$$

$$\sigma_{zz}^{a2}(x, z^{a2}) = \frac{w^{fip2} - w^{m2}}{c^{a2}} E^a + \tau_{,x}^{a2} \cdot \left(\frac{c^{a2}}{2} - z^{a2} \right) \quad (43)$$

$$w^a(x, z^{a2}) = \frac{-\tau_{,x}^{a2} z^{a2} \cdot (z^{a2} - c^{a2})}{2E^a} + \frac{(w^{fip2} - w^{m2}) \cdot z^{a2}}{c^{a2}} + w^{m2} \quad (44)$$

$$u^a(x, z^{a2}) = \frac{\tau^{a2}}{G^a} z^{a2} - \left(\frac{\tau_{,xx}^{a2}}{E^a} \right) \left(\frac{c^{a2} (z^{a2})^2}{4} - \frac{(z^{a2})^3}{6} \right) - \frac{(z^{a2})^2}{2c^{a2}} (w_{,x}^{fip2} - w_{,x}^{m2}) - w_{,x}^{m2} z^{a2} + \left(u_o^{m2} - \frac{d^{m2}}{2} w_{,x}^{m2} \right) \quad (45)$$

Governing equations and solution methods

The governing equations are derived by introducing the constitutive relations (Eqs. 28-33), the stress fields of the adhesive layers (Eqs. 38-45), and the traction separation laws (Eqs. 36-37) into the equilibrium equations (Eqs. 10-19). In addition, they include two additional equations that impose the condition of compatible axial deformations at the interfaces of the adhesive layers. The equations are stated in terms of $u_o^c, u_o^{m1}, u_o^{m2}, u_o^{fip1}, u_o^{fip2}, w^c, w^{m1}, w^{m2}, w^{fip1}, w^{fip2}, \tau^{a1}$, and τ^{a2} . The governing equations, which for brevity are not explicitly presented here, form a set of ordinary nonlinear differential equations. The nonlinearity stems from the geometrical nonlinearity due to intermediate class of deformations and the interfacial nonlinearity due to debonding and the cohesive interface. Due to the nonlinearity, a closed form analytical solution does not exist. In the present work, the governing equations are numerically solved using a multiple shooting algorithm³⁶ with a pseudo-arc-length continuation.

ENERGY RELEASE RATE AND CRACK LENGTH CONTROL MODEL

The classical fracture mechanics consideration of the debonding process uses Griffith's energy criterion for the propagation of the interfacial crack. In light of that criterion, the interfacial crack grows when the energy release rate, G , reaches the critical value G_c . The assessment of the fracture process, which differs from the assessment of the fracture point¹¹, requires finding the combination of load, displacements, and crack length that jointly satisfy the fracture criterion $G=G_c$. To achieve this, the assessment of G requires a structural model for the debonded structure and a procedure for the evaluation of the ERR. The structural model is much simpler than the one used for the

cohesive interface simulation and it only makes a distinction between fully bonded and debonded regions. The length of the debonded regions designates the crack length and its growth designates the debonding process.

In the following sub-sections, the model for the analysis of the debonded structure, the assessment of the ERR, and the assessment of the equilibrium path of the debonding process by means of the fracture criterion are briefly outlined. A similar procedure for the assessment of the debonding path but for geometrically and physically linear problems is presented and discussed in Rabinovitch²⁹. Opposed to that work, the present formulation focuses on the geometrically nonlinear case and the impact of the nonlinearity on the debonding process.

Following the concepts outlined above, as well as the ones outlined in Rabinovitch³⁷ for the geometrically nonlinear analysis of single-sided un-cracked strengthened elements and Rabinovitch and Frostig¹¹ for the evaluation of the ERR in such elements, the modeling of the double-sided strengthened element uses five layers. In that sense, it differs from the cohesive interface model that uses nine layers. The five layers of the model appear in Fig. 2e. The outer layers in the ERR model are the FRP layers. These layers are directly bonded to the concrete or masonry wall with adhesive layers. The same notation used in the previous sections for the different layers is adopted here as well. In addition, the same modeling assumptions are used for the FRP layers, the adhesive layers, the masonry units, and the representation of the cracked joint as an eccentric hinge. The modeling of the structure uses a smaller number of layers (compared with the cohesive interface model). However, it has to make a distinction between fully bonded regions and debonded regions. Furthermore, a refined distinction between debonded regions in which the cracked surfaces are in contact and can transfer normal compression and debonded regions in which the surfaces are fully detached and do not transfer any stress is also required.

Kinematic relation and interfacial conditions

The kinematic relations and the constitutive equations for the masonry wall, the FRP layers, and the adhesive layers are identical to the ones used in the cohesive interface model, Eqs. (1)-(5). However, opposed to the previous model where the presence of an interfacial layer has been taken into account, here it is assumed that the FRP strips are adhesively bonded to the wall. Under this assumption, the compatibility conditions at the interfaces of the adhesive layers are:

$$u^{ai}(x, z^{ai} = 0) = u_o^j - \frac{d_j}{2} w_{,x}^j \quad (46)$$

$$w^{ai}(x, z^{ai} = 0) = w^j \quad (47)$$

$$u^{ai}(x, z^{ai} = c^{ai}) = u_o^k + \frac{d_k}{2} u_{,x}^k \quad (48)$$

$$w^{aj}(x, z^{aj} = c^{aj}) = w^k(x) = w^k \quad (49)$$

For the first adhesive layer ($i=1$), j refers to the first FRP layer ($j=frp1$) and k refers to the masonry or concrete element ($k=c$). For the second adhesive layer ($i=2$), j refers to the masonry or concrete element ($j=c$) and k refers to the second FRP layer ($k=frp2$). In the debonded regions, the damaged interface cannot transfer shear. Hence, the requirements of compatible axial deformations are replaced with the conditions of stress free surfaces. Explicitly, Eq. (46) and/or (48) is replaced with the following two conditions, respectively:

$$\tau_{xz}^{ai}(x, z^{ai} = 0) = 0 \quad (50)$$

$$\tau_{xz}^{aj}(x, z^{aj} = c^{aj}) = 0 \quad (51)$$

If the debonded interfaces are fully detached (debonding without contact), Eqs. (47) or (49) are replaced with

$$\sigma_{zz}^{ai}(x, z^{ai} = 0) = 0 \quad (52)$$

$$\sigma_{zz}^{ai}(x, z^{ai} = c^{aj}) = 0 \quad (53)$$

Equilibrium equations and stress / displacement fields in the adhesive layer

The nonlinear equilibrium equations read:

$$N_{xx,x}^{frp1} + b\tau_{xz}^{a1}(x, z^{a1} = 0) = -\lambda n^{frp1} \quad (54)$$

$$N_{xx,x}^c - b\tau_{xz}^{a1}(x, z^{a1} = c^{a1}) + b\tau_{xz}^{a2}(x, z^{a1} = 0) = -\lambda n^c \quad (55)$$

$$N_{xx,x}^{frp2} - b\tau_{xz}^{a2}(x, z^{a2} = c^{a2}) = -\lambda n^{frp2} \quad (56)$$

$$M_{xx,xx}^c + (N_{xx}^c w_{,x}^c)_{,x} + \frac{bd^c}{2}(\tau_{xz,x}^{a1} + \tau_{xz,x}^{a2}) + b(\sigma_{zz}^{a2}(x, z^{a2} = 0) - \sigma_{zz}^{a1}(x, z^{a1} = 0)) = -\lambda q^c \quad (57)$$

$$M_{xx,xx}^{frp1} + (N_{xx}^{frp1} w_{,x}^{frp1})_{,x} + \frac{bd^{frp1}}{2}\tau_{xz,x}^{a1}(x, z^{a1} = 0) + b\sigma_{zz}^{a1}(x, z^{a1} = c^{a1}) = -\lambda q^{frp1} \quad (58)$$

$$M_{xx,xx}^{frp2} + (N_{xx}^{frp2} w_{,x}^{frp2})_{,x} + \frac{bd^{frp2}}{2}\tau_{xz,x}^{a2}(x, z^{a2} = c^{a2}) - b\sigma_{zz}^{a2}(x, z^{a2} = c^{a2}) = -\lambda q^{frp2} \quad (59)$$

where λ is a proportional loading factor.

The equilibrium equations for the adhesive layers are the same as in the cohesive interface model (Eqs. 20-23). Therefore, in the case of the fully bonded region, the stress and displacement fields of the adhesive layer take the form of Eqs. (38-45) with the superscript "c" replacing "m1" and "m2". In the debonded regions, the shear stress and its derivatives vanish and the out-of-plane normal stresses read:

$$\sigma_{zz}^{ai}(x, z^{ai}) = \frac{E_a(w^j - w^k)}{c^{ai}} \cdot H(w^k - w^j) \quad \begin{matrix} (j=1, j=c \ k=frp1) \\ \text{or} \\ (i=2, j=frp1 \ k=c) \end{matrix} \quad (60)$$

where H is the Heaviside function.

