

REQUIREMENTS FOR SEISMIC-RESISTANT FLAT PLATES: STRENGTH AND DUCTILITY

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SYNOPSIS: Lateral displacement of multi-story flat plate concrete buildings in an earthquake induces moment reversals between columns and slabs. The amplitude of the transferred moment depends upon the story drift, defined as the displacement of one floor relative to the floor above or below. Flat plate buildings must have a lateral force-resisting system that limits the design story drift ratio to 0.025; where the design story drift includes plastic deformation and is defined as the design story drift divided by the distance between the mid-surfaces of the flat plates of two consecutive floors. The moments transferred from the columns to the slabs have to be resisted by flexural and shear reinforcements, whose magnitudes and detailing provide the slabs with the strength and the ductility to undergo the design story drift without failure.

The design of shear reinforcement for the moment transfer in an earthquake, as required by ACI 318, considers either the strength or the ductility, not both. ACI 421.2R-10 recommends and justifies a design procedure for the shear reinforcement providing the strength as specified by ACI 318; in addition, it recommends a minimum amount and extension of shear reinforcement that provides a level of ductility adequate for a design-story drift ratio = 0.025 (the upper permissible level in several codes). The design procedure is presented with examples.

KEYWORDS: column; drift; ductility; earthquake; flat plate; flexure; punching shear; seismic; strength.

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INTRODUCTION

For seismic design, the International Building Code 2006¹ (IBC-06) requires that: $DR_u \leq 0.007$ to 0.025 , depending upon the type of building and its usage; where DR_u is design story drift ratio defined as the difference of displacements, including plastic deformations, between the top and bottom of a story divided by the story height. The present paper concentrates on the punching shear design, assuming that DR_u has been determined.

For seismic design, Section 21.13.6 of ACI 318-11² permits option (a) or (b) for punching shear in slab-column connections. Option (a) is a strength design to resist V_u combined with the unbalanced moment, M_u transferred between the slab and the column under the design displacements to satisfy the requirement of Sections 11.11.7 and 13.5.3.2². Option (b) requires shear reinforcement when the design story drift ratio exceeds the larger of 0.005 and $[0.035 - 0.05(V_{ug}/\phi V_c)]$; these limits are presented by line *abc* in Fig. 1. It also specifies the amount of shear reinforcement and the extent of the shear-reinforced zone. V_{ug} is the factored shear force due to gravity loads; ϕ ($= 0.75$) is the strength reduction factor; V_c is the shear strength provided by concrete in absence of shear reinforcement. The two alternatives are here studied in view of the strength required in Sections 11.11 and 13.5.3.2 of the code².

Option (a)² requires calculation of the shear forces and the moments caused by the drift. ACI 318-11², Chapter 8, permits elastic analysis to calculate the internal forces. (No other method is required for seismic-resistant structures). An elastic analysis of the plane frame idealization in Fig. 2a, subjected to imposed drift ratio DR_e , can be used; where

$$DR_e = DR_u / (C_d / I_E) \quad (1)$$

C_d is a dimensionless coefficient, representing the inherent inelastic deformability of the lateral force-resisting system (LFRS); I_E is the occupancy importance factor; $(C_d/I_E) = 2.67$ to 6.5 (IBC-06¹ or ASCE 7-08³ (Section 9.5.5.7.1)). A similar relationship to Eq. 1 between DR_e and DR_u is given in the National Building Code of Canada 2005, Section 4.1.8.13⁴. The flexural reinforcement at critical sections of the slab must be designed (according to 13.3 of ACI 318-11²) for the moments determined by the analysis of the effect of the drift, combined with the moments due to factored gravity loads. Insufficient flexural reinforcement can invoke premature punching failure^{5,6}.

Section 13.7 of ACI 318-11² specifies the I -values in Figs. 2a and 2b to be used in the elastic analysis of the internal forces due to earthquake-induced drift and due to gravity loads, respectively. In Fig. 2c, the equivalent moment of inertia of the column, I_{ec} accounts for an assumed torsional strip in accordance with Section 13.7.5².

While option (a) gives attention to the strength, option (b) focuses on the ductility. With option (b), the design criterion that permits the absence or requires shear reinforcement – and specifies its amount – is based on experiments that give an empirical relationship between the parameter $V_u/(\phi V_c)$ and the drift ratio capacity. The present paper claims that this relationship is not universal, because it does not include other parameters that affect the drift ratio capacity, including the flexural reinforcement ratio and the column's aspect ratio / flexural rigidity. The paper justifies and recommends a design with a combination of the two options to ensure the strength and the ductility.

RESEARCH SIGNIFICANCE

A study of the major parameters affecting the drift capacity of the connections of columns with flat plates is presented to show a need for a change in the ACI 318-11 code². The section that needs the change and the recommended revision are indicated.

PUNCHING SHEAR DESIGN EQUATIONS

The design equations of ACI 318-11² that will be used in this paper are:

$$v_u \leq \phi v_n \quad (2)$$

$$v_u = \frac{V_u}{b_o d} + \frac{\gamma_v M_u x}{J_c} \quad (3)$$

where v_u = the maximum shear stress; v_n = the nominal shear strength (MPa or psi); b_o = the perimeter length of the shear critical section at $d/2$ from the column face (Fig. 3), or from the outermost peripheral line of shear reinforcement; d = effective depth of slab = the average of distances from extreme compression fiber to the centroid of the tension reinforcements running in two orthogonal directions; γ_v = the fraction of unbalanced moment transferred by eccentricity of shear; x = coordinate of the point at which v_u is calculated; J_c = property of the shear critical section. ACI 421.1R-08⁷ gives equations for J_c and γ_v for shear critical sections of any polygonal shape. Equation 4 or 5 gives, respectively, the nominal shear strength, v_n in the absence or in the presence of shear reinforcement (stirrups or headed shear studs).

$$v_n \equiv v_c = \text{the least of: } \lambda \sqrt{f'_c}/3; [2 + (4/\beta)] \lambda \sqrt{f'_c}/12; [(\alpha_s d/b_o) + 2] \lambda \sqrt{f'_c}/12 \text{ (MPa)} \left\{ \right. \quad (4)$$

$$\text{or } v_n \equiv v_c = \text{the least of: } 4 \lambda \sqrt{f'_c}; [2 + (4/\beta)] \lambda \sqrt{f'_c}; [(\alpha_s d/b_o) + 2] \lambda \sqrt{f'_c} \text{ (psi)} \quad \left. \right\}$$

$$v_n = 0.17 \sqrt{f'_c} + v_s \leq 0.5 \sqrt{f'_c} \text{ (MPa) or } v_n = 2 \sqrt{f'_c} + v_s \leq 6 \sqrt{f'_c} \text{ (psi) Stirrups} \quad (5)$$

$$v_n = 0.25 \sqrt{f'_c} + v_s \leq 0.67 \sqrt{f'_c} \text{ (MPa) or } v_n = 3 \sqrt{f'_c} + v_s \leq 8 \sqrt{f'_c} \text{ (psi) Headed shear studs}$$

$$v_s = A_v f_{yt} / (b_o s) \quad (6)$$

where λ = modification factor reflecting the reduced mechanical properties of lightweight concrete, relative to normal-weight concrete of the same compressive strength; f'_c = the specified compressive strength of concrete; β (≥ 1.0) is the aspect ratio of column; $\alpha_s = 40, 30$ or 20 for interior, edge or corner columns, respectively; A_v = the area of the vertical legs of the shear reinforcement on one peripheral line parallel to the column face; f_{yt} = the specified yield strength of the shear reinforcement; s = the spacing between peripheral lines of the shear reinforcement. At the critical section at $d/2$ from the outermost peripheral line of shear reinforcement, the nominal shear strength is:

$$v_n = 0.17 \lambda \sqrt{f'_c} \text{ (MPa)} = 2 \lambda \sqrt{f'_c} \text{ (psi)} \quad (7)$$

Section 13.5.3.2 of ACI 318-11² requires the provision of flexural reinforcement within a specified width to resist a fraction of the unbalanced moment given by $(1-\gamma_v) M_u$. When shear reinforcement is required in option (b) of Section 21.13.6², it must satisfy Eqs. 8 and 9:

$$v_s \geq 0.29 \sqrt{f'_c} \text{ (MPa)} \left[3.5 \sqrt{f'_c} \text{ (psi)} \right] \quad (8)$$

$$b_s \geq 4h + d/2 \quad (9)$$

where h = the slab thickness; b_s = the distance from the column face to the shear critical section at $d/2$ from the outermost peripheral line of shear reinforcement.

DUCTILITY DEPENDENCE ON FLEXURAL REINFORCEMENT RATIO

A frequently used test setup is shown in Fig. 4a; it represents a specimen of a square column connection with a simply supported square slab subjected to a shear force V . The nominal strength is the smaller of:

$$V_n = v_n b_o d \text{ (Punching shear failure mode)} \quad (10)$$

$$\text{and } V_f = 8 m l_s / (l_1 - c) \text{ (Flexural yield-line collapse mode; Fig. 4b)} \quad (11)$$

$$m = \rho f_y d^2 [1 - 0.59 (\rho f_y / f'_c)] \quad (12)$$

where l_s , l_1 and c are dimensions shown in Fig. 4b; m = the ultimate flexural strength of a strip of the slab of unit width in each of two orthogonal directions; ρ and f_y = the ratio of the top flexural reinforcement in a strip of unit width and its specified yield strength.

The load-deflection graph, e.g. Fig. 4c, is often used to give a ductility index, which is defined as the ratio of the deflection at a point on the descending part of the graph (e.g. point B or C) where the load drops to a specified fraction of its maximum value, to the deflection at a point on the ascending part where the graph ceases to be linear; specifying these two points is not needed in the following discussion.

When the nominal shear strength is sufficiently large, $V_{n1} > V_f$, the load-deflection graph will be OAC (Case I, Fig. 4c), exhibiting large deflection, D_C at which the yield-line fracture pattern in Fig. 4b is developed; the maximum load value in the graph will be V_f that can be predicted by Eq. 11. The ductility index in this case is high. At this stage, the cracks in the vicinity of the column extend deep in the slab; concrete crushing occurs at the compression face and the connection may exhibit secondary failure by punching before extensive plastic deflection.

When the nominal shear strength is small, e.g. a slab without shear reinforcement, $V_{n2} < V_f$, the load-deflection graph extends from O to A, where sudden punching failure occurs at a deflection $D_A \ll D_C$. For such a slab, the lack of ductility can be explained by the limited extent of the zone at which the flexural reinforcement yields. Here, the ductility index is low (Case II).

Yielding of the flexural reinforcement starts well before the development of the full yield-line pattern that can produce collapse (Case I). The ductility index depends upon the extent of yielding of the flexural reinforcement. Consider Case III, where the nominal shear strength, V_{n3} is such that $V_{n2} < V_{n3} < V_f$. This can occur by the provision of adequate and properly detailed shear reinforcement, to prevent failure within the shear-reinforced zone, but the extension of the shear-reinforced zone is limited; the load-deflection curve will be OAB. At point B, punching failure occurs outside the shear-reinforced zone, exhibiting a ductility index somewhere in between the two extremes of Cases I and II. The V -values marked in the graph of Fig. 4c are for a connection having: $c = 250$ mm (9.84 in.); $h = 150$ mm (5.91 in.); $d = 114$ mm (4.49 in.); $l_s = 1.9$ m (74.8 in.); $l_1 = 1.8$ m (70.9 in.); $\rho = 0.0116$; $f'_c = 30$ MPa (4400 psi); $f_y = 400$ MPa (58 ksi). The shear reinforcement considered has: $A_v = 570$ mm² (0.88 in.²); $f_{yt} = 345$ MPa (50 ksi); $s = 85$ mm (3.3 in.); $b_s = 929$ mm (36.6 in.); at $d/2$ from the outermost peripheral line of shear reinforcement, $b_o = 6310$ mm (248 in.).

Appropriately designed shear reinforcement can delay failure until the development of the full yield-line pattern, exhibiting extensive ductility. For this to occur, the shear reinforcement must be designed to ensure that punching shear strengths within and outside the shear-reinforced zone are greater than V_f . The increase of the shear reinforcement beyond what is needed to satisfy these requirements does not change the graph OAC in Fig. 4c. Examples of the three graphs of the typical Cases I, II and III in Fig. 4c can be seen in results of experiments reported in the literature^{8,9}.

From this discussion, it can be seen that in the test setup in Fig. 4a, the ductility would generally depend upon the value of V_f with respect to V_n . Because V_f is mainly dependent upon ρ , it follows that the ductility is also dependent on ρ . Application of two equal and opposite horizontal forces at the tips of the column stubs, combined with V , would transfer unbalanced moment, M between the column and the slab. If V is kept constant at a level below V_n and M is gradually increased, a graph of M versus DR would be similar to one of the graphs of Fig. 4c; however, the yield-line pattern at which flexural failure occurs will be different from that in Fig. 4b. By the same reasoning as above, it can be seen that ρ significantly affects the ductility, represented by the drift ratio capacity, DR_u (the value of DR at maximum M or after its drop by 20%).

DUCTILITY DEPENDENCE ON THE RATIO $V_u/(\phi V_c)$

Several researchers¹⁰⁻¹⁷ have investigated the effect of the ratio $V_u/(\phi V_c)$ on the ductility of slab-column connections transferring V_u combined with cycles of lateral displacements (drifts) of increasing magnitude; in these investigations, the variation of the drift ratio capacity, DR_u versus $V_u/(\phi V_c)$ have been plotted. Kang and Wallace¹⁶ plotted the points $[V_u/(\phi V_c), DR_u]$ of 76 interior column tests; all the columns were square except six. The graph clearly shows the trend that the ductility drops as the ratio $V_u/(\phi V_c)$ increases. However, DR_u does not depend uniquely upon the value of $V_u/(\phi V_c)$; the graph of Kang and Wallace¹⁶ does not indicate how the ductility is affected by other parameters including ρ and the flexural rigidity of the column.

