

Bulletin 81



Punching shear of structural concrete slabs:

Honoring Neil M. Hawkins

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Punching shear of structural concrete slabs: Honoring Neil M. Hawkins



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Technical report

Proceedings of a symposium held in Philadelphia, PA, USA,
on 25 October 2016

Edited by Carlos E. Ospina, Denis Mitchell and Aurelio Muttoni

April 2017

Approval of this bulletin

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State-of-the-art report	Commission
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Any publication that has not met the above requirements will be clearly identified as preliminary draft.

fib Bulletin 81 is published as a technical report and is a collection of contributions to a symposium that was co-sponsored by the *fib* and the American Concrete Institute (ACI). The authors have presented their individual views. Although these contributions have not been discussed in any of the *fib*'s working bodies, the subject matter is highly topical and believed to be of general interest to members of the *fib*.

This bulletin is also published as an ACI Symposium Publication, ACI SP-315.

Cover images: Saw-cuts of inner slab-column connections failed in punching.
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ISSN 1562-3610

ISBN 978-2-88394-121-2

Layout by Laura Vidale on behalf of the *fib* secretariat.

Printed by DCC Document Competence Center Siegmars Käßler e.K., Germany

Foreword

fib Bulletin 81 deals with punching of slabs, which is a relevant issue in the design of reinforced concrete. Punching is one of the most frequent reasons for failure of concrete structures, which underlines the importance of this problem. Flat slabs are used in buildings, bridges and other structures. Punching may result in brittle failure, which requires that special attention be paid to the design of new structures and the assessment of existing ones. Punching and shear capacity of structures is often evaluated using empirical methods, which does not always provide a sufficient level of safety. This bulletin is a result of an international symposium where experts from *fib* and ACI met and exchanged their experience. The papers included in this bulletin provide new experimental evidence and a comprehensive review of analytical and numerical methods that are used to evaluate the structural performance of slab-column connections.

This bulletin provides a review of the performance of slabs under static, dynamic and seismic loading. The individual papers generally show the studied phenomena using experiments and analytical evaluation, which makes it possible to compare results obtained according to the European code (EC2), the ACI code and the international code, the *fib* Model Code for Concrete Structures (MC2010). Particular attention is paid to the so-called size effect (i.e. the nominal load carrying capacity of thicker slabs is lower than that of thin slabs). The arrangement of reinforcement is important for the residual strength after a local failure and robustness, which are instrumental to avoid the progressive collapse of the entire structure after a local failure. It is confirmed that the bottom reinforcement crossing the column is very beneficial to ensure sufficient residual strength and avoid progressive collapse. A paper dealing with the shear capacity of bridge slabs illustrates an extended strip model, which may be used for efficient assessment of bridge slabs under concentrated loads. This bulletin also presents new types of shear reinforcements and shows efficiency of distribution of classical shear reinforcing elements like headed studs. Some papers deal with retrofitting and strengthening of existing slabs. Post-installed shear reinforcing elements may be used for strengthening of existing structures.

This bulletin summarizes several phenomena that influence the performance of slabs sensitive to punching. It is a valuable summary of the state-of-the art knowledge for practicing engineers, academics and also for students. It is also important to appreciate that the opinions and experiences of American and European experts are summarised in one publication.

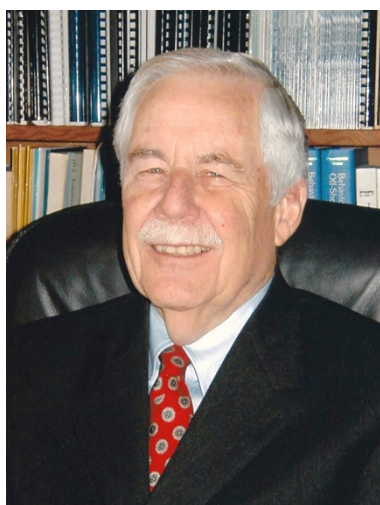
Finally, it is necessary to thank all authors, as well as the main organizers of the symposium (A. Muttoni convener of *fib* WP 2.2.3, and C. E. Ospina and D. Mitchell representing ACI committee 445) for their editing of this extraordinary document. Last but not least, many thanks to Laura Vidale for the preparation of the bulletin for publication.

Jan L. Viték

Chairman of *fib* Commission 2, *Analysis and design*

Preface

fib Bulletin 81 reports the latest information available to researchers and practitioners on the analysis, design and experimental evidence of punching shear of structural concrete slabs. It follows previous efforts by the International Federation for Structural Concrete (*fib*) and its predecessor the Euro-International Committee for Concrete (CEB), through CEB Bulletin 168, *Punching Shear in Reinforced Concrete* (1985) and *fib* Bulletin 12, *Punching of structural concrete slabs* (2001), and an international symposium sponsored by the punching shear sub-committee of ACI Committee 445 (Shear and Torsion) and held in Kansas City, Mo., USA, in 2005.



Prof. Emeritus Neil M. Hawkins

This bulletin contains 18 papers that were presented in three sessions as part of an international symposium held in Philadelphia, Pa., USA, on October 25, 2016. The symposium was co-organized by the punching shear sub-committee of ACI 445 and by *fib* Working Party 2.2.3 (Punching and Shear in Slabs) with the objectives of not only disseminating information on this important design subject but also promoting harmonization among the various design theories and treatment of key aspects of punching shear design. The papers are organized in the same order they were presented in the symposium. The symposium honored Professor Emeritus Neil M. Hawkins (University of Illinois at Urbana-Champaign, USA), whose contributions through the years in the field of punching shear of structural concrete slabs have been paramount.

The papers cover key aspects related to punching shear of structural concrete slabs under different loading conditions, the study of size effect on punching capacity of slabs, the effect of slab reinforcement ratio on the response and failure mode of slabs, without and with shear reinforcement, and its implications for the design and formulation in codes of practice, an examination of different analytical tools to predict the punching shear response of slabs, the study of the post-punching response of concrete slabs, the evaluation of design provisions in modern codes based on recent experimental evidence and new punching shear theories, and an overview of the combined efforts undertaken jointly by ACI 445 and *fib* WP 2.2.3 to generate test result databanks for the evaluation and calibration of punching shear design recommendations in North American and international codes of practice.

Sincere acknowledgments are extended to all authors, speakers, reviewers, as well as to *fib* and ACI staff for making the symposium a success and for their efforts to produce this long-awaited bulletin. Special thanks are due to Laura Vidale for preparing the bulletin for publication.

The editors of fib Bulletin 81:

Carlos E. Ospina (chair of ACI 445 Punching Shear Sub-committee)

Denis Mitchell (ACI 445)

Aurelio Muttoni (chair of *fib* Working Party 2.2.3)



Participants in the 2016 ACI-fib symposium on punching shear of structural concrete slabs.

Standing, L to R: Sagaseta (Surrey, UK), Vollum (Imperial College, UK), Ramos (Nova, Portugal), Fernández Ruiz (EPFL, Switzerland), Kueres (Aachen, Germany), Genikomsou (Queen's, Canada), Walkner (Innsbruck, Austria), Topuzi (Toronto, Canada). Seated, L to R: Lantsoght (Delft, The Netherlands, and Quito, Ecuador), Polak (Waterloo, Canada), Ospina (Houston, USA), Hawkins (U. of Illinois, USA), Muttoni (EPFL, Switzerland), Mitchell (McGill, Canada), Criswell (Colorado State, USA), Alexander (Edmonton, Canada). Not shown: Bažant (Northwestern, USA), Dam (U. of Michigan, USA), Gayed (Calgary, Canada), Hueste (Texas A&M, USA). Photo by ACI staff.

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Shear and moment transfer at column-slab connections

Scott D. B. Alexander

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Abstract

The Strip Model describes a load path for the transfer of vertical shear between a slab and column. The model is easily adapted to design but its application to the analysis of specimens tested under combined shear and moment is less clear. This paper provides a brief description of the Strip Model, updates the model to include size effect, and shows how it can be applied to interior and edge column-slab connections transferring combinations of shear and moment.

Keywords

Columns, connections, punching shear, reinforced concrete, shear strength, slabs, structural design.

1 Introduction

The Strip Model for slab punching shear, originally called the Bond Model (Alexander and Simmonds, 1992), describes an internal distribution for the transfer of vertical load between a two-way slab and column. The model may be considered an extension of the Strip Method of Design (Hillerborg, 1975). The Strip Method allows a designer to define a load distribution that rigorously satisfies equilibrium at all points in a slab and to reinforce the slab for the bending moments that are the consequence of that load distribution. The Strip Method as developed by Hillerborg does not address shear strength.

The Strip Model for slab punching shear is consistent with the Strip Method for flexural design but is focused on a particular problem: the development of an internal load distribution for shear transfer at concentrated loads that does not violate either shear or flexural strength limits at any point. This internal load distribution is derived from subdividing the slab into regions dominated by slender flexural behavior (B-regions) and regions dominated by deep beam behavior (D-regions). The result is a model for shear transfer that can be verified by direct measurement (Alexander et al., 1995).

The distinguishing characteristic of a B- or D-region is the predominant mechanism of moment gradient (i.e. shear transfer). In a slender beam, moment gradient is mostly the result of a varying flexural tension force acting on a more or less constant moment arm. Such behavior is called **beam action**. In a deep beam, moment gradient results from a constant tensile force acting on a varying moment arm. This behavior is called **arching action**.

It is appropriate to describe shear transfer by beam action in terms of an average shear stress acting on the cross-section. A reasonable design strategy for slender members is to limit the average shear stress to some critical value; however, an average shear stress does not model the behavior in a D-region. D-regions are more correctly modeled using strut-and-tie.

Column-slab connections exhibit the characteristics of both B- and D-regions. Tests show that radial arching action is an important mechanism of shear transfer between a slab and a column, suggesting that column-slab connections should be considered D-regions. In the circumferential direction, however, column-slab connections behave more like B-regions.

This paper provides a brief summary of the mechanics of the strip model and extends the model to account for size effect. It then introduces the concepts of non-proportional loading and shows how these are used to describe the transfer of shear moment at a column-slab connection.

2 Strip model for concentric punching

2.1 Internal load distribution

The Strip Model divides the slab into radial strips and plate quadrants, as shown in Fig. 1. No load can reach the column without passing through one of the radial strips. Within each radial strip shear is carried to the column by arching action. This is visualized as a curved arch, with maximum slope at the face of the column as shown in Fig. 2. The quadrants of two-way slab are fundamentally slender flexural elements, which means shear transfer across the boundary between a strip and its adjacent quadrant of plate is through the two-way plate equivalent of beam action.

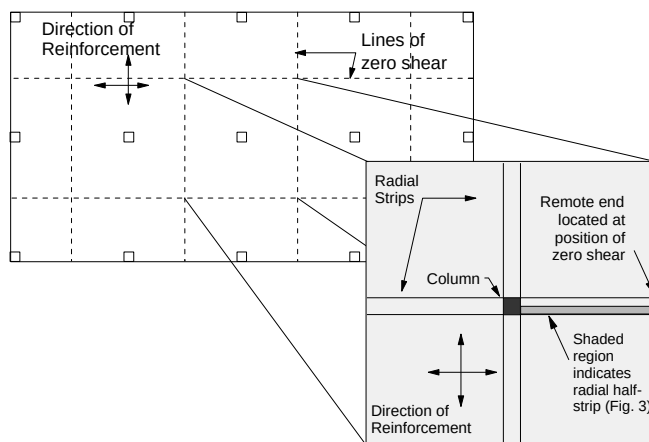


Figure 1: Geometry of Strip Model.

Consider the compression arch shown in Fig. 2. The compression force in the arch is approximately constant throughout. At the column face, the vertical component of the arch accounts for the shear transferred to the column; the horizontal component provides a flexural compression.

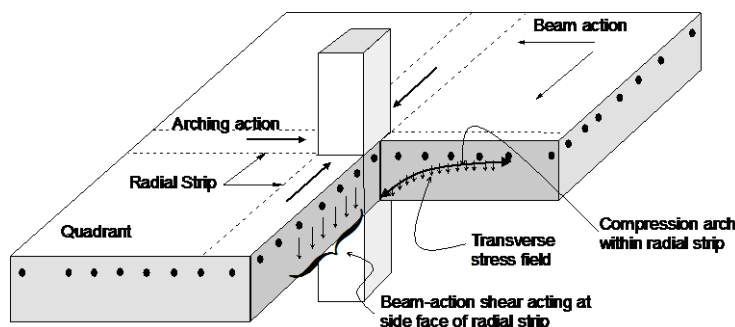


Figure 2: Beam and arching action at column-slab connection.

Moving away from the column, the slope of the arch decreases. Vertical equilibrium of the arch requires that there be a transverse stress field. The transverse stress field is internal and is generated by the two-way plate equivalent of beam action shear acting in a direction perpendicular to the arch. Thus, the model is of the interaction between the slender quadrants of plate and the radial strips acting as deep beams.

Figure 3 shows a free body diagram of half of a radial strip. The half-strip is loaded on its side face by a combination of plate bending moment, m_n , torsional moment, m_t , and shear, v . The strip is supported by a vertical reaction, P_s , at the column-supported end and bending moments, M_{neg} and M_{pos} , at the column and remote ends of the strip, respectively.

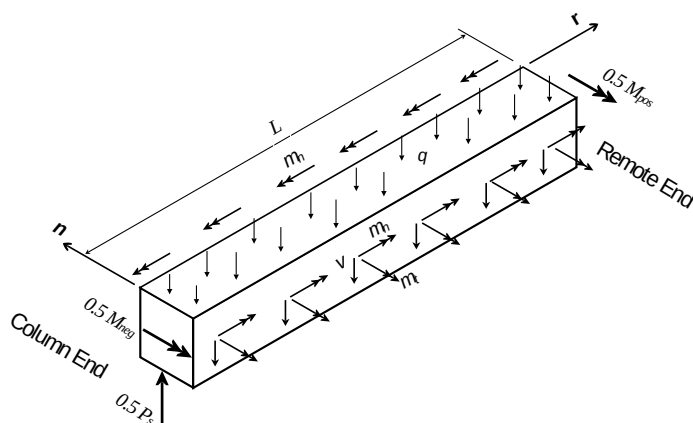


Figure 3: Forces on radial half-strip.

The internal vertical shear at any point along the side face of a radial strip is a function of the gradients of bending and torsional moments at that point. Alexander and Simmonds (1992) and Afhami et al. (1998) examine the various components of this internal shear in some detail to justify the simplified free body diagrams of radial strips at ultimate load, shown in Fig. 4. The loading term, w , is the limiting one-way shear that can be carried by the slab. An internal radial strip, such as those shown in Fig. 1, is loaded on two faces; hence the total distributed line load on the strip is $2w$. At an edge column, a spandrel strip running parallel to the free edge of the slab would be loaded on only one side and so would be subject to a line load of w .