Governing equations

By using the same constitutive relations used in the cohesive interface model for the masonry layer, the FRP layers, and the adhesive materials, the governing equations take the following form:

$$EA^c \cdot (u_{o,xx}^c + w_{,x}^c w_{,xx}^c) - b\tau^{a1} + b\tau^{a2} = -\lambda n^c \quad (61)$$

$$A_{11}^{frp1} \cdot (u_{o,xx}^{frp1} + w_{,x}^{frp1} w_{,xx}^{frp1}) + b\tau^{a1} = -\lambda n^{frp1} \quad (62)$$

$$A_{11}^{frp2} \cdot (u_{o,xx}^{frp2} + w_{,x}^{frp2} w_{,xx}^{frp2}) - b\tau^{a2} = -\lambda n^{frp2} \quad (63)$$

$$EI^c w_{,xxxx}^c - EA^c \cdot \left(u_{o,xx}^c w_{,x}^c + u_{o,x}^c w_{,xx}^c + \frac{3}{2} w_{,x}^c \cdot (w_{,xx}^c)^2 \right) + b \frac{c^{a1} + d^c}{2} \tau_{,x}^{a1} + b \frac{c^{a2} + d^c}{2} \tau_{,x}^{a2} - \frac{bE_a}{c^{a1}} (w^c - w^{frp1}) \cdot H(w^{frp1} - w^c) + \frac{bE_a}{c^{a2}} (w^{frp2} - w^c) \cdot H(w^c - w^{frp2}) = -\lambda q^c \quad (64)$$

$$D_{11}^{frp1} w_{,xxxx}^{frp1} - A_{11}^{frp1} \cdot \left(u_{o,xx}^{frp1} \cdot w_{,x}^{frp1} + u_{o,x}^{frp1} \cdot w_{,xx}^{frp1} + \frac{3}{2} w_{,x}^{frp1} \cdot (w_{,xx}^{frp1})^2 \right) - \frac{(c^{a1} + d^{frp1}) \cdot b}{2} \cdot \tau_{,x}^{a1} - \frac{bE_a}{c^{a1}} \cdot (w^c - w^{frp1}) \cdot H(w^{frp1} - w^c) = \lambda q^{frp1} \quad (65)$$

$$D_{11}^{frp2} w_{,xxxx}^{frp2} - A_{11}^{frp2} \cdot \left(u_{o,xx}^{frp2} \cdot w_{,x}^{frp2} + u_{o,x}^{frp2} \cdot w_{,xx}^{frp2} + \frac{3}{2} w_{,x}^{frp2} \cdot (w_{,xx}^{frp2})^2 \right) - \frac{(c^{a2} + d^{frp2}) \cdot b}{2} \cdot \tau_{,x}^{a2} + \frac{bE_a}{c^{a2}} \cdot (w^{frp2} - w^c) \cdot H(w^c - w^{frp2}) = \lambda q^{frp2} \quad (66)$$

$$u_o^{frp1} - u_o^c - \frac{(c^{a1} + d^{frp1})}{2} \cdot w_{,x}^{frp1} - \frac{(c^{a1} + d^c)}{2} \cdot w_{,x}^c + \frac{\tau_{,xx}^{a1} c^{a1}}{G_a} - \frac{\tau_{,xx}^{a1} \cdot (c^{a1})^3}{12E_a} = 0 \quad (67)$$

$$u_o^c - u_o^{frp2} - \frac{(c^{a2} + d^{frp2})}{2} \cdot w_{,x}^{frp2} - \frac{(c^{a2} + d^c)}{2} \cdot w_{,x}^c + \frac{\tau_{,xx}^{a2} c^{a2}}{G_a} - \frac{\tau_{,xx}^{a2} \cdot (c^{a2})^3}{12E_a} = 0 \quad (68)$$

In the debonded regions, the order of the equations is reduced, the shear stresses and all terms that include τ^{a1} , τ^{a2} , and their derivatives vanish, and Eqs. (67) and (68) are omitted.

Modeling of the interfacial crack, debonding simulation, and equilibrium path

The interfacial crack front is modeled as the transition point between the fully bonded and the debonded region. At that point, the continuity condition requires that the shear stress in the bonded region drops to zero.

The fracture equilibrium path is implicitly defined as follows:

$$G(\lambda, \mathbf{u}, a) - G_c = 0 \quad (69)$$

$$NL(\lambda, \mathbf{u}, a) = 0 \quad (70)$$

where G_c is the fracture energy per unit of area, G is the energy release rate, λ is the loading parameter, $\mathbf{u}$ is the displacement field, a is the crack length, and $NL(\lambda, \mathbf{u}, a)$ is a nonlinear operator that represents the governing equations and the boundary/continuity conditions of the model. The energy release rate is defined as follows:

$$G = -\frac{\partial \Pi}{\partial A} = -\frac{\partial \Pi}{b \partial a} \quad (71)$$

where Π is the total potential energy. G is calculated by means of an analytical or numerical evaluation of the derivative in (71) or using the J integral⁸.

Equations (69) and (70) implicitly define a parametric path in the λ - $\mathbf{u}$ plane with the crack length serving as a parameter. On top of its mathematical interpretation, the physical interpretation of the monotonic growth of a defines the debonding process. Thus, for a given crack length, the load factor λ and the corresponding displacement vector $\mathbf{u}$ that jointly satisfy the governing equation of the model (Eq. 70) and bring the ERR to its critical value (Eq. 69) define a point on the debonding path. Repeating this process for a monotonically growing crack lengths define the behavior of the structure during the debonding process (see for example Carpinteri^{25,26}, Rabinovitch²⁹). In the present case, where the problem is strongly nonlinear, a simple and computationally "cheap" assessment of the equilibrium path in the form used in Rabinovitch²⁹ is not possible. Alternatively, a repeated assessment of the ERR based on the nonlinear solution is required.

DISCUSSION AND A NUMERICAL STUDY

The cohesive interface model and the ERR model present different approaches to the same problem. The handling of the geometrical nonlinearity and the consideration of the presence of the compliant adhesive layer are similar in both models. However, the handling of the debonding problem is different. In the cohesive interface model, it is automatically incorporated in the analysis. In the energy-based model, it is simulated through the ERR criterion by repeating the analysis and using the crack length as a parameter. Although both models require the solution of a set of nonlinear ordinary differential equations, the nonlinearity associated with the cohesive interface model is significantly stronger and the softening characteristics of the cohesive interface require an arc-length type of continuation. The nonlinearity associated with the energy-based model is less problematic (and therefore require simpler continuation methods). On the other hand, the number of times that the nonlinear model is solved may become significant.

In the physical modeling sense, the cohesive interface approach yields a refined model that takes into account the presence of the interfacial layer and a broader spectrum of parameters that control the decohesion process. These parameters include the shear strength of the interface (the peak shear traction in the traction-separation law), the characteristic length scale of the interface, the coupling of the shear and normal traction-separation laws, the properties of the interfacial layer, etc. In the energy-based model, the interfacial layer is not included and all parameters of the decohesion process collapse into the fracture energy G_c . While this may be interpreted as an advantage of the cohesive interface modeling approach, it should be noted that in most practical cases there is some level of uncertainty regarding the parameters of model. The thickness and elastic modulus of the interfacial layer and, particularly, the exact shape of the traction-separation laws and the characteristic length parameter, are difficult

to be assessed (see, for example Gain et al³⁸). Furthermore, due to the enrichment of the model with a range of parameters, the results of the analysis become tied to their particular choice and to the specific shape of the traction-separation laws. For example, Volokh³³ showed that the cohesive interface simulation of a rigid body separation is affected by the shape of the interfacial law, even in the case of equated cohesive strength and work of separation. Alfano³⁴ showed that for some geometries, and in particular ones that involve compact or mode II governed specimens, the cohesive interface parameters affect the numerical simulations. In that sense, the agreement (or discrepancies) between the two different approaches may be further affected by the shape, coupling mechanism, cohesive strength, and other parameters of the cohesive interface laws.

Another aspect in which the two models inherently differ is the dependency on a-priori information regarding the cracking process (also see the discussion in Rabinovitch²⁹). The cohesive interface model automatically reacts to the stress concentrations in the structure and can automatically detect the initiation and growth pattern of the interfacial crack. On the other hand, the energy based model, in which the crack length is used as a parameter, requires a-priori information regarding the cracking pattern, its point on initiation, and the direction of crack growth. Finally, in the form presented here, the energy-based model can handle only one interfacial crack at the time. Handling simultaneous debonding mechanisms, which is possible using the cohesive interface model, is beyond the scope of the energy-based model presented here.

The numerical study mainly aims to characterize the buckling driven debonding process in FRP strengthened masonry elements subjected to flexure and to compare between the cohesive interface model and the energy-based model. The study examines the FRP strengthened masonry specimen illustrated in Fig. 3. The examined specimen approximately simulates a "sub-structure" that is mathematically "taken out" from a full-scale masonry wall strengthened with FRP strips in the vertical direction and subjected to one-way (unidirectional) out-of-plane flexure. In order to focus on the local scale initiation and propagation of the debonding processes, the examined specimen only includes two masonry units and one joint. The specimen is subjected to flexure introduced by controlling the rotations at the edges through an arc-length continuation procedure. The sub-structuring approximation neglects the direct effect of the out-of-plane loads and introduces the bending through the end tractions only. The eccentric hinge that approximately simulates the cracking of the joint and connects the two masonry units is located in the compressive zone 25 mm (0.98 in) away from the middle of the specimen (see Fig. 3).