It is noted that the graph of Kang and Wallace¹⁶ is for 76 tests in which the maximum $V_u/(\phi V_c)$ did not exceed 0.9 and this parameter is greater than 0.85 only for 4 tests. However, practical cases where $V_u/(\phi V_c) > 0.9$ are common. Equation 13 shows that for a non-prestressed flat plate, $V_u/(\phi V_c)$ can exceed 0.90 in practice.

$$V_u/(\phi V_c) = 1.0 \text{ when } \begin{cases} q_u \text{ (kPa)} \cong 3.6 [(c/h) + 0.8] \\ q_u \text{ (psf)} \cong 75 [(c/h) + 0.8] \end{cases} \quad (13)$$

where q_u = the sum of the factored dead and live loads; c = side of square column; h = slab thickness. As example, $c = 305$ mm (12 in.) and $h = 203$ mm (8 in.); Eq. 13 gives $q_u = 8.3$ kPa (173 psf). The weight of 203 mm (8 in.) slab plus super-imposed dead load with a factor of 1.2 can amount to 7.2 kPa (150 psf); this leaves only 1.1 kPa (23 psf) for the factored live load to make $V_u/(\phi V_c) = 1.0$. Equation 13 can be derived assuming: $d = 0.8h$; square panels of span = $35h$; $f'_c = 30$ MPa (4400 psi); $\phi V_c = 0.75 (\sqrt{f'_c}/3 \text{ MPa}) b_o d [0.75 (4 \sqrt{f'_c} \text{ psi}) b_o d]$; $V_u = q_u (35h)^2$; $b_o d = 4(c + 0.8h)0.8h$. By a similar derivation, it can be shown that $V_u/(\phi V_c)$ reaches 1.0 more often in practice in prestressed slabs, where the ratio of span to $h > 35$.

Figure R21.13.6 of ACI 318-11² is a graph illustrating the criterion for requiring shear reinforcement in option (b) of 21.13.6. The graph is the same as the broken line abc in Fig. 1 of the present paper, with the abscissa representing $V_{ug}/(\phi V_c)$ and terminating at 0.75. A possible interpretation is: the graph and option (b) are intended for the case where $V_{ug}/(\phi V_c)$ does not exceed 0.75; this would also limit their validity to the case where shear reinforcement is not needed for pure gravity load and would allow only option (a) in other cases. The interpretation of 21.13.6(b) and Fig. R21.13.6 of the code² in this way is not pursued further here; it is more useful to focus on the design in a general case, without the mentioned restrictions. The examples below show that design of shear reinforcement based on 21.13.6(b)² does not ensure adequate strength for relatively high values of $V_u/(\phi V_c)$ or DR_u .

DUCTILITY DEPENDENCE ON THE ASPECT RATIO OR THE FLEXURAL RIGIDITY OF COLUMNS

As mentioned earlier, there is not sufficient experimental data to give the variation of DR_u with $V_u/(\phi V_c)$ for varying aspect ratio or flexural rigidity of columns. The unbalanced moment, M_u increases with the flexural rigidity of the column; also, the value of γ_v for a rectangular column, is higher compared to a square column. It will be shown below that for any design drift ratio, the unbalanced moment and the maximum shear stress are higher for rectangular columns compared to square columns. Thus, it is logical to expect that punching would occur at a lower drift ratio as the column aspect ratio, c_x/c_y , exceeds 1.0 (Fig. 5); with x being the drift direction.

SHEAR STRESS ASSOCIATED WITH DRIFT

The relationship between $V_u/(\phi V_c)$ and DR_u that satisfies Section 11.11² is here derived. Section 11.11.2² requires shear reinforcement when $v_u > \phi v_n$ (with $v_n = v_c$, Eq. 4). Express v_u at the critical section in Fig. 3 as the sum of the factored maximum shear stress v_{ug} due to gravity loads and v_{uE} due to earthquake; thus, the criterion that permits absence of shear reinforcement is:

$$v_{uE} \leq \phi v_c - v_{ug} \quad (14)$$

v_{uE} can be expressed by Eq. 3:

$$v_{uE} = [V_{uE} / (b_o d)] + (\gamma_v M_{uE} x / J_c) \quad (15)$$

Here the subscript E refers to the effect of the drift induced by earthquake; the effect of the vertical component of ground motion is not included. IBC-06¹ and ACI 318-11² (Section 8.3.1) permit the calculation of V_{uE} and M_{uE} by linear analysis. The frame in Fig. 2a is subjected to an imposed displacement $= DR_e l_c$; DR_e is a fraction of DR_u specified by Eq. 1; l_c is the story height = distance between mid-surfaces of slabs of two consecutive floors. For the column on the axis of symmetry in Fig. 2a, the drift produces no shear force; thus, $V_{uE} = 0$, $V_u = V_{ug}$ and v_{uE} is proportionate to DR_e . The maximum shear stress due to earthquake can thus be expressed as linear function of DR_e or DR_u as:

$$v_{uE} = v_e DR_e \text{ with } DR_e \leq 0.0094 \quad (16)$$

$$v_{uE} = \left(\frac{v_e}{C_d / I_E} \right) DR_u \text{ with } DR_u \leq 0.025 \quad (17)$$

$v_e = \text{constant}$ (MPa or psi), whose value increases with the increase of the flexural rigidity / aspect ratio (c_x/c_y , Fig. 3). The constant v_e represents the maximum shear stress corresponding to $DR_e = 1$. IBC-06¹ specifies a maximum value of $DR_u = 0.007$ to 0.025 and gives values of (C_d/I_E) varying between 2.67 and 6.5; this sets an upper limit of DR_e below 0.94 percent. Thus, IBC-06¹ permits the elastic analysis to calculate the unbalanced moment for $DR_e \leq 0.94$ percent. In option (b) of 21.13.6², $V_{ug} = v_{ug} b_o d$ = the factored shearing force due to factored gravity loads. Substitution of Eq. 17 in Eq. 14 and noting that $v_{ug} = V_u / (b_o d)$ and $V_c = v_c b_o d$, gives the limit of DR_u to permit absence of shear reinforcement:

$$DR_u \leq \phi \left(\frac{v_c}{v_e} \right) \left(\frac{C_d}{I_E} \right) \left[1 - \frac{V_u}{\phi V_c} \right]; \text{ with } DR_u \leq 0.025 \quad (18)$$

Equation 18 is represented in Fig. 5 by a series of straight lines (e.g. ef and eg), whose slope depends on v_e (Appendix A). Line ef in Fig. 5 represents Eq. 18 for the connection of a slab with square columns, $c_x \times c_y = 300 \text{ mm} \times 300 \text{ mm}$ (11.8 in. $\times$ 11.8 in.) (Example 1). For any point above the line, option (a) of 21.13.6² requires shear reinforcement to satisfy Section 11.11². Line eg is for the slab-column connection of Example 1, with the square columns replaced by rectangular columns, $500 \text{ mm} \times 250 \text{ mm}$ (19.7 in. $\times$ 9.8 in.); the slope of the line is changed resulting in a wider zone for which shear reinforcement is required. The slopes of lines ef and eg are calculated in Appendix A. This shows that the shear reinforcement requirement depends on the aspect ratio / the flexural rigidity of the column.

EXAMPLE 1: DESIGN ACCORDING TO OPTION (a) OF 21.13.6, ACI 318-11²

Consider a solid flat plate for an office building with equal span lengths in the x - and y -directions, $L_x = L_y = 6.0 \text{ m}$ (19.7 ft), floor height, $l_c = 3.50 \text{ m}$ (11.5 ft), column size, $c_x \times c_y = 300 \text{ mm} \times 300 \text{ mm}$ (11.8 in. $\times$ 11.8 in.), and slab thickness, $h = 200 \text{ mm}$ (7.87 in.). The floor is designed for a service live load of 2.4 kPa (50 psf), a super-imposed dead load of 1.3 kPa (27 psf), and a self-weight of 4.7 kPa (98 psf). The earthquake-excited motion is in the x -direction, where the structure has six bays (Fig. 2), with many bays in the y -direction. The structure has a LFRS that limits the ultimate inter-story drift ratio, including inelastic deformations, to $DR_u = 0.02$; assume that the ratio $C_d/I_E = DR_u / DR_e = 3.5$. The elastic drift (Eq. 1):

$$DR_e l_c = 0.02(3.5)/3.5 = 0.02 \text{ m (0.79 in.)}$$

It is required to calculate V_u , M_u and v_u at the central column and design the shear reinforcement, if required. Other data are: concrete cover = 20 mm (0.79 in.); $f'_c = 30.0 \text{ MPa}$ (4350 psi); $E_c = 24.6 \text{ GPa}$ (3580 ksi); the top flexural reinforcement is composed of 16 mm (0.63 in.) bars, spaced at 150 mm (5.91 in.). The effective slab depth, $d = 200 - 20 - 16 = 164 \text{ mm}$ (6.46 in.). If required, use headed shear stud reinforcement with $f_{yt} = 345 \text{ MPa}$ (50 ksi).

ACI 318-11² requires seismic design for: $(1.2D + E + 0.5L)$. The symbol E in the load combinations represents the effect of drift, excluding the effect of the vertical component of ground motion. The factored gravity load = 8.4 kPa (175 psf); corresponding $V_{ug} = 314$ kN (70.6 kips) and $M_{ug} = 0$. From an elastic analysis of the equivalent frame in Fig. 2a, subjected to lateral elastic displacement, $DR_e l_c = 20$ mm (0.79 in.), the shearing force and unbalanced moment at the central interior column are: $V_{uE} = 0$ and $M_{uE} = 117$ kN-m (1035 kip-in.). Appendix A determines the moments of inertia of the members of the equivalent frames used in the analysis.

Punching shear strength design is required for $V_u = 314$ kN (70.6 kips) combined with $M_u = 117$ kN-m (1035 kip-in.). Properties of the shear critical section at $d/2$ from the column face (Appendix B of ACI 421.1R-08⁷) are: $b_o = 1.856$ m (73.07 in.); $x = 232$ mm (9.13 in.); $\gamma_v = 0.4$; $J_c = 10.92 \times 10^{-3}$ m⁴ (26.24×10^3 in.⁴). The maximum shear stress is (Eq. 3): $v_u = 2.03$ MPa (294 psi). This value is greater than $\phi v_c = 1.37$ MPa (199 psi); Eq. 4); thus, shear reinforcement is required.

Because v_u is less than $0.5\phi\sqrt{f'_c} = 2.05$ MPa (298 psi), $s \leq 0.75d = 123$ mm (4.84 in.). Use eight rails of headed shear stud reinforcement, arranged as shown in Fig. 6a, having nominal diameter = 12.7 mm (1/2 in.), $s = 120$ mm (4.72 in.), $s_o = 65$ mm (2.56 in.) and $A_v = 1013$ mm² (1.57 in.²); Eqs. 5 and 6 give: $v_s = 1.58$ MPa (216 psi); $\phi v_n = 2.21$ MPa (320 psi) $> v_u = 2.03$ MPa (294 psi). Thus, the chosen A_v and s are satisfactory. At $d/2$ from the outermost peripheral line of studs, $b_o = 4148$ mm (163.3 in.); $x = 657$ mm (25.9 in.); $\gamma_v = 0.4$; $J_c = 139.2 \times 10^{-3}$ m⁴ (334.4×10^3 in.⁴) and $v_u = 0.68$ MPa (98 psi). This stress value is less than the factored concrete strength, $\phi v_n = 0.17\phi\sqrt{f'_c} = 0.69$ MPa (99 psi) (Eq. 7) indicating that the extension of the shear-reinforced zone is adequate (Fig. 6a).

Flexural reinforcement to resist $(1 - \gamma_v) M_{uE}$ has to be provided in the slab and detailed according to 13.5.3.2².

EXAMPLE 2: DESIGN ACCORDING TO OPTION (b) OF 21.13.6, ACI 318-11²

For the column-slab connection in Example 1, $V_u = 314$ kN (70.6 kips), $V_c = 556$ kN (125 kips), $V_u/(\phi V_c) = 0.75$ and $DR_u = 0.02$. The point $[V_u/(\phi V_c), DR_u]$ lies above the bilinear limit abc in Fig. 1; thus, shear reinforcement is required. The shear reinforcement has to satisfy Eqs. 6, 8 and 9. For eight rails of 12.7 mm (1/2 in.) studs,

$$s \leq \frac{A_v f_{yt}}{b_o (0.29\sqrt{f'_c})} = \frac{1013(345)}{1856(0.29\sqrt{30})} = 119 \text{ mm (4.7 in.)}$$

Use $s_o = 65$ mm (2.6 in.) and $s = 115$ mm (4.53 in.). The shear-reinforced zone should extend such that $b_s \geq 4h + (d/2) = 4(200) + (164/2) = 882$ mm (34.7 in.); this extension is achieved by eight studs on each rail as shown in Fig. 6(b). Because, the shearing force and moment transferred between the column and the slab due to earthquake are not calculated, the design with option (b) does not verify that the flexural reinforcement or the shear strength satisfies 13.5.3.2 or 11.11.7².

COMPARISON OF DESIGNS ACCORDING TO THE OPTIONS OF SECTION 21.13.6²

Equation 18 is represented in Fig. 1 by a typical line eg , whose slope depends on the column's aspect ratio / flexural rigidity; a connection, without shear reinforcement, represented by a point of $[V_u/(\phi V_c), DR_u]$ that falls within the shaded zones satisfies option (b) of 21.13.6², but lacks the strength required by Section 11.11².

From Figs. 6(a) and 6(b), it can be seen that Eq. 9 requires unnecessarily large extension of the shear-reinforced zone. It is impossible to find a case that requires this extension by a strength design based on Section 11.11².

