

Reliability Basis for Some Mexican Codes

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Synopsis: A strategy is suggested for the evolution of building codes towards their ideal format; reliability optimization. Due to practical constraints the forthcoming generations of codes will probably be formulated in terms of more restricted concepts such as central safety factors or prescribed reliabilities. Explicit applications of optimization criteria are feasible at present as alternate procedures, at least for some special problems. Derivation of simplified methods for evaluation of the reliability of structural assemblages is suggested with the double aim of implementing a central-safety factor code and of setting the stage for the more advanced generations of codes to come.

Keywords: beams (supports); building codes; columns (supports); continuous beams; earthquake resistant structures; economics; failure; frames; loads (forces); optimization; probability theory; reinforced concrete; reliability; safety factor; statistical analysis; strength; structural analysis; structural design.

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Evolution of building codes and of similar documents in Mexico -- and, it would seem, in other countries as well -- is haphazard and inefficient for lack of a well defined goal: we do not know how the ideal code should look. It is time we attempted formulating the ideal. If we succeed we shall have a rational basis for proposing and scheduling the research, education and discussion needed to propose a sequence of changes in our building codes.

There are several reasons why existing codes do not constitute the ideal. The most obvious is that the safety factors, allowable stresses, seismic coefficients and other basic numbers contained in these codes have been established intuitively. There might be room for believing that, despite their dubious origin, these numbers happen to be almost what they should, they happen to lead to near optimum designs because of the power of engineering judgment. Any doubts that this could be the case are quickly dispelled as soon as we look at the coefficients' insensitivity to variables that are certainly important. Thus, our codes call for the same load factor in cantilever slabs and beams as in continuous beams while experience suggests that the former should be designed for higher load factors and an elementary reliability analysis confirms this contention, particularly when we take into account the usual discrepancies in effective depth for top steel between structural drawings

and reality. Ground-story columns are designed to have the same probability of failure as penthouse columns. Other deficiencies stem from inadequate knowledge of the probability distributions of loads and actions on structures in general, of structural responses, of criteria of failure and of the consequences of failure. The situation stands in sharp contrast with recent advances in structural analysis.

A comparison of current Mexican codes and design manuals (1,2) with those of other countries shows that the former come slightly closer to the ideal documents in certain aspects despite their being more backward in others. Relatively advanced provisions include those concerning the variability of unit weights and of the discrepancies between nominal and actual dimensions of structural and nonstructural elements; design live loads as functions of tributary area; the use of "accidental" eccentricities, to be added to the computed values, for longitudinal forces in the design ^{of} columns, for wind forces and for story shears due to earthquake effects on buildings; dynamic magnification factors to be applied to statically computed seismic-shear eccentricities; reductions in some nominal cross-sectional dimensions for design of reinforced concrete members, and explicit recognition of the variability of concrete strength, as a function of the type of mix control, for establishing stress reduction factors. Backward provisions are not worth mentioning.

Despite lack of definition of the ideal code the foregoing assets of Mexican codes were probably the outcome of a vague and fragmentary image of what that document should be. A defense of those features in existing codes would be based mostly on arguments that have served to justify the use of split factors in American and European codes, plus some reasoning that has already found its way into the literature(3,4) and hence would serve little purpose here. Rather, the present paper will aim at describing and discussing what the writers visualize as the ideal code, at suggesting one way in which pres-

ent codes could evolve into the ideal and at illustrating various provisions that the intermediary building codes would contain.

TOWARD THE IDEAL BUILDING CODE

What should be the purpose of building codes? -- The usual answer is "To protect public safety; soundness of investment in buildings is entirely the owner's concern." Out of oversimplification this view is quite erroneous; it does not even reflect the contents of present building codes; witness the inclusion of clauses that refer to serviceability. Clearly the main purpose of building codes should be to guard the interests of society, which are much wider than a fixed level of safety. Soundness of investments does concern society and hence should be reflected in building codes.

Even if we assumed that codes should do no more than protect public safety, the allowable level of safety would remain a moot question. Within reasonable bounds we cannot design an absolutely safe structure, one with zero probability of failure. Now, under stochastically stationary conditions, if the probability of failure is finite over any finite period of time, it is one over an infinitely long time span. It follows that every structure must fail unless it is demolished. Under the circumstances allowable levels of safety are meaningful only in the context of optimization.

On the other hand it may seem objectionable that a code should prohibit the construction of excessively safe or luxurious buildings. It might be argued that the investor should be free to spend his capital as he wishes so long as safety is not excessively endangered. If the argument were valid, codes should not contain provisions tending to limit such matters as deflections and crack widths. Capitalist countries do protect their economies by various means: they forbid the exodus of capital or the entrance of small tomatoes; they may and should, equally, try to prevent the burying of capital. It would be consistent with accepted practices, therefore, to set limits pre-

venting the construction of overly safe or unnecessarily expensive buildings.

From the viewpoint of society a convenient objective function to maximize equals the expected present value of the benefits derived from the existence of the building minus the initial cost and the cost of damage, maintenance and failure. As a function of a design parameter, x , the benefits may vary as shown in Fig. 1. (The slow decrease with x is realistic when this parameter is, say, the diameter of reinforced concrete columns, since an increase in column size brings about a reduction in rentable area and perhaps in rentability. The benefits are practically independent of many other design parameters, such as concrete strength or steel yield stress.) As illustrated, we must subtract from the benefits the initial cost and the expected present values of the losses.

The figure also shows, by means of dashed lines, the objective function and its components, as functions of x , for a hypothetical owner. Conceivably the benefits derived from the structure's existence may be the same for the owner as for society, since in a free market the benefits derived from occupying the building are measured by the rent that the public is willing to pay therefor, and the owner will usually try to make this rent as high as possible. Conceivably, also, the initial cost may be the same for both subjects, society and owner, when that is relatively small. It is worth appreciably more to the owner when this cost is so high as to approach his total capital (including his fortune, productive power and credit worth) because of the typical nonlinear relation between money and utility. The losses due to failure may, however, be worth much less to the owner than to society if failure is not expected at an early age of the building, since the owner may contemplate an early sale and, when he does not, there remains the fact that his expected lifespan is comparatively short. Hence, the owner's objective function may attain its maximum for a smaller x than for society. Alternatively the oppo-

site may be true, as shown by the dotted curve, if the owner has a preference for the monumental or a special aversion toward the possibility of failure.

Even when the design may not be optimal the structure is worth erecting from society's or the owner's viewpoints if and only if the corresponding objective function is positive. Hence, the building code should forbid the construction of buildings outside of this range.

It may happen, as in the figure, that there is an overlap of values of x within which it is worth erecting the building both to society and to the owner. Because of the latter's freedom the tendency will be to make the design approach his optimal solution. A compromise decision may, however, be reached owing to the engineer's and architect's divided loyalties, toward society and to their client.

The owner is a member of society and the potential satisfaction of his whims should have an influence on the shape of society's objective function. However, the building under consideration will be judged by at least thousands of the owner's contemporaries and by his and their descendants. Taking this judgment as basis for decision(5), the weight of the owner's preferences becomes negligible insofar as society's objective function is concerned, although those preferences are decisive in shaping the owner's objective function.

What we have said about a single design parameter applies equally to the rest of the structural parameters that fall within the designer's control. In general, then, rather than a plane graph, representation would demand a hyperspace; and many of the relations would not fit the smooth types of curves illustrated in the figure but would be jagged and discontinuous. The essence of the present arguments, however, is not affected thereby.

Negative values of the objective function mean that investment in other operations is more advisable than erection of the building being contemplated.

A poor architectural solution, an inefficient structural setup, the un-

wise use of luxurious materials or of structurally inadequate materials or the adoption of a building shape that is objectionable to the urban community may lead to a societal objective function that is nowhere positive, such as curve 1 in Fig. 2; that building should not be built. Or it may lead to a function similar to curve 2, which allows only a small tolerance about society's optimal design. The opposite situation may lead to a function such as curve 3: benefits to society are so great over such a wide range of the design parameter in question that the owner should^{be} allowed to build the structure practically as safe or unsafe as he wishes.

Judgment about these and all other pertinent questions affecting society's objective function should ideally be settled as a consequence of public debate. Such a procedure would, however, be totally impractical. The only operative way to achieve comparable results is the setting up of an interdisciplinary committee appointed by popular election. Its sentences would be sanctioned by the possibility of reelection or removal and by public debate of its basic criteria. In the interest of efficiency the committee would pass judgment only on sufficiently important projects and on marginal instances of lesser projects; most of the latter would be approved or rejected on the basis of general rules.

In brief, then, a building code should include criteria for constructing society's objective function taking into account the most obvious variables that govern benefits, initial cost and expected losses. A committee modifies this curve as the result of its interpretation of the utility of the project to present and future generations. As a result, when there may be net positive utility, a set of recommended values of the design parameters is issued together with a set of allowable extreme values of these parameters. If there is an overlap with the range that is convenient to the owner, a compromise solution, near optimal for both owner and society, is finally produced by the

engineer and the architect.

Up to this point, mention has been made only of the function of building codes in norming some of the relations between society, the owner and the designer. It is also proper that the same documents norm some relations between owner and builder, at least insofar as these relations may influence the benefits to society. To a limited extent this is done in present codes when they are made to contain clauses that refer to construction tolerances in geometry and in mechanical properties of materials. Codes as a rule are deficient in stipulating what should be done when such tolerances are not met. Rational bases are apparently lacking for the fixing of the tolerances and of the actions that should be forthcoming when the tolerances are violated. The problem can be understood by devoting attention first to these actions. The most elementary and drastic measures that are ordinarily taken consist in either condoning, strengthening, or demolishing and rebuilding. These actions are sometimes self defeating or wasteful to owner, builder and society. A first approximation to a rational approach can be based on the premise that the utility to the owner should not be affected by differences between the design and the building as built. If the contractor behaves then so as to maximize his own utility, the utility of society will be approximately maximized.

From the appearance of things it seems that neither authorities nor investors nor perhaps professionals are ready for the ideal type of code. Evolution is essential for its eventual production and acceptance. But it is equally essential that the goal of the evolutionary process be at least cursorily outlined, as has been done here. This permits computing a bonus that the builder should receive from the owner, or a penalty that should be applied, as a function of how well the specified strengths and geometrical properties were met. The approach has been developed for concrete strength under the assumption that the disturbances are the combination of time independent, ran-

dom-magnitudo loads and a generalized Poisson process, such as may correspond to the combination of gravity forces and earthquake or wind disturbances (6).