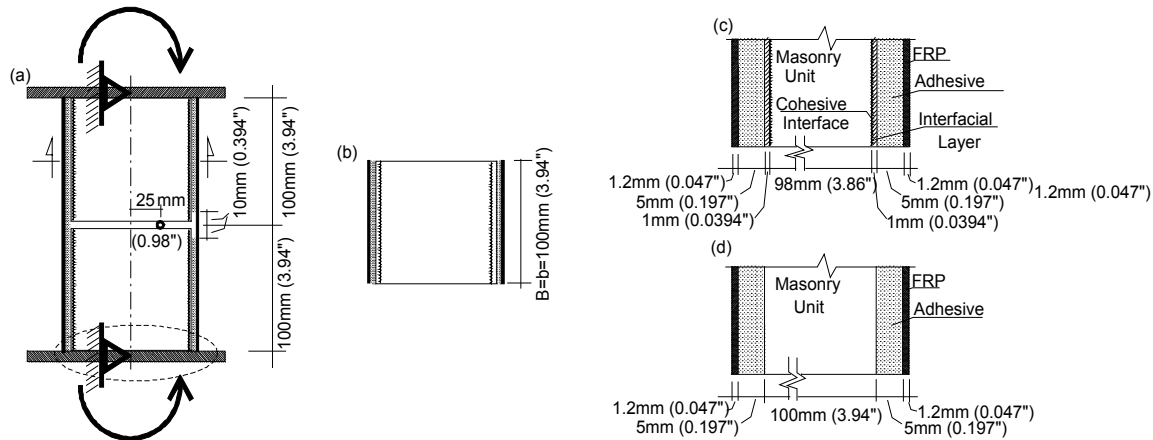


Figure 3– Numerical example (a) geometry and loading; (b) cross section; (c) cohesive interface model (d) ERR model

The elastic and mechanical properties of the various components of the examined specimen are listed in Table 1. Due to the absence of specific experimental data, the parameters of the cohesive interface are estimated based on typical values of the shear strength and fracture energy of the materials involved. The relatively low value assigned to the fracture energy $G_c = \phi_n$ reflects on the poor bonding conditions associated with the outer face of existing masonry elements.

Since the ERR simulation requires a-priori information about the propagation of the crack and since the method in the form presented here it is limited to debonding processes that involve a single interfacial crack, this example focuses on the debonding process of the compresses FRP strip only. For that purpose, the ERR model assumes that

the tensed FRP strip is fully bonded and that it does not debond during the process. In the cohesive interface model, the debonding of the tensed strip is neutralized by limiting the traction separation law of the relevant interface to the linear range and by increasing its initial stiffness by a factor of 10. It is also noted that the analysis takes advantage of the symmetry and that an initial imperfection in the form of uniformly distributed loads acting on the masonry units and the compressed FRP strip in opposite directions is introduced. Finally, it is noted that the examined specimen includes a pre-existing 20 mm long debonded region near the cracked joint. In order to further simplify the analysis, the ERR model assumes that all debonded regions are not in contact.

Component	Elastic and Mechanical properties
Masonry unit	$E=20,000$ MPa (2,900 ksi)
Interfacial masonry layer	$E=200$ MPa (29 ksi)
FRP	$E=120,000$ MPa (17,404 ksi)
Adhesive	$E=2,000$ MPa (290 ksi) $G=800$ MPa (116 ksi)
Cohesive interface	$\phi_n=G_c=58.3$ J/m ² (0.333 lbf/inch); $\delta_n=0.01$ mm (0.394*10 ⁻³ inch); $T_{t,max}^{cz}(\Delta_n^c=0)=5$ MPa (0.725 ksi)

Table 1– Elastic and mechanical properties

The numerical results of the two analyses are presented in terms of the magnitude of the global bending moment and the global axial force that evolves due to the arching effect versus the end rotation in Figs. 4a and 4b, respectively. Both analyses reveal a global "generalized load-deflection" pattern that involves a limit-point type of behavior and a snap type of instability. This includes a shift from a stable debonding growth to an unstable one and reflects on the potentially brittle and unstable debonding mechanism in the strengthened element. The shift from the stable to the unstable behavior is also observed in the crack length versus rotation curves that are shown in Fig. 4c. These curves reveal that in the cohesive interface model, the critical crack length equals 0.127L whereas in the ERR model it equals 0.140L mm. The differences between the two (which may further increase or decrease due to the dependency on the particular choice of the cohesive interface parameters) are attributed to the softened process zones that develop at the edges of the debonded region in the cohesive interface model. This softened zone slightly "relaxes" the condition for the compressed strip at the ends of the debonded region. The ERR model, on the other hand, is governed by an abrupt shift from the debonded to the bonded region and therefore a more "firm" transition into the buckled region.

Beyond the critical crack length, both models indicate that the debonding process becomes unstable. In order to characterize the factors that contribute to the instability, the curves of the out-of-plane deflections of the masonry specimen and the FRP strips at the middle of the strengthened specimen (i.e. at the cracked joint) versus the end rotations are examined in Fig. 5. The results for the compressed FRP strip appear in Fig. 5a. A zoom on the initial phase of the deflections of all components (compressed FRP strip, tensed FRP strip, masonry units) appears in Fig. 5b. Both figures clearly indicate that up to the limit point, the out-of-plane deflections generally follow the global ones induced by the end rotation of the masonry units with only minor deviations that are attributed to the flexibility of the adhesive layer and the cohesive interface. Beyond the critical crack length, the out-of-plane deflections of the compressed FRP strip significantly deviate from the ones of the substrate and start to grow rapidly. This is a direct outcome of the buckling of the compressed strip and the geometrical nonlinearity. Due to these effects, the compressed FRP strip wrinkles and forms a type of 1D "blister" across the debonded region. During the buckling driven unstable crack growth, which is theoretically involved with a reduction in the end rotation and the global tractions, the out-of-plane deflections of the compressed strip keep growing. This observation further clarifies the contribution of the buckling and the geometrical nonlinearity to the instability of the crack growth process and designates the wrinkling of the compressed strip as one of its driving forces.

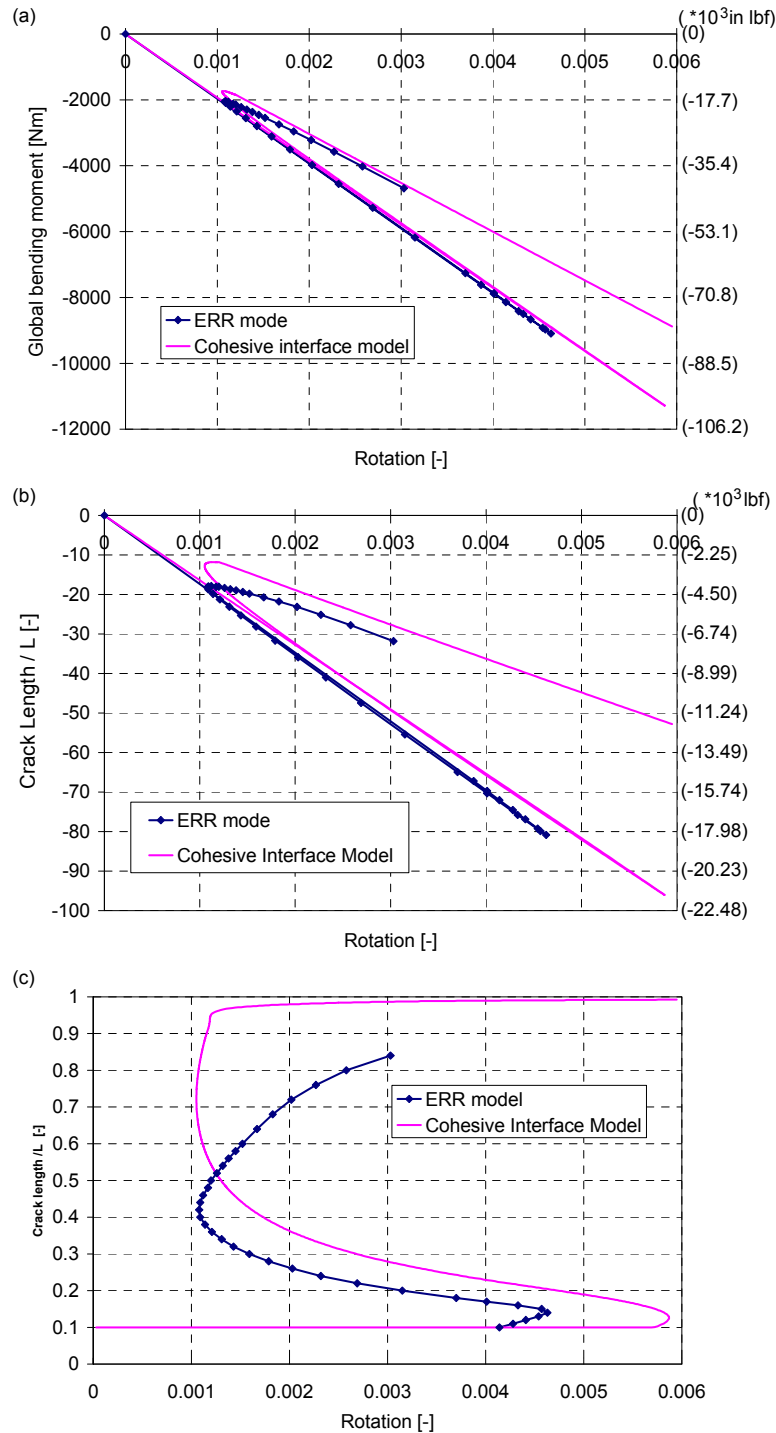


Figure 4– Debonding process in a small scale cracked masonry specimen strengthened with double-sided FRP systems: (a) Global axial force Vs. end rotation; (b) Global bending moment Vs. end rotation; (c) Length of the debonded region (normalized with respect to the length of the specimen) Vs. end rotation