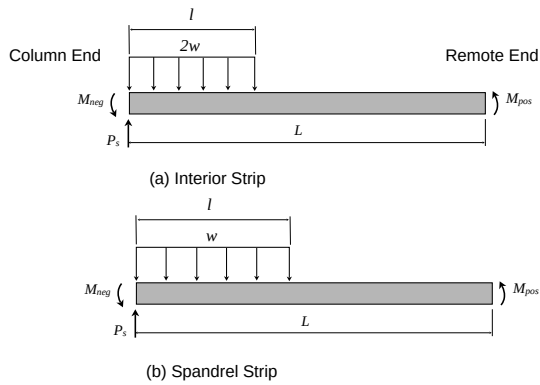


Figure 4: Simplified loading on radial strip.

The flexural strength of the radial strip, M_s , is the sum of the negative and positive flexural capacities, M_{neg} and M_{pos} , at the ends of the strip. The loaded length of the strip is l and the total load carried by one strip is P_s , termed the **nominal capacity** of a strip. Equilibrium for an internal radial strip requires that:

$$l = \sqrt{\frac{M_s}{w}} \quad (1)$$

$$P_s = 2\sqrt{M_s w} \quad (2)$$

The corresponding equations for a spandrel strip are:

$$l = \sqrt{\frac{2M_s}{w}} \quad (3)$$

$$P_s = \sqrt{2M_s w} \quad (4)$$

The total load that can be delivered to the column is simply the summation of the individual strip contributions.

$$P_{col} = \sum P_s \quad (5)$$

While the load distribution described above is certainly a simplification of reality, it is nevertheless realistic. The radial strips are parallel to the stiff directions of the reinforcing mat. Alexander et al. (1995) and Afhami et al. (1998) show that bending moment gradients perpendicular to the side faces of radial strips, estimated from strain gauge measurements, account for the measured column reaction over a considerable range of loading. Using non-linear finite elements, Afhami et al. (1998) show that yielding of flexural reinforcement through the column results in a stepped distribution of internal shear along the side face of a radial strip, consistent with what is shown in Fig. 4.

Earlier work (Alexander and Simmonds, 1992) examines a limited data set of 116 concentrically loaded interior column-slab connections from eight sources, six with simply supported edges and two with rotationally restrained edges. The flexural capacities are strictly limited to those of the strip of slab defined by the width of the column.

$$M_{neg} = \rho_{neg} \times f_y \times j \times c \times d^2 \quad (6a)$$

$$M_{pos} = \rho_{pos} \times f_y \times j \times c \times d^2 \quad (6b)$$

$$j = 1 - \frac{\rho f_y}{1.7 f'_c} \quad (6c)$$

$$M_s = M_{pos} + M_{neg} \quad (6d)$$

Consistent with the assumption of slender flexural behavior, the loading term w is taken as the unit one-way shear strength of the slab. Based on ACI 318-14 (ref), w is given by:

$$w = d \times 0.17 \sqrt{f'_c \text{ (MPa)}} = d \times 2 \sqrt{f'_c \text{ (psi)}} \quad (7)$$

where d is the flexural depth of the slab and f'_c is the specified concrete strength.

Alexander and Simmonds (1992) report an average ratio of test load to calculated load of 1.29 with a coefficient of variation of 12.3%. On the same body of test results, the ratio of test load to the load predicted by ACI code is 1.56 with a coefficient of variation of 26.5%.

To illustrate a typical set of calculations used to analyze these tests, consider a hypothetical interior column-slab punching test specimen. The slab measures 2.33 m (7 ft) square, centered on a 400 mm (15.75 in) square column. The slab has 1% top reinforcement each way based on an average flexural depth of 130 mm (5.12 in). The yield strength of the reinforcement is 400 MPa (58 ksi) and the concrete strength is 30 MPa (4350 psi). The slab is loaded on its perimeter. There is no rotational restraint at the boundary (i.e. $M_{pos} = 0$).

From Equations (6a) through (6d):

$$M_s = 1\% \times 400 \text{ MPa} \times \left(1 - \frac{1\% \times 400 \text{ MPa}}{1.7 \times 30 \text{ MPa}}\right) \times 400 \text{ mm} \times (130 \text{ mm})^2 = 24.9 \text{ kN} \cdot \text{m}; (18.4 \text{ ft} \cdot \text{kips})$$

$$\text{From Equation (7), } w = 130 \text{ mm} \times 0.17 \times \sqrt{30 \text{ MPa}} = 121 \text{ N/mm}; (8.29 \text{ kips/ft}).$$

From Equation (2), $P_s = 2\sqrt{24.9 \times 121} = 110 \text{ kN} (24.7 \text{ kips})$. Because there are four strips framing into the column, Equation (5) gives a total load of 440 kN (98.8 kips).

Note that even though the top reinforcing mat will have two values of d , one for each layer of reinforcement. The average value of d , measured to the middle of the mat, is used for all calculations. One could consider the different values of d but this is needlessly complicated. The larger value of d for flexure always pairs with the smaller value of d for the loading term, w , and vice versa.

2.2 Size effect

A shortcoming of the data set used by Alexander and Simmonds (1992) is the limited range of flexural depth. The deepest slab had a flexural depth of 200 mm (7.9 in). Forty-two of the tests were on small-scale specimens with flexural depths less than 55 mm (2.2 in).

To address this shortcoming, a subset of 257 specimens from the ACI 445 Punching Shear Databank (Ospina et al., 2011) is used. The subset has results from 38 separate sources. Specimens are simply supported (i.e. $M_{pos} = 0$), steel reinforced, with either square or circular columns. While the dataset still contains a large number of small-scale specimens (39 with flexural depth 65 mm [2.6 in] or less), it has better representation of deeper slabs (29 with flexural depths over 140 mm [5.5 in]).

Figure 5 shows distributions of the ratio of test load to calculated load with respect to flexural depth. In Fig. 5(a) the calculated loads are nominal capacities based on the ACI code. The average ratio of test load to calculated load is 1.45 and the coefficient of variation is 25.4%. The results show some size effect. As flexural depth increases, the ratio of test load to calculated load decreases.

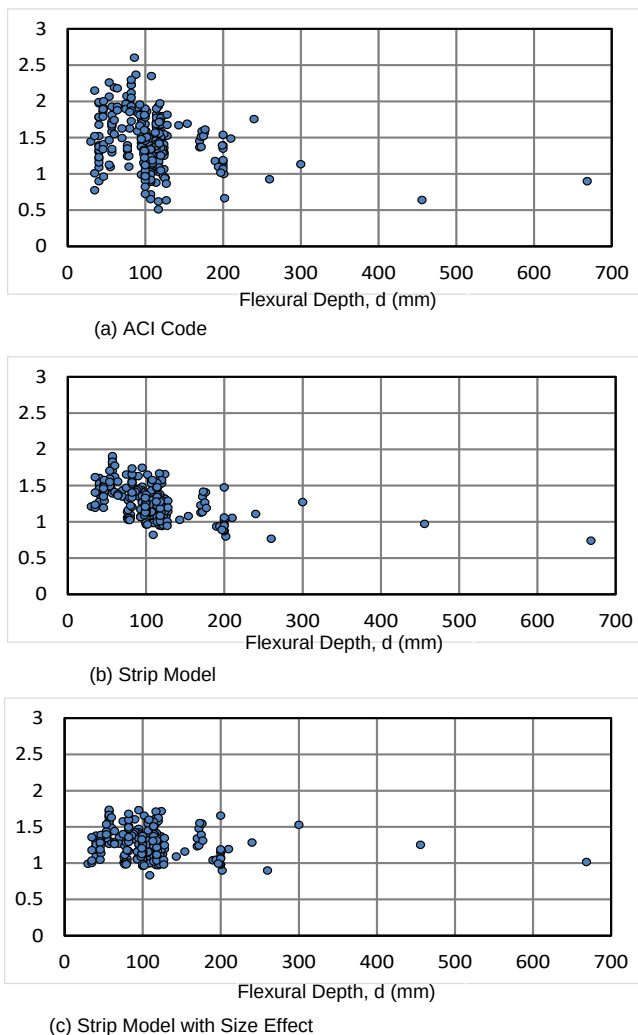


Figure 5: Ratio of test load to calculated load: concentric tests.

Figure 5(b) shows the same test results relative to calculated loads based on the Strip Model with no size effect considered. Here the average ratio of test load to calculated load is 1.24 with a coefficient of variation of 16.2%. While considerably less scattered than the results using the ACI code, there is still a noticeable trend to decreasing strength with increasing flexural depth.

To incorporate size effect with the Strip Model, one need only calculate the loading term, w , using an expression for one-way shear value that accounts for size. It is beyond the scope of this paper to present a detailed treatment of size effect. Instead, a simpler approach will be used. A size factor, $(a/d)^\lambda$, where a is a reference depth, d is the flexural depth of the slab, and λ is a fraction, will be incorporated in the loading term w .

The reference depth, a , is set to 100 mm (4 in). There are a couple of reasons for this choice. First, it is convenient. Second, it is approximately equal to the depth of the thinnest slabs that could still be considered full-scale.

Fujita et al. (2002) assess several methods that account for size effect in one-way shear and concluded that two separate populations were represented in their body of test results. For normal and medium strength concrete, they propose $\lambda = 1/4$. For high strength concrete specimens, they propose $\lambda = 1/2$. Since the expression for P_s places any size factor used in the expression for w under a second root sign, it follows that two-way shear is somewhat less sensitive to size than one-way shear. A compromise value of $\lambda = 1/3$ is used here. Revising the expression for w results in:

$$w = d \times 0.17 \sqrt{f'_c} \times \sqrt[3]{\frac{100 \text{ mm}}{d}} \quad (8)$$

Figure 5(c) shows results ratios of test load to calculated load using the revised expression for w [Equation (8)] that accounts for size effect. The average ratio of test to calculated load for these 257 specimens is 1.24 and the coefficient of variation is 14.0%.

Some comments on the Strip Model:

- The Strip Model shows how a one-way shear limit combined with localized arching behavior is sufficient to explain two-way shear. A magnified "two-way" limiting shear stress is not necessary.
- With the loading term, w , held constant, the loaded length, l , increases with increasing flexural strength. In effect, the additional flexural capacity supports a larger "critical section."
- As presented above, the Strip Model has in no way been calibrated to the tests it is being used to analyze. The flexural strength of the radial strip is strictly that which can be attributed to a strip of slab with width defined by the supporting column. The loading term, w , is the one-way shear per unit width that one would use for a one-way slab. All the terms used in the Strip Model to calculate a two-way punching capacity are derived from one-way tests.
- Because the Strip Model makes use of only conventional flexural strengths and a one-way shear strength, it allows a unification of design treatment for one and two-way systems.
- The Strip Model does not describe a failure mechanism for a column-slab connection. It describes a load path that does not violate strength limits for flexure or one-way shear. With sufficient ductility in the system, the load predicted by the Strip Model is a lower bound of the capacity of the structure.

3 Connections transferring shear and moment

3.1 Proportional and non-proportional loading

To apply the Strip Model to the more general case of an interior connection under shear and moment requires a modified approach that does not rely on the level of symmetry usually found in concentric load tests. Most concentrically loaded test specimens reported in the literature have, more or less, four axes of symmetry in the distribution of slab curvature around the column. As is observed by Afhami et al. (1998), the loading is such that the nominal capacity of each radial strip is developed. This situation is called **proportional loading**. While most concentric load tests in the literature have this level of symmetry, most column-slab connections in practice do not.

Non-proportional, concentric loading occurs when the radial strips in one direction do not each their nominal capacity. The load is still concentric if there is no moment transferred to the column but the distribution of slab curvature around the column has only 2 axes of symmetry. The skew axes are no longer axes of symmetry. In this case enhanced radial strips will develop in one direction at the expense of strip development in the other. The limiting case, illustrated in Fig. 6, occurs when fully enhanced strips, each carrying a load P_{ss} , develop in one direction. The fully enhanced strip is called a super strip. Because radial strips do not develop in the direction perpendicular to the super strip, load transfer on the side face (c_1 face) of the column is limited to the one-way shear limit. The maximum total load transferred to the column with this load distribution is given by:

$$P_{col} = (\sum P_{ss}) + 2 \times w \times c_1 = 2 \times (P_{ss} + wc_1) \quad (9)$$

This changed loading pattern requires some revision to the flexural capacity of the super strip. With non-proportional loading, the negative moment capacity of the strip need not be limited to the reinforcement within the strip. The radial super strip acts something like a T-beam in negative moment, with its "stem" defined column dimension perpendicular to the strip, c_2 and with a "top flange" wider than c_2 .

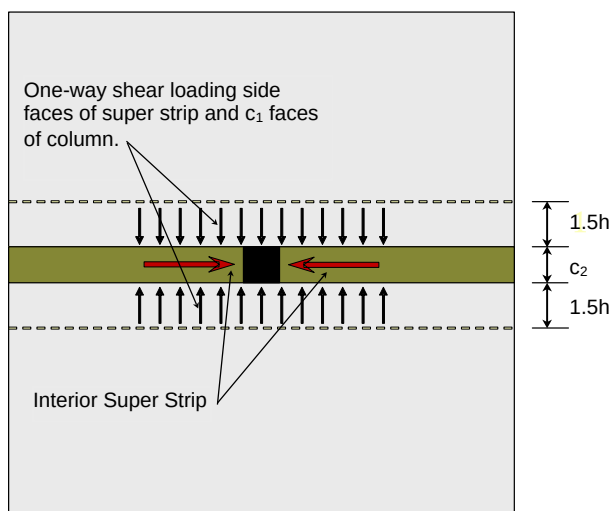


Figure 6: Non-proportional, concentric loading at interior connection.

Consistent with usual practice, it is assumed that top reinforcement within 1.5 times the slab thickness on either side of the column (i.e. within $c_2 + 3h$) is effective as top reinforcement for the radial super strip. Equation (6a) becomes:

$$M_{ss,neg} = \rho_{ss,neg} \times f_y \times j_{ss} \times c_2 \times d^2 \quad (10)$$

$$\text{where } \rho_{ss,neg} = \rho_{neg} \times \frac{(c_2 + 3h)}{c_2}$$

The factor j_{ss} is modified to account for flexural compression stresses framing into the side faces of the column joint. Alexander and Simmonds (2003) show that the effective width of the compression block associated with a wide band of flexural reinforcement is $c_1 + c_2$. For a super strip framing into a square or circular column, the gives:

$$j_{ss} = 1 - \frac{\rho \times f_y}{2 \times 1.7 \times f'_c} \quad (11)$$

The basic equations for loaded length and the total load carried are conceptually unchanged.

$$l_{ss} = \sqrt{\frac{M_{ss}}{w}} \quad (12)$$

$$P_{ss} = 2\sqrt{M_{ss}w} \quad (13)$$

Applying Equation (9) to the data set of 257 concentrically loaded test specimens results in an average test to calculated load of 1.24 with a coefficient of variation of 15.6%. In other words, the load path defined for non-proportional loading accounts for essentially the same load as proportional loading in the case of an interior column under concentric load. The virtue of considering this alternative load path is that it is more easily adapted to test specimen under eccentric loading.