A more realistic and ambitious formulation would take into consideration changes in utility to society, as brought about by discrepancies between construction drawings and the actual structure as well as the effects of stimuli, such as bonuses and penalties to the contractor. The specification of tolerances would result from the calculation of the economic advantages in accountancy derived from the omission of bonuses and penalties when the discrepancies were sufficiently small; requirement for strengthening, or demolishing and rebuilding as the case may be, would be the natural outcome of extremely high penalties.

A strategy must now be devised to design the evolutionary process toward the ideal code. In so doing we should keep in mind the educational function of codes. This comes about through the need which everyone has to study in order to apply the code as well as through its incorporation in text books and in the didactic material of courses in the undergraduate, graduate and continuing-education curricula. But the steps from one code to the next should not be so great as to demand a discouragingly high effort. And a successful evolution requires that design aids be available when a new code is adopted. The fact that not all engineers adapt at the same rate makes it most desirable to stipulate alternative procedures in building codes in such a way that the most advanced criteria in one edition become the least advanced in the next version of the code. Experience with the introduction of ultimate strength is eloquent in this respect and suggests the most appropriate rate of innovation in each country.

Whatever format is adopted as a first step in the evolutionary process we shall not escape the need to specify nominal or characteristic values of loads and material strengths, and indeed the ideal code ought to contain

changes to this effect. This is a consequence of the function of codes in regulating relations between owner, designer and builder, or of establishing bases for fixing responsibilities, which in itself may be desirable, and it is most useful as a means toward optimization for society. However, the nominal values constitute no more than point estimates of the variates, and formulating design expressions and optimization criteria in terms of those values in addition to other parameters of the distribution leads to unnecessarily complicated expressions. This is already apparent in Cornell's second-moment format (7) despite its not dealing with optimization. Hence, it is advisable to keep both types of variate characterization separate; one of these should use nominal values to establish responsibilities, bonuses and penalties, while the other should be expressed in terms of expected values and standard deviations or coefficients of variation so as to produce sufficiently simple expressions.

There is a second advantage in the use of nominal values of which one would not like to be deprived. This refers to their place in defining the most significant portion of the probability distributions of interest, as nominal values lie near the lower tail of strengths and near the upper tail of loads. The traditional statistical criteria of parameter estimation tend, in some sense, to minimize the discrepancies between the actual probability distribution and the one derived from the estimated parameters throughout a wide, central range of the variable in question. For design purposes one would like to give greater weight to the fit in the range of exceptionally low strengths and in that of exceptionally high loads. A.L.L. Baker's way of computing the standard deviation, in which he only takes into account strengths smaller than the mean (8), is an intuitive step in this direction. The same is true of code requirements and specifications worded in terms of "minimum guaranteed" or nominal strengths. The most desirable criterion for estimation of preba-

bility distributions is that which minimizes losses due to discrepancies with the actual distribution; in other words, it is an optimization criterion. Such a criterion can be developed within a Bayesian framework.

Independently of how the parameters of the probability distributions are estimated, there remains the advantage of dealing with central rather than nominal values in formulating design and optimization expressions. This will still be the case when the time comes to use probability distributions requiring more than two parameters for their characterization.

STRATEGY FOR EVOLUTION

It seems wise to adopt the following order in the development of a sequence of codes ever approaching the ideal format.

1. A first-order second-moment format, much like the one advanced by Cornell but using central load factors and not necessarily tied to a distribution-free formulation but in some cases adopting specific families of probability distributions for loads and resistances. This step would introduce the option of varying the load factors as a function of the uncertainties in the parameters involved and the importance of the structure and its cost sensitivity to conservatism in design. Yet, it would deal implicitly only with member reliability or critical-section reliability.
2. An explicit treatment of reliabilities opening the door for the calculation of structural reliability and recognition of the influence of such matters as statical indeterminacy on the probability of failure. This would promote familiarity with probabilities of failure.
3. Open dealing with initial costs and consequences of failure, permitting the introduction of an optimization format with all the characteristics described in the foregoing section.

The first step would be taken retaining the present formats as alterna-

tive criteria of design. Each additional generation of codes would retain the preceding format as alternative. In the process one would expect theoretical advances in the prediction of structural behavior the implications of which would be incorporated in successive editions of the code. Moreover, each step would be taken simultaneously with the publication of design aids such as tables, graphs and computer programs.

We are now witnessing the initial stages of this evolutionary process in Mexico.

THE FIRST STEP: FIRST-ORDER SECOND-MOMENT FORMAT

The Format

Cornell's proposed format can be succinctly described as follows.

Means and coefficients of variation of loads and strengths are defined so as to include statistical deviations in material strengths and in load intensities and reflect other sources of uncertainty as well. Control and construction inaccuracies, errors in strength prediction formulas and imperfect analysis and modeling of load actions and structural responses are explicitly incorporated in the evaluation of these coefficients of variation. Nominal or characteristic values of load intensities, material strengths and member dimensions are adopted as working variables. In the process of code writing, a distribution-free approximate reliability measure that depends only on means and variances of loads and strengths is expressed in terms of central safety factors. Approximate expressions based on a suggestion by Lind (9) are then used to derive split safety factors in terms of nominal values, and nominal values of functions of several random variables in terms of the nominal values of these variables.

Ang and Amin (10) have made a strong case for a format that uses nominal strength and load values in conjunction with an "ignorance factor." This format is appealing because the safety factor is relatively insensitive to the

probability distributions of the variates, given a probability of failure. A similar approach is used in Mexico's existing codes. Nevertheless it has not been adopted in their revision owing mainly to two reasons. First, as was pointed out, use of nominal values for design is objectionable because it gives rise to a complicated treatment of uncertainties in loads, geometry, material strength, methods of analysis and design criteria and because those values are expected to do two things: serve as basis for design and guide in the application of legal sanctions; since the most appropriate values for one purpose are not necessarily the most adequate for the other, adjustments and concessions generally have to be made. Second, the ignorance factor is not connected in a simple manner with probabilities of failure; consequently its use does not lend itself to overt treatments of structural-assemblage reliability and of optimization, which should appear in subsequent code versions.

Thought has also been given to a variation of the split-factor formats, based on a suggestion by A.L.L. Baker (8). Suppose that all quantities governing stability were known deterministically except strength. For a given probability of failure we would replace every quantity with its actual value, save strength, which we would divide by an appropriate safety factor, say γ_R . Now suppose that only the loads were uncertain. To attain the same probability of failure we would replace all variables with their actual values, save the load, which we would multiply by an appropriate load factor, say γ_L . If, finally, both strength and load were uncertain, each with the same probability distribution as in the corresponding case of the two we have referred to, we would design for the same failure probability as in the foregoing cases by using an overall safety factor γ , which approximately satisfies the expression

$$\gamma - 1 = (\gamma_R - 1) + (\gamma_L - 1) \quad (1)$$

At least for failure probabilities not much smaller than 10^{-5} and with coefficients of variation of loads and strengths of up to 0.4, this criterion

yields fairly accurate results, usually on the conservative side, as shown in Fig. 3.

Owing mostly to the difficulties posed by this format when attempting to deal explicitly with the reliability of structural assemblages and with optimization, it too was rejected as basis for the first step in the evolutionary process.

Our proposed first step lies closer to Cornell's format than to any other, but the central safety factor has been chosen as the design parameter. It is specified in terms of the coefficients of variation mentioned and of a parameter that depends on the desired safety level. Since no nominal loads and strengths have to be defined, writing of consistent safety specifications is relatively simple. Still, as in more conventional approaches, tolerances and quality control procedures deserve special attention, aimed at assigning clearly defined professional responsibilities. The few control tests that are feasible in most practical cases do not suffice to evaluate means and variances (much less density functions) of the corresponding variables with sufficient accuracy. Hence, minimum values of strengths, or of average strengths of a number of specimens, should be specified in such a manner as to minimize the consequences of errors in the estimation of the parameters mentioned.

Let R and S denote the strength and the load action at a critical section, respectively. Let $Y = \ln R - \ln S$ and suppose that safety is specified in such a way that the mean value of Y is at least β times its standard deviation. Then,

$$E(Y) = E(\ln R) - E(\ln S) = \beta \sqrt{\text{var } \ln R + \text{var } \ln S} \quad (2)$$

If we adopt a first order approximation for means and variances (11), there results the following expression for the minimum allowable central safety factor.

$$n = \frac{E(R)}{E(S)} = \exp(\beta \sqrt{V_R^2 + V_S^2}) \quad (3)$$

Here V_R and V_S are the coefficients of variation of strength and load, respectively.

Equation 3 can also be obtained as an approximation to the solution in the case that loads and strengths have lognormal distributions; the exact expression for n would then be

$$n = \sqrt{\frac{1 + V_R^2}{1 + V_S^2}} \exp \left[\beta \sqrt{\ln[(1 + V_R^2)(1 + V_S^2)]} \right] \quad (4)$$

where β is the argument of the normal complementary distribution function that corresponds to the specified failure probability. The first factor is almost equal to 1, $\ln(1 + V_S^2) \approx V_S^2$ and $\ln(1 + V_R^2) \approx V_R^2$, since V_R and V_S are much smaller than 1. Hence Eq. 2.

Figure 4 shows the variation of the safety factors computed through this equation and under other assumptions regarding the statistical distributions of strengths and loads for several coefficients of variation and failure probabilities ordinarily accepted and for coefficients of variation of loads and strengths not much greater than 0.3. Eq. 3 provides safety factors in agreement with those derived from several reasonable assumptions concerning the families of the distribution functions of the variables in question. Only the curves corresponding to normal distributions of loads and strengths depart from the general trend and from Eq. 3.

In some cases the applicability of given families of statistical distributions of loads and strengths may be recognized and specified by the code. Failure probability charts as that in Fig. 5 may then be supplied, with the recommendation that the central safety factor be obtained by equating the corresponding failure probability to that implicit in Eq. 3, which in the range $2 < \beta < 6$ is given approximately by the following equation.

$$\beta_F = 460 \exp(-4.3 \beta) \quad (5)$$

In its simplest conception, a code framed in this format would ignore,

perhaps with some exceptions, the relationship between the reliabilities of individual sections and that of the structural system. For highly indeterminate structures having several members in parallel it is usually conservative to take the overall probability of failure equal to the highest individual-section failure probability. The criterion is, however, not always conservative. For example, the probability of failure of a statically determinate system is higher than the maximum individual-member failure probability. Hence, the allowable individual-section or individual-member failure probabilities in a code of this type should be fixed in such a way that they lead to acceptable designs for the most unfavorable difference to be expected in practice between individual and overall probabilities of failure. Alternatively, the safety parameter β should be made to depend not only on the importance of the structure and the consequences of its failure but also on the type and location of the member and on its influence on safety against the most significant failure modes.