The above discussion highlights the qualitative agreement between the cohesive interface and the ERR analyses. On the other hand, from the quantitative point of view, the comparison reveals several discrepancies (which, as mentioned above, may somewhat increase or decrease depending on the specific parameters of the cohesive interface). Particularly, it reveals that the limit point rotation estimated by the cohesive interface model is larger than

the one predicted by the ERR model. The crack lengths attributed by the two analyses to the limit point are rather close ($0.127L$ in the cohesive interface model and $0.140L$ in the energy balance model) but the limit point rotation in the cohesive interface model is about 25% larger than the one in the ERR model. Correspondingly, the magnitudes of the global bending moment and the global axial forces attributed by the cohesive interface model to the limit point are also larger. This effect is due to the differences in modeling and mainly to the presence of the interfacial layer, a feature that is only included in the cohesive interface model. Although this layer is less stiff than the adjacent masonry substrate, it still tends to slightly stiffen the debonded region and to postpone the initiation of the debonding mechanism.

The comparison between the results of the two models in terms of the post-limit-point out-of-plane deflections and crack growth rates also reveal differences. The differences in the out-of-plane deflections are attributed to the conditions at the edges of the cracked region. Once the debonding process has started, the two models impose different conditions at the edges of the debonded region. In the cohesive interface model, the response of the cohesive interface to tractions yields a process zone near the crack front. The process zone tends to loosen the constraints imposed by the masonry substrate on the bonded FRP strip. In the ERR model, the transition between the unbonded and the bonded regions is abrupt. As a result, the compressed strip is better attached to the bonded side of the masonry substrate. Due to this difference in modeling, the deflections observed in the cohesive interface analysis after the debonding has started are larger than their counterparts in the ERR model.

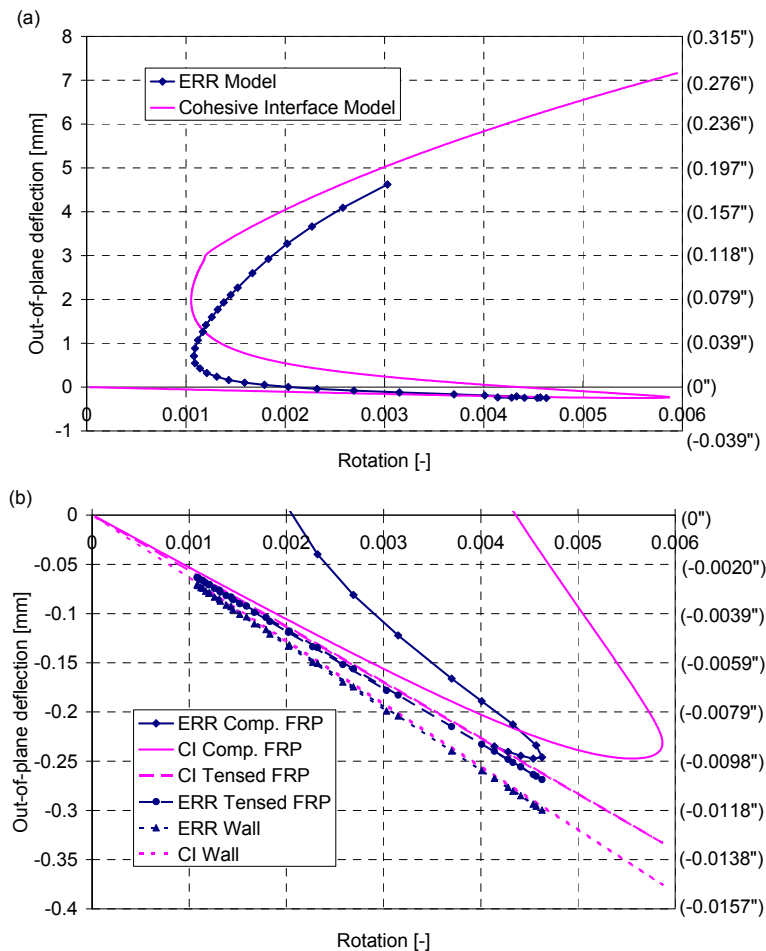


Figure 5– out-of plane deflections versus end rotation in a small scale cracked masonry specimen strengthened with double-sided FRP systems: (a) deflection of the compressed FRP strip; (b) deflections of all components – zoom plot on the initial response phase.

The above feature of the cohesive interface model also contributes to the differences between at the initial phase of the response, before the delamination crack nucleates and before the debonding process enters the unstable regime. Fig. 5b indicates that in the cohesive interface model, the relative out-of-plane displacement between the compressed FRP strip and the masonry substrate grows faster than in the ERR model. This is an outcome of the additional flexibility introduced by the cohesive interface and its gradual softening under the effect of the interfacial stresses.

Another aspect of the global response in the form it appears in Figs. 4 and 5 is the post-limit-point generalized load-deflection behavior. After the interfacial debonding enters the unstable phase, the load and the end rotation monotonically reduce while the crack length and the out-of-plane deflection monotonically grow. This process, which is theoretical by nature and located on the unstable part of the equilibrium path, stops at a rotation of about 0.001 where the cracking mechanism stabilizes and the load starts to grow again. This stabilization point is also located on the unstable branch and, therefore, it is also theoretical in nature. In reality, it is expected that the unstable cracking process would be governed by a snap-back or snap-through behavior and that if physical stabilization could occur it would happen after the snap process and at much larger crack lengths.

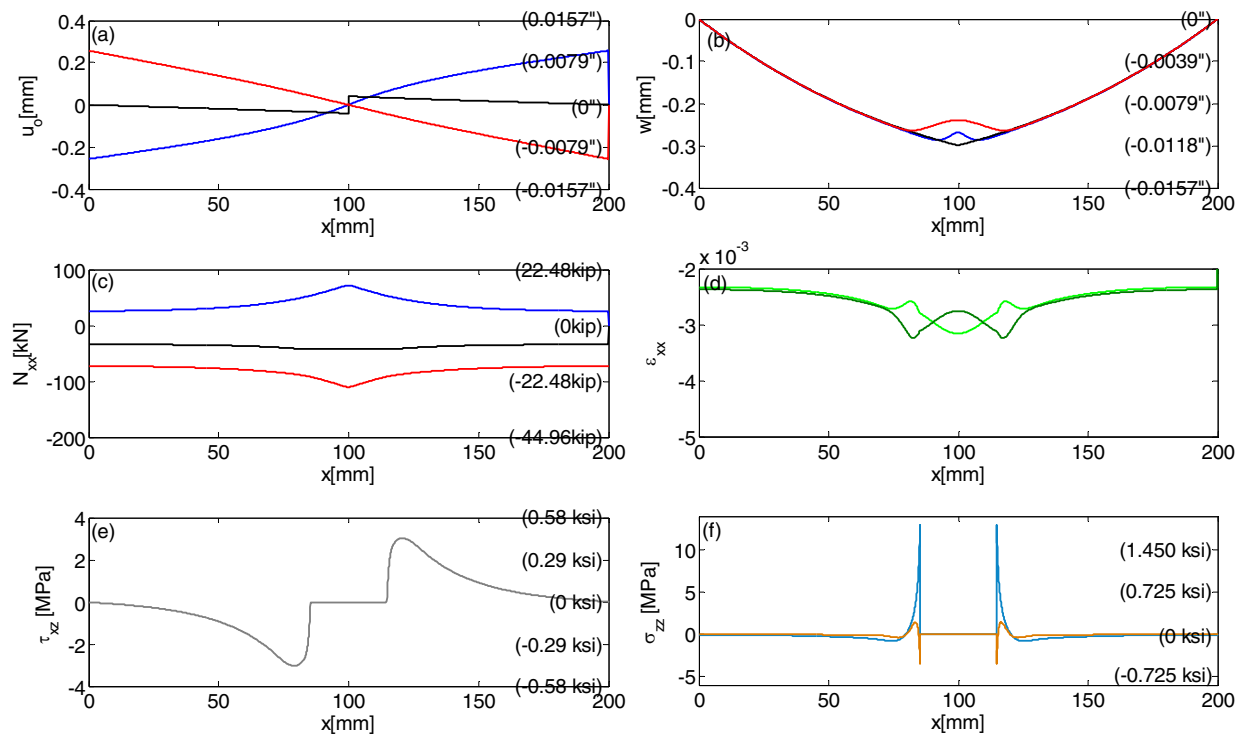


Figure 6– Results of the ERR model at the limit point: (a) axial displacements; (b) out-of-plane displacements; (c) axial forces; (d) axial strains in compressed FRP strip; (e) shear stresses in the adhesive layer (compressed strip); (d) Normal stresses at the interfaces of the adhesive layer (compressed strip) (Legend: — Masonry units; — tensed strip — Compressed strip; — inner face of the compressed strip — outer face of the compressed strip; — shear stress — adhesive-**FRP** interface; — adhesive-masonry interface.)