3.2 Interior connections

The Strip Model allows the designer to assess independently the load coming into each face of a connection. Analogous to a shear failure in one of four beams framing into a column, punching occurs when one of these faces is overloaded. The net moment transferred to the connection is a consequence of the loading but not particularly relevant to the shear on any particular side of the connection. For example, in a continuous system with unequal spans, it is possible to design a connection that is not concentrically loaded yet transfers no moment to the column.

In contrast, for a test on an isolated column-slab connection, moment transfer is often the only measure of the degree to which one side of the connection is more heavily loaded. The following shows how the non-proportional load distribution developed above can be adapted to test specimens that meet at least one of two criteria: (1) the slab has the same reinforcing mats top and bottom and/or (2) the loading eccentricity is low enough to avoid load reversal.

Figure 7 shows a side view of an interior connection specimen under non-proportional, eccentric load. The load is treated as a combination of symmetric and anti-symmetric components. Punching occurs when one side of the connection reaches its limiting load.

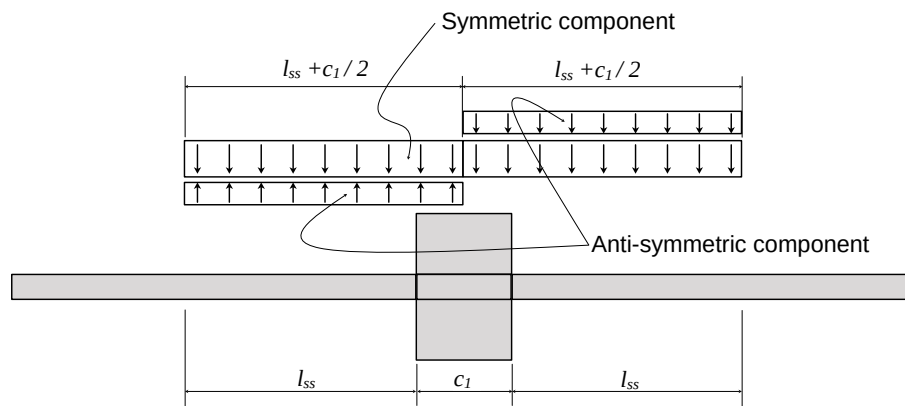


Figure 7: Simplified shear and moment transfer at interior connection.

The overall length of loading for the concentric component of load is $2l_{ss} + c_1$. The antisymmetric loading components are each distributed over half of this length, or $l_{ss} + c_1/2$. From equilibrium, the load distribution described in Fig. 7 results in:

$$\frac{P_{ult}}{2} + \frac{M_{ult}}{r} \leq P_{ss} + w \times c_1 = 2 \times w \times r \quad (14)$$

where $r = l_{ss} + c_1/2$

The length r is analogous to a radius of gyration for the connection. The Strip Model makes this quantity dependent on the flexural reinforcement near the connection. The equivalent quantities in the ACI code are derived from geometric properties of a fixed critical section that is independent of reinforcement.

Tests by Hanson and Hanson (1968) and Hawkins et al. (1989) satisfy at least one of the two criteria listed earlier. Table 1 provides details for the 37 specimens considered here.

Hanson and Hanson (1968) tested 17 column-slab connections. Of these, seven specimens were of interior column-slab connections with solid slabs (no holes near the column) tested under combined shear and moment. All specimens had the same layout of reinforcement, top and bottom. Two loading methods are represented here. Type I loading involved the application of equal upward and downward line loads to the slab on either side of the column to produce high moment with little net vertical load transfer. Type III involved a single line load on one side of the column.

Hawkins et al. (1989) presents results from 30 tests of interior connections under eccentric loading. Point loads, controlled to produce a fixed eccentricity, were applied around the perimeter of the slab.

Table 2 provides test results as well as predicted failure loads using both the Strip Model [Equation (14)] and the ACI code. Both analyses give reasonable results although the limited size of the data set makes any statistical comparisons less compelling.

Table 1: Interior connections under shear and moment: description of specimens

Hanson and Hanson (1968)							
Mark	f'_c (MPa)	f_y (MPa)	c_1 (mm)	c_2 (mm)	h (mm)	d_{ave} (mm)	ρ (%)
A1	30.3	366	152	152	76.2	57.2	1.64
A2	31.4	377	152	152	76.2	57.2	1.64
B7	33.0	355	305	152	76.2	57.2	1.64
C8	32.9	412	152	305	76.2	57.2	1.64
A12	33.3	373	152	152	76.2	57.2	1.64
B16	30.5	341	305	152	76.2	57.2	1.64
C17	36.1	342	152	305	76.2	57.2	1.64
Hawkins, Bao, and Yamazaki (1989)							
6.00AH	31.4	473	305	305	152	120.7	0.60
9.60AH	30.7	416	305	305	152	117.5	0.96
14.00AH	30.4	421	305	305	152	114.3	1.40
6.00AL	22.8	473	305	305	152	120.7	0.60
9.60AL	29.0	416	305	305	152	117.5	0.96
14.00AL	27.1	421	305	305	152	114.3	1.40
7.30BH	22.2	473	305	305	114	82.6	0.73
9.50H	19.9	473	305	305	114	82.6	0.95
14.20BH	29.6	416	305	305	114	79.4	1.42
7.30BL	18.2	473	305	305	114	82.6	0.73
9.50BL	20.1	473	305	305	114	82.6	0.95
14.20BL	20.6	416	305	305	114	79.4	1.42
6.00CH	52.5	473	305	305	152	120.7	0.60
9.60CH	57.3	416	305	305	152	117.5	0.96
14.00CH	54.9	421	305	305	152	114.3	1.40
6.00CL	49.7	473	305	305	152	120.7	0.60
14.00CL	47.8	421	305	305	152	114.3	1.40
6.00DH	30.1	473	305	305	152	120.7	0.60
14.00DH	31.8	421	305	305	152	114.3	1.40
6.00DL	28.7	473	305	305	152	120.7	0.60
14.00DL	24.3	421	305	305	152	114.3	1.40
10.20FHI	26.0	421	305	305	152	114.3	1.02
10.20FHO	33.9	421	305	305	152	114.3	1.02
14.00FH	31.3	421	305	305	152	114.3	1.40
6.00FLI	26.0	473	305	305	152	114.3	0.60
10.20FLI	18.1	421	305	305	152	114.3	1.02
10.20FLO	26.6	421	305	305	152	114.3	1.02
9.60GH2	24.7	416	406	203	152	117.5	0.96
9.60GHO.5	26.3	416	203	406	152	117.5	0.96
9.60GH3	27.1	416	457	152	152	117.5	0.96

Table 2: Interior connections under shear and moment: test and calculated results

Hanson and Hanson (1968)											
	Test results		Calculated quantities			Strip Model results			ACI code results		
Mark	M_{col} (kN·m)	P_{col} (kN)	w kN/m	M_{ss} (kN·m)	r (mm)	M_{pred} (kN·m)	P_{pred} (kN)	T/P	M_{pred} (kN·m)	P_{pred} (kN)	T/P
A1	22.4	5.75	63.0	7.02	410	20.1	5.2	1.11	14.9	3.8	1.50
A2	24.3	4.81	64.0	7.22	412	20.9	4.1	1.16	15.3	3.0	1.59
B7	35.8	4.90	65.7	6.86	475	28.8	3.9	1.24	27.3	3.7	1.31
C8	31.5	5.62	65.6	11.03	486	29.7	5.3	1.06	28.4	5.1	1.11
A12	20.5	26.9	66.0	7.19	406	17.2	22.5	1.19	13.2	17.4	1.55
B16	27.4	34.4	63.1	6.57	475	21.9	27.6	1.25	21.1	26.5	1.30
C17	24.8	31.6	68.7	9.30	444	21.1	26.9	1.17	23.8	30.3	1.04
Hawkins, Bao, and Yamazaki (1989)											
6.00AH	90.6	169.8	105.5	30.7	692	61.2	115	1.48	82.4	154	1.10
9.60AH	97.9	187.2	102.5	40.4	780	71.5	137	1.37	77.7	149	1.26
14.00AH	110.4	205.9	100.1	55.4	896	87.6	163	1.26	75.0	140	1.47
6.00AL	32.7	244.3	89.8	30.3	734	25.9	193	1.27	31.8	238	1.03
9.60AL	34.6	257.6	99.6	40.3	789	31.5	234	1.10	34.7	258	1.00
14.00AL	43.5	319.6	94.4	55.0	915	36.3	267	1.20	32.6	240	1.33
7.30BH	39.1	79.8	68.9	14.6	612	31.8	64.9	1.23	39.2	80.1	1.00
9.50H	45.5	94.5	65.2	18.5	686	35.8	74.3	1.27	36.9	76.5	1.23
14.20BH	51.1	102.1	77.4	22.7	694	44.0	87.9	1.16	43.3	86.5	1.18
7.30BL	12.8	130.2	62.3	14.4	633	11.8	120	1.08	13.7	139	0.94
9.50BL	16.6	142.2	65.5	18.5	684	15.6	134	1.06	16.4	140	1.01
14.20BL	15.3	162.2	64.6	22.1	737	14.3	152	1.07	13.4	142	1.14
6.00CH	95.3	186.8	136.3	31.0	629	66.8	131	1.43	105	205	0.91
9.60CH	113.3	218.0	140.0	41.1	694	80.9	156	1.40	106	204	1.07
14.00CH	133.6	252.7	134.4	56.9	803	98.5	186	1.36	100	190	1.33
6.00CL	36.9	273.7	132.6	31.0	636	31.9	237	1.16	47.2	350	0.78
14.00CL	49.5	362.4	125.5	56.6	824	42.4	311	1.17	43.4	318	1.14
6.00DH	65.9	134.2	103.1	30.6	697	58.7	119	1.12	77.8	158	0.85
14.00DH	98.9	195.7	102.3	55.5	889	86.1	170	1.15	74.9	148	1.32
6.00DL	24.5	232.7	100.9	30.6	703	22.9	218	1.07	29.8	283	0.82
14.00DL	34.9	282.1	89.5	54.5	933	32.6	264	1.07	28.8	233	1.21
10.20FHI	74.2	153.8	92.5	40.7	816	66.7	138	1.11	66.4	138	1.12
10.20FHO	90.1	183.6	105.6	41.2	777	71.2	145	1.27	76.3	156	1.18
14.00FH	102.7	206.4	101.5	55.5	892	85.1	171	1.21	73.8	148	1.39
6.00FLI	27.2	227.3	92.5	27.4	696	22.9	192	1.19	29.0	243	0.94
10.20FLI	26.9	240.2	77.2	39.8	870	24.0	214	1.12	23.1	206	1.17
10.20FLO	35.1	290.2	93.6	40.8	812	28.3	234	1.24	29.6	245	1.19
9.60GH2	82.4	165.8	91.9	34.7	817	67.4	136	1.22	66.1	133	1.25
9.60GHO.5	89.8	182.7	94.8	45.5	794	66.1	135	1.36	69.9	142	1.28
9.60GH3	83.6	165.4	96.2	32.1	807	69.6	138	1.20	67.7	134	1.23
Average								1.20	Average		1.17
COV (%)								9.0	COV (%)		17.0

Some comments on interior connections transferring shear and moment:

- On average, ratios of test load to predicted load are about the same, with the Strip model producing an average of 1.20 and the ACI code producing an average of 1.17.
- The Strip Model provides safe predictions for all tests, with a minimum ratio of test load to predicted load (T/P) of 1.06. The corresponding minimum for the ACI code is 0.78. This is a large enough error on the unsafe side to be of some concern.
- The ACI code tends to overestimate the strength of more lightly reinforced specimens. All but one of the unsafe predictions are for specimens with 0.6% flexural reinforcement.
- The concept of non-proportional loading is easily adapted to design. As a design tool, the Strip Model becomes a guide for the lateral distribution of reinforcement.

3.3 Edge connections

For an edge column-slab connection with no edge beam, there are three radial strips that may develop: two spandrel strips and one interior strip. By considering how these strips are loaded, one can develop an interaction diagram for shear and moment transferred to the column; however, it will be seen that for many practical design cases, developing a full interaction diagram is unnecessary.

An additional factor to consider at an edge connection, that was not a concern for an interior connection, is the close proximity of the free edge of the slab. This will require an assessment of the anchorage of reinforcement perpendicular to the free edge.

Anchorage of reinforcement perpendicular to the free edge — with increasing eccentricity of load, the interior radial strip becomes a super strip. The negative reinforcement of this super strip will include bars perpendicular to the free edge but outside the column. Triangular-shaped wings, as shown in Fig. 8, project $1.5h$ from either side of the column. Any force generated by bars within the wings contributes to the transfer of moment to the column by flexure; however, the proximity of the free edge limits the available length for anchorage of reinforcement perpendicular to the free edge.

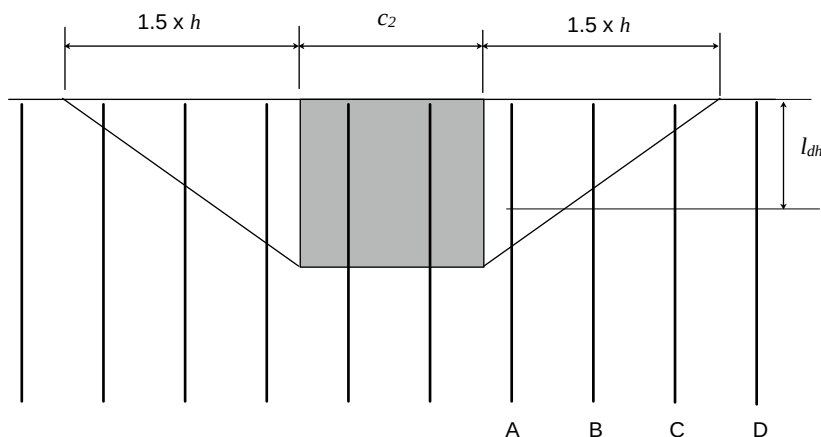


Figure 8: Bar development at an edge column-slab connection.

It is assumed here that all top bars perpendicular to the free edge will be hooked. For most circumstances, the development length for a hook, l_{dh} , is roughly half that for a straight bar. It follows that the mechanical anchorage provided by a hook develops half the strength of the bar. So, any hooked bar within $c_2 + 3h$ can develop at minimum half of its strength. How much more it can develop depends upon its embedment within the column or the triangular wings.

Bars perpendicular to the free edge but within the column will usually develop their full strength. They have maximum embedment and very favorable confinement within the column (This may not be the case for a roof slab). Bars outside of the column can still develop their full force if they have sufficient embedment beyond the diagonal line that defines the wing.