Approximate procedures for evaluating system reliabilities from those of members, as exemplified below and in another paper by the writers (11), are probably worth introducing during the first step in the proposed evolution of building codes.

Second-Moment Theory For Loads

Cornell and Peir (12) suggest that the combined action of dead and live loads at a critical section be expressed as

$$S_0 = \phi_{AO}(\phi_{DO}S_{DO}' + \phi_{LO}S_{LO}') \quad (6)$$

where S_{DO}' and S_{LO}' are respectively the computed effects of dead and live load, ϕ_{AO} is a random factor that accounts for inaccuracies of structural analysis, while ϕ_{DO} and ϕ_{LO} , also random, take care of discrepancies between the effects of actual loads and those of their idealized representation. The computed values S_{DO}' and S_{LO}' are obtained from

$$S_{i0}^I = w_{0i} \int \alpha_i f_0 \, dA, \quad i = D, L \quad (7)$$

where $w_{0i}\alpha_i$ is the load intensity and f_0 is the influence function for S_{i0}^I . The intensities of dead and live load are taken as stochastically independent.

The statistical correlation between the values of either variable at different locations is approximately taken into account by assuming $w_i = \alpha_i w_{0i}$, where α_i and w_{0i} are independent variables, and only the former is a function of location.

For practical applications, all the variability in dead load may be ascribed to w_{0D} . When dealing with live load, one must specify the distribution of its intensity at any instant, from which ideally its maximum effects during a given time period should be computed through application of extreme-value statistics, reasonable assumptions on the number of live load applications or increments, and on the correlation between the corresponding intensities. Alternatively the distribution of a conventional maximum probable intensity may be specified, without making reference to a time period, or a set of simple rules may be provided for passing from the distribution of the instantaneous value of the load effect to that of its maximum probable value.

The mean value of S_{i0}^I is

$$\bar{S}_{i0}^I = \bar{w}_{0i} \int f_0 \, dA \quad (8)$$

and its coefficient of variation, $V(S_{i0}^I)$, may be obtained from the following approximate expression,

$$V(S_{i0}^I) = \left[\frac{V^2(\alpha_i)}{A} u^2 + V^2(w_{0i}) \right]^{1/2} \quad (9)$$

where A is the tributary area for the load action of interest, and u^2 is the ratio of the mean value of f_0^2 to the square of the mean value of f_0 , both taken throughout area A . For most practical cases where $f_0 > 0$ throughout A , u may be taken equal to 1.2 (13). If it is of interest to determine the variability of the total live load acting on area A , $f_0 = 1$, and Eq. 9 may be applied, with $u = 1$. Mitchell and Woodgate (14) present the results of a survey

on live loads on office buildings in England. Their results may be approximated by Eqs. 8 and 9, with $\bar{w}_{OL} = 65 \text{ kg/m}$, $\alpha_L = 1$, $V(w_{OL}) = 0.45$, and $V(\alpha_L) = 1.4$.

It is advisable to run more extensive field observations of instantaneous and maximum live loads, since there is some contradiction between the few available reports on the subject (14-18).

The probability that any given area have no live load over an appreciable length of time is much greater than the relevant probabilities of failure. Hence, when the influence ordinates are positive over some zones and negative in others, the distribution of the maximum effect^{can}_A be computed with good accuracy under the assumption that the load is acting only on the portions of the tributary area that have the same sign and that the statistical distribution of its intensity at any location is the one which corresponds to its instantaneous value. Then the mean and coefficient of variation of the maximum S'_L are, respectively,

$$\bar{S}'_{LO} = \bar{w}_{OL} \int f_0 \, dA \quad (10)$$

and

$$V(S'_{LO}) = \left[\frac{V^2(\alpha_L) u^2}{\sum A} + V^2(w_{OL}) \right]^{1/2} \quad (11)$$

where the summations cover every portion of tributary area where f_0 has the same sign, and u is replaced with 1.2.

In theaters, stadiums, temples and other gathering places, furniture is permanent or nonexistent. In the former case it may be assimilated to dead load. Live load is made up entirely of the weight of persons and of the dynamic effects produced by movements of the assembly. In this case w_{OL} may be assumed as having two possible values: zero, with probability $1 - p$, and its conditional mean value given that the load is applied, with probability p . Since most of the variability will be associated with w_{OL} and not with α_L , the

coefficients of variation of the instantaneous or maximum effects will be less sensitive to the tributary area than in the cases of office or apartment buildings.

Now consider the combined effects of earthquake, dead, and live loads. It is assumed that the latter is applied just after construction, sampling from the population of its instantaneous values. The following paragraphs deal with the conditional distribution of an action on a critical section under the assumption that an earthquake of intensity y has occurred. In a later section a model is derived for evaluating the reliability function of the structure under a given failure mode.

Let action S be given as the sum of S_O and S_A which are respectively the effects of permanent (including live) and accidental loads. Let ϕ_{AA} , ϕ_{OA} and ϕ_{LA} be defined in the same way as ϕ_{AO} , ϕ_{DO} and ϕ_{LO} but in correspondence with the effects of accidental loads. Thus, ϕ_{AA} accounts for inaccuracies of seismic analysis, and ϕ_{DA} and ϕ_{LA} for those due to dead and live load idealization. As a rule, $\phi_{DO} \neq \phi_{DA}$ and $\phi_{LO} \neq \phi_{LA}$.

The effect of permanent load is given by Eq. 6. That produced by earthquake is

$$S_A = y\phi_{AA}(\phi_{DA}S_{DA}^i + \phi_{LA}S_{LA}^i) \quad (12)$$

where

$$S_{iA}^i = w_{O1} \int \alpha_1 f_A dA \quad (13)$$

and f_A denotes the influence function for the contribution of earthquake effects to the action of interest. For convenience, the resulting action, S , may be expressed as $\sum_i \sum_j S_{ij}$ where

$$S_{ij} = y_1 \phi_{A1} \phi_{j1} w_{Oj} B_{j1}, \quad i = O, A; j = D, L \quad (14)$$

$y_1 = 1$, $y_2 = y$ and $B_{j1} = \int \alpha_j f_1 dA$. The mean of S is

$$\bar{S} = \sum_i \sum_j \bar{y}_i \bar{\phi}_{A1} \bar{\phi}_{j1} \bar{w}_{Oj} \bar{B}_{j1}, \quad i = O, A; j = D, L \quad (15)$$

and its variance is

$$\text{var } S = \sum_i \sum_j \text{cov}(S_i, S_j) \quad (16)$$

where $\text{cov}(S_i, S_j)$ is approximately given by

$$\text{cov}(S_i, S_j) \approx \sum_k \sum_\ell \bar{S}_{ik} \bar{S}_{j\ell} \left[\rho(\phi_{A1}, \phi_{AJ}) V(\phi_{A1}) V(\phi_{AJ}) + \rho(\phi_{k1}, \phi_{\ell j}) V(\phi_{k1}) V(\phi_{\ell j}) \right. \\ \left. + \rho(w_{0k}, w_{0\ell}) V(w_{0k}) V(w_{0\ell}) + \rho(B_{k1}, B_{\ell j}) V(B_{k1}) V(B_{\ell j}) \right] \quad (17)$$

where $\rho(\cdot, \cdot)$ denotes coefficient of correlation and $V(\cdot)$ denotes coefficient of variation.

According to the assumptions stated, the correlation coefficients of all the pairs of values in the second member of Eq. 17 are either zero or one with the exception of $\rho(B_{k1}, B_{\ell j})$. Letting $\mu_{ij}'' = \int f_1 f_j dA$ and $\mu_i' = \int f_1 dA$, this coefficient is zero if $k \neq \ell$, and equals $\mu_{ij}'' / \sqrt{\mu_i' \mu_j'}$ if $k = \ell$.

The coefficient of variation of B_{k1} is

$$V_{B_{k1}} = \frac{V\alpha_k}{A} \frac{\sqrt{\mu_{11}''}}{\mu_1'} \quad (18)$$

For applications to design practice, μ_{0A}'' , μ_{00}'' and μ_{AA}'' must be computed for a set of conditions. When dealing with building frames it would suffice to present curves of those parameters for a few columns and girders at various locations.

Second-Moment Theory For Strengths

In its simplest version, the second-moment format deals separately with strengths in individual failure modes of critical sections. As suggested by Cornell (7), the coefficient of variation V_R accounts for uncertainties in material properties, in construction procedures and in the strength prediction equations. Its computation makes use of first-order approximations for means and variances. This requires the derivation of analytical expressions for the strengths of interest.

Consider a simply reinforced concrete beam and suppose that it can fail only in flexural tension. Its yield moment capacity can be approximated by

the expression

$$M_y = \phi p b d^2 f_y (1 - 0.59 \phi f_y / f'_c) \quad (19)$$

where ϕ is the equation error (a random variable taken as independent of the other variables in Eq. 19), p is the percentage of reinforcement, f_y is the steel yield stress, and b and d are respectively the cross-sectional width and effective depth. According to the first-order theory the coefficient of variation of M_y is equal to $(V_\phi^2 + V_M^2 + V_C^2)^{1/2}$, where V_ϕ is the coefficient of variation of ϕ , associated with errors in strength prediction formulas, and V_M and V_C measure material and construction uncertainties respectively:

$$V_M^2 = \left(\frac{1 - 1.18\bar{q}}{1 - 0.59\bar{q}} \right)^2 V_{F_y}^2 + 0.35 \left(\frac{\bar{q}}{1 - 0.59\bar{q}} \right)^2 V_{f'_c}^2 \quad (20)$$

$$V_C^2 = \left(\frac{1}{1 - 0.59\bar{q}} \right)^2 V_d^2 + 0.35 \left(\frac{\bar{q}}{1 - 0.59\bar{q}} \right)^2 V_b^2 \quad (21)$$

where $q = \phi f_y / f'_c$, $F_y = p b d f_y$, the V 's stand for coefficients of variation and $\bar{q}$ is the expectation of q .

Member or ~~system~~ system resistance to some generalized actions, when failure involves more than one critical section or when more than one failure mode is involved, can also be dealt with within the second-moment format, as shown by the examples that follow.

Consider a span of a continuous beam subjected to distributed dead and live load. Design provisions should intend to control deflections and cracking and to attain adequate safety against failure of the beam as a plastic mechanism. Deflection-control provisions lead to the required values of stiffnesses, and cracking control requirements are expressed partially in terms of the ratio of the expected elastic moment to the expected yield strength at each individual critical section, while safety against mechanism failure has to be expressed in terms of generalized concepts, such as the bending moment at midspan of a simply supported beam; we shall refer to this

generalized force as the isostatic moment.