To satisfy the requirement: $v_u \leq \phi v_n$ (Eq. 2), the designer can limit v_u by increasing the slab thickness or augmenting v_n by increasing f'_c or providing shear reinforcement. In the design with option (b) of 21.13.6², the amount of shear reinforcement is prescribed (by Eq. 8) and the calculation of v_u is not done. The prescribed shear reinforcement of Eq. 8 may not be sufficient to bring v_n to the required level. Section 11.11.7² sets an upper limit for v_n (Eq. 5 or 6) that cannot be exceeded regardless of the amount of shear reinforcement. Again, by not using Eq. 2 in the design, there is no guarantee that the upper limit for v_n is respected. In summary, the shortcomings of 21.13.6(b)² are:

- It may miss to indicate the need for shear reinforcement, or require shear reinforcement having inadequate v_s and extend the shear-reinforced zone farther than needed.
- It may miss to indicate the need to increase the slab thickness or f'_c .
- It does not enable the design of the flexural reinforcement required to resist $(1-\gamma_v) M_u$ (Section 13.5.3.2²).
Thus, the design with option (b) does not insure that all the requirements of Sections 11.11.7 are satisfied.

EXAMPLE 3: APPLICATION OF 21.13.6² TO A RECTANGULAR EDGE COLUMN

Consider the edge column-slab connection of the same structure of Example 1, with all columns having rectangular cross-section, $c_x \times c_y = 500 \text{ mm} \times 250 \text{ mm}$ (19.7 in. $\times$ 9.8 in.) subjected to the same gravity load. Find the upper limits of DR_u permitted in absence of shear reinforcement by options (a) and (b) of Section 21.13.6². The solution of this example, given below, will show that the upper limits are 0.61 percent with option (a) and 1.13 percent with option (b). This large difference indicates that option (b) cannot safely be a substitute of option (a).

For the edge column-slab connection, the internal forces due to factored gravity loads are: $V_{ug} = 168 \text{ kN}$ (37.8 kips) and $(M_{ug})_O = 44 \text{ kN-m}$ (389 kip-in.); where M_{uO} is the unbalanced moment about an axis through the column's centroid, O (Fig. 3b). The critical shear stress occurs for a drift in the negative x -direction; elastic analysis of the equivalent frame in Fig. 2a, subjected to lateral elastic displacement in this direction, $DR_e l_c$ (Eq. 1), gives at the edge column the shearing force, $V_{uE} = 2400DR_u \text{ kN}$ (540 DR_u kips) and the unbalanced moment, $(M_{uE})_O = 7800DR_u \text{ kN-m}$ (69000 DR_u kip-in.). Calculate the maximum shear stress, v_u due to the factored internal forces of the combination $(1.2D + E + 0.5L)$: $V_u = (168 + 2400DR_u) \text{ kN}$ ((37.8 + 540 DR_u) kips), combined with $M_{uO} = (44 + 7800DR_u) \text{ kN-m}$ ((389 + 69000 DR_u) kip-in.). The corresponding unbalanced moment at the centroid of the shear critical section at $d/2$ from the column (Fig. 3b) is: $M_u = M_{uO} + V_u x_O$.

Properties of the shear critical section at $d/2$ from the column face (Fig. 3b) are: $b_o = 1.578 \text{ m}$ (62.13 in.); $x_O = -117 \text{ mm}$ (-4.61 in.); $x_A = -367 \text{ mm}$ (-14.4 in.); $x_B = 215 \text{ mm}$ (8.46 in.); $\gamma_v = 0.423$; $J_c = 9.629 \times 10^{-3} \text{ m}^4$ (23.13 $\times 10^3 \text{ in.}^4$) (ACI 421.1R-08⁷). The value of $DR_u = 6.1 \times 10^{-3}$ would bring v_u to its permissible limit without shear reinforcement, $\phi v_c = 1.37 \text{ MPa}$ (199 psi); Eq. 4). Thus, option (a) of 21.13.6² requires shear reinforcement for $DR_u > 6.1 \times 10^{-3}$. On the other hand, with $V_{ug}/(\phi V_c) = 168/(472 \phi) = 0.475$, the design with option (b) of 21.13.6² would permit absence of shear reinforcement with $DR_u \leq 0.035 - 0.05 [V_{ug}/(\phi V_c)] = 11.3 \times 10^{-3}$.

SEISMIC DESIGN RECOMMENDED BY ACI 421.2R-10¹⁸

ACI 421.2R-10¹⁸ presents design examples of interior, edge and corner column-slab connections according to option (a) of 21.13.6, ACI 318-11². In addition, based on referenced experimental data, ACI 421.2R-10¹⁸ recommends for ductility a minimum amount of shear reinforcement specified by Eq. 20 and an extent of the shear-reinforced zone according to Eq. 21; the minimum shear reinforcement is recommended only when V_u or DR_u is relatively high (Eq. 19).

$$V_u/(\phi V_c) \geq 0.4 \text{ or } DR_u \geq 0.035 - 0.05 [V_u/(\phi V_c)] \text{ with } DR_u \geq 0.015 \quad (19)$$

$$v_s \geq 0.25 \sqrt{f'_c} \text{ (MPa)} \left[3 \sqrt{f'_c} \text{ (psi)} \right] \quad (20)$$

$$b_s \geq 3.5d + d/2 \quad (21)$$

The design in Example 1 (Fig. 5a) satisfies Eq. 20; with two additional peripheral lines of shear reinforcement, Eq. 21 would be satisfied.

To account for the degradation of concrete shear resistance caused by the reversals of the unbalanced moment, ACI 421.2R-10¹⁸ recommends calculating v_n by Eq. 22, in which the first term is half the corresponding term in Eq. 5:

$$\begin{aligned} v_n &= 0.083 \sqrt{f'_c} + v_s \text{ (MPa)} \text{ or } v_n = \sqrt{f'_c} + v_s \text{ (psi)} \quad \text{Stirrups} \\ v_n &= 0.125 \sqrt{f'_c} + v_s \text{ (MPa)} \text{ or } v_n = 1.5 \sqrt{f'_c} + v_s \text{ (psi)} \quad \text{Headed shear studs} \end{aligned} \quad (22)$$

In lieu of Eq. 22, it may be possible to reduce the first term in Eq. 5 by a factor dependent upon DR_u (a proposal that needs further research).

CAUTIONARY NOTE

The design for punching by option (b) of 21.13.6² is simple to apply, because the shear force and the unbalanced moment associated with the drift of the structure is not considered; at the same time, the option does not ensure satisfying the punching shear or the flexural strength of Section 11.11 or 13.5.3.2². Note that the calculation of M_u is still required to verify that $M_u \leq M_f$, where M_f is the unbalanced moment that transforms the slab into a flexural collapse mechanism. Gesund and Goli¹⁹ give the yield-line patterns and the equations for M_f .

NEED FOR CODE CHANGE

Section 21.13.6 of ACI 318-11² permits option (a) or (b) for the seismic punching shear design of the connections of flat plates and columns. Option (a) ensures the strength required in Section 11.11 of the same code; while option (b) is based on the effect of the parameter $V_{ug}/(\phi V_c)$ on the drift capacity. To ensure strength and ductility, it is here recommended to combine the two options: For strength, satisfy 11.11.7.1 and 13.5.3.2; for ductility, require shear reinforcement having minimum v_s and b_s when $V_u/(\phi V_c)$ or DR_u exceeds a specified limit (e.g. see values recommended in ACI 421.2R-10¹⁸). Also, to account for the degradation of concrete shear resistance caused by the reversals of the unbalanced moment, the contribution of concrete to v_n , in the presence of shear reinforcement, may need to be revisited (e.g. Eq. 22).

CONCLUSIONS

Calculation of the shearing force and the unbalanced moment transferred at a column-slab connection in multi-story buildings due to gravity loads and drift (due to any lateral load) is demonstrated by examples. In an earthquake, the connection must be ductile to withstand drift reversals in the plastic range without loss of its punching shear strength. The ductility, expressed in terms of the design inter-story drift ratio, DR_u , depends upon $V_u/(\phi V_c)$, the flexural reinforcement ratio and the column's aspect ratio / flexural rigidity. An empirical graph of the parameter $V_u/(\phi V_c)$ versus DR_u obtained by tests investigating the effect of this parameter does not ensure the strength for all cases.

The seismic design options (a) and (b) of 21.13.6, ACI 318-11² are studied to show that option (b) may not satisfy the strengths required in 11.11.7 and 13.5.3.2². An example is presented to show that the limit of DR_u , at which shear reinforcement is required, is substantially different with the two design options. Revisions of Section 21.13.6² and its commentary are proposed to ensure that flat plate-column connections have the strengths required in the code². In addition, for ductility, a specified minimum amount of shear reinforcement is proposed when $V_u/(\phi V_c)$ or DR_u is relatively high.

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LIST OF SYMBOLS

A_v	=	area of shear reinforcement on one peripheral line parallel to column face
b_o	=	length of perimeter of shear critical section
b_s	=	distance between column face and outer shear critical section
C_d	=	dimensionless coefficient representing the inherent deformability of lateral force-resisting system
c_x, c_y	=	size of rectangular column measured in the x and y directions
d	=	effective depth of slab; average of distances from extreme compression fiber to centroids of tension reinforcements running in two orthogonal directions
DR_e, DR_u	=	design elastic and inelastic inter-story drift ratio
E_c	=	concrete modulus of elasticity
f'_c	=	specified compressive strength of concrete
f_y, f_{yt}	=	specified yield strength of flexural or shear reinforcement
h	=	slab thickness
I_E	=	occupancy importance factor
I_{ec}	=	equivalent moment of inertia of column, accounting for torsional strip(s)
J_c	=	property of shear critical section
l_c	=	distance between mid-surfaces of slabs of two consecutive floors
L_x, L_y	=	span lengths in the x - and y -directions
m	=	ultimate flexural strength of a slab strip of unit width
M_u	=	factored unbalanced moment transferred between slab and column
M_{uO}	=	factored unbalanced moment about an axis through column's centroid O (Fig. 3b)
s	=	spacing between peripheral lines of shear reinforcement
s_o	=	spacing between first peripheral line of shear reinforcement and column face
v_c	=	nominal shear strength provided by concrete in absence of shear reinforcement
v_e	=	constant defined in Eq. 16 (of units MPa or psi); it depends upon the dimensions of the column and the slab
v_n	=	nominal shear strength at shear critical section
v_s	=	nominal shear strength provided by shear reinforcement
v_u	=	maximum shear stress due to factored forces
V_c	=	nominal shear strength provided by concrete in absence of shear reinforcement
V_u	=	factored shearing force
V_{ug}	=	factored shear force due to gravity loads
x, y	=	coordinates of point on perimeter of shear critical section with respect to centroidal principal axes, x and y
α_s	=	dimensionless coefficient equal to 40, 30, and 20 for interior, edge, and corner columns, respectively
β	=	ratio of long side to short side of column cross section
γ_v	=	fraction of unbalanced moment transferred by eccentricity of shear
λ	=	modification factor reflecting the reduced mechanical properties of lightweight concrete, relative to normal-weight concrete of the same compressive strength
ϕ	=	strength reduction factor
ρ	=	ratio of flexural reinforcement in a slab strip of unit width

APPENDIX A—SUMMARY OF CALCULATIONS INVOLVED IN PREPARING FIGS. 1 AND 5

The end-rotational stiffness, K_c is the moment at one column end to rotate it with an angle = 1 and $\Sigma K_c = 2K_c$ = the sum of the moments at the ends above and below the slab-column joint. Equation A1 or A2 gives K_c for the frame in Fig. 2a or 2b, respectively. The equations can be derived by the method of column analogy or other structural analysis methods²⁰.

Frame in Fig. 2a:

$$K_c = \frac{E_c I_c}{l_c} \cdot \frac{6}{[1 - (h/l_c)]^3} ; \Sigma K_c = 2K_c \quad (\text{A1})$$

Frame in Fig. 2b:

$$K_c = \frac{E_c I_c}{l_c} \left[\frac{1}{1 - (h/l_c)} + \frac{3}{[1 - (h/l_c)]^3} \right] ; \Sigma K_c = 2K_c \quad (\text{A2})$$

Figure 5, line *ef*

Properties of the members in the equivalent frames in Figs. 2a and 2b, with column size 300 mm×300 mm (11.8 in.×11.8 in.), and using N-m and Pa for the values of K and E , respectively, are:

Fig. 2a: $I_s = 2.00 \times 10^{-3} \text{ m}^4$ ($4.805 \times 10^3 \text{ in.}^4$); $I_{sj} = 2.22 \times 10^{-3} \text{ m}^4$ ($5.324 \times 10^3 \text{ in.}^4$); $I_c = 0.68 \times 10^{-3} \text{ m}^4$ ($1.634 \times 10^3 \text{ in.}^4$); K_t (N-m) = $1.62 \times 10^{-3} E_c$ (N/m²); ΣK_c (N-m) = $2.76 \times 10^{-3} E_c$ (N/m²); $I_{ec} = 0.25 \times 10^{-3} \text{ m}^4$ ($0.601 \times 10^3 \text{ in.}^4$).