Consider the reinforcement layout in Fig. 8. With this layout, a maximum of seven bars will contribute to the negative moment of an interior super strip. This is determined as follows:

- The bars (2) within the column will develop fully.
- Bar A is expected to develop fully since its embedment past the diagonal line defining the wing is greater than l_{dh} .
- The length of embedment of bar B beyond the diagonal is approximately $0.75 l_{dh}$. It will develop approximately 88% of its full strength. Mechanical anchorage of the hook accounts for half of the bar strength. Straight bond accounts for 75% of the remaining half of the bar strength.
- Bar C has embedment of approximately $0.25 l_{dh}$. It should develop approximately 62% of its strength, made up by 50% from the mechanical anchorage of the hook and 25% of the remaining 50% by straight bond.
- Bar D is outside the band and not expected to develop.
- Summing up: 2 wings $\times$ (1 + 0.88 + 0.62) = 5 bars anchored in the triangular wings plus 2 bars within the column for a total of 7 bars.

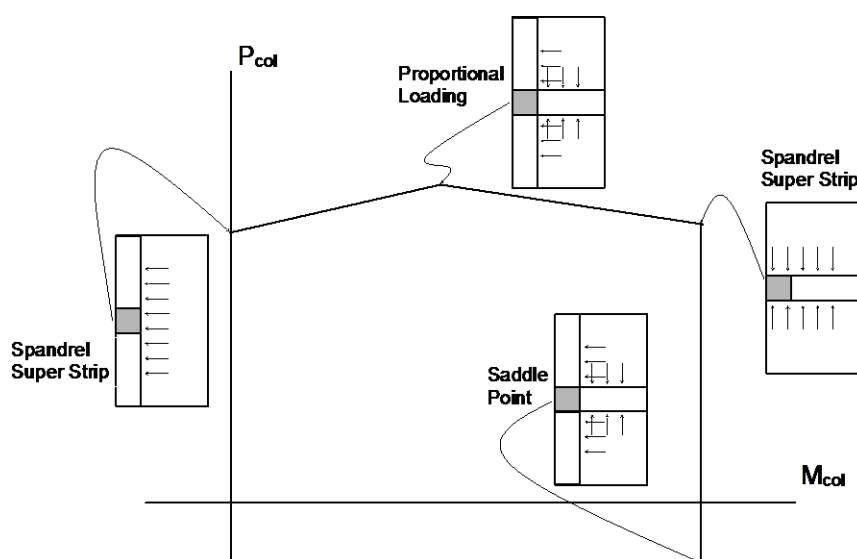


Figure 9: Conceptual interaction diagram for edge column.

Shear-moment interaction — Figure 9 shows a conceptual interaction diagram at an edge connection with moment transfer about an axis parallel to the free edge. Four points define the interaction: (1) Proportional Loading, (2) Interior Super Strip, (3) Spandrel Super Strip, and (4) Saddle Point.

1) Proportional Loading (PL)

Proportional loading occurs when each radial strip reaches its nominal capacity as defined by Equation (2) for the interior strip and Equation (4) for the spandrel strips. The total vertical load transferred to the column is simply the summation of the three nominal strip loads. The net moment about the centroid of the column is purely a function of the interior strip.

$$M_{col} = M_{neg} + P_s \times \frac{c_1}{2} \quad (15)$$

2) Interior Super Strip (ISS)

With increasing moment, reinforcement outside the column acts to reinforce an interior super strip. Equation (9) is used to estimate the total vertical load transferred to the column, with due consideration of the anchorage of bars perpendicular to the free edge. The moment about the centroidal axis of the column is:

$$M_{col} = M_{ss,neg} + P_{ss} \times \frac{c_1}{2} \quad (16)$$

3) Spandrel Super Strip (SSS)

As the moment transferred to the column decreases, the spandrel strips must carry a larger fraction of the total load. Reinforcement within a band of $c_1 + 1.5h$ is assumed effective. The load on each spandrel super strip is:

$$P_{ss} = \sqrt{2M_{ss}w} \quad (17)$$

Because there is no interior radial strip, there is no net moment transferred to the column. The total load transferred to the column is:

$$P_{col} = 2 \times P_{ss} + w \times c_1 \quad (18)$$

4) Saddle Point (SP)

Under extreme lateral load, there can be uplift in the spandrel concurrent with gravity loading on the interior strip. At the column-slab connection, the interior strip is in negative bending while the spandrel strips are in positive bending. The deformed shape is a saddle point.

How much uplift can develop in the spandrel strip is debatable. Here it is assumed that the full capacity of the spandrel strip under proportional loading can be developed in uplift. This might be a little optimistic but great precision is not called for here; this loading combination is more a creation of laboratory testing than field experience.

The moment transferred to the column is the same as for the case of the interior super strip. The total load transferred to the column is the difference between the gravity load carried by the interior super strip and the uplift load carried by two spandrel strips at nominal capacity.

Comparison to test results — Results from three separate investigations are examined here. Single-column tests reported by Zaghlool (1971) and by Hanson and Hanson (1968) cover a wide range of shear and moment combinations, providing some experimental justification for the interaction shown in Fig. 9. The two connections reported in Afhami et al. (1998), from a full-scale, two-panel specimen with rotational restraint to model flexural continuity, are much more representative of the edge column-slab connections in a real structure.

The test program by Zaghlool included four geometrically similar specimens, all with the layout of reinforcement shown in Fig. 10(a). The column was 267 mm (10.5 in) square. The slabs were 152 mm (6 in) thick with a flexural depth measured to the middle of the reinforcing mat of 121 mm (4.75 in). All reinforcing bars were US #4 (12.7 mm diameter). The average concrete strength was 33.9 MPa (4920 psi) and the average yield strength of the reinforcement was 465 MPa (67.4 ksi). The slabs were simply supported on three sides with prescribed combinations of vertical and lateral load applied through the column.

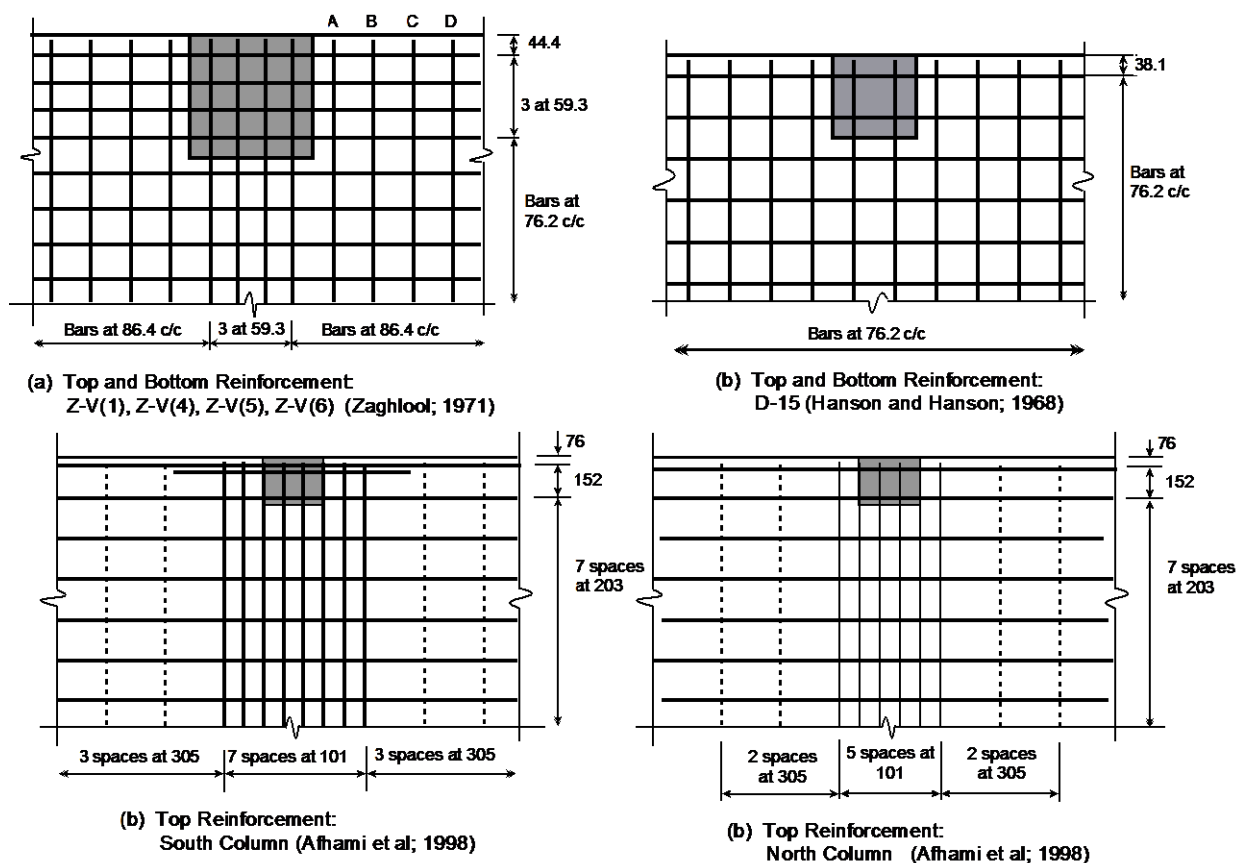


Figure 10: Top slab reinforcement for selected edge column tests.

Figure 11 shows test results compared to the calculated interaction diagram. Table 3 lists calculated values needed to generate the interaction. For all points on the interaction, the loading term, w (Eq. 8), is equal to 110 kN/m (7.54 kips/ft). Two of the interaction points are discussed in more detail below.

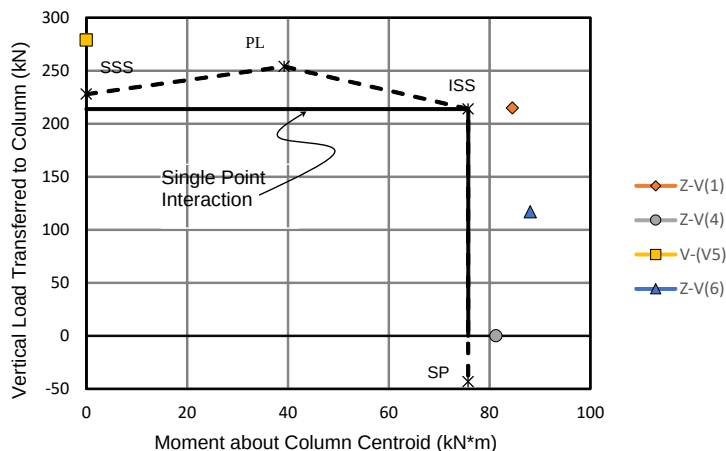


Figure 11: Interaction for tests by Zaghlool.

Table 3: Shear-moment interaction for tests by Zaghlool (1971) and Hanson and Hanson (1968)

	Point	Interior Strip					Spandrel Strip					Interaction Values	
		N_{bar}	$A_{s,neg}$ (mm ²)	M_s or M_{ss} (kN*m)	P_s or P_{ss} (kN)	One-way (kN)	N_{bar}	A_s (mm ²)	M_s or M_{ss} (kN*m)	P_s or P_{ss} (kN)	One-way (kN)	P_{col} (kN)	M_{col} (kN*m)
Zaghlool	PL	4.00	506	24.9	104.5	-	4	506	24.9	73.9	-	252.2	38.8
	ISS	8.87	1122	54.3	154.3	-	-	-	-	-	29.3	212.9	74.8
	SSS	-	-	-	-	29.3	7	882	44.2	98.4	-	226.2	0.0
	SP	8.87	1122	54.3	154.3	-	4*	506*	24.9*	-73.9*	-	6.6	74.8
Hanson and Hanson	ISS	4.85	344	6.23	39.9	-	-	-	-	-	9.7	59.3	9.27
	SP	4.85	344	6.23	39.9	-	2*	142*	2.80*	-18.9*	-	2.1	9.27

* For spandrel strips at saddle point of interaction, bottom mat reinforcement is engaged. Spandrel strip carries uplift (negative) load.

For the condition of Proportional Loading (PL), the interior strip and the two spandrel strips are each reinforced by four bars. As a result, $A_{s,neg} = 506 \text{ mm}^2$ (0.784 in^2) for both of these. Based on the average flexural depth of 121 mm (4.76 in), both interior and spandrel strips have the same flexural capacity, $M_s = 24.9 \text{ kN} \cdot \text{m}$ (18.4 ft·kips). Equation (2) predicts a capacity of 104.5 kN (23.5 kips) for the interior strip and 73.9 kN (16.6 kips) for each of the spandrel strips. This gives a total load of 252.2 kN (56.7 kips). Equation (16) gives a total moment transferred to the column of 38.8 kN · m (28.6 ft·kips).

With the development of an Interior Super Strip (ISS), reinforcement perpendicular to the free edge is anchored to the column. The hook development length for these bars is estimated at 153 mm (6 in). Following the procedure outlined above and in Fig. 8, it is estimated that 8.87 bars are effective (100% of the four bars through the column, 100% of bar A, 88% of bar

B, and 55% of bar C). Bar D is considered too remote to be effective. Summing up the effective bars, the total area of reinforcement for the interior super strip is 1122 mm^2 (1.74 in^2) and the corresponding value of M_{ss} is $54.3 \text{ kN} \cdot \text{m}$ ($40.0 \text{ ft} \cdot \text{kips}$). Equation (13) gives a value of P_{ss} equal to 154.3 kN (34.7 kips). The spandrel column faces each carry a one-shear of 29.3 kN (6.59 kips).

It is worth noting that a simplified, square interaction defined by the ISS interaction point would appear to provide a reasonable approximation of the overall interaction diagram.

Hanson and Hanson (1968) report one edge column-slab specimen tested at high eccentricity. The concrete strength was 31.2 MPa (4525 psi). The slab was 76 mm (3 in) thick with an average flexural depth of 57 mm (2.25 in). The column was 76 mm (3 in) square. Slab reinforcement is shown in Fig. 10(b). All bars were US #3 (9.5 mm diameter) with a yield strength of 366 MPa (53 ksi). The column extended above and below the slab, supported by the laboratory floor at the lower end and laterally braced at the upper end. A single line load was applied to the slab.

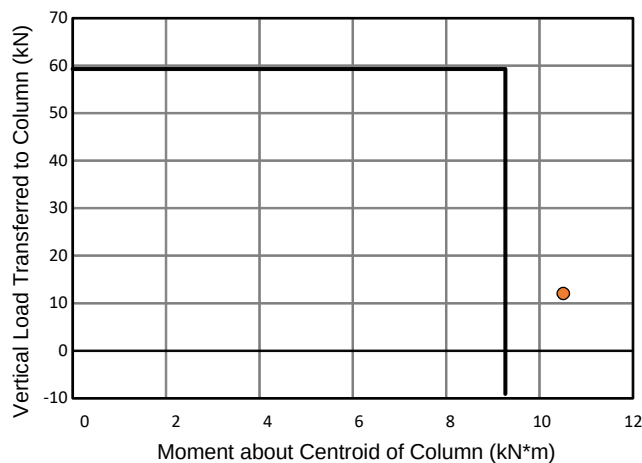


Figure 12: Interaction for test by Hanson and Hanson.