Figure 6 shows some of the failure modes that would involve formation of a mechanism at a given span. The upper portion deals with the most frequent case when plastic hinges are formed at the ends and at midspan. The generalized strength is denoted by the isostatic moment at failure, M_0 , which is a linear combination of M_l , M_r and M_c , respectively the bending strengths at the left and right supports and at midspan. The lower portion illustrates two cases when the failure mechanism does not involve plastic hinges at one or both span ends. M_l and M_r are smaller than the critical section strengths, M_l^* and M_r^* , since the former are limited respectively by the loads on the cantilever at the left and by the loads and the strength of the span lying to the right of the one under study. At the expense of some preliminary work, it is possible to derive simplified criteria for estimating means and variances (and indeed probability distribution functions) of M_l and M_r , accounting for all relevant failure mechanisms. Yet, in most cases the assumption that plastic hinges form at the ends and center of the span is acceptable. Hence, the treatment that follows is confined to this case.

Let S be the isostatic moment of the acting loads. According to Eqs. 6-8 the mean of S is equal to the isostatic moment computed under the assumption that dead and live loads are equal to their mean values. Also,

$$V^2(S) = V^2(S_{i0}) + \frac{\sum_i [V^2(S_{i0}^2) + V^2(S_{i0}^2)] \bar{S}_{i0}^2}{(\sum_i \bar{S}_{i0}^2)^2}; \quad i = D, L \quad (22)$$

where $V(S_{i0}^2)$ is given by Eq. 9.

Let the generalized strength be denoted by $R = \sum_j a_j N_j$, where $j = l, r, c$, and the a_j 's are deterministic constants. Approximately, $a_l = a_r = 0.5$ and $a_c = 1$. From Eq. 19,

$$R = \sum_j a_j \phi_j A_{sj} f_{y_j} d_j (1 - 0.59q_j) \quad (23)$$

The variability in f_{y_j} is the result of among lot and within lot varia-

tions. Correlation between the corresponding values at pairs of critical sections depends on the relative contributions of the two sources of uncertainty. This may be accounted for if f_{y0} is taken as $\alpha_j f_{y0}$, where both factors are random and mutually independent. It is assumed that $\phi_i \alpha_i d_i$ is independent of $\phi_j \alpha_j d_j$ for $i \neq j$. Let $k_j = M_j/M_0$, and make use of the approximation $\bar{k}_j \approx \bar{M}_j/\bar{M}_0$. The mean and coefficient of variation of R may be obtained from the following equations,

$$\bar{R} = \sum_j \alpha_j \bar{M}_j \quad (24)$$

$$v^2(R) = v^2(f_{j0}) + \frac{1}{(\sum \alpha_j k_j)^2} \left[\sum (\alpha_j^2 k_j^2 (v^2(\phi_j) + v^2(\alpha_j) + v^2(d_j))) \right] \quad (25)$$

Accordingly, second moments of generalized loads and strengths may be used in practical design in order to attain a more consistent safety measure than that resulting from separate analysis of individual critical sections.

SIMPLIFIED RELIABILITY THEORY

We stated that progress toward consistency, made possible by adoption of a second-moment format, can be enhanced through implementation of simple procedures for estimating means, variances and, when possible, probability distributions of generalized system properties, and hence for evaluating reliabilities of structural assemblages. As shown in the sequel, such additions are feasible in the forthcoming generation of codes, even though they require some preliminary committee work, ranging from simulation studies to construction of graphs and design aids.

When dealing with simple systems, having only one failure mode, load and strength can be defined each by a single parameter. Failure probabilities may then be computed as follows,

$$p_F = \int F_R(s) f_S(s) ds \quad (26)$$

Here, F_R , the cumulative probability distribution function of strength, and f_S , the probability density function of load, reflect all sources of uncer-

tainty described above. Applicable families of distributions may be specified for various conditions, and a set of design aids and graphs such as Fig. 5 may be furnished.

Most practical problems involve several failure modes, either at the same or at different critical sections. The corresponding loads and strengths may have different degrees of statistical correlation.

In the general case, the evaluation of member and system reliabilities must account for all significant failure modes. Approximate evaluations of reliability are feasible if R_i and S_j are expressed in the following simplified manner,

$$R_i = A_i R_0 \quad (27)$$

$$S_j = B_j S_0 \quad (28)$$

where R_0 , S_0 , A_i and B_j ($i = 1, 2, \dots, m$; $j = 1, 2, \dots, m$) are all mutually independent. Their means and variances may be obtained if means, variances and covariances of loads and strengths are available (19).

If the cost of failure is independent of the corresponding mode, only the total probability of that event is of interest. Such is the case in statically determinate structures, where m failure modes, all assumed to lead to collapse, are identified in correspondence with the number of critical sections.

Letting $N_0 = R_0/S_0$, $\nu = N_0/\bar{N}_0$, $Y_i = B_i/(A_i \bar{N}_0)$, the probability of failure is

$$p_F = \int \bar{F}_{Y_1}(u) f_\nu(u) du + \int \bar{F}_{Y_2}(u) F_{Y_1}(u) f_\nu(u) du + \dots \\ \dots + \int \bar{F}_{Y_m}(u) \prod_{i=1}^{m-1} F_{Y_i}(u) f_\nu(u) du \quad (29)$$

where $F_{Y_i}(\cdot)$ is the cumulative probability distribution function of Y_i , $\bar{F}_{Y_i}$ is $1 - F_{Y_i}$, and $f_\nu(\cdot)$ is the probability density function of ν . Often all the Y_i 's have the same distribution; then Eq. 29 lends itself to graphical representation. Otherwise, simple approximations, such as exponential functions,

may be adopted for the F 's and $\bar{F}$'s in the integrands, and graphs drawn to permit term by term computation of p_F according to Eq. 29. When the correlation between loads or strengths is not negligible, only the first few terms will contribute significantly to p_F .

The foregoing analysis can be applied also to the computation of total failure probabilities at sections potentially critical in more than one mode (19). Its application, combined with a generalized definition of load and strength, gives one solution to the problem of safety of reinforced concrete columns.

Let the interaction diagram of a reinforced concrete column be defined as two curves of given form and random parameters (G_c and G_t in Fig. 7), respectively representing the conditions for failure in compression and in tension. The acting load is the resultant of an axial force, S_p , and its moment S_M , divided by the expected value of the cross-section depth. Let $S = \sqrt{S_p^2 + S_M^2} / \bar{t}$, where $\bar{t}$ is the expected effective depth. In a similar manner, the strength R will be taken as the distance from the origin to the intersection of the line joining O and A with the interaction diagram. Since both the load eccentricity and the location of the balanced point are unknown, two intersections, A' and A'' , respectively with G_c and G_t are possible, and we cannot beforehand state which failure mode will govern, particularly when the expected load eccentricity is approximately equal to the value it would have at the expected balanced point. Algebraic expressions for the safety margins with respect to both modes can be obtained, and hence first-order estimates can be made of their means, variances and covariance. Applicability of Eq. 29 is then obvious.

Some cases may be envisaged for which the cost of failure depends on the mode. For instance, a reinforced concrete, simply supported beam can fail in flexural compression or tension at midspan or in diagonal tension at either

end. Failure in flexural tension is ductile. It warns of its impending occurrence, allowing one to take measures that prevent excessive damage. In these cases not only the total failure probability but the individual contribution of each mode is also of interest. Suppose that the failure modes are mutually exclusive. When all loads S_j are proportionally increased, mode i occurs if $R_i < S_i$ and $R_i/S_i < R_j/S_j$ for all $j \neq i$. Hence, the probability that failure take place in precisely the i th mode is

$$p_i = \int F_{Z_i}(u) F_{X_j}(u) f_{\mu_i}(u) du \quad (30)$$

where $Z_i = (R_0/S_0)/E(B_i/A_i)$, $X_j = (B_j/A_j)/E(B_i/A_i)$, and $\mu_i = (B_i/A_i)/E(B_i/A_i)$.

In earthquake resistant design of tall buildings on weak soil it is sometimes valid to assume that only two failure modes are possible: excessive lateral distortion of the ground story, and foundation failure produced by overturning moment. Usually the force-deflection curve for lateral deformation of the ground story is characterized by a ductile range, while overturning failure is of the brittle or sudden type. Three different generalized strengths are defined: R_y is the yield deflection of the ground story, $R_u = \mu R_y$ (μ and R_y are independent) is the lateral deflection at which collapse takes place, and R_M , independent of α and of R_y , is the overturning capacity (overturning moment that would cause failure). In correspondence with these strengths, three maximum earthquake effects are also defined: S_e = ground story deformation of an elastic structure having the same mechanical properties as those of the one under study for small deformations; S_u = inelastic deformation of ground story; S_M = overturning moment. These responses are related to each other and to earthquake intensities through the following equations,

$$S_e = B_e y \quad (31)$$

$$S_u = B_u B_e y \quad (32)$$

$$S_M = B_M B_e y \quad (33)$$

where the B_i 's are mutually independent and y is earthquake intensity.

Every time an earthquake occurs the following events are possible.

- a) The structure yields laterally, preventing failure by overturning moment. Eventually, the structure may collapse if $S_u > \alpha R_y$.
- b) Overturning moment failure occurs before lateral yielding.
- c) Responses remain within the elastic range.

The analytical conditions expressing the failure events are as follows.

- a) Failure by excessive inelastic deformation if $\alpha R_y < B_u B_e y$ and $R_M/S_M > R_y/S_e$.
- b) Failure by overturning moment if $R_M < B_M B_e y$ and $R_M/S_M < R_y/S_e$.
- c) Survival otherwise (includes yielding previous to condition a).

The probability of failure by excessive deformation whenever an earthquake occurs is, then,

$$p_u = \int P\left\{\frac{B_e B_u y}{\alpha} > z\right\} P\left\{\frac{R_M}{B_M} > z\right\} f_{R_y}(z) dz \quad (34)$$

that of failure by overturning moment is

$$p_M = \int P\{B_e y > z\} P\{R_y > z\} f_{R_M/B_M}(z) dz \quad (35)$$

and the corresponding mean rates of failure per unit time are, respectively,

$\lambda_u = \lambda_0 p_u$ and $\lambda_M = \lambda_0 p_M$, where λ_0 is the mean number of earthquakes per unit time.

OPTIMUM-RELIABILITY FORMULATION: ITS PRESENT AND FUTURE

We have identified the optimum-reliability format as ideal. Yet there are many apparent objections to its adoption. Most of these can be classified under one of two headings: computational difficulties, and insufficient knowledge of structural behavior, failure laws and consequences of failure (19). Computational obstacles can be circumvented after considerable implementation work, some of which is in process.