In order to gain more insight into the behavior of the debonding specimen, into the similar aspects of the predictions of the two models, and into the differences between them, the distributions of the deflections, stress resultants, stresses, interfacial tractions, and stains along the specimen are studied. The results of the ERR model appear in Fig. 6 and the ones of the cohesive interface model appear in Fig. 7. Both sets of results refer to the limit point.

The comparison between Figs. 6 and 7 indicates that the distributions of the axial and out-of-plane displacements (Figs 6a-b,7a-b) reveal a reasonable agreement between the two models. Figs. 6a, 7a show the axial discontinuity due to the eccentric hinge and Figs. 6b, 7b show the "blister" due to buckling of the compressed FRP strip. The

global bending mechanism of the FRP patched specimen and the effect of the localized buckling are also reflected by the distributions of the axial stress resultants in the various components (Figs 6c, 7c) and the strains in the compressed FRP strip (Figs 6d, 7d). The distributions of the axial resultants show that the presence of the joint locally increases the tensile force in the tensed FRP strip. The increased tension is balanced by compressive forces in the masonry units and in the compressed FRP strip. This effect is well observed in both models and with a reasonable quantitative agreement. Fig. 7c also indicates that the axial forces that develop in the interfacial layers are not significant and they do not notably alter the stress distribution in the specimen. The strain distributions (Figs 6d, 7d), on the other hand, reveal some more noticeable differences. In this case, the increased localized displacements attributed by the cohesive interface model to the compressed FRP strip and the "relaxed" conditions at the edges of the debonded regions end up with slightly increased strains in the debonded regions.

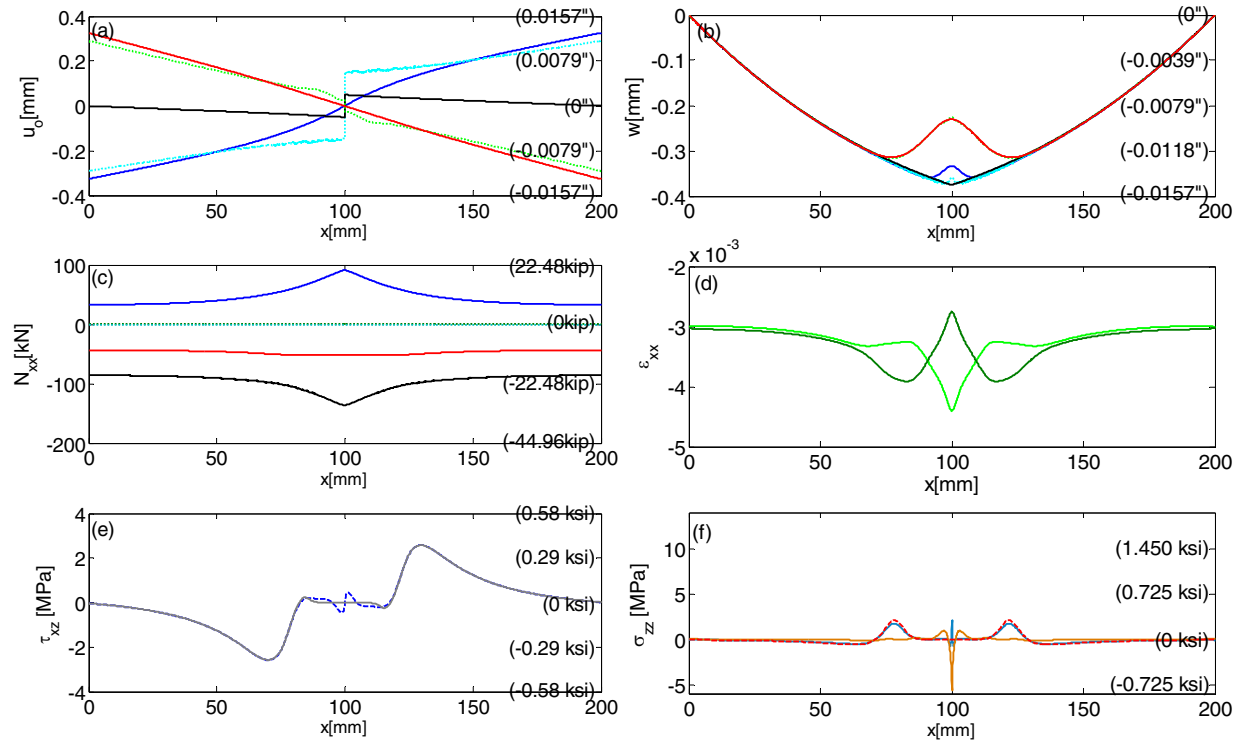


Figure 7– Results of the cohesive interface model at the limit point: (a) axial displacements; (b) out-of-plane displacements; (c) axial forces; (d) axial strains at the compressed FRP strip; (e) shear stresses in the adhesive layer (compressed FRP strip); (d) Out-of-plane normal stresses at the interfaces of the adhesive layer (compressed strip) (Legend: — Masonry units; — tensed strip — Compressed FRP strip; — inner face of the compressed FRP strip — outer face of the compressed strip; — shear stress; - - - shear tractions across the interface; — adhesive —FRP interface; — adhesive-masonry interface - - - normal tractions across the interface.)

The distributions of the shear stresses in the adhesive layer that bonds the compressed FRP strip and the shear tractions across the cohesive interface appear in Figs. 6e and 7e. These curves reveal a good agreement between the two analyses as well as between the shear stresses in the adhesive layer and the shear traction across the cohesive interface. On the other hand, the out-of-plane normal stresses at the interfaces of the adhesive layer that bonds the compressed strip and the tractions across the adjacent cohesive interface that appear in Figs. 6f and 7f reveal some discrepancies. In the cohesive interface model, the stresses at the adhesive masonry interface near the crack front are close to the normal tractions across the cohesive interface. This implies that the presence of the interfacial layer does not significantly alter the interfacial stress field. On the other hand, the magnitudes of peeling stresses in the ERR model near the edges of the cracked region are larger than the ones in the cohesive interface model. The lower values in the cohesive interface model are attributed to the coupling effect of the cohesive interface. In this model, the evolution of significant shear tractions across the interface reduces its peeling strength and the interfacial debonding progresses under combined but relatively lower level of stresses. In the ERR model, this coupling is not

taken into account and the interfacial conditions that satisfy the fracture criterion involve higher magnitudes of peeling stresses. It is also observed that in the cohesive interface model, the presence of the interfacial layer allows for the development of the localized stress concentration in the adhesive layer near the cracked joint. This effect, which is expected to trigger failure of the interfacial layer and to decay, is not observed in the ERR model.

SUMMARY AND CONCLUSIONS

The problem of localized buckling driven debonding in flexural members strengthened with FRP layers bonded on their tensed and compressed faces has been investigated. This problem is particularly relevant to flexural (and in some cases shear) dynamic strengthening applications where the bending moment changes its sign and the exposure of the FRP reinforcement to compression is inevitable. Masonry and reinforced concrete walls strengthened to resist cyclic loading are probably the most common examples of this scenario.

Due to the nature of the debonding problem, its analytical and/or numerical handling has to be conducted under the umbrella of fracture mechanics. In the present work, two fracture mechanics based analytical approaches have been compared. The first approach has used the nonlinear fracture concept of the cohesive zone and has implemented a cohesive interface pseudo-layer in a high order analytical model. The second approach has adopted the classical fracture mechanics concept of the energy release rate and has combined a high order analysis with the energy criterion for fracture. In both models, the effect of the geometrical nonlinearity and buckling of the compressed layer and its role as the driving force for the debonding process have been incorporated.

The two analytical/numerical fracture mechanics models have been used for the comparative study of the debonding process in a small-scale specimen comprised of two masonry units and a cracked joint and strengthened with two externally bonded FRP layers. The numerical results have captured many aspects of the debonding mechanism including its initiation as a stable process and the impact of buckling of the compressed FRP strip on its transition into an unstable process. This type of behavior, which is governed by a limit point followed by a snap-through or snap-back and theoretical stabilization after accumulation of a significant interfacial damage, has been well detected by both models. In practical terms, this type of behavior reflects on the possibility to detect the initiation of the debonding during its initial stable phase but also on the brittle and, in many cases, problematic nature of the crack growth through its unstable phase.

The comparison between the two modeling approaches has revealed a good agreement in terms of the characterization of the stability features of the debonding process and, particularly, in quantifying the critical crack length beyond which the process becomes unstable. This value, which is well captured by the two modeling approaches, is critically important for the assessment of the safety of the FRP strengthened element. On the other hand, the different modeling approaches and the differences in the fundamental representation of the debonding phenomena (as well as the dependency of the cohesive interface model on a range of parameters) have lead to some discrepancies in the numerical characterization of other aspects of the response. In that sense, the two modeling approaches can balance and complement each other in a joined effort for the assessment of the stability characteristics, resilience (in the sense of post-failure survivability), and safety of the externally strengthened element.