Figure 12 shows the test result plotted against a simplified interaction. The relevant part of the interaction is constructed by considering two points: the Interior Super Strip (ISS) and the Saddle Point (SP). Table 3 summarizes various calculated quantities. For both points, from Equation (8), $w = 63.8 \text{ kN/m}$ (4.37 kips/ft).

The hook development length is estimated to be 120 mm (4.7 in). Following the procedure outlined previously, the maximum reinforcement that can be developed perpendicular to the free edge is estimated at 4.85 bars (2 column bars @ 100%, 2 bars 38 mm (1.5 in) from column face @ 92.5%, and 2 bars 114 mm (4.5 in) from column face @ 50%).

From Equation (10):

$$M_{ss,neg} = A_{s,neg} \times f_y \times j_{ss}d = 346 \text{ mm}^2 \times 365 \text{ MPa} \times 0.863 \times 57 \text{ mm} = 6.23 \text{ kN} \cdot \text{m} \quad (4.59 \text{ ft} \cdot \text{kips}).$$

$$\text{Equation (13) gives: } P_{ss} = 2\sqrt{M_{ss}w} = 2\sqrt{6.23 \times 63.8} = 39.9 \text{ kN} \quad (8.97 \text{ kips}).$$

From Equation (17):

$$M_{col} = M_{ss,neg} + P_{ss} \times \frac{c_1}{2} = 6.23 + 39.9 \times \frac{.152 \text{ m}}{2} = 9.27 \text{ kN} \cdot \text{m} \text{ (6.84 ft} \cdot \text{kips)}.$$

Equation (9) applied to the edge column gives:

$$P_{col} = (\sum P_{ss}) + 2 \times w \times c_1 = 39.9 + 2 \times 63.8 \times 0.152 = 59.3 \text{ kN (13.3 kips)}.$$

At the saddle point of the interaction, the interior super strip is still active. An uplifting, proportionally loaded spandrel strip replaces the one-way shear on each spandrel face of the column. For each of these spandrel strips, M_s is estimated to be 2.80 kN · m (2.07 ft·kips) and P_s to be an upward 18.9 kN (4.25 kips). The net moment transferred to the column is unchanged at 9.27 kN · m (6.84 ft·kips). The net load transferred to the column is 39.9 kN – 2 × 18.9 kN = 2.1 kN (0.47 kips).

Afhami et al. (1998) reports test results from a full-scale, two-panel slab and column structure encompassing two edge column-slab connections (north and south) and one interior column-slab connection and measuring approximately 10 m (32.8 ft) by 4.3 m (14.1 ft) in plan. The concrete strength and steel yield strength are reported at 34.8 MPa (5050 psi) and 420 MPa (60.9 ksi), respectively. The edge columns measured $c_1 = 254$ mm (10 in) and $c_2 = 305$ mm (12 in). Hydraulic jacks with load-distributing frames simulated a uniformly distributed load (16 equal point loads in each panel). Rotational restraint was provided along the long edges (east and west) of the specimen.

The edge connections were designed for different fractions of the panel moment, resulting in the two reinforcing layouts shown in Figs. 10(c) and 10(d). All bars parallel to the free edge are 15M ($A_{bar} = 200 \text{ mm}^2$ or 0.31 in^2). The dashed lines perpendicular to the free edge represent unhooked 10M bars ($A_{bar} = 100 \text{ mm}^2$ or 0.155 in^2), added for crack control but not expected to contribute to ultimate strength. For the south connection, the solid lines perpendicular to the free edge represent hooked 15M bars. For the north connection, the solid lines perpendicular to the free edge represent hooked 10M bars.

Two results are reported for the north connection. Failure of the two-panel structure first occurred by punching at the interior column-slab connection. Just before this, there were visible signs of distress at the north connection indicating that it too was near failure. The load and moment carried at the north connection when the interior connection punched is reported as the first result.

After punching of the interior connection, slab supports were adjusted so that the two edge column-slab connections could be tested to failure. These re-tests produce the result for the south connection and the second result for the north connection.

Figure 13 shows simplified interaction diagrams for both the north and south columns. The interior super-strip is the only interaction point that need be considered to assess these tests. Calculated values are summarized in Table 4. The loading term, w , from Equation (8) is equal to 110 kN/m (7.54 kips/ft).

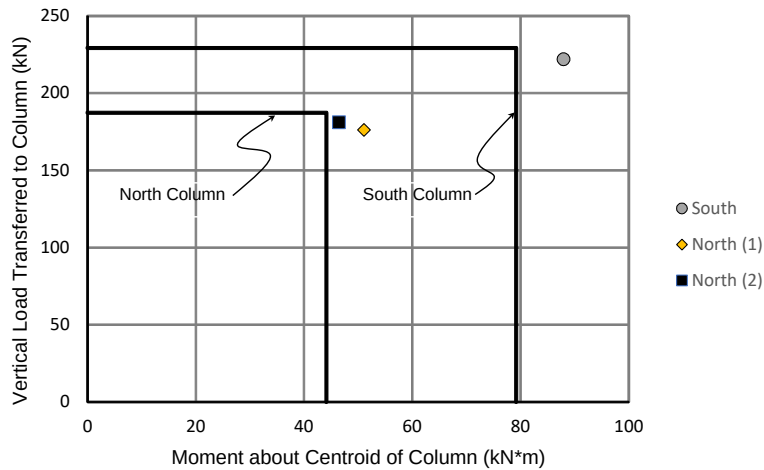


Figure 13: Interaction for tests by Afhami et al.

Table 4: Shear-moment interaction for Tests by Afhami et al. (1998)

	Point	Interior Strip					Spandrel Strip One-way (kN)	Interaction Values	
		N_{bar}	$A_{s,neg}$ (mm ²)	$M_{ss,neg}$ (kN*m)	$M_{s,pos}$ (kN*m)	P_{ss} (kN)		P_{col} (kN)	M_{col} (kN*m)
North	Interior SS	6	600	27.3	15.3	136	27.5	191	44.5
South	Interior SS	6.9	1380	57.1	12.2	173	27.5	228	79.0

The hook development lengths for 15M and 10M bars are estimated to be 190 mm (7.48 in) and 134 mm (5.28 in), respectively. For the south connection, the total number of effective 15M top bars reinforcing the interior super strip is estimated to be 6.9 (four bars through the column, 87% of the first bar outside the column, 58% of the second bar outside the column). For the north connection, all six 10M hooked bars perpendicular to the free edge are considered fully effective.

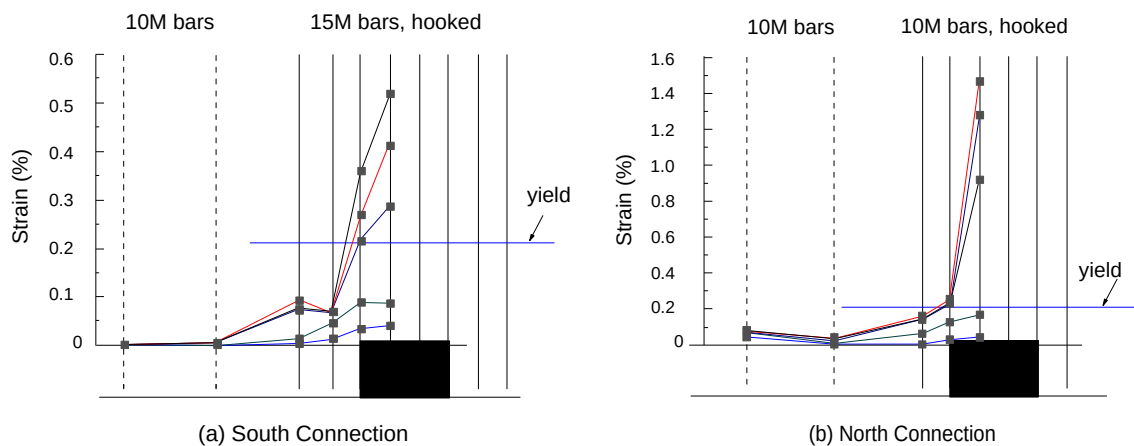


Figure 14: Measured strains in slab top reinforcement at edge column.

Figure 14 shows measured strains in reinforcement perpendicular to the free edge for both the north and south connections. In both cases, the measured strains are in reasonable agreement with the predicted effectiveness of the bars.

Because the slab was continuous, the flexural capacity of the Interior Super Strip (ISS) includes a contribution, $M_{s,pos}$, from bottom steel. The edge columns were designed to take different fractions of panel moment and the bottom reinforcing mats were adjusted accordingly. The values of $M_{s,pos}$ for the north and south panels are 15.5 kN·m (11.4 ft·kips) and 12.2 kN·m (9.00 ft·kips), respectively.

Some comments on edge column-slab connections:

- The Strip Model analysis results shown in Figs. 11 through 13 are in good agreement with test results.
- For combinations of negative moment (tension in top steel) and gravity load transferred to a column, there is no substantive interaction between moment and shear. The ISS point on the interaction diagram is all that need be considered. This is likely not true for load cases that involve reversals of either load or moment transfer.
- An important design issue at the edge column-slab connection is the anchorage of reinforcement so that it will be effective in transferring moment to the column by flexure. Placing reinforcement within a band $c_2 + 3h$ is by itself not sufficient to ensure full development.
- Reinforcing for any load case that increases the demand for transfer of negative moment to the column will automatically increase the shear capacity of the connection.

4 Conclusion

The Strip Model describes a load path for the transfer of vertical load from a slab to a column. The various mechanisms of load transfer assumed to be acting within the load path are consistent with observations of tests. The model fully explains two-way behavior using capacities that can be determined from tests of one-way members.

The load path described in the Strip Model is easily adapted to a variety of test configurations. The calculated capacity of the connection using the Strip Model agrees well with test results for a variety of loading conditions.

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Size effect on punching shear strength of RC slabs with and without shear reinforcement

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Abstract

Comparison of various design codes reveals major differences among the design provisions for punching shear, especially with respect to the size effect. This indicates the need for deeper analysis of the existing test data supplemented by realistic finite element (FE) analysis. This study presents a refined statistical analysis of the ACI-445 database comprising 440 punching shear tests, and an FE analysis based on concrete microplane model M7 calibrated by test data. Computer filtering of the database is used to create data subsets in which the averages of secondary variables, such as the steel ratio and shape parameters in subsequent size intervals, are almost constant. The resulting trend of the mean punching shear strength v_c clearly reveals that the slope of the diagram of $\log v_c$ versus $\log d$ is milder, but not much milder, than $-1/2$, and that the trend does not disagree with the theoretically well justified energetic size effect law (endorsed for shear failures by ACI Committee 446). A new design equation with a size effect factor, emulating previous equations in many respects, is proposed. The equation is verified and calibrated by nonlinear least-square multivariate regression of the database, with weights compensating for the crowding and scarcity of data in various parts of the range. The size effect and other trends are also verified by finite element fitting of selected data series with a broader range. The size effect factor validated here can be applied to improve any design equation missing the size effect, including a plastic limit analysis equation, provided it fits the small-scale test data well (for which the size effect factor is defined as 1).

Keywords

Design codes, failure, finite element analysis, optimum data fitting, punching shear, quasibrittle fracture, reinforced concrete, size effect, statistical analysis.

1 Introduction

Although concrete is not a plastic material, the shear strength of beams and slabs has traditionally been analyzed according to plastic limit analysis. In this analysis, as well as in elasticity with a strength limit, the nominal strength, v_c , of geometrically similar structures, is independent of structure size d . This is the case of no size effect. But if the structural failure is due to fracture or localization of cracking damage, which is typical of concrete, v_c decreases with d , which is called the size effect. The size effect is of two kinds:

1) statistical, due to material strength randomness, which occurs when a macrocrack initiates dynamically from one of many possible places of random strength, as described by the weakest link model and Weibull theory; and

2) energetic (i.e., deterministic), which occurs when a large crack grows in a stable manner prior to reaching the maximum load (in which case the material randomness affects only the coefficient of variation of v_c and the size effect is due to increase of energy release rate with structure size). The shear failures of reinforced concrete are generally of Type 2, which is the type occurring in structures with a deep notch or a deep stress-free crack formed stably before reaching the maximum load (while the Type I size effect is that which occurs in structures that fail right at crack initiation from a smooth surface) (Hoover and Bažant, 2014).

The plastic limit analysis, which underlies the concrete design codes, gives realistic results for relatively small structures used in most laboratory testing. The reason is that the size l_0 of the fracture process zone is in concrete very large—about 0.5 m (compared to about 1 μ m for metals or fine grained ceramics). This implies that, at maximum load, the distributed cracking cannot localize into one dominant crack prior to maximum load, and thus the size effect must be negligible for small structures. The size effect becomes strong only when the structure is sufficiently larger than l_0 . Consequently, extending the current code provisions to large sizes is relatively easy—it suffices to multiply the v_c value according to the current code, based on limit analysis, by the proper size effect factor, which is about 1 for small structure sizes. Here it is proposed how to do it for punching shear.

The ACI-445C database, refined for punch failures, is used in this study; see Fig. 1a-d (Ospina et al., 2011). The figure shows the nominal punching shear strength

$$v_c = \frac{V_c}{b_0 d} \quad (1)$$

normalized by the mean concrete strength f_c as a function of the slab depth d , in comparison with the curves of $\log v_c / \sqrt{f_c}$ (for Eurocode $\log v_c / f_c^{1/3}$) versus $\log d$ according to four design codes. Note that the strength in Eq. (1) is not the design strength, since the material safety factors are not included.

The database for the punch failures, repeated in all four diagrams in Fig. 1, contains the results of 440 tests reported in 60 experimental studies conducted in many laboratories throughout the world (Ospina et al., 2011). The slabs had square, rectangular, circular or octagonal simply supported boundaries. The depths, d , in the database range from 30 to 668 mm (1.18 to 26.3 in.); the mean concrete strength f_c from 8 to 118 MPa (1160 to 17114 psi), and the longitudinal reinforcement ratio ρ from 0.1% to 7.3%. The data points in Fig. 1 show very high scatter. But the scatter is due not only to inevitable randomness but also, and largely, to inevitable sampling bias.

The objectives of this study are to:

- 1) filter the inevitable sampling bias from the existing experimental database (Ospina et al., 2011) to reveal the basic size effect trend;
- 2) propose an improved equation for v_c that includes the size effect in order to avoid excessive or insufficient safety factors (Bažant and Yu, 2006); and
- 3) calibrate and validate the size effect by weighted multivariate regression.