The existence of areas of ignorance about structural behavior actually constitutes a reason in support of an explicit optimum-reliability format:

even if initially the situation will force the adoption of numerical values of some parameters so that they lead to designs that do not depart markedly from accepted practice, gaps in our knowledge will become manifest and point out the topics in which research is most desirable. This has already happened in the course of implementing work in progress. Moreover, there have been other instances in which it has paid to reverse the apparently logical order of things: modern stress analysis, dynamic analysis of earthquake and wind effects, and so on.

Overt reliability optimization should become standard practice once reliability oriented computer languages constitute a reality. But short term applications of optimum reliability theory should probably involve handling of a small number of primary parameters (load factors, story shears, etc.) in terms of which near optimum solutions would be derived, while the values of other design parameters (resisting bending moment at a section, member capacity under axial load, etc.) would be chosen to hold fixed ratios to the primary variables. Such recommended fixed ratios would be associated with given types of structural systems and could result from the work of safety committees.

Extensive use of optimization criteria will not entirely eliminate the need to specify the values of some uncertain parameters in building codes, at least for a long time to come. Among these stand out the consequences of failure of some kinds of structural members. For example, it is uncertain how many floors will be lost above a column that fails and how far the failure may propagate to neighboring columns. At least for some time the estimate of these costs should not be left up to the designer.

Some types of structural systems will be encountered so often that it will pay for code writers to obtain optimum reliability solutions for a wide range of conditions. The results would be converted into explicit expressions for failure probabilities or safety factors. Accordingly, the ideal code

might include lists of these quantities as well as detailed specification of objective functions.

In the very near future, optimization may have a place as an alternate procedure in codes with the purpose of improving some recommendations contained in second-moment or specified-reliability formats. Matters that would properly be covered in the alternate section of such codes include the specification of optimum nonstructural damage, intensity of vibrations, seismic design intensities, wind design velocities, differential settlements, reliability allocation to members of statically determinate systems, the safety of roofs for storage rooms and several others. These questions lend themselves to simplified treatment within the optimum reliability frame.

Consider a continuous beam that must support the roof of a large assembly hall. It is assumed that the proposed code specifies the optimum ratios between the allowable failure probabilities in the potential modes; diagonal tension, and flexural tension and compression. Since the cost of failure in flexural tension is much lower than in the other modes and the cost of increasing safety against this mode of failure is not greatly different as against failure in the other two modes, the corresponding failure probability should be the highest. Practical applications of optimization can be made in terms of this probability and of the assumed ratios of it to the failure probabilities in the other modes. To this end the expected cost of failure must be adjusted to allow for the probabilities of failure in other modes and for their corresponding costs.

In the first step, means and variances of the generalized loads on each span (M_0 in Fig. 6) are computed. Then an assumption is made concerning the relative ordinates of the law of variation of the resisting moment along the beam. This step is similar to the decision on the location of plastic hinges and on the ratios between their capacities in limit design.

Under some fairly general conditions the location of the plastic hinges could be taken as known. Failure probabilities of each span would be obtained from consideration of Eqs. 22-25 together with recommended assumptions about the form of the probability distributions of M_0 and R . The expected cost of failure would be equal to the sum of the contributions of all the spans. Let us assume that all spans are designed for the same safety factor, n . The negative terms of the objective function (expected initial cost and cost of damage) would equal

$$U = C_0 + n \sum_i C_i + \sum_i H_i p_i(n) \quad (36)$$

where H_i is the expected cost of failure of the i th span, $p_i(n)$ the corresponding probability of failure for a safety factor n , and C_i the rate of variation of initial cost with the central safety factor for the same span.

If we differentiate Eq. 36 with respect to $\bar{n}$ and the result is equated to zero, Eq. 37 is obtained, under the assumptions that loads and strengths have lognormal distributions and that $C = C_0 + \sum_i c_s k L R / \bar{f}_y \bar{d}$, where c_s is cost of reinforcement per unit cross-sectional area and unit length, L is span length, k is a constant ranging approximately from 0.3 to 0.4, and the other variables were defined previously.

$$c_s \sum_i \frac{k_i L_i}{\bar{f}_y \bar{d}_i} = \frac{1}{n} \sum_i \frac{\varphi(Z_i)}{M_{0i} \sigma_i} \quad (37)$$

Here, $\varphi(\cdot)$ is the normal probability density function,

$$Z_i = -\frac{1}{\sigma_i} \left[\ln \sqrt{\frac{1 + V^2(M_{0i})}{1 + V^2(R_i)}} + \ln n \right] \quad (38)$$

and

$$\sigma_i^2 = \ln \left[(1 + V^2(R_i)) (1 + V^2(M_{0i})) \right] \quad (39)$$

Equation 37 has to be solved by trial and error. Variations of this procedure may be contemplated for those cases that call for a different safety factor in each span.

Some situations can be envisaged where the cost of failure depends appreciably on the value of the live load acting at the instant of collapse.

Bridges and balconies of auditoriums are good examples. Let R = strength, S_D = dead load effect, S_L = live load effect and $Q = R - S_D$. At failure, $Q = S_L$. The mean and coefficients of variation of Q are respectively

$$\bar{Q} = \bar{R} - \bar{S}_D \quad (40)$$

and

$$V^2(Q) = \frac{\text{var } R + \text{var } S_D}{(\bar{R} - \bar{S}_D)^2} \quad (41)$$

In the following, optimization is carried out with respect to $\bar{Q}$, with $V(Q)$ given. The optimum value of $\bar{R}$ is obtained from that of $\bar{Q}$ through use of Eq. 40. If $V(R)$ is assumed fixed, $\text{var } R$ is not known beforehand. Hence, several cycles of trial and error may be necessary. Suppose that the cost of failure is a linear function of the live load acting on the tributary area of the member under study: take it to be $H_1 + H_2 S_L$, where H_1 and H_2 are constants. The objective function is

$$U = -C_1 - C_2 \bar{Q} - H_1 P_F - H_2 \int F_Q(s) f_{S_L}(s) s \, ds \quad (42)$$

where the integrand in the last term accounts for the fact that the cost of failure is a function of S_L .

Differentiation with respect to $\bar{Q}$ leads to

$$C_2 + H_1 \frac{\partial P_F}{\partial \bar{Q}} + H_2 \left(\frac{\partial}{\partial \bar{Q}} \int F_Q(s) f_{S_L}(s) s \, ds \right) \quad (43)$$

At least for the cases when both Q and S_L are normally or lognormally distributed, closed form expressions can be obtained for all the terms in Eq. 43. For instance, in the second case there results the following optimization condition:

$$\begin{aligned} \frac{C_2}{H_2} = & \frac{H_1}{H_2 \bar{S}_L} \frac{\bar{S}_L / \bar{Q}}{\sqrt{2\pi} \sqrt{\sigma_Z^2 + \sigma_X^2}} \exp \left[-\frac{1}{2} \left(-\frac{\ln(\bar{Q}/\bar{S}_L)}{\sqrt{\sigma_Z^2 + \sigma_X^2}} \right)^2 \right] + \\ & + \frac{\bar{S}_L / \bar{Q}}{\sqrt{2\pi} \sqrt{k_1 \sigma_Z \sigma_X}} \exp \left[\frac{1}{2} \left(\frac{k_2^2}{k_1^2} - k_3 \right) \right] \quad (44) \end{aligned}$$

In this equation,

$$\sigma_X^2 = \ln \left[1 + v^2(S_L) \right] , \quad \sigma_Z^2 = \ln \left[1 + v^2(Q) \right] \quad (45)$$

$$k_1 = \frac{1}{\sigma_X^2} + \frac{1}{\sigma_Z^2} , \quad k_2 = \frac{\bar{Z}}{\sigma_Z^2} + \frac{\bar{X}}{\sigma_X^2} , \quad k_3 = \frac{\bar{Z}^2}{\sigma_Z^2} + \frac{\bar{X}^2}{\sigma_X^2} \quad (46)$$

$$\bar{X} = \ln \bar{S}_L - \frac{1}{2} \ln \left[1 + v^2(S_L) \right] , \quad \bar{Z} = \ln \bar{Q} - \frac{1}{2} \ln \left[1 + v^2(Q) \right] \quad (47)$$

A comparison of the optimum safety factor obtained according to this criterion and from the one which assumes that the expected cost of failure equals $(H_1 + H_2 \bar{S}_L) p_F$ is shown in a separate paper (19), together with additional simplified formulations of optimum reliability problems.

CLOSING REMARKS

The time is ripe in Mexico and, it is felt, in other countries as well, to define the characteristics which the ideal code should have. This will permit setting up a strategy of research, education and code writing to replace the present haphazard process.

In view of the purposes of structural design, the ideal code should be one of reliability optimization. Disguising it in a more palatable way would call for an excessive number of obscuring, complicated and sometimes contradictory adjustments. Hence, the ideal format is one of explicit optimization. However, an attempt to introduce such a format at present would meet with serious opposition. Adoption of the ideal code will also demand changes in procedures and even in the organization of building departments.

Aside from matters of tradition and resistance to change, the main apparent objections to the immediate adoption of this type of building code stem from computational difficulties and from our insufficiency of knowledge concerning conditions of failure and their consequences. The former can be circumvented through the development of design aids, including charts, simple approximate expressions, computer programs and reliability oriented computer languages. The latter calls for more research of a selective nature. And

there is also need for an educational process.

A realistic strategy would begin by producing codes in a first-order second-moment format, followed by codes in a specified-reliability format and a third generation of documents coming close to the ideal. Each code would contain a more advanced approach as an alternate.

Some special problems in structural design already lend themselves to a relatively simple formulation of reliability optimization, as exemplified in this and in other recent papers. Introduction for this approach for those problems as an alternate in building codes now is possible and advisable. By proceeding openly, the need for research in some aspects of structural behavior and structural economics and for certain design aids will become manifest; the educational process will begin and, surely, some committees will engage in a worthwhile endeavor to make the ideal code materialize.