Fig. 2b: Same properties as the frame in Fig. 2a except: $I_s = 4.00 \times 10^{-3} \text{ m}^4$ ($9.610 \times 10^3 \text{ in.}^4$); $I_{sj} = 4.43 \times 10^{-3} \text{ m}^4$ ($10.643 \times 10^3 \text{ in.}^4$); ΣK_c (N-m) = $1.79 \times 10^{-3} E_c$ (N/m²); $I_{ec} = 0.32 \times 10^{-3} \text{ m}^4$ ($0.769 \times 10^3 \text{ in.}^4$). Elastic analysis of the frame with spans $L_x = 6.0 \text{ m}$ (19.7 ft), $E_c = 24.6 \text{ GPa}$ (3580 ksi), subjected to $DR_e = 1.0$, $DR_e l_c = 1.0(3.5) = 3.5 \text{ m}$ (11.5 ft), gives at the central column an unbalanced moment = 20.2 MN-m ($14.888 \times 10^3 \text{ kip-ft}$) and shearing force = 0. Properties of the critical section at $d/2$ from the column periphery (according to Reference 7): $\gamma_v = 0.4$; $J_c = 10.92 \times 10^{-3} \text{ m}^4$ ($26.24 \times 10^3 \text{ in.}^4$); $v_c = \sqrt{f'_c}/3 = 1.83 \text{ MPa}$ (265 psi) (Eq. 4); the maximum shear stress corresponding to $DR_e = 1$ is (Eq. 3):

$$v_e = \frac{0.4(20.2)(0.232)}{10.92 \times 10^{-3}} = 171.5 \text{ MPa} \quad (24.871 \text{ ksi})$$

Thus, the equation of line *ef* is (Eq. 18):

$$DR_u = 0.75 \left(\frac{1.83}{171.5} \right) (3.5) \left(1 - \frac{V_u}{\phi V_c} \right) = 0.0280 \left(1 - \frac{V_u}{\phi V_c} \right)$$

Figures 1 and 5, line *eg*

Line *eg* is for the same structure in Example 1 but with columns $c_x \times c_y = 500 \text{ mm} \times 250 \text{ mm}$ (19.7 in.×9.8 in.). Properties of the members of the equivalent frame in Fig. 2a are: $I_s = 2.00 \times 10^{-3} \text{ m}^4$ ($4.805 \times 10^3 \text{ in.}^4$); $I_{sj} = 2.18 \times 10^{-3} \text{ m}^4$ ($5.232 \times 10^3 \text{ in.}^4$); $I_c = 2.60 \times 10^{-3} \text{ m}^4$ ($6.247 \times 10^3 \text{ in.}^4$); K_t (N-m) = $3.40 \times 10^{-3} E_c$ (N/m²); ΣK_c (N-m) = $10.6 \times 10^{-3} E_c$ (N/m²); $I_{ec} = 0.63 \times 10^{-3} \text{ m}^4$ ($1.514 \times 10^3 \text{ in.}^4$). Subjecting the frame to the same $DR_e l_c = 3.5 \text{ m}$ (11.5 ft) gives at the central column an unbalanced moment = 38.9 MN-m ($28.69 \times 10^3 \text{ kip-ft}$) and shearing force = 0. Properties of the critical section at $d/2$ from the column periphery (according to Reference 7): $\gamma_v = 0.458$; $J_c = 23.0 \times 10^{-3} \text{ m}^4$ ($55.258 \times 10^3 \text{ in.}^4$); the maximum shear at $x = 0.332 \text{ m}$ (13.1 in.) is (Eq. 3):

$$v_e = \frac{0.458(38.9)(0.332)}{23.0 \times 10^{-3}} = 257.2 \text{ MPa} \quad (37.29 \text{ ksi})$$

Equation of line eg (Eq. 18) is:

$$DR_u = 0.75 \left(\frac{1.83}{257.2} \right) (3.5) \left(1 - \frac{V_u}{\phi V_c} \right) = 0.0186 \left(1 - \frac{V_u}{\phi V_c} \right)$$

Effect of cracking

In the above calculations, the effect of cracking is accounted for by reducing the gross moment of inertia by factors 0.5 and 1.0 for I_s and I_c , respectively. ACI 318-11², Sections 13.5 and R13.5.1.2 require the use of a factor between 0.25 and 0.5 for the slab and factor 1.0 for the columns. R13.5.1.2 of ACI 318-11² states that the multiplier for I_s should be greater for prestressed construction. ACI 421.2R-10¹⁸ recommends the use of a factor 0.5 for the slab to avoid underestimating the internal forces transferred between the slabs and their supporting columns.

The slope of the line eg in Fig. 1 would increase if smaller reduction factors are used for the slab and column. Repetition of the calculations with the factors 0.25 and 0.7 for I_s and I_c , respectively changes the slopes of the line eg from 0.0186 to 0.0271. This reduces the shaded zones in Fig. 1 without changing the conclusion from the discussions presented in the paper. Note that the use of underestimated multipliers in this application errs in the unsafe side; it can permit absence of shear reinforcement where shear reinforcement is necessary.

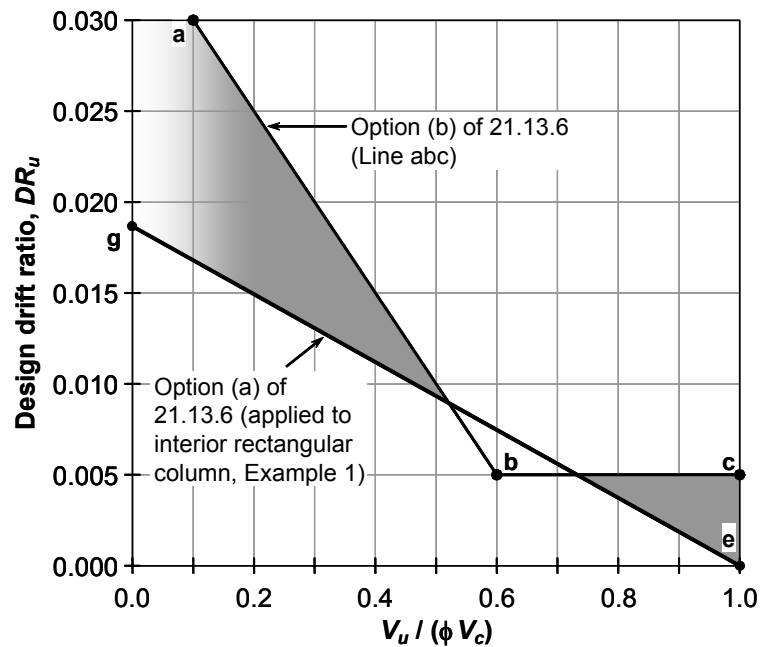


Figure 1—Illustration of the criterion for requiring shear reinforcement according to option (a) compared with option (b) of Section 21.13.6, ACI 318-11².

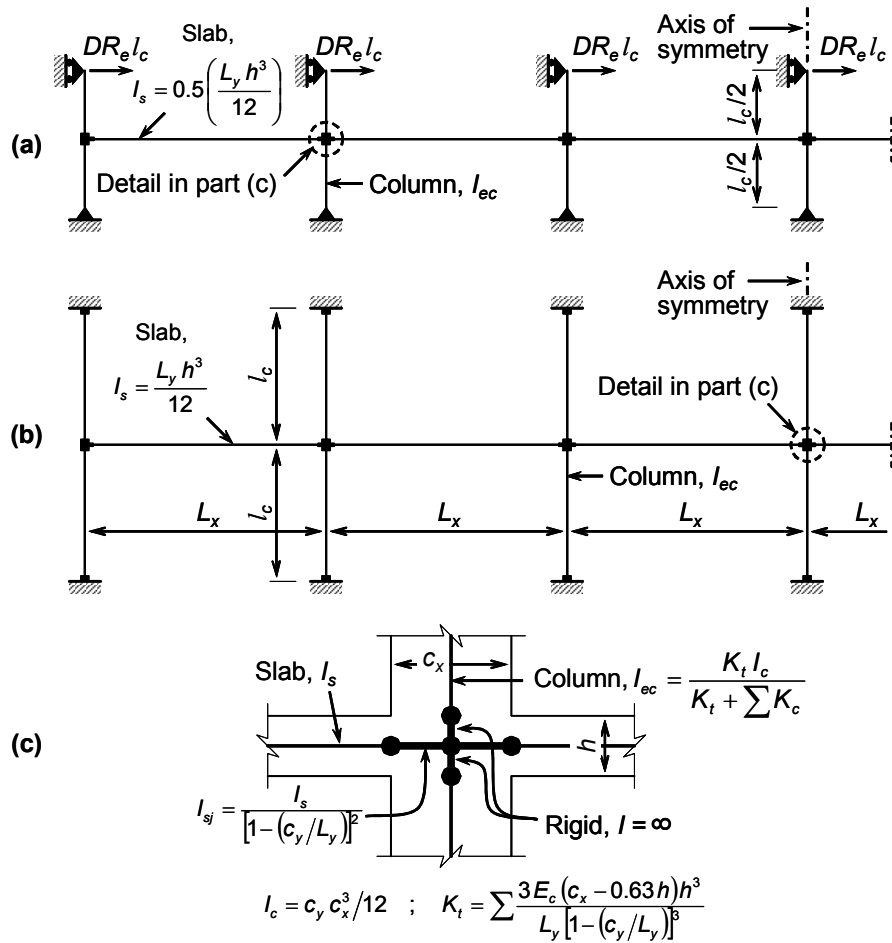


Figure 2—Plane frame idealizations of flat plate structures according to Sections 13.5 and 13.7 of ACI 318-11²: (a) equivalent frame for analysis of effect of drift; (b) equivalent frame for analysis of effect of gravity load; (c) detail of the connections in (a) and (b).

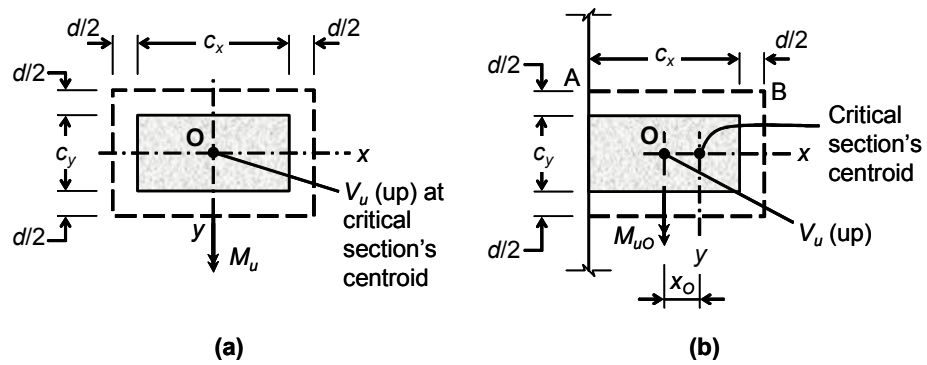


Figure 3—Shear critical section, at $d/2$ from column face, assumed in accordance with ACI 318-11² at: (a) interior column; (b) edge column.

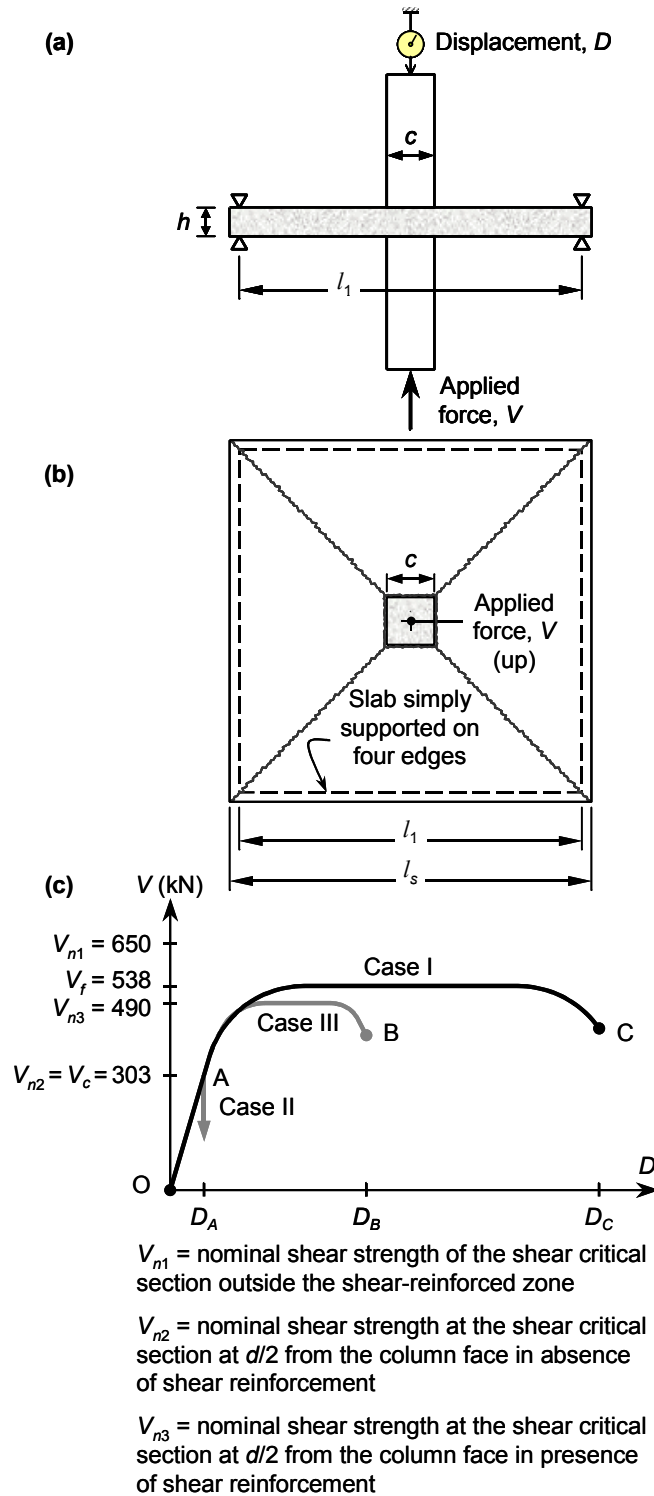


Figure 4—Effect of flexural reinforcement ratio on ductility: (a) simply supported square slab-interior column connection; (b) yield-line fracture pattern; (c) typical graphs of shearing force, V versus displacement, D . (Note: $1 \text{ kN} = 224.8 \text{ lb}$)

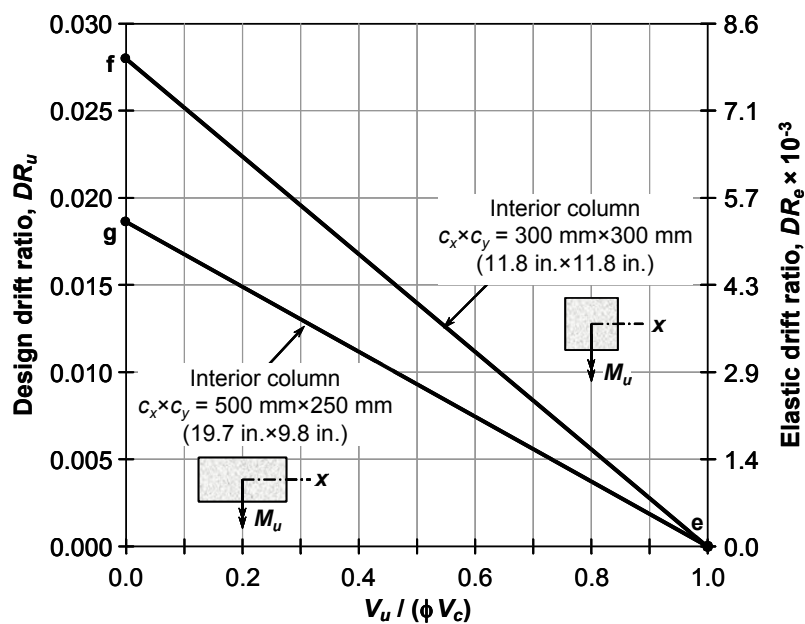


Figure 5—Requirements of shear reinforcement according to Chapter 11², applied to flat plate-rectangular column connections of Example 1. $DR_u/DR_e = (C_d/I_E) = 3.5$.