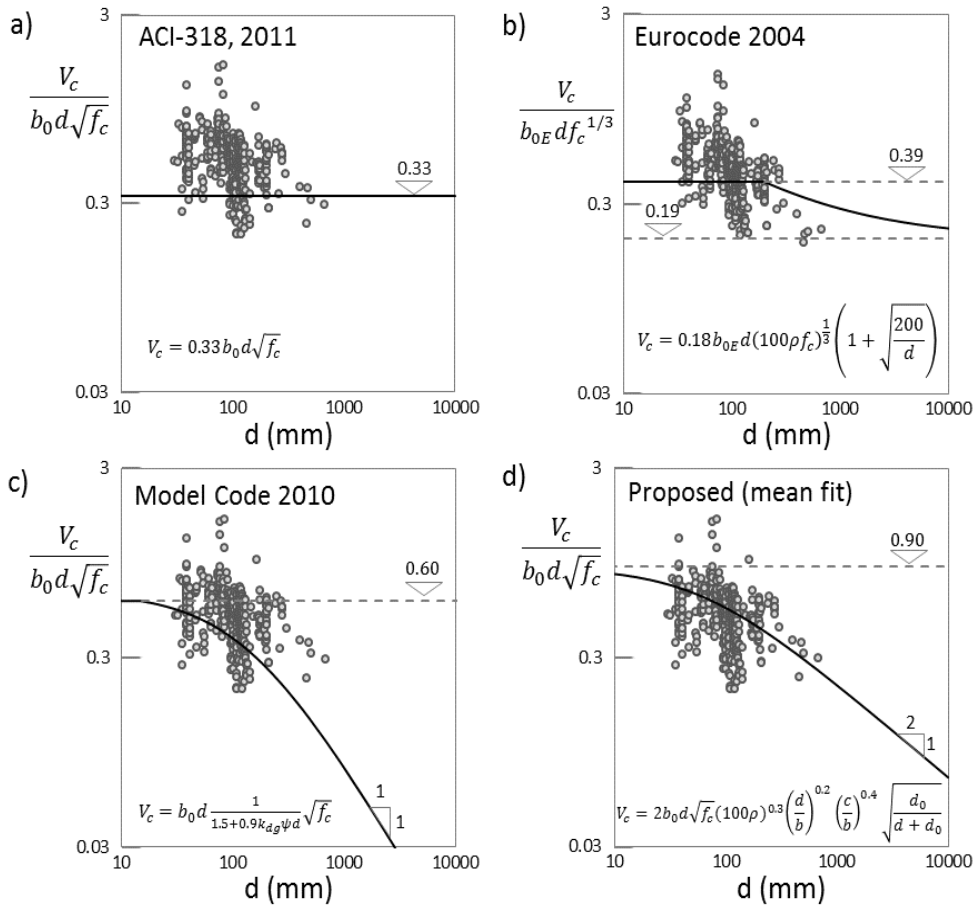


Figure 1: Normalized strength comparisons among the provisions of a) ACI (x12 for psi), b) Eurocode (x5.25 for psi), c) fib Model Code (x12 for psi), and d) proposed (x12 for psi).

2 Punching shear in current design codes

For the punching load capacity, V_c , due to concrete, the current standard ACI-318 specifies the formulas (ACI, 2011):

$$V_c = \lambda b_0 d \sqrt{f_c} \quad \text{for no shear reinforcement} \quad (2)$$

$$V_c = \lambda b_0 d \sqrt{f_c} + A_{sw} f_{yw} \frac{d}{s_w} \quad \text{with shear reinforcement} \quad (3)$$

where f_c is in MPa for SI (or in psi for USC units); b_0 = control perimeter of loading area (or column); A_{sw} = cross sectional area of one shear reinforcement layer around the column; f_{yw} = yield strength of shear reinforcement; $\lambda = 1/3$ for SI units (4 for USC units) if there is no shear reinforcement, $1/4$ if the shear reinforcement is provided by studs (3 for USC units), and $1/6$ by stirrups (2 for USC units); s_w = distance between shear reinforcement layers.

The Eurocode 2 (2004) specifies the formula:

$$V_c = \lambda \xi (100 \rho f_c)^{1/3} b_0 d \quad \text{for no shear reinforcement} \quad (4)$$

$$V_c = 0.75 \lambda \xi (100 \rho f_c)^{1/3} b_0 d + A_{sw} f_{yw,e} 1.5 \frac{d}{s_w} \quad \text{with shear reinforcement} \quad (5)$$

$$\xi = 1 + \sqrt{\frac{200}{d}} \quad (6)$$

where ρ = flexural reinforcement ratio, and $\lambda = 0.18$ (or 5 for USC units), which must be considered to have the dimension of $(\text{MPa})^{2/3}$ if f_c is in MPa (note in Eq. 5 that, in case of shear reinforcement, λ is multiplied by 0.75).

The *fib* Model Code 2010 (“MC2010”; *fib* [2013]) specifies the formula:

$$V_c = k_\psi \sqrt{f_c} b_0 d \quad \text{for no shear reinforcement} \quad (7)$$

$$V_c = k_\psi \sqrt{f_c} b_0 d + \sum A_{sw} \sigma_{sw} \quad \text{with shear reinforcement} \quad (8)$$

where σ_{sw} = stress in shear reinforcement and

$$k_\psi = \min \left(0.6, \frac{1}{1.5 + 0.9 k_{dg} \psi d} \right) \quad (9)$$

where k_ψ must be considered to have the dimension of $\sqrt{\text{MPa}}$ if f_c is in MPa; d_g = maximum aggregate size in mm, and $k_{dg} = 32/(d_g + 16 \text{ mm})$; ψ is the slab rotation defined in CSCT (Muttoni and Schwartz, 1991) for different cases; for the most practical case $\psi = 1.5(r_s/d)(f_{yd}/E_s)$ in r_s = distance to zero radial moment and can be taken as 0.22 times the slab span. It should also be stated that there are 4 levels of approximations to calculate the slab rotation, ψ , and the simplest one, which is the LoA I, is used in here. The accuracy of the MC2010 provision depends, of course, on the accuracy of the rotation calculations.

These three code formulas are compared (for the case of no shear reinforcement) in Fig. 1a-c with the raw (unfiltered) ACI-445C database, in which the measured values of v_c divided by $\sqrt{f_c}$ for ACI and MC2010, and by $f_c^{1/3}$ for Eurocode, are plotted versus d in logarithmic scales. For ACI-318, the formulas give a horizontal line, i.e., there is no size effect. For Eurocode 2004, the size effect term ξ in Eq. (6) approaches a horizontal asymptotic slope for $d \rightarrow \infty$. For MC2010, there is size effect given by factor k_ψ in Eq. (7), which has, for $d \rightarrow \infty$, an asymptote proportional to d^{-1} , i.e., an asymptote of slope -1 in a log-log plot. Fig. 1d shows the size effect proposed here, discussed later. In plotting these size effect curves, all the parameters other than d had to be fixed, and they were fixed at values equal to the average values in the database.

3 Filtering of database to remove bias in secondary variables

Consider that the size range is subdivided into several intervals of constant width in $\log d$ (five in Fig. 2). The averages of steel ratio ρ and ratios b/d and c/b calculated separately for each size interval, should ideally be about the same for all the size intervals (Bažant and Yu, 2008). But, as seen in the rows III to V of column 1 (c.1) in Fig. 2, the averages of ρ , b/d , c/b , vary significantly through the intervals; ρ decreases with increasing size even by an order of magnitude and b/d decreases more than 3 times. Also note in Fig. 2a, row II, that the interval averages for the entire database (shown solid diamond points) are highly scatter and do not show a clear trend.

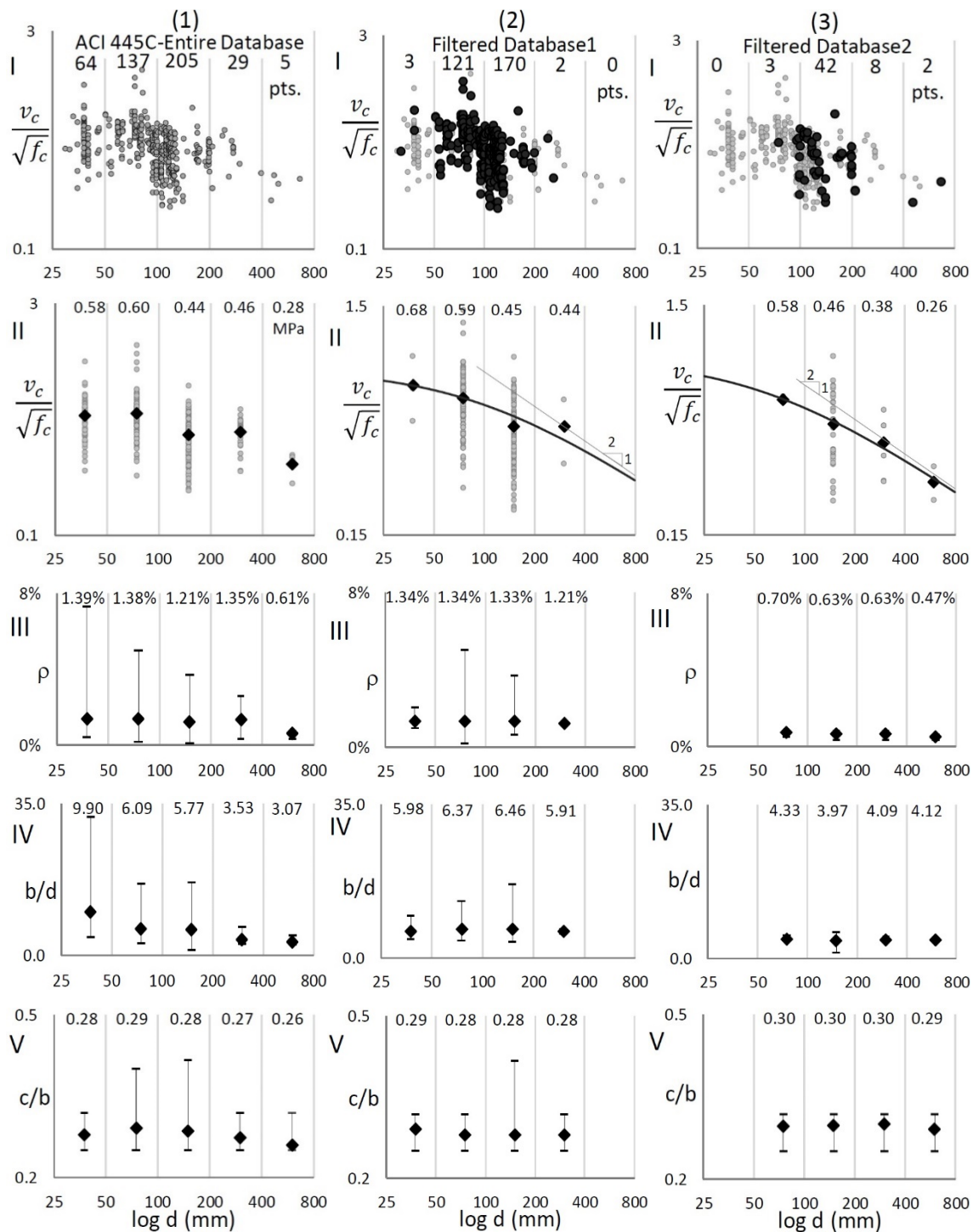


Figure 2: Column 1) entire database; columns 2–6) filtered database and mean values of longitudinal reinforcement ratio (ρ), aspect ratio (b/d), shape factor (c/b) and their variations inside the size intervals; ($v_c = P/(b_0 d \sqrt{f_c})$) vs. d plots are in log scale; P = maximum punching load; b = perimeter of the column; b_0 = control perimeter of the column according to ACI; d = slab effective thickness; f_c = concrete compressive strength (for cylindrical specimens); c = side length (or diameter) of the column (figure continues on next page).

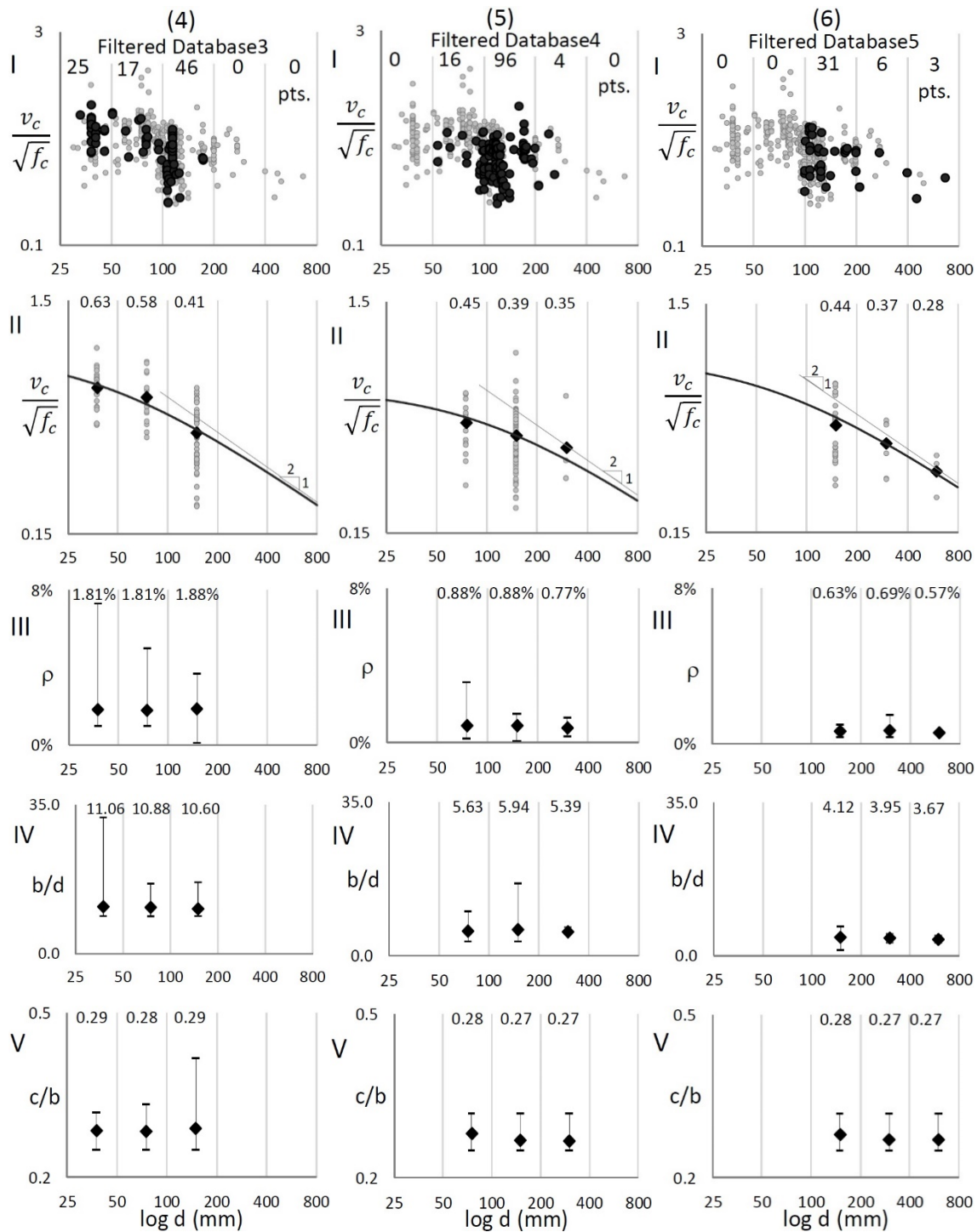


Fig. 2 (cont.): Column 1) entire database; columns 2–6) filtered database and mean values of longitudinal reinforcement ratio (ρ), aspect ratio (b/d), shape factor (c/b) and their variations inside the size intervals; ($v_c = P/(b_0 d \sqrt{f_c})$) vs d plots are in log scale; P = maximum punching load; b = perimeter of the column; b_0 = control perimeter of the column according to ACI; d = slab effective thickness; f_c = concrete compressive strength (for cylindrical specimens); c = side length (or diameter) of the column).