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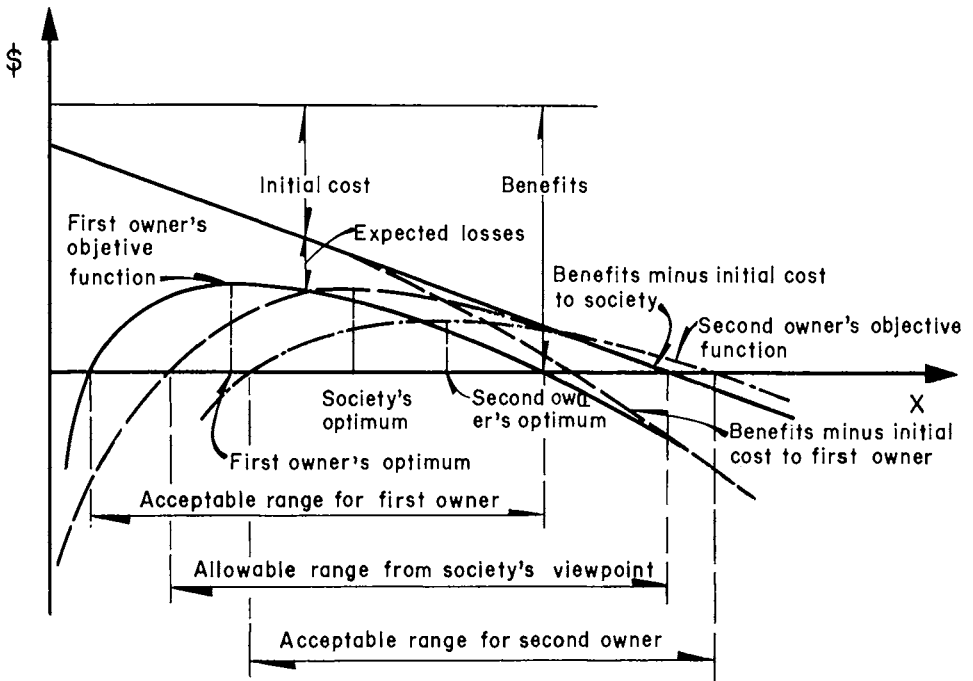


Fig 1 Objective functions for society and two different investors

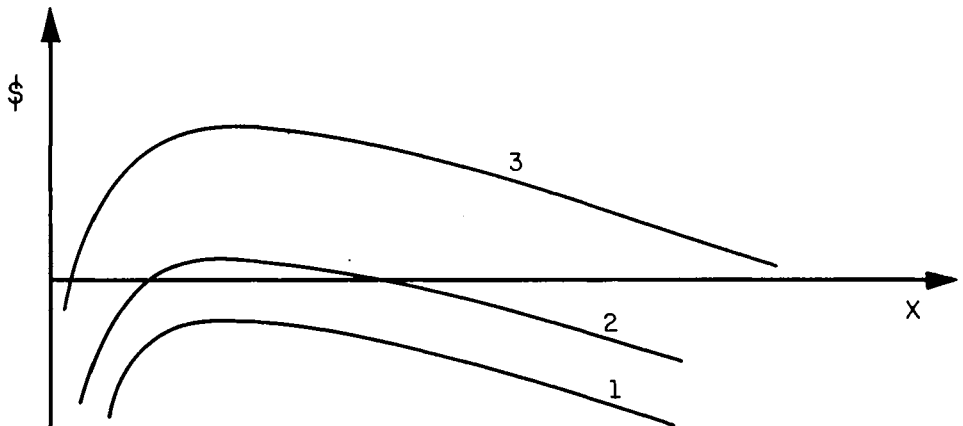


Fig 2 Three societal objective functions

Curve	Load	Strength	V_S	Symbol
1	Normal	Normal	0.1	————
2			0.3	
3	Lognormal	Lognormal	0.1	-----
4			0.3	
5	Normal truncated	Lognormal	0.1	- - - -
6			0.3	
7			0.4	
8	Extreme type I	Lognormal	0.1	- - - -
9			0.3	
10			0.4	

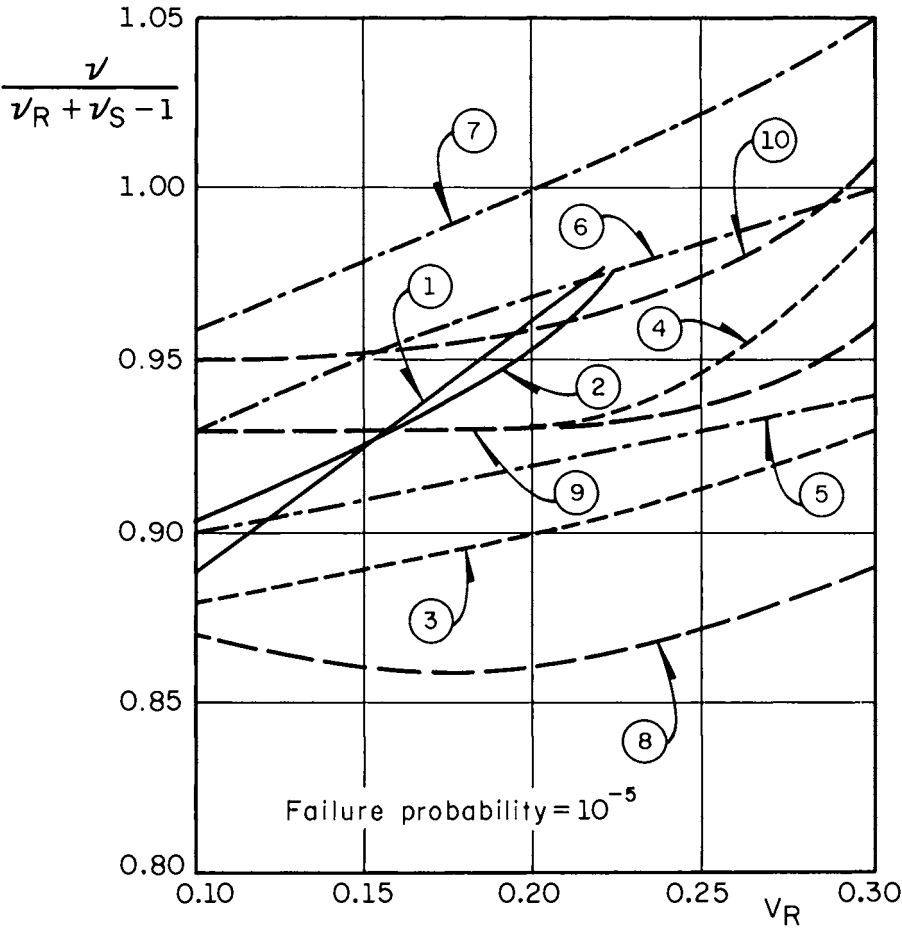


Fig 3 Split safety factors in Baker's version

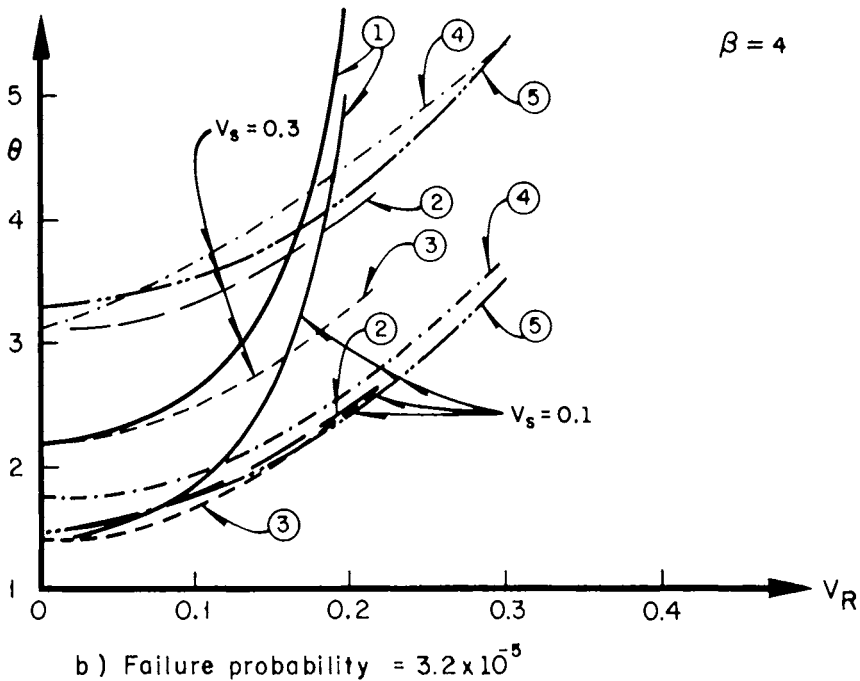
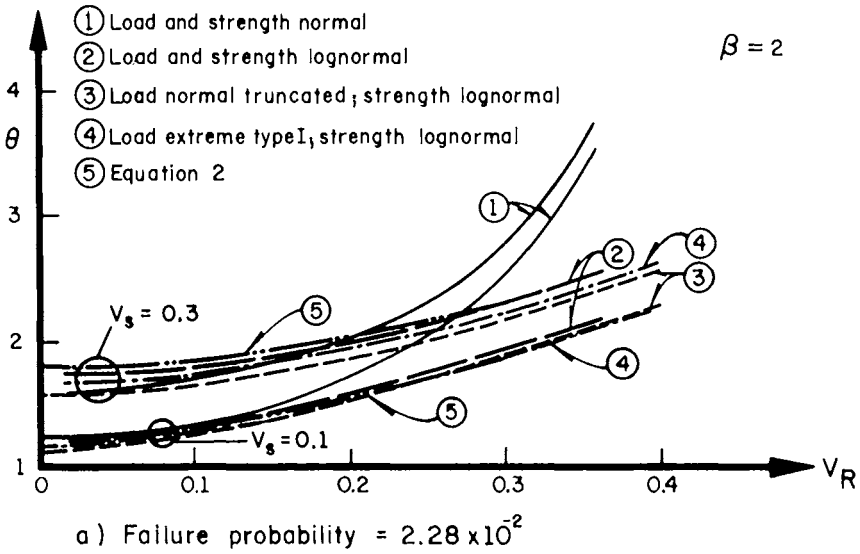