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Fracture Characteristics of Notched Concrete Beams Shear-strengthened with CFRP Sheets Subjected to High Temperature

Yail J. Kim¹, Amer Hmidan², Kyoung-Kyu Choi³, and Siamak Yazdani⁴

¹ Department of Civil Engineering, North Dakota State University, Fargo, ND, USA, jimmy.kim@ndsu.edu

² Department of Civil Engineering, North Dakota State University, Fargo, ND, USA, amer.hmidan@ndsu.edu

³ School of Architecture, Soongsil University, Seoul, Korea, kkchoi@ssu.ac.kr

⁴ Department of Civil Engineering, North Dakota State University, Fargo, ND, USA, frank.yazdani@ndsu.edu

SYNOPSIS:

This paper presents the residual fracture characteristics of concrete beams shear-strengthened with carbon fiber reinforced polymer (CFRP) sheets subjected to high temperature ranging from 25°C [77°F] to 200°C [392°F] for three hours. The beams are intentionally unreinforced to focus on strengthening effects. Three levels of initial shear deficiency are simulated by notching the beams: a_0/h ratios = 0.25, 0.38, and 0.50 where a_0 is the notch depth and h is the beam height. Ancillary tests are conducted to examine the temperature-dependent residual material properties of concrete, CFRP, and epoxy-concrete interface. Empirical equations are then generated to predict such properties for a potential modeling purpose. The interfacial fracture capacity is significantly affected by the level of temperature exposure, including two different failure modes such as failures at the concrete substrate and within the adhesive layer. CFRP debonding, associated with diagonal tension cracking of the concrete, is the primary source of the failure of the strengthened beams. High temperature exposure above the glass transition temperature (T_g) of the adhesive accelerates the bond failure. Energy dissipation of the strengthened beams is influenced by the elevated temperatures. The contribution of the CFRP to compliance variation is, however, not significant.

Keywords: carbon fiber reinforced polymer (CFRP), interface, repair, shear, strengthening, temperature

ACI member **Yail J. Kim** is an Assistant Professor in the Department of Civil Engineering at North Dakota State University, Fargo, ND. He is a member of ACI Committees 342 (Evaluation of Concrete Bridges and Bridge Elements), 345 (Concrete Bridge Construction, Maintenance, and Repair), and 440 (Fiber Reinforced Polymer Reinforcement). His research interests include the application of advanced composite materials for concrete structures, performance evaluation of constructed facilities, and development of intelligent structural systems. He is a licensed Professional Engineer in the Province of Ontario, Canada.

Amer Hmidan is a Ph.D. student in the Department of Civil Engineering at North Dakota State University, Fargo, ND. His research interest includes the rehabilitation of deteriorated structural members and damage mechanics.

ACI member **Kyoung-Kyu Choi** is an Assistant Professor in the School of Architecture, Soongsil University, Seoul, Korea. He received his BE, MS, and PhD in architecture from Seoul National University, Seoul, Korea. His research interests include shear strength and seismic design of reinforced concrete structures.

Siamak Yazdani is a Professor in the Department of Civil Engineering at North Dakota State University, Fargo, ND. His research interests include damage mechanics and constitutive modeling of concrete-like materials.

INTRODUCTION

Rehabilitation of deteriorated concrete structures with fiber reinforced polymer (FRP) composite sheets is becoming an accepted method in the structural engineering community¹. FRP-strengthened members show many advantages such as less susceptibility to corrosion, significant increase in load-carrying capacity, and good chemical and fatigue resistance^{2,3}. Although the relatively high material cost of FRP may be an initial problem, low labor and long-term maintenance expenses counterbalance it. The acceptance of FRP in the rehabilitation industry is thus increasing. Practical codes and design manuals are continuously being updated⁴⁻⁶. Carbon FRP (CFRP) is widely used for external strengthening applications because of its high modulus and strength⁷. CFRP sheets may be adhesively bonded to the sides of a concrete beam to complement the shear resistance of existing steel stirrups. The load-resisting mechanism by CFRP in shear is more complicated than that in flexure. Bank⁸ addressed the reasons of such complexity in terms of the mechanics of shear behavior and the geometric condition of concrete members.

A challenge in CFRP applications may take place due to the potential hazard of fire when structural members in a building is strengthened. The performance of structural adhesives is very susceptible to high temperature because their material characteristics change⁹. The critical temperature of an adhesive is called *glass transition temperature*, T_g . The efficacy of CFRP-strengthening is, accordingly, dependent upon the level of temperature exposure. Although research endeavors concerning the fire safety of CFRP-strengthened concrete members have been recently made¹⁰, there still exists limited information in this research area. More experimental and analytical investigations are required to promote the promising strengthening method and contribute to the development of design guides and code provisions. This paper presents an on-going research program concerning the performance of concrete members shear-strengthened with CFRP sheets subjected to high temperature. A concise review on literature relevant to the research is also provided. The experimental study reported here examines the effect of initial shear deficiency (i.e., damage prior to strengthening) on the residual behavior of unreinforced concrete beams strengthened with CFRP subjected to elevated temperatures. The emphasis of the current work is on evaluating the temperature-dependent fracture characteristics of test specimens, including interfacial fracture energy, energy dissipation, and compliance.

RESEARCH SIGNIFICANCE

Fracture characteristics of concrete beams strengthened with CFRP sheets in shear are of interest because the failure of such structural members is brittle. Investigations based on a fracture mechanics perspective can provide adequate understandings of the shear behavior of the members, including interfacial fracture energy and the variation of compliance. In spite of considerable amounts of experimental results on the behavior of concrete beams strengthened with CFRP, there has been no study with regard to their shear behavior subjected to high temperature. This paper generates important experimental data to address the identified research needs.

BACKGROUND

Shear strengthening with CFRP sheet

The brittle shear failure of reinforced concrete structures is more critical than ductile failure induced by insufficient flexural capacity. CFRP-strengthening can improve the shear resistance of constructed members. Externally bonded CFRP sheets effectively preclude the development of diagonal tension cracks (or shear cracks) in a shear critical zone. While various shear strengthening schemes have been reported¹¹, two-side wrapping and U-wraps are commonly used in practice. The presence of CFRP shear reinforcement also precludes the premature debonding of longitudinal CFRP sheets, if any, by providing end anchorage. According to Bakis et al.¹, quantifying the shear resistance of a member strengthened with CFRP is extremely difficult. Most existing literature is thus based on empirical equations¹². Simplified design methods such as the effective strain approach are presented in practice manuals⁶. The most widely accepted expression to estimate the shear resistance of a CFRP-strengthened reinforced concrete member is given in Eq. 1:

$$V_n = V_c + V_s + V_f \quad (1)$$

where V_n is the nominal resistance of the member; V_c is the resistance by the concrete; V_s is the resistance from the steel stirrups; and V_f is the CFRP resistance. ACI Committee 440 (ACI440.2R-08) adopts the method proposed by Khalifa et al.¹³ to calculate the CFRP contribution, V_f . Typical failure modes of externally bonded shear reinforcement are the rupture or debonding of the CFRP¹⁴, as shown in Fig. 1. Such failure modes are dominated by a number of factors: bond strength and schemes, anchorage methods, configuration of CFRP termination points, axial stiffness of CFRP, tensile strength of concrete, and the orientation of fiber directions^{1,2,7}. Further details on shear strengthening with CFRP are available in a literature survey¹¹.

Interfacial behavior of CFRP bonded to concrete substrate

Bond is the most important means to transfer shear force from concrete to CFRP. Two-part epoxy adhesives are widely used for bonding CFRP sheets to concrete substrate. Surface preparation of the concrete is necessary to improve bond prior to bonding the sheets. Numerous research results have been reported on the bond behavior of CFRP for concrete members¹⁵. Bond failure of the CFRP is induced by a combination of shear and normal stresses near the termination point of the CFRP or at a discrete crack location¹⁶. Interfacial failure of CFRP-strengthened concrete members can be categorized into two cases: failure of the concrete substrate or failure within the adhesive layer. The substrate failure is observed in most cases, whereas the adhesive layer failure occurs when the CFRP-concrete interface is deteriorated. Typical sources of interface deterioration are environmental effects or fire, which can change the material characteristics of adhesives^{17,18}.

Behavior of CFRP materials at elevated temperatures

CFRP is basically a polymeric material. High temperature exposure can thus change its constituent properties. Bisby et al.¹⁰ performed a literature review on the behavior of FRP composites subjected to elevated temperatures. The effect of temperature was not significant on the strength of carbon fibers up to almost 600°C [1112°F]. CFRP composites, however, demonstrated substantial decreases in tensile strength and modulus with increasing temperature because of the thermal behavior of a resin matrix. The review highlighted the need for experimental observations on the residual properties of FRP composites in post-fire conditions.

Fracture mechanics approach

Most existing research results on CFRP-strengthening are based on a strength-based approach. Limited investigations are available from a fracture mechanics point of view. Qiao and Xu¹⁹ evaluated the fracture energy of CFRP sheets adhesively bonded to a concrete substrate using a beam test. Figure 2 shows the conceptual definition of interfacial fracture energy. They reported that the interfacial fracture was not a function of loading rates, but was dominated by the properties of resins and concrete substrate. Coronado and Lopez²⁰ performed an experimental program to examine the fracture properties of epoxy-concrete interface. Substrate conditions were found to be a contributing factor to the fracture energy (G_f) of the interface. Numerical models have been developed to predict the bond failure of CFRP using interfacial fracture energy²¹⁻²⁴. All of these studies indicate that interfacial fracture energy is a critical parameter for CFRP-debonding and varies with bond conditions. Very limited research has been reported on the fracture characteristics of CFRP-concrete interface at elevated temperatures.