In previous work (Bažant and Yu, 2008), a filtering program was developed to delete gradually, in an unbiased way (without human intervention), the extreme points in each size interval, so as to achieve a nearly uniform variation of the secondary parameters throughout the size intervals. The resulting interval averages, obtained with this procedure, are shown, by the solid diamond points in rows III–V of c.2-5. As seen from the diamond points in c.2-5 of row II, a clear size effect trend has emerged. The filtering lead to clear trends of size effect on $v_c/\sqrt{f_c}$. As seen, the filtering (with objective 1) leads to a clear size effect trend (and so would the filtering with objective 2). The trend invalidates the size effect in Eurocode, as well as the size effect in MC2010. There is no hint of a slope steeper than $-1/2$, and certainly not a slope approaching -1 , as implied by MC2010.

But there are also some drawbacks. The main one is that the filtered database may have lost too many points from the original database. This could be remedied in future studies by creating many database subsets with different levels of secondary parameters. Another is that most of the slab sizes in the filtered databases are below the sizes in practical use. These drawbacks are avoided by multivariate regression analysis, which is discussed next.

4 Proposed shear strength equation

Based on the previous experience reviewed in Ospina et al. (2011) and on the energetic size effect law, which was shown to be generally applicable to quasibrittle failures and was endorsed by ACI Committee 446 (Bažant et al., 2007), the following equation is proposed:

$$V_u = V_c + V_s, \quad V_c = b_0 d v_c \quad (10)$$

$$v_c = v_0 \theta, \quad v_0 = \lambda \sqrt{f_c} (100\rho)^{0.3} \left(\frac{d}{b}\right)^{0.2} \left(\frac{c}{b}\right)^{0.4} \quad (11)$$

$$\theta = \frac{1}{\sqrt{1+d/d_0}} \quad (12)$$

$$V_s = \lambda_s A_{sw} f_{yw} \left(\frac{d}{s_w}\right) \left(\frac{c}{b}\right)^{0.3} \quad (13)$$

Here v_0 = value of v_c for vanishing size d ; θ = size effect factor (Bažant [1984], Bažant and Planas [1997]) unanimously endorsed for shear failures by ACI Committee 446 (Bažant et al., 2007); d_0 = transitional size (empirical, 60 mm according to database regression); V_u = ultimate punching capacity (or maximum load); V_c , V_s = ultimate punching capacities due to concrete and to shear reinforcement, respectively; ρ = flexural reinforcement ratio calculated for two-way slabs as the geometric average of reinforcement ratios for each direction, i.e., $\rho = \sqrt{\rho_x \rho_y}$; A_{sw} = cross sectional area of one shear reinforcement layer around the column; s_w = spacing between shear reinforcement layers around the column, measured in radial direction; λ = empirical constant = $2.0 \sqrt{\text{MPa}}$ in SI (or $24.1 \sqrt{\text{psi}}$ in USC units); λ_s = empirical dimensionless constant = 0.9 for both in SI and American units. The exponent values 0.2, 0.3 and 0.4 of (d/b) and (c/b) were obtained by least-square optimization of the fit of entire database, as described later.

5 Verification and calibration by weighted multivariate regression for slabs without shear reinforcement

The maximum load in punching shear is reached only after stable growth of a large crack, and the location of the crack tip line at maximum load is determined by mechanics. The volume of concrete where mechanics permit the crack tip to be located is much too small compared to the volume of structure. Thus, although the material strength and fracture energy are random, the possible crack tip locations are so restricted by mechanics that a large size-dependent volume fraction of the structure cannot be sampled by the structure (Bažant, 2002).

Therefore, the size effect cannot be statistical. Rather, it must be energetic, i.e., deterministic, and must follow the size effect law of quasibrittle fracture mechanics (Bažant [1984], Bažant and Planas [1997], Bažant [2002]) as expressed by the size effect factor θ in Eq. (12). The analytical form of this factor is here supported by the fits of mean trends of the filtered subsets of database; see the solid curves Fig. 2, row II, c.2-6.

By contrast, the size effects in Eurocode 2004 and in MC2010, seen in Fig. 1b-c, would not fit the interval means in Fig. 2 row II, c.1-6, at all. Besides, the finite asymptotic value of the Eurocode size effect (Fig. 1b) is theoretically impossible, according to fracture mechanics. So is the asymptotic slope of -1 in Fig. 1c in the size effect of MC2010 (which is shared with the previous Swiss design code). This slope even violates thermodynamic restrictions.

The results of the weighted regression of the entire database are shown in Fig. 3a, in which the database points represent the values of normalized shear strength v_c divided by v_0 , where v_c are the measured punching strength values and v_0 are the values from the optimized fit by Eq. (11). This is another way to eliminate the effect of variation of secondary variables with d , but, of course, it meets success only if a realistic form of the formula, with a low coefficient of variation of errors, is found. Fig. 3b (in which the solid diamond points show the interval averages) shows that the ACI-446 size effect factor θ (solid curve) matches the data trend well (Bažant et al., 2007). The transition to asymptotic slope -1/2 was for shear failures experimentally verified also by Ruiz et al. (2015), and was supported by a different type of analysis.

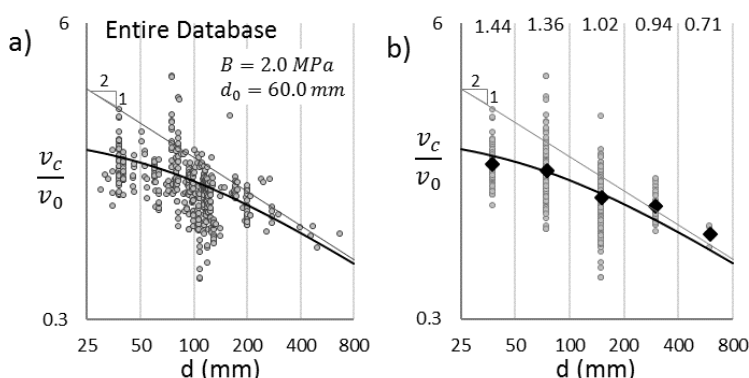


Figure. 3: a) Illustration of the size effect fit on entire database, normalized by the secondary variables found by multivariate regression, b) size-effect fit for the normalized mean strengths of subdivided data into size intervals.

A comment should be made on the invisible capacity reduction (or ‘understrength’) factors which are embedded in the design equations of the design codes, to provide additional (covert) safety. These factors scale down the design equation from the mean fit of data to the lower margin of the data cloud (Bažant and Yu, 2006). To conform to this practice, the value of v_c given by Eq. (10) would have to be scaled down, in this case by a factor of about 2.1 (however, it should be noted these hidden safety factors create a problem for probabilistic structural analysis; if the mean of the test data is not known to the analyst, the probabilistic analysis is meaningless).

6 Microplane FE analysis of test series with different slab sizes

There nevertheless exist a few punching test series where the size was varied significantly (Fig. 4a-d). The data from each of these test series have been optimally fitted by a finite element (FE) program with a realistic damage constitutive law. In this program, the constitutive law was the microplane model M7 (Bažant and Caner, 2013) (which is the latest and most realistic in a series of microplane models for concrete developed at Northwestern University). Model M7 is robust, always convergent, and has been successfully used in dynamic problems with $> 3 \cdot 10^7$ unknowns. Here all the simulations are conducted with commercial software ABAQUS, in which model M7 is introduced in VUMAT as a user’s material subroutine. To avoid spurious mesh sensitivity, the crack band model is used as the localization limiter (Bažant and Oh, 1983). To avoid some loss in accuracy due to scaling of the post peak (Červenka et al., 2005), a constant element size, h_c (approximately equal to double the maximum aggregate size) is used in all the present computations. Linear hexahedral or tetrahedral elements are chosen. The adjustable parameters of M7 sufficed to match the main material properties.

After calibration by fitting of the available punching test data of each test series (Bažant and Cao [1987], Regan [1986], Guandalini et al. [2009], Li [2000]), model M7 was used to calculate the values of v_c/v_0 for various sizes. Compared to the highly scattered individual test data shown by circles in Fig. 4, the calibrated FE results, shown by the x-points in the figure, have the advantage of no scatter. Overall, Fig. 4a-d demonstrate a significant transitional size effect on the punching shear strength, agreeing with the size effect law and with finite element simulations based on microplane model M7.

Although the shear reinforcement of slabs does not enhance ductility, it changes in the cracking pattern and significantly elevates the load and deformation capacity because it intersects and suppresses the conical fracture produced by the punching load from the column. There is a variety of transverse reinforcement types, such as stirrups, studs and assorted profiles, and a variety of reinforcement layouts around the column. This increases uncertainties in predicting the punching load capacity.

Like in shear of beams, the contribution of concrete and the contribution of steel reinforcement, calculated under the assumption of yielding, are considered to be additive (Elgabry and Ghali, 1990) (Eq. 3). However, it is not guaranteed, and is in fact doubtful for thick slabs, that the steel would actually yield at maximum load rather than later in post peak softening. Moreover, the failure of shear reinforcement can take different forms, such as the punch failure outside the shear reinforced area, delamination, and concrete crushing. Investigation of these diverse forms and their effects on the size effect curve is beyond the scope of this study.

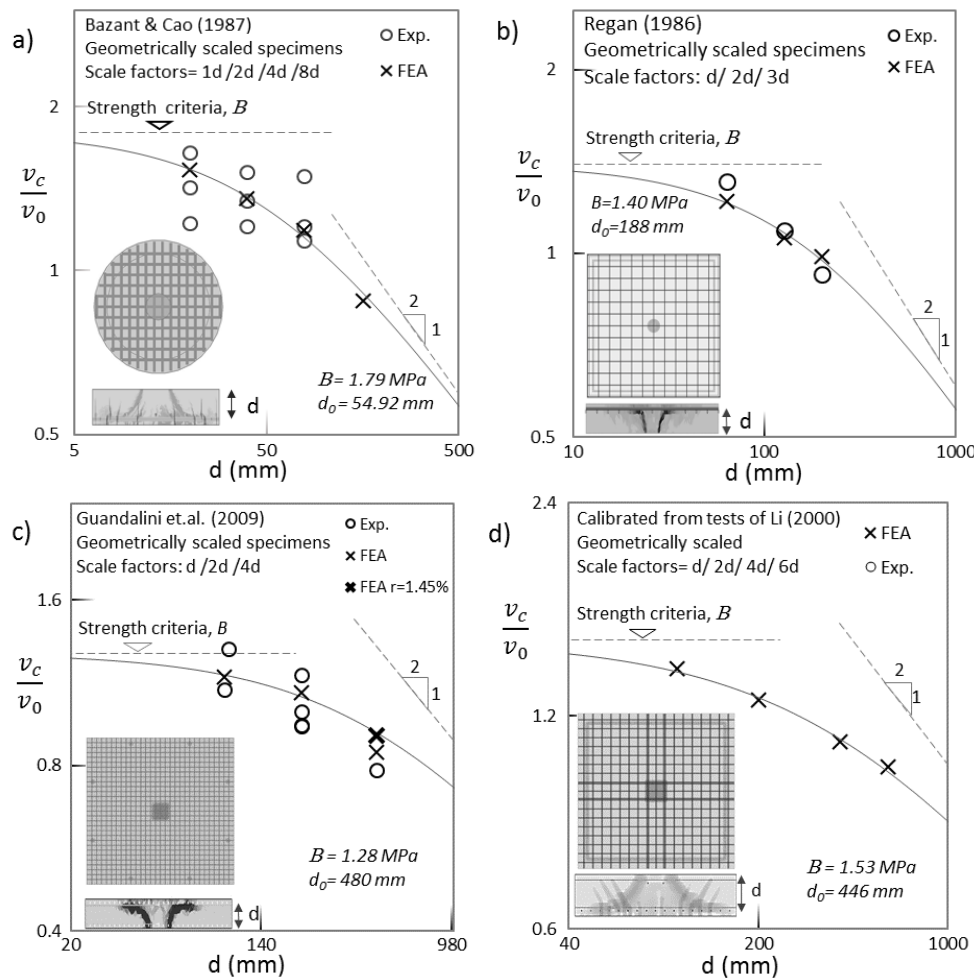


Figure 4: Size effect analysis of tests by a) Bažant and Cao (1987), b) Regan (1986), c) Guandalini et al. (2009) and created specimen with 1.45% rein. ratio for the size effect convenience, and d) FEA of calibrated test data from Li (2000), and a sample of corresponding fracture patterns for every sets obtained from FEA (left bottoms).

Unfortunately, there are no tests clearly demonstrating the size effect for slabs with shear reinforcement. Lips et al. (2012) and Birkle (2004) tested slabs with and without shear reinforcement but the size range was too narrow compared to inevitable scatter. To reveal the size effect, the shear reinforcement ratio, which is defined as $\rho_s = A_{sw}/bs_w$, should be constant, but in these and other tests, it varied with d , and so did the aspect ratio and even the concrete strength (here A_{sw} = shear reinforcement area in the first layer from the column face, b = column perimeter, and s_w = radial distance between the shear reinforcement layers).

Figure 5a-b shows the predicted size effect simulations calibrated from the tests by Birkle (2004) and Lips et al. (2012) for the slabs with and without shear reinforcement (solid and empty cross points in the Figs). As shown in Fig. 5, the effect of studs is to shift the size effect curve upwards. Note also that the studs, with the ACI-type layout (Fig. 5), greatly increase the punching capacity.