Fig 4 Safety factors as functions of coefficients of variation

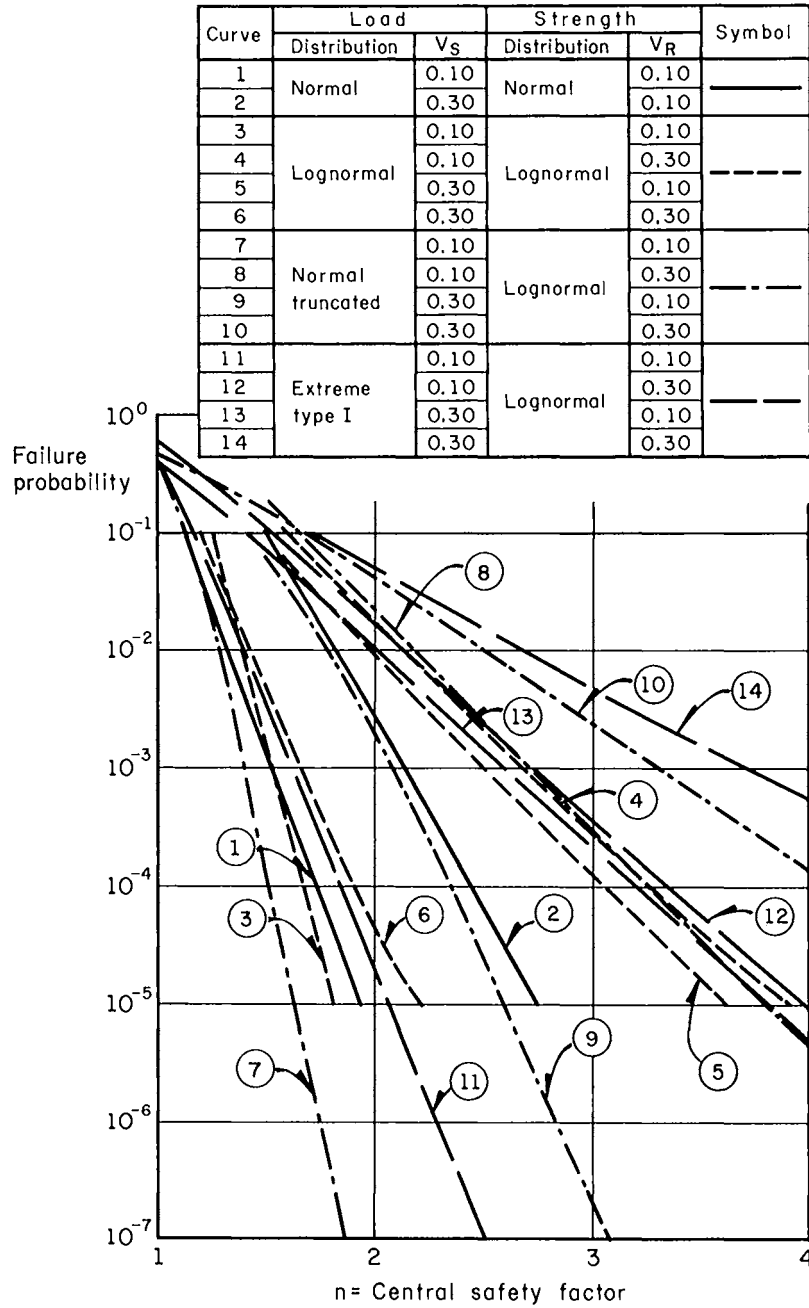
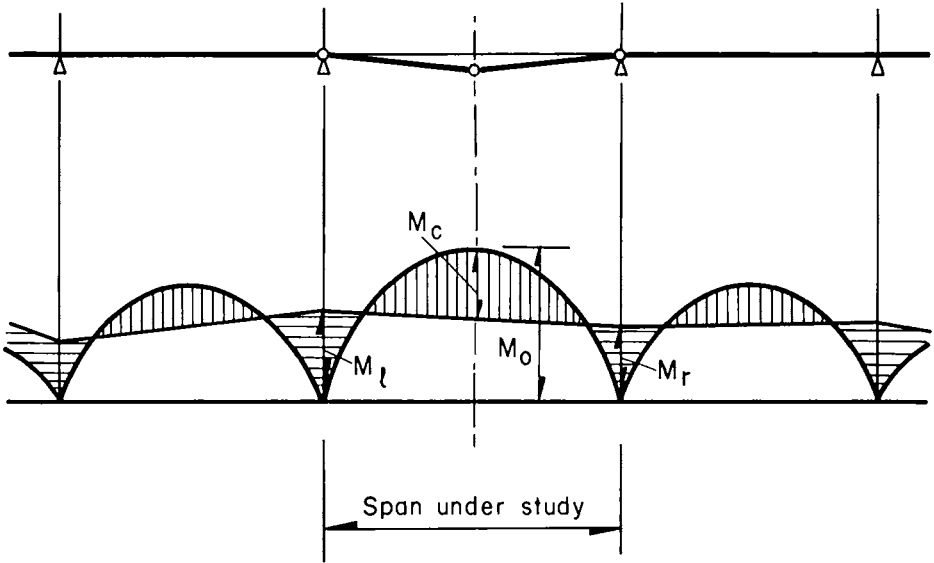
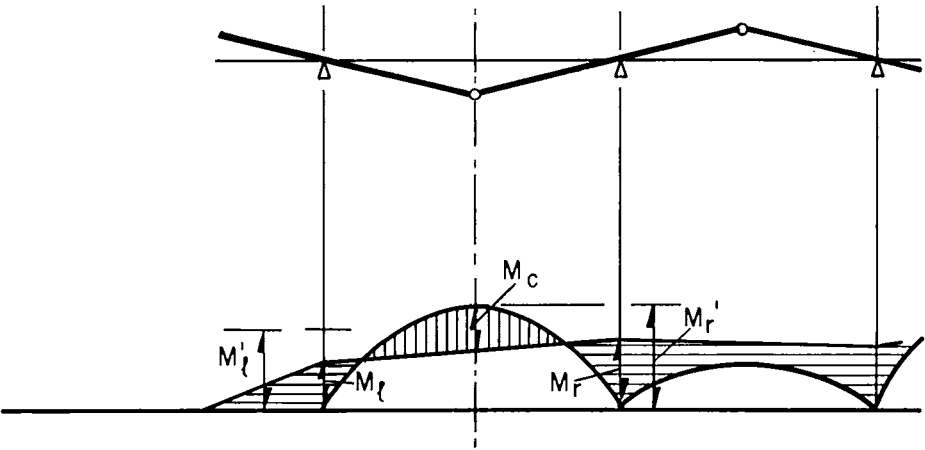


Fig 5 Failure probabilities for several families of distributions



a) Plastic hinges at span ends and at midspan



b) End moments limited by loads and strengths of adjacent spans

Fig 6 Failure mechanisms of continuous - beam span

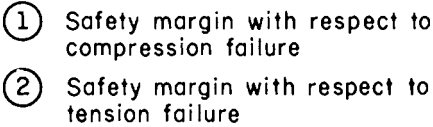


Fig 7 Concepts defining safety of reinforced concrete column

Statistical Definition and Combination of Loads

By J. Ferry Borges and M. Castanheta

Synopsis: A statistical treatment of the problem of combination of different types of loadings involves a suitable definition of each of the loads to be combined. In the idealization adopted each load is assumed constant during given intervals of time, and variable from interval to interval according to an uncorrelated stochastic process. Further, variability from structure to structure and variability in time are assumed to be equivalent.

An algorithm is deduced for the combination of different loadings with different numbers of repetitions, which is simplified for the regions with small densities of probability. Assuming a linear relationship between loads and load-effects, the multivariate density of probability expressed in terms of loads is transformed into a density of probability in terms of load-effects.

The use of a program allowing to compute the density of probability of the load-effect vectors, when the distribution functions of the loads are of normal and of extreme types, is exemplified for a simple case. The quantitative definition of load factors and of load-effect factors is discussed. It is shown how two-variable problems can be reduced to one-variable problems and how preceding studies about the latter can be used.

The results obtained give a clear understanding on how to perform the combination of loads and on how to choose the load factors. It is also shown that the design rules at present included in the codes, based on intuition alone, are not adequate. However, as it is difficult to recommend design rules of general application, a compromise between accuracy and simplicity is necessary.

Keywords: earthquakes; failure; live loads; loads (forces); probability theory; reinforced concrete; safety factor; static loads; statistical analysis; structural design; variability; wind pressure.

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1 - INTRODUCTION

The most convenient way for studying structural safety consists in defining and comparing in separate, loadings and resistances.

The different types of loads acting on a structure vary in time and in space. For studying the combinations of loads these variabilities have to be duly considered.

Usually safety is checked not directly in terms of loads and resistances but by comparing the load-effects due to combinations of loads – loading load-effects – and the load-effects defining the resistance of the different structural elements – resisting load-effects.

The vectorial character of the load-effects must be emphasized. This is particularly true in concrete structures for which the several components of the resisting load-effects are very closely interrelated.

2 - IDEALIZATION OF LOADS

When dealing with problems of structural safety in which loading and resistance are expressed in function of one variable alone, loading is simply defined by a distribution function $F(s)$, which indicates the probability of the load intensity being smaller than s during the lifetime of the structure.

When combining several types of loads, their variability in time has to

be considered and the problem becomes more involved. Variability in time can be idealized by different means. In the present paper constancy during given intervals of time and variability from interval to interval according to a uncorrelated stochastic process are assumed. However, in a first approximation the results obtained can also be applied to the other types of idealizations. (1)

The intervals of time in which the load intensity is constant are called elementary intervals; their duration depends on the type of loading. The number of independent repetitions of the elementary intervals during the life time of the structure is inversely proportional to their duration. Table 1 indicates for some types of loads the assumed durations of the elementary

Table 1 - Duration and number of repetitions
of different types of loads

Load	Duration of elementary interval	Number of independent repetitions r	Remarks
Permanent	50 years	1	
Live load on buildings	5 years	10	Duration depends on the occupancy type
Wind	1 hour	$50 \cdot 10^3$	
Earthquakes	30 seconds	$50 \cdot 10^6$	

intervals and the corresponding number of independent repetitions for a 50-years lifetime of the structure.

When discussing load variability, both variability from structure to structure and from time interval to time interval should be considered. For studying the combination of loads the following basic assumption is adopted:

"Variability from structure to structure and variability in time are

equivalent". Accordingly, the probability of observing loads less than a given value during t elementary intervals of time in s similar structures equals the probability of observing loads less than the given value in one structure during st elementary intervals of time.

The distribution function of the load intensity for the elementary intervals being given, the distribution function of the maximum load intensity during the lifetime of the structure is the extreme distribution of the largest of r samples taken from the initial distribution, fig. 1. Assuming

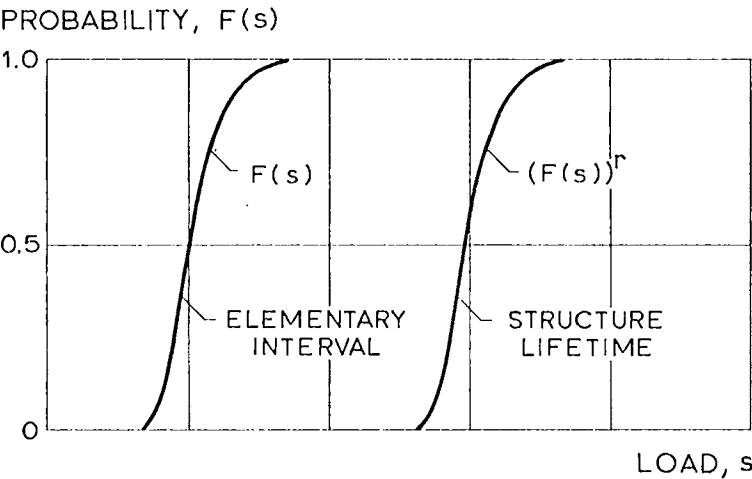


Fig. 1 - Distribution functions of load intensity for the elementary interval and the structure lifetime

the distributions to be of extreme type and adopting for the ordinates an extreme instead of a linear scale, the distribution function indicated in fig. 1 become parallel straight lines, fig. 2.