EXPERIMENTAL PROGRAM

Material properties

The specified compressive strength of concrete was 40 MPa [5,800 psi]. According to cylinder tests [$\Phi 75 \times 150$ mm [$\Phi 3 \times 6$ in.] Fig. 3(a)], the average 28-day strength at room temperature was 41.7 MPa [6,050 psi]. It should be noted that the room temperature was assumed to be 25°C [77°F] for this research. Concrete cylinders were exposed to elevated temperatures ranging from 50°C [122°F] to 200°C [392°F] for three hours in a thermal chamber and cooled down (details are described in the *Elevated temperature* section). The cylinders were then tested in compression to determine temperature-dependent residual strength, as listed in Table 1. The identification code of the test cylinders indicates the degree of temperature exposure and repetition. For instance, specimen C150-2 means the cylinder was subjected to 150°C [302°F] and was the second repetition. The effect of high temperature on compressive strength of the concrete was not significant: the difference between 25°C [77°F] and 200°C [392°F] was 6.0% (Table 1). Thermal spalling of the concrete was not observed. The strengthening material was a unidirectional CFRP sheet (equivalent thickness $t = 0.165$ mm) having a tensile strength of 3,800 MPa [550 ksi] with corresponding modulus of 227 GPa [33 msf]²⁵. A two-part epoxy adhesive was used to bond the CFRP to a concrete substrate. The ultimate tensile strength of the adhesive was 54 MPa [7.8 ksi] and corresponding modulus was 3 GPa [435 psi] with a glass transition temperature (T_g) of 71°C [160°F]²⁶. CFRP coupons [17 mm [0.7 in.] wide $\times$ 250 mm [10 in.] long, Fig. 3(b)] were made and monotonically tensioned to obtain the residual strength and modulus of the CFRP exposed to elevated temperatures. Table 2 summarizes test results, including similar identification codes to those in concrete cylinders. The tested tensile strength decreased with temperature: a 21.0% decrease was observed, on average, when the temperature increased from 25°C [77°F] to 200°C [392°F]. It should be noted that the tested strength was lower than the manufacturer's property possibly due to stress concentrations during the mechanical test. The tensile modulus of the CFRP was obtained within a strain range between 1,000 and 3,000 microstrains²⁷. The modulus also decreased with temperature while the values were comparable to the manufacturer's modulus. Such a decreasing trend is attributed to the degradation of the resin matrix that has resulted in inadequate stress transfer between fibers.

Interface test

An interface test was conducted to examine the performance of the adhesive bonded to concrete. A total of 21 specimens were prepared (Table 3). One test specimen consisted of three concrete prisms (50 mm [2 in.] wide $\times$ 50 mm [2 in.] deep $\times$ 120 mm [4.7 in.] long), as shown in Fig. 4(a). Prior to applying the adhesive, the surface of the concrete was grit-blasted to enhance bond. The fabricated test specimens [Fig. 3(c)] were cured for a minimum of one week and heated up with a temperature range between 50°C [122°F] and 200°C [392°F]. The specimens were then mechanically loaded until failure occurred [Fig. 4(a)]. The interface test was expected to provide the interfacial fracture energy of adhesively bonded concrete subjected to elevated temperatures. It is important to note that the failure of CFRP-strengthened concrete beams takes place either in the concrete substrate or within the adhesive layer, depending upon the level of interface degradation^{14,17}.

Beam test

Seventy two concrete beams were cast with dimensions of 45 mm [1.8 in.] wide $\times$ 165 mm [6.5 in.] deep $\times$ 330 mm [13.0 in.] long, as shown in Figs. 3(d) and 4(b). The beams were intentionally unreinforced to focus on the effect of CFRP-strengthening at elevated temperatures. The level of initial shear deficiency was simulated by notching the beams [Fig. 3(e)]. Three different notch depth-to-beam height ratios (a_0/h) were used for the present study, namely, $a_0/h = 0.25, 0.38$, and 0.50 for notch depths of 40 mm [1.6 in.], 62 mm [2.4 in.], and 83 mm [3.3 in.], respectively. Table 4 provides a summary of the test beams. The identification code indicates the presence of CFRP (U = unstrengthened and R = strengthened), shear deficiency ratios (025, 038, and 050 representing $a_0/h = 0.25, 0.38$, and 0.50 , respectively), temperature exposure (25, 50, 75, and so on meaning 25°C [77°F], 50°C [122°F], 75°C [167°F], respectively), and the number of repetition (1, 2, and 3). The same surface preparation was conducted as in the case of the interface test specimens to improve bond between the CFRP and concrete [Fig. 3(f)]. One ply of CFRP sheet was bonded to both sides of the prepared beams, as shown in Fig. 4(b). When bonding the sheets, the notch of each beam was protected with a temporary filler (piece of paper) so that the notch was free from the adhesive. After curing of the adhesive, the strengthened beams were heated up to 200°C [392°F] for three hours and cooled down at room temperature. The residual behavior of the conditioned beams was measured by load and displacement at midspan using a built-in load cell and the stroke of the loading head [Fig. 4(b)]. The loading rate was 1.0 mm/min.

Elevated temperature

The heating equipment used for the research was an electric furnace. All specimens were heated to designated temperatures from 50°C [122°F] to 200°C [392°F] for three hours, passing through the T_g of the adhesive (71°C [160°F]), and cooled down gradually at room temperature, as shown in Fig. 5. During the heat-up process, the temperature was regularly checked with a digital thermometer. Such an experimental procedure is widely accepted when an electric heating equipment is used^{18,28}. The temperature range and exposure time selected here were reasonable to study the thermal response of CFRP-strengthened concrete members, given that CFRP layers are generally located inside an insulation material in practice. Kodur et al.²⁹ reported that CFRP sheets might be exposed to a temperature over 200°C [392°F] after three hours in a fire if the strengthened members were insulated.

RESULTS AND DISCUSSION

Interfacial behavior and fracture energy

Table 3 summarizes the interface test results. The failure load of the specimens decreased with increasing temperature. For example, the specimens tested at room temperature failed at an average load of 31.7 kN [7.1 kip], whereas those subjected to 200°C [392°F] failed at an average load of 6.7 kN [1.5 kip]. It was noticed that the failure strength of the interface test specimens showed a wide scatter range, as shown in Table 3. This is possibly attributed to the facts that i) the amount of the bonding agent used was not the same for all test specimens, which is a common issue in wet-lay-up applications of CFRP sheets and ii) material degradation of the adhesive induced by the temperature exposure was rather random in the bonded area because of the non-uniform degradation of the materials in such an environment. Despite such experimental scatters, the capacity decrease was obvious when the temperature exceeded the T_g of the adhesive, based on the fact that the failure load of the specimens subjected to temperatures over 100°C [212°F] was consistently lower than 35% of the control capacity (Table 3). Selected load-displacement responses of the interface test are shown in Fig. 6. All specimens exhibited a similar trend of gradual increases in displacement with increased load; however, their incremental rate was significantly influenced by the level of temperature exposure. The residual stiffness of the specimens exposed to a temperature below T_g [Fig. 6(a)] was considerably higher than that of the specimens subjected to a higher temperature [Fig. 6(b)]. Such an observation can be explained by their different failure modes. The failure of the specimens exposed to a temperature below and above T_g occurred at the level of concrete substrate and within the adhesive layer, respectively, as shown in Fig. 7. The specimens with the degraded adhesive properties hence exhibited lower stiffness responses. The interfacial fracture energy of the specimens was obtained using the definition shown in Fig. 2:

$$G_f = \int \tau(\delta) d\delta \quad (2)$$

where τ is the average shear stress of the bonded area and δ is the displacement of the interface. It is worthwhile to note that actual shear stresses along the interface are nonlinear, whereas averaged stresses are widely used for engineering applications. A trapezoidal rule was used to numerically integrate the average shear stress versus displacement curve of each test specimen. Although wide scatters were observed (Fig. 8), the best-fit line can be reasonably used to represent the fracture characteristics of the epoxy-concrete interface subjected to elevated temperatures within a range between 25°C [77°F] and 200°C [392°F]. It is believed that such an empirical model is particularly useful for those who are interested in numerical modeling.

Load-displacement behavior of CFRP-strengthened beams

Figure 9 shows the load-displacement behavior of the strengthened beams subjected to 50°C [122°F] and 200°C [392°F], representing temperatures below and above T_g of the adhesive. The response was essentially linear up to the peak load irrespective of the temperature exposure. The stiffness of the beams was influenced by the temperature. This observation was mainly due to the change in concrete properties such as the initial shear damage (notch size) and its progression, rather than the contribution of the CFRP (details are discussed in the *Compliance variation* section). It should be noted that the stiffness degradation mechanism presented here is very complicated because the shear behavior of a notched concrete beam is not readily predictable (most concrete codes adopt empirical equations for shear resistance of concrete) and bond deterioration of the CFRP due to elevated temperatures also partially influenced the mechanism (even though not significant). Further refined modeling may provide a better understanding of the displacement issue of the shear-strengthened beams subjected to elevated temperatures.