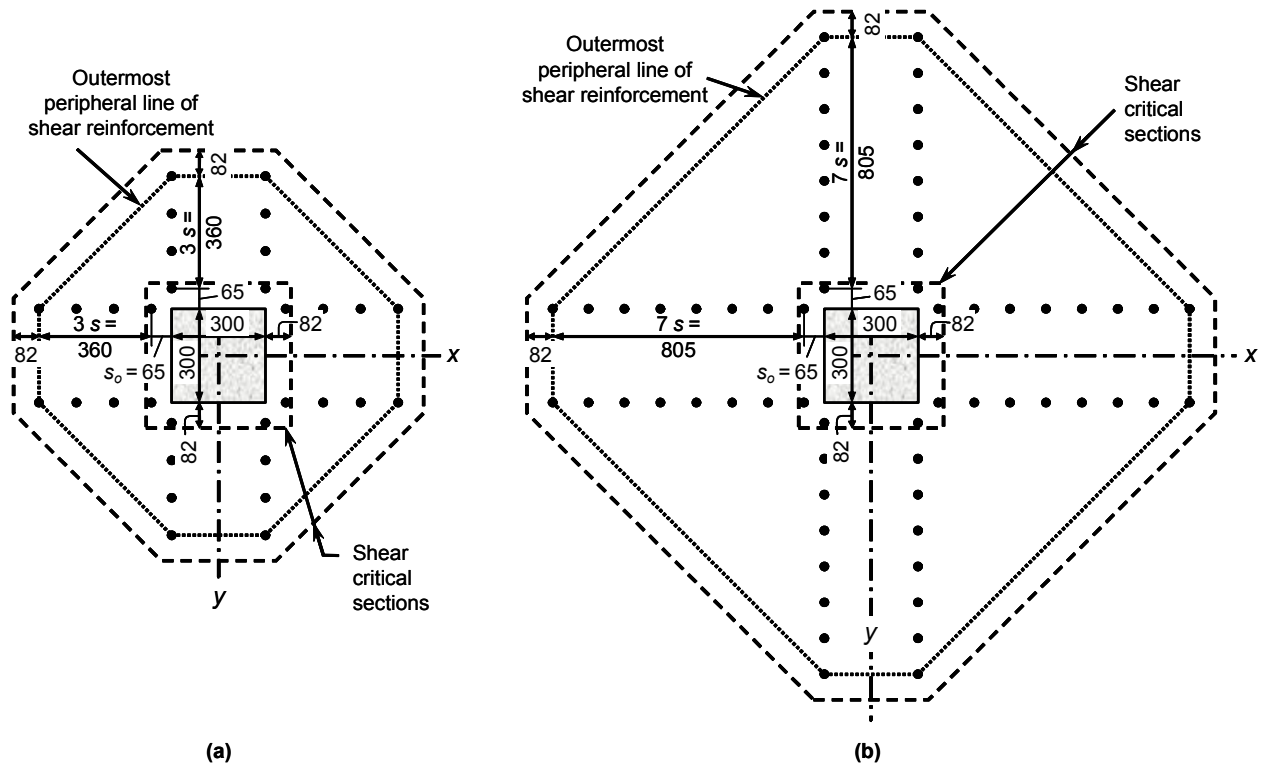


Figure 6—Alternative designs of shear reinforcement permitted by 21.13.6 of ACI 318-11², 12.7 mm studs: (a) design according to option (a), Example 1; (b) design according to option (b), Example 2. All dimensions are in millimeters. (Note: 1 mm = 0.04 in.; 1 mm² = 1.55 × 10⁻³ in.²)

Un-bonded Post-Tensioned Slabs Development and Repair Systems using CFRP

By: Pinaki Chakrabarti, Uksun Kim and David Naish

Synopsis: Un-bonded post-tensioned slabs were developed and principally flourished in the USA since the mid 1950's. The continuous un-bonded one-way post-tensioned slabs became popular due to their predominant use in parking structures all over the country which is true even now. Two-way un-bonded post-tensioned slabs, mainly flat-plates and flat-slabs gained popularity since mid 1960's. The use of a Banded System of placement of un-bonded post-tensioning tendons, introduced in the early 1970's, made flat-plate and flat-slabs more competitive. Flat-plate/slab and shear-wall system became and remains very popular for mid-rise and hi-rise buildings. A brief discussion on the development of un-bonded post-tensioned slabs and its relevance in current design and construction is made in **part-1 of this paper**. Many of these slabs are now between 30 to 60 years old. These structures need routine repairing and retrofitting work. Existing methods are labor intensive and expensive.

An alternative method could be repair work with composite materials. Use of composites (mainly CFRP) as a repair material for concrete structures is becoming very common. Most of the repair procedures are based on researches with reinforced concrete specimens and in some cases with pre-tensioned specimens. Research work using un-bonded post-tensioned specimens, especially two-way slabs is practically non-existent.

A testing program was developed with the goal of finding the cracking and ultimate strength behavior of un-bonded post-tensioned slabs (before and after repair with CFRP) with different boundary conditions. A total of six slabs were tested. In the first phase two two-way simply supported un-bonded post-tensioned slabs were tested. In these tests CFRP repair configurations were varied. In one case CFRP was placed across the cracks in another case it was orthogonal (parallel to the edges). In the second phase, four more slabs were tested (three one-way slabs and one two-way slab). One one-way slab had two ends fixed, another one had one end fixed and one end simply supported and the third one had both ends simply supported. The CFRP placement configuration in these three slabs varied. CFRP was placed across the cracks in the supports and mid spans. The fourth slab was a two-way slab with simple support. In this case the CFRP repair configuration was similar to the first slab (but CFRP had 2 inches/5 cm overlaps). Sketches of different cracking patterns and CFRP configurations are shown inside. The repair and testing of slabs is discussed in **part-2 of this paper**.

Key Words: Development, Un-bonded post-tensioning, Existing Structures, Continuous Slab, Flat-Plate/Slab, Repair System, CFRP

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Part-1: UN-BONDED POST-TENSIONED SLABS DEVELOPMENT and DISCUSSION

INTRODUCTION

Un-bonded post-tensioned slabs are different from the traditional reinforced concrete slabs in many ways. These slabs are relatively thin, there is no contact between the steel pre-stressing strands and concrete, the strands can be placed in many different configurations and they need end anchors, generally the strands do not reach their ultimate strength even at the collapse stage of the slabs and generally a small amount of normal reinforcement is used to prevent crack propagation. Poured in place un-bonded flat-plates/slabs and a shear walls or bonded (grouted) post-tensioned beams and un-bonded one-way slabs and shear walls make very effective structural systems. Pre-stressed structures now compete very well with the traditional steel, concrete, and wood structures and they are taking a sizeable market share. In the beginning, only small steel wires were used for pre-stressing [29]. Gradually, rods and strands were developed. The seven wire stress relieved strands are most popular now. At one time, almost all major structures were pre-tensioned, or post-tensioned grouted structures. In Europe there were no un-bonded pre-stressed structures. Most early applications of pre-stressed elements were in the form of pre-cast pre-tensioned beams. In large bridges a poured-in-place post-tensioned grouted system was used and in segmental bridges a grouted post-tensioned system was used. Un-bonded pre-stressed structures came into existence when attempts were made to apply pre-stressing in poured in place continuous slabs. This occurred in the USA because grouting of a large number of individual strands was labor intensive (expensive in the USA) and not necessary.

New structural systems where the strands were left un-bonded were developed. Before the formal design methods were established, pre-stressed flat plates were constructed on ground and were lifted by lift slab method in California in 1955 [4], Photograph-1. Professor T. Y. Lin introduced the load balancing system in the early 1960's [5], and at the same time Scordelis and Lin had experimented with the application un-bonded strands in slabs [1][2][3]. The un-bonded post-tensioned slab system was very suitable for long span one-way slabs for parking structures and two-way flat plates for commercial buildings. It was used in many lift slab construction in the early days. In the early 1960's the ACI code did not have any clear guidance on the design of pre-stressed structures not to speak of the application of un-bonded system. It was realized that with un-bonded post-tensioning, at ultimate stage, few large cracks instead of several small distributed-cracks develop at the maximum moment zone. It was also realized that the stresses in the un-bonded post-tensioning tendons remained much lower than their ultimate capacities, at the collapse stage of the specimen. In 1972 Lin and Thornton have developed the use of secondary moments for the calculation of ultimate moments [9] in continuous structures. ACI code introduced three sets of equations in 1971 for calculating the design ultimate stress (f_{ps}) of the un-bonded tendons [31]. To prevent large cracks and to control crack widths, use of a minimum percentage of normal steel was made compulsory. In 1983 three more equations for f_{ps} (a slight modification of the previous three) were introduced in the ACI code for thin

members like slabs (l/d ratio larger than 35) [32]. In 1985, Naaman presented a review and recommendation for partial pre-stressing, where he discussed some issues related to un-bonded pre-stressing [13]. Chakrabarti suggested new equations for f_{ps} [21] in 1995 and made statistical comparison with ACI equations and other suggested equations [23] in 1997.

Since the mid 60's, the primary use of un-bonded post-tensioning in the USA can be seen in the flat-plates and flat-slabs for residential and commercial buildings [8], Photograph-3-4-5, and one-way continuous slabs for long span parking structures, Photograph-2. In slab like structures, where strands are not placed in big groups, an un-bonded post-tensioning system is used to avoid grouting of large number of strands, a procedure which is labor intensive and expensive. The un-bonded strands allow large deformation without reaching ultimate strength, and if properly detailed it may prevent total collapse of the building in case of a very large earthquake. In the early stages of development, heavily greased paper wrapped strands were used in the un-bonded post-tensioning system. At present strands covered with plastic tubes are generally used.

CURRENT DESIGN REQUIREMENT in the USA

The values for allowable elastic stresses, ultimate stresses and minimum reinforcement for pre-stressed members are given in the chapter 18 of ACI-318-2008 code. However, the following regulations apply only to the un-bonded post-tensioning systems (which are predominantly one-way or two-way slabs):

Permissible elastic stresses in concrete in flexure – In case of two-way slabs, the allowable tension stress at the extreme fiber cannot exceed $6\sqrt{f'_c}$ (in other cases up to $12\sqrt{f'_c}$ is allowed if deflection is permissible using I_{cr}).

At present, according to ACI code, the permissible ultimate stresses in the un-bonded post-tensioning tendons (f_{ps}) shall have the lowest value among the three equations given below. The second set is generally applicable to slabs.

$$f_{ps} = f_{se} + 10,000 + f'_c / (100 \rho_p) \text{ (psi) ...1)}$$

$$f_{ps} = f_{se} + 10,000 + f'_c / (300 \rho_p) \text{ (psi) ...1a)}$$

$$\text{or, } f_{ps} = f_{se} + 60,000 \text{ (psi) ...2)}$$

$$\text{or, } f_{ps} = f_{se} + 30,000 \text{ (psi) ...2a)}$$

$$\text{or, } f_{ps} = f_{py} \text{ (psi) ... 3)}$$

$$\text{or, } f_{ps} = f_{py} \text{ (psi) ...3)}$$

$$\text{For } l/h \leq 35 \text{ ...4)}$$

$$\text{For } l/h > 35 \text{ (more applicable to slabs) ...4a)}$$

$$(1 \text{ inch} = 2.54 \text{ cm and } 1 \text{ psi} = 0.0069 \text{ MPa})$$

Where, f'_c is the concrete stress in psi, f_{se} is the effective stress in the pre-stressing steel, ρ_p is the percentage of pre-stressing steel in the slab or beam cross section, f_{py} is the yield stress of the pre-stressing steel, l is the span of the beam, and d_p is the depth (generally average depth is used in two way systems) of the center of pre-stressing steel from the compression face. These equations do not include the mild-steel that are present in the members [21].