In the following variable loads are idealized by extreme distributions of type I,

$$F(s) = \exp (- \exp (- \alpha (s-u))) \dots\dots\dots 1)$$

or of type II,

$$F(s) = \exp (- (ks)^{-\beta}) \dots\dots\dots 2)$$

Both types of distributions have two parameters only; α and u in the former case, k and β in the latter.

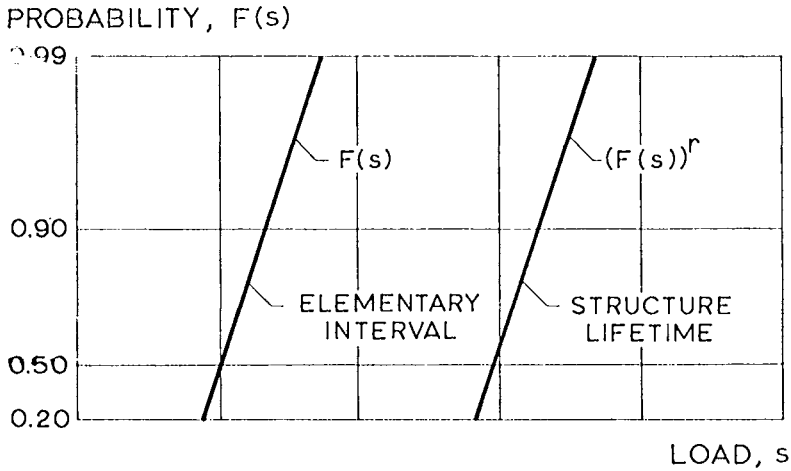


Fig. 2 - Distribution functions of load intensity in extreme scale

3 - COMBINATION OF LOADS

Consider a structure under n independent loads defined by a load vector S with components $s_1, \dots, s_n$. Assume the number of independent repetitions of each loading during the lifetime of the structure to be $r_1 \leq r_2 \leq \dots \leq r_n$, respectively. Condition $r_1 = 1$ means that s_1 is a permanent load.

It can be shown (1) that the probability of the load vector S falling in the domain $ds_1 \dots ds_n$ at least once during the lifetime of the structure is given by

$$d F_{r_1, \dots, r_n}(S) = 1 - \left[1 - d F_1(s_1) \left(1 - \left[1 - d F_2(s_2) \left(1 - \left[\dots \right]^{r_3/r_2} \right)^{r_2/r_1} \right]^{r_1} \right) \right] \dots \dots \dots 3)$$

This expression is obtained by generalizing the expression for one

variable

$$d F_{r_i}(S) = 1 - \left[1 - d F_i(s_i) \right]^{r_i} \dots\dots\dots 4)$$

This last expression, generalized for several variables having the same number of repetitions, is used by Paloheimo (2).

In the region of special interest, where the probability of occurrence of the loads is small, expression 3) can be approximated by

$$d F_{r_1, \dots, r_n}(S) \approx r_n f_1(s_1) f_2(s_2) \dots f_n(s_n) ds_1 ds_2 \dots ds_n \dots\dots\dots 5)$$

or by

$$f(S) \approx r_n f_1(s_1) f_2(s_2) \dots f_n(s_n) \dots\dots\dots 6)$$

The effects of the most repeated load being known, it is preferable to substitute distribution $f_n(s_n)$ by the distribution of the largest or alternatively, the smallest of s_n , for a number of repetitions r_n/r_{n-1} . Thus, in the case of a combination of for instance three types of loads, one of which permanent

$$d F_{r_1=1, r_2, r_3}(S) \approx d F_1(s_1) \left(1 - \left[1 - d F_2(s_2) d F_{r_3/r_2}(s_3) \right]^{r_2} \right) \dots\dots\dots 7)$$

where F_{r_3/r_2} is the extreme distribution obtained from $F_3(s_3)$ for a number of repetitions r_3/r_2 .

Again for combinations of loads having a small probability of occurrence, expression 7) can be approximated by

$$f(S) \approx r_2 f_1(s_1) f_2(s_2) f_{r_3/r_2}(s_3) \dots\dots\dots 8)$$

or by

$$f(S) \approx f_1(s_1) f_{r_2}(s_2) f_{r_3/r_2}(s_3) \dots\dots\dots 8')$$

4 - TRANSFORMATION OF LOADS INTO LOAD-EFFECTS

Assuming the relationship between loads and load-effects to be linear the load-effect vectors Q are expressed in function of the load vector S by

$$Q = A S \dots\dots\dots 9)$$

A being the transformation matrix.

The number of components of loads and load-effects being n and m respectively, three cases can occur: $n < m$, $n = m$, $n > m$.

For $n = m$, A is a square matrix of order n . If so, the density of probability $f(Q)$ according to the theory of random variables is given by

$$f(Q) = f(S) \frac{\partial(s_1 \dots s_n)}{\partial(q_1 \dots q_n)} \dots\dots\dots 10)$$

where $f(S)$ is the density of probability of the load vector S , and the Jacobian has the value

$$\frac{\partial(s_1 \dots s_n)}{\partial(q_1 \dots q_n)} = \frac{1}{|A|} \dots\dots\dots 11)$$

The variables s_i in 10) are expressed in function of q_i by the relationship

$$S = A^{-1} Q = C Q \dots\dots\dots 12)$$

The results above can be easily generalized for $n < m$ and $n > m$. If $n > m$, an integration has to be performed in the domain of variation of $n-m$ variables. Accordingly, in the case of a two-component load-effect and a three-component load, the density of probability $f(q_1, q_2)$ is given by

$$f(q_1, q_2) = \frac{1}{|A|} \int f(s_1 = c_{11} q_1 + c_{12} q_2 + c_{13} s_3, s_2 = c_{21} q_1 + c_{22} q_2 + c_{23} s_3, s_3) ds_3 \dots\dots\dots 13)$$

where $f(s_1, s_2, s_3)$ is the joint density of probability of loads. Adopting for $f(s_1, s_2, s_3)$ the value given by expression 8')

$$f(q_1, q_2) \approx \frac{1}{|A|} \int_{-\infty}^{+\infty} f_1(s_1 = c_{11}q_1 + c_{12}q_2 + c_{13}s_3) f_2(s_2 = c_{21}q_1 + c_{22}q_2 + c_{23}s_3) f_3(s_3) ds_3 \quad \dots 14)$$

The expression above can be computed by analytical means if the functions are normal distributions but not if some of distributions are of extreme type. In this case, however, a numerical computation can be easily performed in an electronic computer (+).

The computation of the above integral is exemplified by considering the simple structure represented in fig. 3 under the loads given in Table 2.

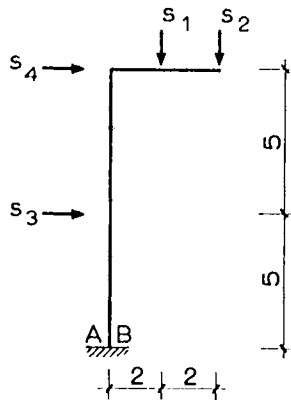


Fig. 3 - Structure acted by four types of loads

The densities of probability of the load-effects acting on section AB for the combinations of loads: permanent + live; permanent + earthquake; permanent + live + wind, and permanent + live + earthquake are presented in figs.4 to 7.

(+) The authors gratefully acknowledge the collaboration of Mr. Ferreira Pinto, Assistant Professor of the University of Luanda, for writing the program for this computation.

Table 2 - Type and parameters of the statistical distribution of loads

Type of loading	Number of repetitions r_i	Basic distribution		Extreme distribution for r_i repetitions	
		Type	Parameters	Parameters	Characteristic value
s_1 permanent	1	Normal	$\bar{s}_1 = 50.0$ $\sigma_{s_1} = 5.0$	—	58.2
s_2 live load	10	Extreme type II	$k_2 = 0.2$ $\beta_2 = 4.0$	$k_2 = 0.112$ $\beta_2 = 4.0$	18.7
s_3 wind load	50 10 ³	Extreme type II	$k_3 = 1.0$ $\beta_3 = 6.0$	$k_3 = 0.165$ $\beta_3 = 6.0$	10.0
s_4 earthquake load	50 10 ⁶	Extreme type II	$k_4 = 100.0$ $\beta_4 = 3.0$	$k_4 = 0.271$ $\beta_4 = 3.0$	9.9

LOAD-EFFECT, q_2 , AXIAL FORCE

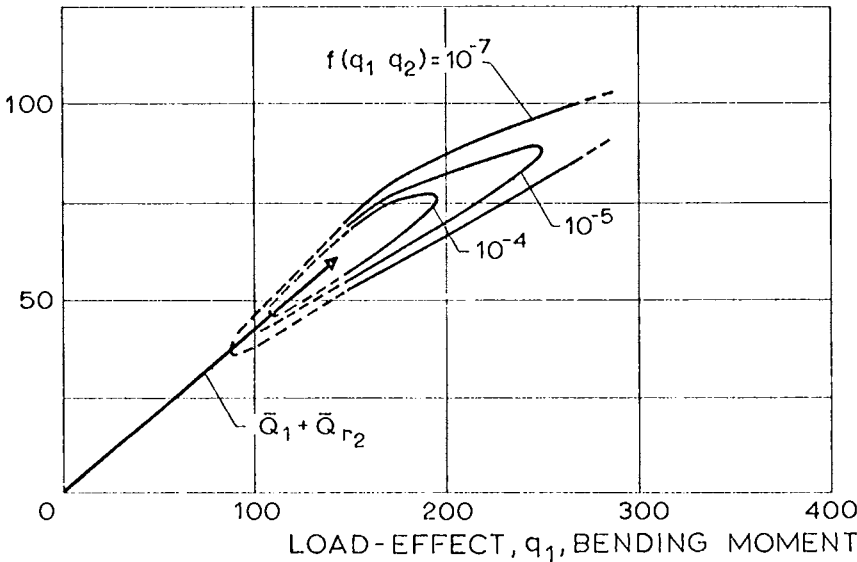


Fig. 4 - Combination of permanent and live load