Failure mode and energy dissipation

All unstrengthened beams showed a typical shear failure mode. A crack initiated at the tip of the notch and diagonally propagated toward the loading point with increased load, as shown in Fig. 10(a). This failure mode was independent of the level of the initial damage (a_0/h ratios). For the strengthened beams, debonding of the CFRP was first observed along the notch and progressed upward when the load increased [Fig. 10(b)]. In the meantime, a diagonal tension crack developed and propagated as in the case of the unstrengthened beams [Fig. 10(a)]. The CFRP-debonding was accelerated with temperature. This experimental observation was supported by the failure load of the strengthened beams, as summarized in Table 3; for example, the load-carrying capacity of the strengthened beams approached that of the unstrengthened beams when the temperature increased. The energy dissipation of the strengthened beams exposed to temperatures below and above the T_g of the adhesive is given in Figs. 11 and 12. The dissipated energy was obtained from the area under the load and displacement curve of each test specimen up to the peak load (Fig. 9). The post-peak behavior was not accounted for in this research because there was no strengthening effect beyond the peak load due to debonding of the CFRP. The energy dissipation of the beams tended to decrease when the level of shear damage (a_0/h ratios) increased regardless of the presence of the CFRP. However the dissipated energy of the unstrengthened control beams was less than 5 kNmm [44 lb-in.], which was consistently lower than that of the strengthened beams, as shown in Figs. 11(a) through (d). The energy dissipation of the strengthened specimens was not significantly influenced by the temperature up to T_g , while the dissipation tended to be more controlled by the level of initial damage (a_0). The beams subjected to temperatures above T_g exhibited somewhat different responses, as shown in Fig. 12. A noticeable drop in the energy dissipation was observed with increased a_0/h ratios. For example, an average drop of 60.9% was noticed for the beams subjected to 125°C [257°F] when the a_0/h ratio increased from 0.25 to 0.50 [Fig. 12(b)]. Such a trend, however, was not pronounced when bond of the CFRP significantly deteriorated, as shown in Fig. 12(d).

Compliance variation

The variation of compliance (inverse of stiffness) of the test beams is given in Figs. 13 and 14, depending upon the level of temperature exposure. The compliance of the unstrengthened beams increased by 167%, on average, when the a_0/h increased from 0.25 to 0.50 [Fig. 13(a)]; while, corresponding increases of the strengthened beams subjected to 25°C [77°F], 50°C [122°F], and 75°C [167°F] were 158%, 188%, and 161%, respectively [Figs. 13(b) through (d)]. The contribution of temperatures above T_g was lower than that below T_g (i.e., increases in compliance were 149%, 153%, 143%, 160% for 100°C [212°F], 125°C [257°F], 150°C [302°F], and 200°C [392°F], respectively, from $a_0/h = 0.25$ to 0.50). These observations indicate that CFRP-strengthening did not substantially influence the compliance of the beams subjected to elevated temperatures, which can be attributed to the fact that the contribution of the thin CFRP sheets to the global compliance was not significant.

SUMMARY AND CONCLUSIONS

This paper has addressed an on-going research program concerning the residual shear behavior of CFRP-strengthened unreinforced concrete beams subjected to elevated temperatures with emphasis on experimental fracture characteristics. Temperature-dependent material properties were presented, including epoxy-concrete interfacial responses. The strengthened beams had three sizes of notch depth to represent the initial shear deficiency of existing concrete members. Investigation parameters included interfacial fracture energy, energy dissipation, and compliance variation, depending upon the level of temperature exposure. The following is concluded.

- The interfacial fracture capacity was significantly influenced by elevated temperatures even though a wide range of experimental scatters was observed. Two types of failure modes were noticed depending upon the level of temperature exposure such as the failure near the concrete substrate and the failure within the adhesive layer. An empirical equation to estimate the temperature-dependent interfacial fracture energy of an epoxy-bonded concrete substrate was provided for a potential modeling purpose within the investigation range given here.
- The shear-strengthened beams failed in a brittle manner and the response was essentially linear up to the peak load. The stiffness of the test beams was influenced by concrete properties and the level of initial shear deficiency. CFRP-debonding was the primary failure mode of the strengthened beams. The level of debonding was affected by temperature exposure. Energy dissipation of the strengthened beams was dominated by elevated temperature and shear deficiency. The thermal response of the CFRP was not a

critical factor for the compliance of the strengthened beams. An updated version of the ACI440.2R-08 document may consider such findings.

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Fracture Characteristics of Notched Concrete Beams Shear-strengthened with CFRP Sheets Subjected to High Temperature

Table 1— Concrete strength in compression depending upon temperature

ID	Test strength (MPa [psi])		ID	Test strength (MPa [psi])		ID	Test strength (MPa [psi])	
	Each	Ave.		Each	Each		Each	Ave.
C25-1	41.7 [6,050]	41.7 [6,050]	C100-1	40.8 [5,920]	40.8 [5,920]	C200-1	36.6 [5,310]	39.2 [5,690]
C25-2	41.2 [5,980]		C100-2	42.1 [6,110]		C200-2	41.2 [5,980]	
C25-3	42.1 [6,110]		C100-3	39.1 [5,670]		C200-3	38.8 [5,630]	
C50-1	41.2 [5,980]	41.8 [6,060]	C125-1	38.4 [5,570]	40.4 [5,860]	$f_c(T) = 42.428 \text{ Exp } (-0.0004T)$ (metric)		
C50-2	44.1 [6,400]		C125-2	41.2 [5,980]				
C50-3	40.1 [5,820]		C125-3	41.7 [6,050]				
C75-1	42.6 [6,180]	41.2 [5,980]	C150-1	37.1 [5,380]	40.0 [5,800]	$f_c(T) = 6204.3 \text{ Exp } (-0.0002T)$ (US customary)		
C75-2	39.1 [5,670]		C150-2	42.8 [6,210]				
C75-3	41.7 [6,050]		C150-3	39.3 [5,700]				

$f_c(T)$ = compressive strength of concrete at temperature T

Table 2— CFRP properties in tension depending upon temperature

Table 2. Material properties at room temperature depending upon temperature									
ID	Test strength (MPa [ksi])		Test modulus (GPa [msi])		ID	Test strength (MPa [ksi])		Test modulus (GPa [msi])	
	Each	Ave.	Each	Ave.		Each	Ave.	Each	Ave.
F25-1	1,743 [252]	2,073 [300]	252 [36]	254 [36]	F100-1	1,320 [191]	1,549 [224]	224 [32]	233 [33]
F25-2	1,902 [275]		260 [37]		F100-2	1,484 [215]		244 [35]	
F25-3	2,575 [373]		250 [36]		F100-3	1,843 [267]		232 [33]	
F50-1	1,503 [217]	1,819 [263]	240 [34]	245 [35]	F150-1	1,836 [266]	1,566 [227]	235 [34]	222 [32]
F50-2	1,842 [267]		244 [35]		F150-2	1,519 [220]		220 [31]	
F50-3	2,113 [306]		250 [36]		F150-3	1,342 [194]		210 [30]	
F75-1	1,754 [254]	1,935 [280]	235 [34]	238 [34]	F200-1	2,100 [304]	1,637 [237]	218 [31]	215 [31]
F75-2	2,117 [307]		240 [34]		F200-2	1,467 [212]		220 [31]	
F75-3	2,041 [296]		- ^a		F200-3	1,343 [194]		208 [30]	
$f_f(T) = 2008.3 \text{ Exp } (-0.001T)$ (metric); $f_f(T) = 298.40 \text{ Exp } (-0.0008T)$ (US customary) $E_f(T) = 257.07 \text{ Exp } (-0.009T)$ (metric); $E_f(T) = 37.454 \text{ Exp } (-0.0005T)$ (US customary)									

^a: experimental error

$f_f(T)$ = tensile strength of CFRP at temperature T ; $E_f(T)$ = CFRP modulus at temperature T

Table 3— Summary of interface test

ID	Ultimate load (kN [kip])		ID	Ultimate load (kN [kip])		ID	Ultimate load (kN [kip])	
	Each	Ave.		Each	Ave.		Each	Ave.
I25-1	19.2 [4.3]	31.7 [7.1]	I100-1	14.7 [3.3]	10.8 [2.4]	I200-1	4.9 [1.1]	6.7 [1.5]
I25-2	49.1 [11.0]		I100-2	5.8 [1.3]		I200-2	9.8 [2.2]	
I25-3	26.7 [6.0]		I100-3	11.9 [2.7]		I200-3	5.4 [1.2]	
I50-1	29.5 [6.6]	33.7 [7.5]	I125-1	4.2 [0.9]	9.4 [2.1]	$P_u(T) = 38.576 \text{ Exp}(-0.011T)$ (metric)		
I50-2	23.3 [5.2]		I125-2	14.4 [3.2]				
I50-3	48.3 [10.8]		I125-3	9.7 [2.2]				
I75-1	12.2 [2.7]	19.2 [4.3]	I150-1	5.1 [1.1]	7.0 [1.6]	$P_u(T) = 10.427 \text{ Exp}(-0.006T)$ (US customary)		
I75-2	15.4 [3.4]		I150-2	11.4 [2.6]				
I75-3	30.1 [6.7]		I150-3	4.6 [1.0]				

$P_u(T)$ = ultimate load of the interface test specimen at temperature T