The above equations, especially the second set which are applicable to slabs, are very conservative, a fact which was observed in some of the experimental investigations conducted [21, 23]. According to ACI-318-08 post-tensioning steel stress at transfer condition is approximately $0.7f_{pu}$ (189 ksi /1304MPa for 270 ksi /1863MPa strands). If the effective stress is considered as $0.6f_{pu}$, which is very common, the stress is 162 ksi / 1118 MPa. Adding 30 ksi /207 MPa the maximum limit becomes 192 ksi / 1325 MPa, which is almost the same as the transfer stress. Another new observation is – that a small amount of CFRP attached at the bottom side of a beam could increase the f_{ps} of un-bonded PT member to a much larger value [27, 28].

MINIMUM BONDED REINFORCEMENT

A minimum amount of mild steel has to be placed in the flexural members with un-bonded pre-stressing steel. Typically, $A_s = 0.004A$, where A_s is the area of mild steel, and A is the area of tension zone pre-compressed before loading. For two-way flat plates, if the elastic tension stress in the positive moment zone is less than $2\sqrt{f'_c}$ (psi) no bonded mild steel is required. When the stress at the positive moment zone exceeds $2\sqrt{f'_c}$ (psi) the amount of bonded

mild steel shall be $A_c = N_c / (0.5 f_y)$, where N_c is the tensile force due to dead load and live load, and f_y is the yield stress of the bonded mild steel (not more than 60 ksi / 414 MPa). This provision of ACI code needs some change. Based on the observation of several partially pre-stressed (post-tensioned) beam tests it is recommended that there must be some mild steel at the possible tension zones of slabs everywhere [16],[20]. Additional mild steel has to be placed in strategic places where a “banded system” is applied in the design of flat plates/slabs. According to ACI code, in the negative moment areas (at column heads), minimum bonded mild steel shall be, $A_s = 0.00075 A_{cf}$ where, A_{cf} is the cross sectional area of the column strip [32]. Since larger spans are supported by relatively thinner un-bonded post-tensioned slabs, and banded PT-systems are used, it is suggested that the reinforcement at the column head be increased instead of using the minimum values.

ADVANTAGES OF UN-BONDED POST-TENSIONED SLABS

Un-bonded post-tensioned slabs have many advantages. These are: a) No field grouting is necessary, hence more economical. b) They are suitable for slabs, where a large number of single strands are to be placed. c) Future repair is possible by re-stressing or replacing the strands. Repair with FRP is possible, as discussed later in part-2. d) Continuous monitoring of pre-stressing forces is possible if necessary. e) Strands with low pre-stressing stress loss coefficients are now available. f) It is also suitable for the work with exposed strands (for external retrofit, cable stay). g) It is suitable for large seismic load resistance (normally the cracks would close because there would be no yielding of strands). h) A slab much thinner than the conventional reinforced concrete slab would be required (reducing the weight of the building – hence the seismic loads).

There are some disadvantages also – like lower allowable ultimate strand capacity by code (in comparison with bonded strands), larger local cracks at high moment points possible (hence additional mild steel is required by code- however, considering the concept of partial pre-stressing this is good for the structural member), need for anchorages and special care for corrosion protection. Extra caution must be taken for strand placement in typical continuous flat-plate/slab systems, especially where openings are present. Special precautions have to be taken for loss prevention, restraints, seismic loading and corrosion protection [15] [17].

However, advantages outweigh the disadvantages. Un-bonded post-tensioned one way slabs for buildings and parking structures and two-way slabs for buildings are very popular. Placing shear walls at suitable locations a very good lateral load resisting farming system can be created.

UN-BONDED POST-TENSIONED ONE-WAY CONTINUOUS SLAB SYSTEMS

In continuous members, the same tendon placed in a parabolic shape provides both negative and positive moment capacity. Generally slab thicknesses are relatively small. With proper detailing, cutting of strands at intermediate locations, extra anchorage, or overlap can be minimized. Because of shallow drapes, tendons can be continued over many spans without much friction loss. Generally 6 bays with total 120'-0" (36.6 m) of 6 inch (15.24cm) slab spanning 20'-0" inch (6.10 m) in each bay, can be stressed with one end pull. By providing minimum amount of mild steel, sufficient ductility, prevention of single large crack and deflection control can be achieved. Typical applications are found in residential buildings, commercial buildings, and long-span parking structures. Continuous slabs in one direction, and pre-cast pre-tensioned beams, or poured-in-place bonded post-tensioned beams, or steel beams in the other direction can make the total floor system. There are many field items that must be considered. However, strict inspection for maintaining the drapes of the un-bonded tendons and proper placement of the tendons are probably the two most important field items. A long-span, poured-in-place, pre-stressed parking structure (by Chakrabarti) is shown in Photograph-2. The structure is in Long Island, New York, completed in 1968. In the early 1960's, there was no specific requirement of bonded mild steel, which was introduced later, and this is still a matter of further research.

UN-BONDED POST-TENSIONED TWO-WAY SLAB SYSTEMS

Probably it was Guyon who first recognized, in the early 1950's, the possibility of two-way slab pre-stressing [29]. Scordelis and Lin did several tests of two-way un-bonded post-tensioned slabs in the mid 1950's [1] [2] [3]. Several un-bonded post-tensioned lift-slabs were constructed in California around that time [4], Photograph-1. In most cases the development of the formal design procedure lagged behind the actual construction. Some of the laboratory tests were done in Australia, by Rozvany and Hampton [6], and by Botchie and Russell [7] in the mid 1960s. While lift-slab method lost its popularity, actual poured-in-place two-way post-tensioned flat-plats and flat-slabs became more

popular. Since the mid 1960's many such structures were built in the USA. With the help of un-bonded post-tensioning long spans were achieved. Typically an l/h ratio of 45 was used (not a code recommendation yet). In other words, for a 30'-0" x 30'-0" (9.14m x 9.14m) interior bay of an office building only 8" (20.32 cm) thick flat-plate was used. Where slab shear at the column was high, a column capital was provided to solve the problem. Where loads were heavier, or deflections had to be controlled, flat-slabs with drop-heads were used. The flat-slabs with drop-heads have several advantages. They are stiffer and they provide larger moment and shear capacities at column heads (Photograph-3). A large number of flat-plate/slab buildings were designed and constructed in New York, Virginia, Washington D.C., and Connecticut in the late sixties and early seventies (Photograph-4 and Photograph-5 "with banded PT system", by Chakrabarti). Typically, a post-tensioned flat-plate and shear wall provided an excellent structural framing system for high-rise and mid-rise buildings. In non-seismic, low wind areas shear-walls were not required.

In the early stage of development of post-tensioned flat-plates, placements of the un-bonded strands were made according to the ACI-code recommendation for flat-plates with mild steel reinforcement. The un-bonded strands were placed according to the code moment distribution in the column strips and middle strips. The high-strength steel strands were continuous and the drapes were adjusted at the support lines and at the mid-span. One of the biggest problems was the placement of continuous strands with given eccentricities. The task of two-way placement of strands became like a basket weaving. For a real big job, the construction cost went up. One such building is shown in Photograph-3. A new system of placement of the strands was introduced, called "Banded" system. In this system all the strands, in one direction, were placed in a narrow band along the center of column strip, and in the other direction strands were uniformly distributed with equal spacing. The tendon placement looked like the placement in a one-way continuous slab in one direction and continuous beam in the other direction. The Banded system was attempted in the Watergate building at Washington D.C. in the early seventies by Thornton. Almost at the same time, banded system was used in a seven-story building by Chakrabarti, in Danbury, Connecticut and in a twenty story building in Reston, Virginia, Photograph-4 and 5 respectively. During 1970-75, Chakrabarti/Nawy tested slabs with "banded" tendon distribution and other tendon distributions. The specimens were flat-plates. It was observed that the slab with "Banded" tendon placement was equally efficient as the slabs with traditional two-way tendon placement, and uniformly distributed tendon placement. These slabs were almost identical in terms of deflection control and ultimate strength [11] [14] [25]. Smith and Burns worked on post-tensioned flat-plate column joint in 1974 [10]. Burns and Hemakom tested 1/3 scale post-tensioned flat plates with banded post-tensioning in 1977 [12]. The Banded system is now the standard system for the tendon placement in flat-plates and flat-slabs. Un-bonded post-tensioned flat slabs can also be used for deep mat foundations. One such construction is shown in Photograph-6.

Design Ultimate Stress Equation (f_{ps}) - The ACI design ultimate stress equations for un-bonded pre-stressing tendon are given above. Other five equations proposed by several researchers can be found in references [21] [23].

PARTIAL PRE-STRESSING

The placement of bonded mild steel is mandatory in the current ACI code. That makes it all the more rational to use the principles of partial pre-stressing. The tests (by Chakrabarti etc.), using simple span beams of different span/depth ratios, different effective pre-stressing stress, and different shapes show that the ratio of the percentage of un-bonded pre-stressing steel and mild steel has substantial effect on the load carrying capacities and particularly on the ductility of beams [16][20]. Typically 67% un-bonded pre-stressing and 33% mild steel is one of the optimum combinations, where high ductility and sufficient strength can be achieved. Haralji and Kanj have also worked in the similar areas [18]. It is a good practice to keep the Partial Pre-stressing Ratio (PPR) and the combined Reinforcing Index (ω) within certain limits for the design of structural members with un-bonded post-tensioning. A suitable guideline, at least in a commentary form in the code, would be helpful [15].

SPECIAL CONSIDERATION in DESIGN-DETAILING-CONSTRUCTION

Design and Detailing –Spot checks by hand shall be done at all of the critical areas, of a floor system (or other structures), especially, when design software are used for the design work. The criteria and methodology shall be verified. Design engineer's approval of shop drawings must be obtained prior to fabrication and installation of post-tensioning. Pre-stressing provides an active force in the structure. Over-design may cause blowouts. The slabs with un-equal spans, openings, at the end spans, at wall and cross beam intersections must be detailed at the early stages of design. The design engineer must verify the actual material properties of the pre-stressing steel and the anchors. Attachments of the slabs and beams with the retaining walls, stair and elevator-towers create undesirable stress in

the system. Improper placement of the shear walls and transfer beams may cause significant pre-stress loss in the slab systems. An experienced engineer must verify structural detailing drawings related to post-tensioning.

Construction – An “inspection criteria” of the post-tensioning part of the project must be written before the bidding process. The criteria shall be further revised depending on the particular post-tensioning system to be used. Complete inspection must be done (preferably by the design engineer) before the concrete is poured. Any deviation observed in the field, must be reported directly to the design engineer. Most important inspection items are, number of tendons, their location, and the tendon profiles. The high point (at the face of column) and the low points (at mid span) of the profile must be correct. Past experiences indicate that most often the high points of the tendons are not maintained at the right height. Without adequate support at the high points of the tendons, the resulting profile will create reverse curvature over an extended length. The tendons shall not be stressed before the concrete reaches the specified strength. A qualified stressing crew must measure and record the elongations of the tendons during the stressing of the tendons. The tendons shall not be cut or burned, until the stressing-records are verified by the design engineer (or his representative).

SUMMARY and COMMENTS

a) Un-bonded post-tensioned slabs are very suitable for modern buildings. It is a stronger slab system with less weight. b) In un-bonded post-tensioned slab systems, continuous monitoring of the pre-stressing forces is possible throughout the life of the structure if necessary. c) Un-bonded slabs are suitable for providing additional factor of safety and ductility against very large transient lateral forces. It is unlikely that the stress in the pre-stressing steel will near its ultimate stress. d) In two-way flat plates, “banded system” makes easy field placement (more economical) and there is no significant strength reduction. e) Flat plate/slab design using “banded system” is not yet clearly addressed in the code. f) A clear recommendation, in the code, for partial pre-stressing will be helpful. g) In continuous slabs (mostly much shallower than normal beams) secondary moment calculation can be avoided. Instead ACI recommendation of 10% redistribution of moment (ACI-318-08 13.6.7) can be used to simplify the design work.

Part-2: REPAIRING and RETROFITTING of UN-BONDED POST-TENSIONED SLABS with CFRP

INTRODUCTION

Research Significance- Over the last fifty years a large number of structures have been built with one way and two way un-bonded post-tensioned slabs. Many of these slabs need to be repaired or retrofitted. A Fiber Reinforced Polymer (especially CFRP) repairing and retrofitting system is very suitable for this purpose. Much investigation on strengthening of pre-stressed member using FRP laminates was performed in recent decades (Meier, U., Kaiser, H., 1991, Tomas et al 1995, Tadros et al 1998). Extensive research on repair and retrofit of un-bonded pre-stressed structures has been conducted at California State University, Fullerton since 1995. All the tests mentioned above were focused on the study of pre-stressed beams. In this paper tests of un-bonded post-tensioned one-way and two-way slabs and their repair with CFRP are reported. This is the first batch of test specimens. Further testing with different variables would be necessary. Further tests with un-bonded post-tensioned one way slabs (continuous) and two-way flat-plates have to be done. FRP repairs and retrofits for concrete beams and walls have become very common. Someday, FRP could be a very common material to use for the repair and retrofit of un-bonded post-tensioned structures. Research work will evolve according to the industry demand.

Present Research- The size, thickness, post-tensioning and boundary conditions were chosen based on the industry trend, existing codes and standards and available testing facilities. In the present research, a total of six un-bonded post-tensioned slab specimens were tested. Two of the specimens were done in phase-1. These were simply supported two-way slabs with two-way post-tensioning (specimen PTS-1 and PTS-2). Another one (PTS-6) was done in Phase-2. Three one-way slabs with different boundary conditions were completed in phase-2 (specimen PTS-3, PTS-4 and PTS-5). Each was repaired with CFRP using different patterns and then retested. Comparisons were made between the control specimens and the repaired specimens.

Objective- The objectives of this research project were: i) to find the ultimate strength of un-repaired and repaired slabs, ii) to observe the general behavior of the slabs (during the test) before and after the application of CFRP, iii)

to understand the relationship between internal reinforcement (mild steel and un-bonded post tensioning) and externally bonded CFRP, iv) to observe and record crack propagation, strain, load, and deflection during testing and to comply with ACI 318-02 code and ACI 440.2(R)-02 design guide.

MATERIALS

Concrete: Quality concrete of approximately 5500 psi (28 MPa) compressive strength was used. Concrete mixes were prepared according to ASTM C 94-65. **Post-Tensioning Steel:** 16-1/4" Ø 7-wire (0.635cm) strands were used in each direction of the slabs (ASTM A-416 with nominal $f_{pu} = 270$ ksi ie 1863MPa). **Plastic tubing:** The individual pre-stressing strands were inserted through 9/64 inch (0.36 cm) diameter plastic tubes. This process eliminated any bonding between the wires and the concrete. **Mild Steel/Wire-mesh:** Nominal yield strength 60 ksi (414 MPa ASTM A-185) - Placed at the top of each of the two-way-slabs. In one-way slabs, #3 bars were used. **CFRP and Adhesive:** Local Company for Fiber Reinforcement Systems (CF130 and CF160 High Tensile Carbon).

SPECIMEN PREPARATION and STRENGTHENING

Three two-way simply-supported, un-bonded post-tensioned slabs (PTS-1, PTS-2 and PTS-6) were tested with uniformly distributed external loading. The slabs were retested after the repair (PTS-1CR and PTS-2CR and PTS6-CR). The slabs repaired with CFRP are indicated as "CR". The 9'-0½" × 9'-0½" (2.77 m × 2.77 m) slabs had mild steel reinforcements along with un-bonded post-tensioning strands each-way, figure-1 and photograpg-7. The two-way slabs were repaired using different strengthening schemes. Diagonal repair scheme was used for PTS-2CR, (perpendicular to cracks), figure-4, photograph-8 and photograpg-9. Orthogonal repair schemes were used in PTS-1CR and PTS-6R (parallel to slab edges), figure-3. In PTS-6CR the CFRP sheets had 2" (5.08 cm) over-lap. The slab PTS-3 was a one-way slab with both ends fixed, PTS-4 was a one-way slab with one end fixed and PTS-5 was a simply supported one way slab. Pre-stressed strands and reinforcements for all the slabs are shown in Table-1. The one-way slabs (PTS-3CR, PTS-4CR and PTS-5CR) were strengthened with straight CFRP sheets parallel to the span and across the cracks. The CFRP sheets were placed also at the bottom tension zone at the support (which extended to 1/4th clear span), figures- 5, 6 and photograph-10. Details of CFRP materials are given in Table-2.

TEST SETUP and LOADING CRITERIA

Loading was gradually increased at approximately 1 psi load increment. The ultimate load was determined using one of the following three criteria: 1) Excessive cracks were observed visually, 2) Deflection reached close to $L / 120$, and 3) Post-Tensioning forces reached close to 80% of the ultimate load. During the testing- load, deflection, strain (in mild steel), and change in post-tensioning forces were recorded. Same criteria were used for the repaired slabs.

TEST RESULTS and DISCUSSION

Unlike bonded pre-stressed or plain reinforced members, the pre-stressing strands in the un-bonded post-tensioned members did not reach their ultimate strength. As the slabs started to crack heavily, the stress in the mild steel started to increase and became close to the yield stresses. Ultimate loads of the repaired test specimens, strengthened with CFRP composites, were always higher than the ultimate loads of the control specimens. Test results and comparisons are given in the following tables.

The failure loads calculated for the specimens are summarized in Table 3. Good agreement was reached between the measured loads and the estimated loads of control specimens calculated by Yield Line Theory. However, the measured collapse loads of all repaired test specimens were lower than the loads estimated using yield line theory (at the declared ultimate load of slab CFRP did not reach its ultimate strength). The test result confirmed that the analysis of repaired slabs by yield line theory was not conservative. The critical yield line pattern developed in the control specimens could not propagated in the repaired specimens because of the presence of the CFRP across the crack lines. At the time the test was stopped for the repaired specimens, the slabs were in a semi-elastic condition. Clear rigid plate movement along the crack line could not happen. Most likely severe crushing would have happened at the bottom surface along the Yield Lines if the load was further increased (which was not done for safety

reasons). However, in all cases ultimate loads of the repaired specimens were considerably larger than the corresponding ultimate loads of the control specimens. It was observed that effective placement of CFRP (as in PTS-2CR) can further increase the load carrying capacity.

DEFLECTION and CRACKING of CONTROL SPECIMENS and STRENGTHENED SLABS

In general, the control slabs behaved linearly during the first stages of load. Once the slabs cracked the load-deflection curve showed bi-linear behavior, Figure 7 to 12. The second stage of linear behavior of the cracked elastic slab continued until the yielding of bonded non-pre-stressed mild steel began. As the flexural tensile cracks increased in number and width, the load-deflection curves started to show tri-linearity and further slope reduction (not seen in the repaired slabs). In all cases the repaired slabs were stiffer and they had less deflection. As expected, diagonal cracking modes were observed in the two-way control slabs (PTS-1, PTS-2 and PTS-6), figure-2 and photograph-8. Straight line cracks were observed in all of the one way slabs at high moment points (PTS-3, PTS-4 and PTS-5) figure5, 6 and Photograph-10. In general, cracks were not visible on the top surface of the repaired slabs during testing because of the presence of CFRP. In some of the exposed areas of the repaired slabs, the old cracks opened up by insignificant amount. The ultimate loads of control and repaired specimens are given in Table-3. The deflection values of control specimens and repaired specimens at a specific load are shown in Table-4.

CONCLUSIONS AND COMMENTS

1. The post-tensioned concrete slabs, repaired with CFRP fabric (bonded to the tension surfaces), gained considerable strength. Flexural capacity of the slabs strengthened with CFRP composite materials increased by 42%, 88% and 55% for the two-way slabs PTS-1CR, PTS-2CR and PTS-6CR respectively and 68%, 46% and 44 % for the one-way slabs PTS-3CR, PTS-4CR and PTS-5CR respectively.
2. The measured deflection ratios of the control post-tensioned slabs and repaired slabs, at a specific load indicate considerable increase in stiffness for the repaired slabs. The deflection ratios reduced further as the loads were increased more.
3. Slabs repaired with effective CFRP configuration showed larger load carrying capacities. The placement configuration was most effective when the CFRP fibers were perpendicular to the crack lines.
4. Strengthening of the slabs with the CFRP reduced the crack widths at the high moment region at all load level. These slabs were stiffer than the control specimens even at a much higher load. Yield line theory used to calculate the collapse load of the control specimens provided good correlations with test results. For the repaired slabs, estimated ultimate loads obtained by using Yield Line Theory, were larger than that were obtained by measurement. Additional testing would be necessary to further investigate this item.
5. Test results indicated better serviceability conditions for the repaired slabs. Flexural capacity of the slabs significantly increased due to the presence of the CFRP sheets. No de-bonding of the CFRP sheets was observed at the ultimate loading stage.
6. Large increases in flexural strengths were observed for the repaired one-way post-tensioned concrete slabs as indicated in 1). However the results indicate that different end support conditions of one-way slabs also caused variations in performance.
7. As a final statement it can be said that both serviceability and flexural strength improved considerably in all of the slabs repaired with CFRP.

RECOMMENDATIONS

Further testing of un-bonded post-tensioned slabs repaired with FRP is necessary. The suggested variables are: geometry, continuity, repair of the top part of the slab (possibly with FRP rods inside grooves) and boundary conditions. Repaired slabs may be taken to the total collapse stage, if possible, to realize full capacity. Different CFRP placement configurations must be considered and optimum configurations are to be identified. Additional safety precautions must be taken in the tests of un-bonded post-tensioned slabs to avoid mishaps (because the strands have to remain un-bonded).

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Table 1- Summary of Reinforcement Parameters and Concrete Strengths **

Specimens	Pre-stressing Steel (A_{ps}), in ² /in	Mild Steel in Tensile zone (A_s), in ² /in	Mild Steel in Compression zone (A_s), in ² /in	f'_c (psi)
PTS-1	16-1/4" Strands each way (0.00554)	0.0100 (4×4-4×4)	0.0024 (6×6-10×10)	5931
PTS-2	16-1/4" Strands each way (0.00554)	0.0024 (6×6-10×10)	0.0024 (6×6-10×10)	5963
PTS-3	16-1/4" Strands (0.00554)	0.0265 (25-#3 at top and 9-#3 at bottom at each end)	0.0024 (6×6-10×10)	5726
PTS-4	16-1/4" Strands (0.00554)	0.0276 (26-#3 at top and 9-#3 at bottom at each end)	0.0024 (6×6-10×10)	5959
PTS-5	16-1/4" Strands (0.00554)	0.0287 (27-#3 at top and 9-#3 at bottom at each end)	0.0024 (6×6-10×10)	5362
PTS-6	16-1/4" Strands each way (0.00554)	0.0024 (6×6-10×10)	0.0024 (6×6-10×10)	5223

(** 1in=2.54cm; 1psi=0.0069MPa)

Table 2- Properties for CFRP**

Carbon Fiber	Ultimate Strength (ksi)	Design Strength (ksi)	Tensile Modulus (ksi)
CF130 High Tensile Carbon	620 (slab 1CR, 2CR, 5CR and 6CR)	550	33,000
CF160 High Modulus Carbon	620 (slab 3CR and 4CR)	550	33,000

* The information above was obtained from *Manufacturer's Composite Strengthening System Design Guide*.
CF160 (7.14 k/in) has twice the thickness of CF130 (3.57 k/in).

(*1in=2.54cm; 1psi=0.0069MP)

Table 3- First-Crack Loads -Ultimate Loads and Comparisons **

Specimens	First-crack loads (Measured) q_1 (psi)	Ultimate loads (Measured) q_u (psi)	q_u/q_1	Ultimate loads (Repaired) / Ultimate loads (Control)
PTS-1	6.35	6.50	1.02	-
PTS-2	4.80	5.20	1.08	-
PTS-3	5.65	6.50	1.15	-
PTS-4	4.00	5.40	1.35	-
PTS-5	3.50	5.20	1.49	-
PTS-6	3.50	5.50	1.57	-
PTS-1CR	5.50	9.20	1.67	9.2/6.5 = 1.42
PTS-2CR	5.20	9.80	1.88	9.8/5.2 = 1.88
PTS-3CR	6.70	10.90	1.36	10.9/6.5 = 1.68
PTS-4CR	4.90	7.90	1.61	7.9/5.4 = 1.46
PTS-5CR	4.40	7.48	1.70	7.48/5.2 = 1.44
PTS-6CR	6.20	8.50	1.37	8.5/5.5 = 1.55

(**1in = 2.54cm; 1psi = 0.0069MPa)

Table 4 - Comparison of Measured Deflections

at Center of Slabs at 5.2 psi **

Specimens	Ext. loads q (psi)	Δ repaired / Δ control
PTS-1	5.2	1
PTS-1CR	5.2	0.79
PTS-2	5.2	1
PTS-2CR	5.2	0.49
PTS-3	5.2	1
PTS-3CR	5.2	0.70
PTS-4	5.2	1
PTS-4CR	5.2	0.76
PTS-5	5.2	1
PTS-5CR	5.2	0.40
PTS-6	5.2	1
PTS-6CR	5.2	0.52

(**1in = 2.54cm; 1psi = 0.0069MPa)

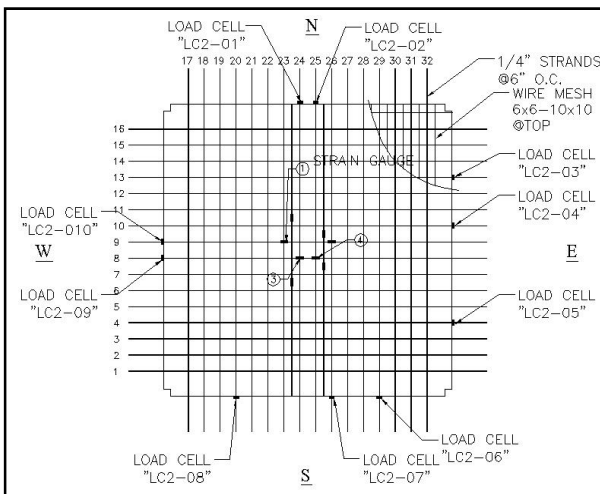


Figure 1- Typical Placement of Strands & Wire Mesh
(1 inch = 2.54 cm)

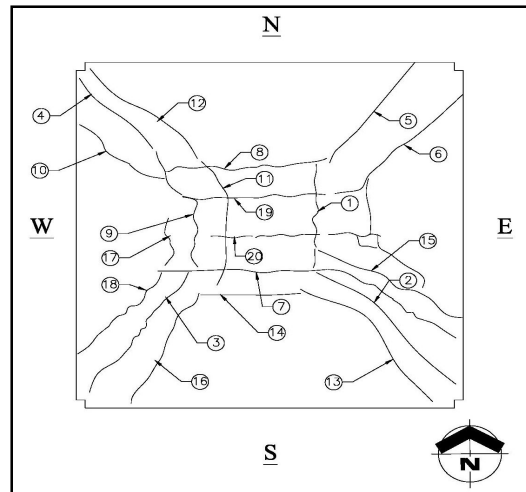


Figure 2- Cracking of PTS-1 (PTS2 & PTS6 similar)
(1 inch = 2.54 cm)

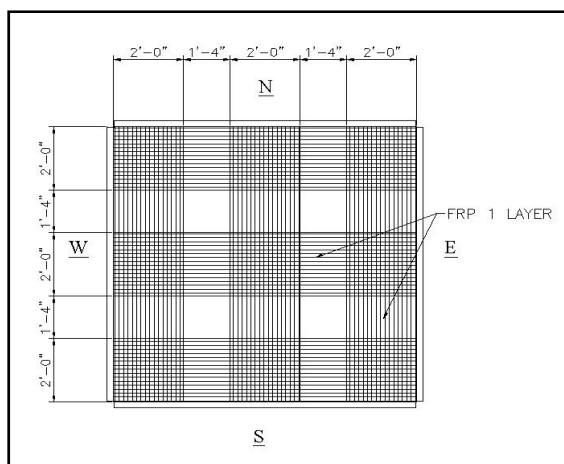


Figure 3- Orthogonal CFRP placement (PTS-1CR)
(1 inch = 2.54 cm)

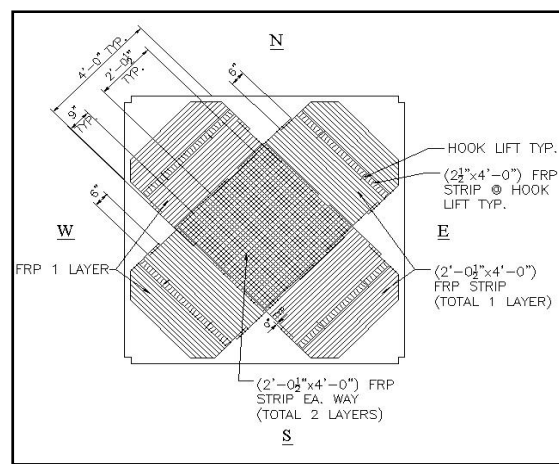


Figure 4- Diagonal CFRP Placement (PTS-2CR)
(1 inch = 2.54 cm)

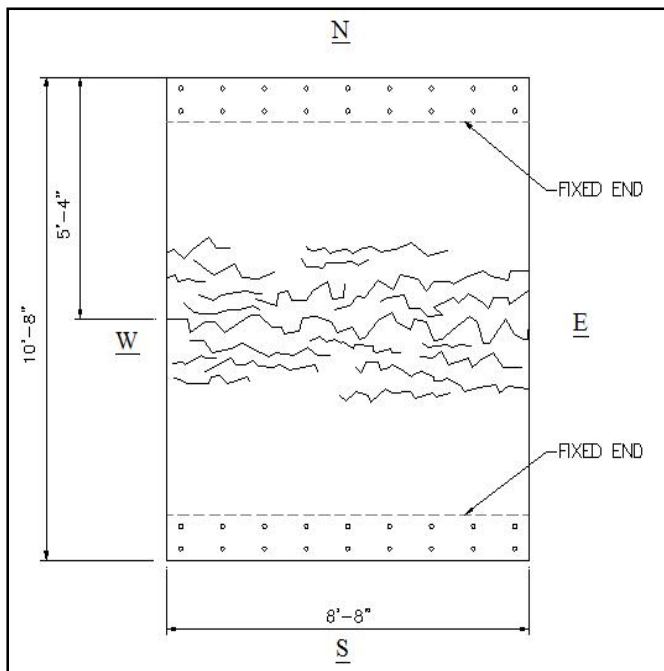


Figure 5 - Crack Line in Control Specimen PTS-3
(1inch = 2.54 cm)

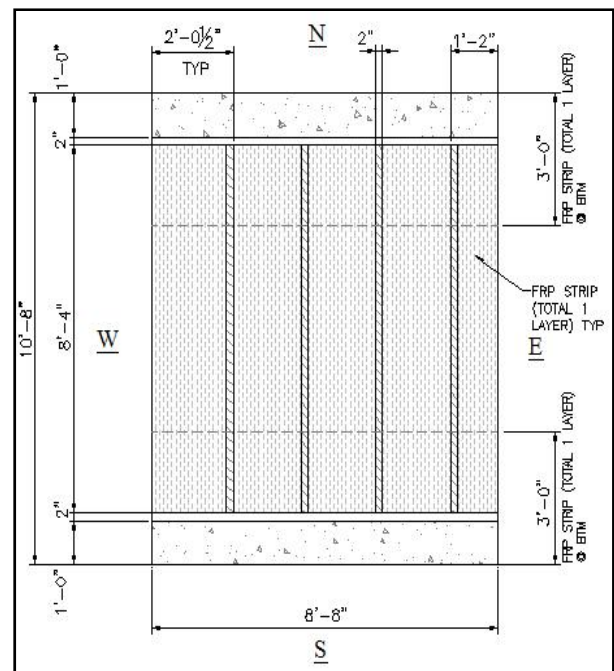


Figure 6 - CFRP Placement on PTS-3CR
(1 inch = 2.54 cm)

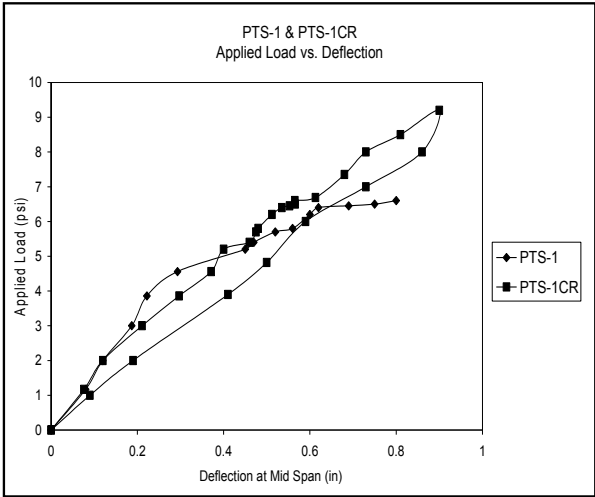


Figure 7 - Distributed pressure load – displacement PTS-1 vs. PTS-1CR (1in = 2.54cm; 1psi = 0.0069MPa)

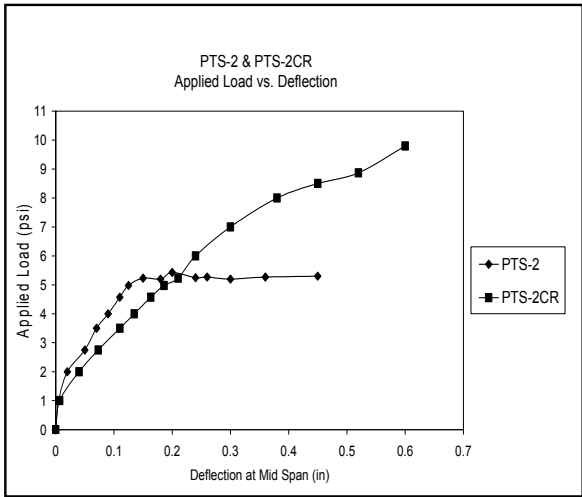


Figure 8 - Distributed pressure load – displacement PTS-2 vs. PTS-2CR (1in = 2.54cm; 1psi = 0.0069MPa)

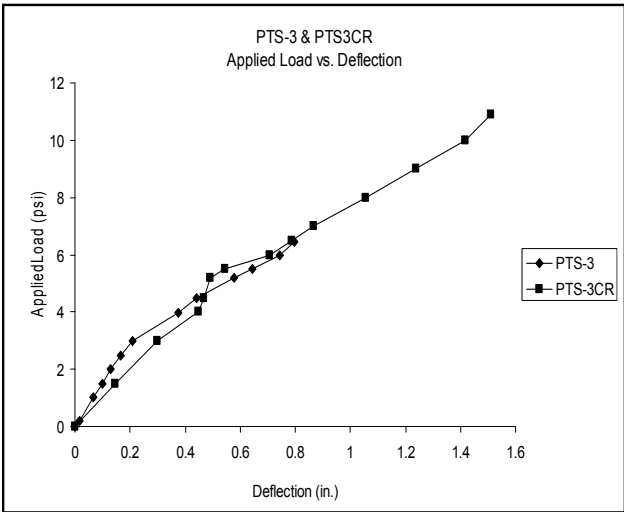


Figure 9 - Distributed pressure load – displacement PTS-3 vs. PTS-3CR (1in=2.54cm; 1psi=0.0069MPa)

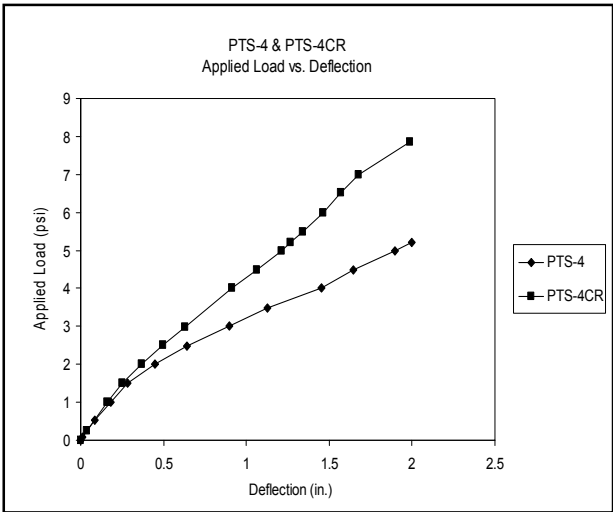


Figure 10- Distributed pressure load – displacement PTS-4 vs. PTS-4CR (1in=2.54cm; 1psi=0.0069MPa)

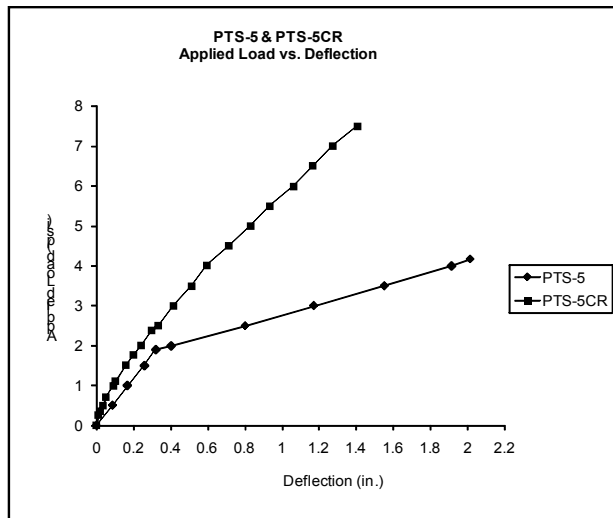


Figure 11 - Distributed pressure load – displacement
PTS-5 vs. PTS-5CR (1in = 2.54cm; 1psi = 0.0069MPa)

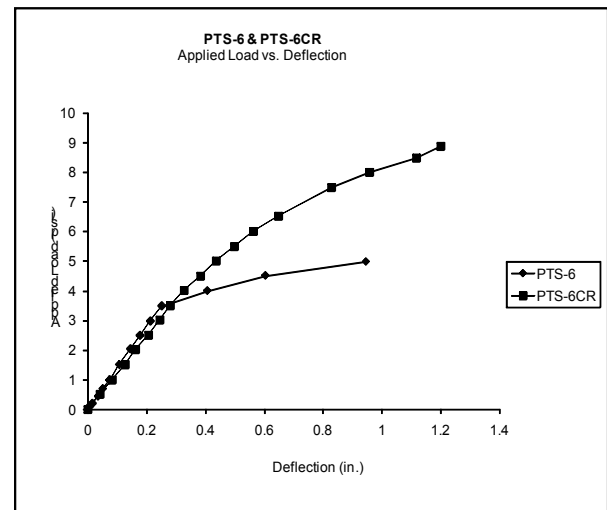


Figure 12- Distributed pressure load – displacement
PTS-6 vs. PTS-6CR (1in = 2.54cm; 1psi = 0.0069MPa)



Photo1- Two-way Post-Tensioned Lift-Slab
Long Beach, CA – mid 1950's



Photo 2 - Parking Structure (Un-bonded PT one-way Slab) Hempstead, Long Island, NY -1968



Photo 3- Hi-Rise (38-story) Un-bonded Post-Tensioned Flat Plate with drop head - 3rd Ave, NY, NY, 1969



Photo 4- Un-bonded Post-Tensioned (Banded System) Flat Plate Office Building, Danbury, Conn 1971



Photo 6- Un-bonded Post-Tensioned Flat Plate Mat Foundation, Washington DC, 1969-70



Photo 5- 20-story Office Building- Un-bonded Post-Tensioned Flat Plate (Banded System) and Hotel Building- One-way un-bonded post-tensioned slab, Reston, Virginia 1972



Photo 7- Form- Post-Tensioned-Strands- Reinforcements
(at CSUF)

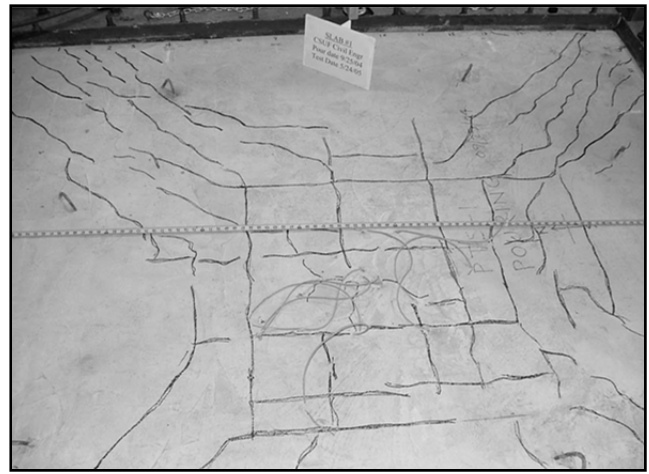


Photo 8- PTS-1(PTS-2&6 similar) Cracking-Ult-Load
(at CSUF)



Photo 9- Diagonal Layout of CFRP Sheets on PTS-2CR
(at CSUF)



Photo 10- PTS-3 Cracking-Ultimate Load
(at CSUF)