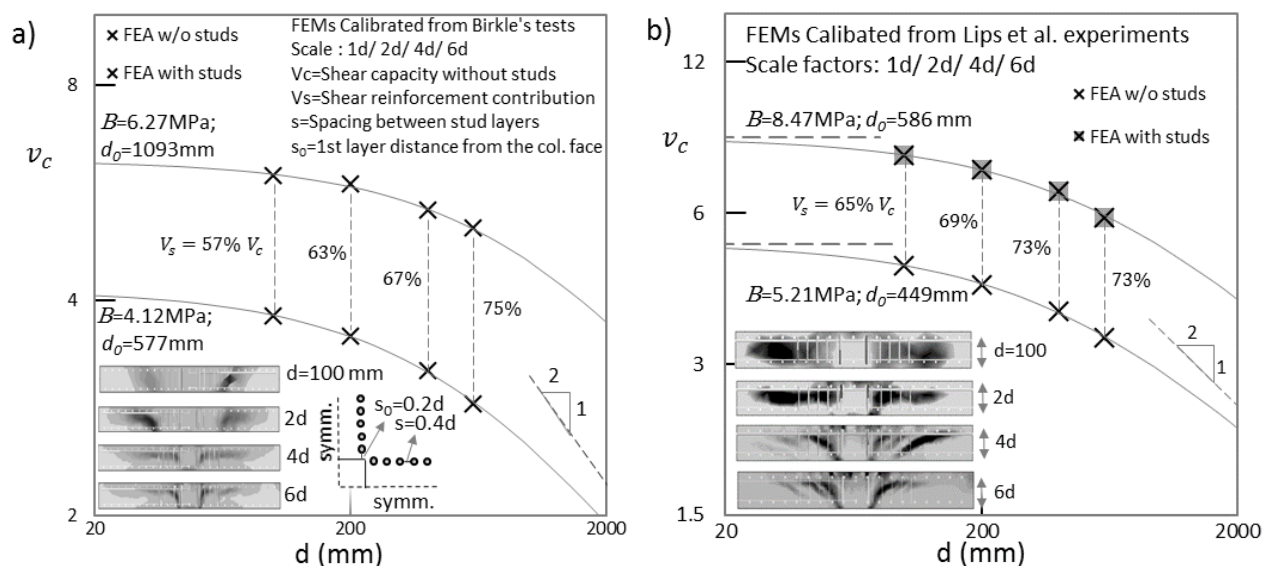


Figure 5: a) Size effect regression curves of FE models (with and without studs) calibrated by tests of Birkle (2004), b) Size effect regression curves of FE models (with and without studs) calibrated by tests of Lips et al. (2012).

7 Conclusions and key observations

- 1) The data trend revealed by filtering confirms that there is a significant size effect on the nominal punching shear strength v_c of concrete. This trend reveals that: a) there is no indication of a sudden slope change on the size effect curve (unlike Eurocode 2004); b) the average slope of the data trend of $\log v_c$ vs. $\log d$ is milder, though not much milder, than $-1/2$; c) there is no hint of a slope approaching -1 , not even steeper than $-1/2$; and d) the data are compatible with the energetic size effect law.
- 2) The least-square multivariate regression of the ACI-445C database with weights countering uneven data distribution indicates that the proposed formula with the ACI-446 size effect factor gives a relatively good fit.
- 3) Including the effects of geometry (d/b , c/b) and of the longitudinal and shear reinforcement ratios in the test data fitting broadened the support of the analytical model. Nevertheless, the extensive previous justifications of the size effect factor θ , obtained and validated here, lend further credence to the model.

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Behavior and performance levels of reinforced concrete slab-column connections

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Abstract

Information is summarized on the behavior, evaluation and design of slab-column connections and flat plate/flat slab systems for three levels of structural performance: strength, strength combined with ductility, and strength combined with ductility and robustness.

Keywords

Concrete design codes, disproportionate collapse, ductility, flat plates, flat slabs, performance levels, punching shear, robustness, slab-column connections

1 Introduction

A primary structural design requirement for two-way flat plate and flat slab reinforced concrete systems is to assure satisfactory performance in shear of the slab regions adjacent to and surrounding the columns. In the so-called punching shear failure mode the column punches through the slab in the vicinity of the column perimeter. The failure surface is an inverted truncated pyramidal to conical shape defined by an inclined crack extending upward from the intersection of the column perimeter and the bottom of the slab. Punching shear requirements, along with control of flexural deflections, very often set the minimum adequate thickness of the slab, and have a large impact on the economics of the slab system.

Although the punching failure issue may appear to be simple, the factors affecting the strength and performance of a slab-column connection are many. The punching failure mode is strongly related to the flexural behavior of the slab, especially in the region near the column. Although several models have been proposed to approximate observed behaviors, most research studies of the punching shear problem have been based on experimental tests and empirical fits to those results. Unlike the situation for a beam in shear, it is difficult to define the punching shear problem as involving a well-described isolated element, as it often governs the strength and ductility of the entire slab system of which it is a part, even when current design provisions are satisfied. Local failures around slab-column connections can also be the initiating event for excessive disproportionate damage and progressive collapse.

Designers can have different expectations for the performance of slab-column connections, depending upon the functions of the slab, the loading conditions, and the evolution of design expectations. Consistent with general trends in structural design, initial design requirements were concerned with strength only. The development of seismic design was among the factors leading to increased design attention to ductility as well as strength. The ability of a structure to survive local failures or distress without experiencing disproportionate damage is now receiving increased attentions, especially for critical and iconic structures.

The purpose of this paper is to examine the primary behavior of a reinforced concrete slab and the associated slab-column connection regions for the three primary performance levels, along with an overview of the research work and design provisions for each of these performance levels.

1.1 Performance levels for slab-column connection regions

The performance levels are: (1) strength, (2) strength with ductility, and (3) robustness. Over time, these performance levels have generally been addressed in design provisions in this same order.

The design objective for the strength performance level is that significant flexural distress should precede any punching event. The design is to be controlled by flexure rather than shear. This was the performance expectation first adopted in slab-column research and the formulation of design provisions. Test results showed that flexural and shear response are related, and that the separation of failure modes into either shear or flexure is simplistic. A punching failure usually occurs even if the slab system first experiences significant distress in flexure. The performance of the slab-column connection can limit the attainment of the full flexural strength of the slab system.

Ductility is desired in structural systems for several reasons, including as a visual warning of distress. Ductility has become of greater significance with the advent of seismic design procedures and with our understanding of the need for larger deformations to permit redistribution of moments and attainment of the full flexural capacity of a slab.

Robustness (resiliency) is the ability of a structure to survive after the occurrence of a traditional failure at one or more critical locations. At the robustness performance level, this initial failure, which might be due to an excess large local loading (including explosive loadings), a construction error, or other shortcoming, should not lead to disproportionate damage. The residual failed area and the region around it should be able to redistribute the forces involved so that progressive collapse does not occur.

1.2 ACI and ASCE technical committees addressing slab shear and slab-column connections

The impact that slab-column connection regions can have on the structural performance of a slab system has resulted in several ACI technical committees addressing different portions of the overall punching shear phenomenon, sometimes without much coordination. This has now resulted in some duplication of effort, some conflicting recommendations (Ghali et al., 2015; ACI-ASCE Committees 352 [2011], 421[2010], and 426 [1974]), and an overall complicated evolution of committee jurisdiction for the topic of shear in slabs.

Historically, the ACI design provisions for punching shear originated, along with provisions for beam shear, with the technical committee that addresses the shear performance of elements and which is now designated as ACI-ASCE Committee 445, Shear and Torsion (previously ACI-ASCE 326 and 426). The basic two-way slab shear strength provisions in the ACI 318-14 Code largely reflect the strength performance level. The design provisions are intended to assure that punching shear is not the initial mode of distress for a two-way slab system. Ductility, shear reinforcement, and the interaction of flexural and shear modes are among the topics also addressed by ACI-ASCE 445. It has long been noted that for gravity loads, tests of slab systems and multi-panel slab models have shown that punching often occurs soon after the computed flexural strength has been reached and some yielding or flexural distress has been observed. A question increasingly being asked is how much slab deflection and slab-column connection ductility is needed before a “so-called” secondary failure, a flexure-driven punching failure, limits the slab performance.

The performance of slab-column connections can be considered a logical extension of the extensive work on beam-column connections carried out by ACI/ASCE Committee 352, Joints and Connections in Monolithic Concrete. This committee’s work on beam-column connections addresses joint ductility and seismic design concerns, with emphasis on the confinement reinforcement and anchorage of flexural reinforcement in the connection core, the volume common to the beam and the column. Beyond the column face, the Committee 352 provisions are heavily dependent upon the beam shear provisions. The primary performance level of interest in the Committee 352 work is that of ductility (together with strength). The portions of the connection area addressed by the Committee 352 work for slabs differs substantially from that for beam-columns in that most of the connection area of interest is outside of the connection core, the volume common to the slab and the column. Because of its confinement by the slab adjacent to the column, the punching shear mode is effectively a connection, rather than a member, phenomenon.

As ductility became a performance level more commonly expected, and a better understanding of the behavior of shear reinforcement in slabs was developed, ASCE-ACI Committee 421, Design of Two-Way Slabs, also became involved in shear performance and the use of shear reinforcement in slabs. The performance and economy of two-way slabs is very much influenced by slab punching shear. Overall slab thickness and thus slab concrete volume and slab dead load—both affecting overall economy—, are very often set by slab shear requirements. Effective slab shear reinforcement provides the potential to allow for thinner slabs and provide the more ductile connection area needed for the moment redistribution and flexural deflections associated with general yielding. The development of effective shear reinforcement for slabs has been an active topic for this committee (ACI-ASCE 421 [2008, 2011]).

The post-punching failure performance of slabs, the robustness performance level, has received some attention in ACI Committees and in ACI Code 318, notably by the inclusion of the structural integrity provisions, 4.10 of ACI 318-14, which requires “reinforcement and connections shall be detailed to tie the structure together effectively and to improve overall structural integrity”. The robustness performance level for slabs is currently being addressed by an ASCE/SEI technical committee and a related standards committee. The ASCE/SEI Technical Committee on Disproportionate Collapse committee report emphasizes the behavior of steel and reinforced concrete beam and column frame buildings, but does address two-way slabs among the many other systems considered. The related Standards Committee

has produced several internal drafts of its standard. Through this future standard and the products of the technical committees, it is reasonable to expect that future building codes will also address the robustness performance level, especially – but not exclusively – for critical and iconic structures.

As described above, the several technical committees addressing the slab-column connection area performance in shear have related but somewhat different interests based on the performance level(s) of interest. Also, the physical size of the slab region involved in the slab shear design considerations changes with the performance level. At the strength level, the problem can be somewhat isolated to the local slab-column region. As ductility becomes of more interest, the region of interest expands to a greater slab region around the connection. Considerations of robustness, as affected by the slab-column connection performance, can involve the entire slab system.

This paper discusses the ways in which the slab-column punching shear phenomenon differs from, and compares with, the simpler beam flexural shear problem. It then provides an overview of the research work, primary findings, and major U.S. design provisions for each of the three performance levels – strength, ductility, and robustness.

2 Comparison of beam and slab behavior in shear

The shear strength and behavior of flexural members have been extensively researched around the world, with much less convergence in the design provisions for beam shear than for beam flexural behavior. Although the usual behaviors associated with beam shear strength (one or more critical shear cracks usually forming as an extension of flexural cracks, aggregate interlock, doweling, shear transfer in the compression zone above the shear crack, the contributions of any stirrups/shear reinforcement, etc.) are reasonably well understood, the design provisions for beam shear differ significantly around the world.

The addition of another dimension for two-way slab shear relative to that for one-way beams adds additional behavioral complications, even though both beam and slab shear depend on the same material properties, have the same basic shear transfer mechanisms (aggregate interlock, doweling, transfer in the compression zone, shear reinforcement, etc.) and display a size effect. Some of the major differences and complications for the slab-column region in shear relative to those in beams, and previously noted by ACI-ASCE Committee 426 (1974), include:

1. Critical shear crack location and orientation: The surface of the punching shear failure extends around the column, while the flexural crack pattern in the slab is, except for a crack around the column face, predominantly in a radial pattern, following in most cases the slab tension reinforcement layout. Thus, the critical shear crack in a slab generally does not develop when a flexural crack turns over to form an inclined shear crack. Rather, it forms from diagonal tension in a direction normal to the adjoining flexural cracks. The slab shear crack formation is thus more similar to web-shear cracking in beams than the more usual flexure-shear cracking.

2. Restraint of critical shear crack opening by moment acting perpendicular to the shear crack: The flexural stiffness of the slab in the radial direction provides a restraint on the overall deflection and curvature of the slab region near the column, which restrains the opening of any slab shear crack, thus helping to maintain the aggregate interlock along the

shear crack. Such an action does not exist in beams. This beneficial constraint on the inclined crack opening decreases as flexural yielding spreads outwards from the column resulting in a strong dependence of punching shear resistance on the slab flexural conditions in the regions near the column.

3. Limits on the critical shear crack location: The punching shear failure location is tightly constrained to a location very near the column face, as the critical perimeter increases with distance from the column face, unlike conditions for a constant width beam.

4. Distribution of horizontal forces across the slab width: For a typical beam, one without significant along-member applied forces (prestressing) or aggregate interlock along the shear crack, the compression force above the shear crack is set by statics as equal to the flexural tension force. In a slab, the distribution of these horizontal flexural forces can vary across the width of the slab and statics alone does not dictate the force in the compression zone above the shear crack. For interior slab-column connections transferring small amounts of moment, the flexural tensions on either side of the potential failure cone are essentially balanced, allowing the shear crack to more easily extend to the slab compression surface rather than assuring a compression zone above the crack.

5. Formation of in-plane compression forces in a slab: Lightly reinforced flexural members expand along their length after cracking, as the mid-depth of the member is on the tension side of the neutral axis. In a slab system, this expansion in the higher moment areas near the column is constrained by the less stressed slab areas further away, resulting in axial compressive forces in the region near a column. A similar constraint can be provided by slab supports and boundary conditions, and in-plane compressive forces develop when flexural reinforcement is not in the direction of the applied moment (ACI-ASCE Committee 426, 1974). The mechanisms for generating in-plane compressions in a slab system which tend to increase shear strength and reduce ductility are absent or much reduced for beams, and are greatly reduced relative to those for a slab system in the isolated beam-column connection test.

6. Benefits of biaxial stress conditions on material strengths: The benefits of biaxial compression (and tension) on material strength exists with the two-way bending action of the slab. Although there may be significant benefits of this biaxial stress condition, especially in the slab regions adjacent to the corners of rectangular columns, it is yet unclear how significant these biaxial loading effects are in determining slab shear strength and overall performance.

7. Loading intensity varying with location around the column face: For beams of usual width-to-depth ratios, the implied assumption is that conditions, including loading, are uniform over the width of the beam. In contrast conditions in a slab can vary significantly around the shear critical section. The doubly-curved “inverted bowl” shape of the deflected slab results in significantly more of the vertical load transferred at the slab-column connection flowing toward the corners of the column. Vertical strain gauges placed in the center region of the column face show much less strain than at column corners. Slab regions near the column corners are both more heavily loaded and likely stronger (in part, due to the biaxial effects noted above) than regions near the center of the column face.

8. Transition of slab shear behavior to beam shear behavior: Another behavior related to unequal conditions around the critical shear perimeter is due to the effects of the ratio of slab depth to column face dimension. As the slab becomes thinner relative to the column size, the portion of the slab adjacent to the center of the column face experiences

conditions more similar to that of a beam, or at least to the one-way shear behavior of a slab framing into a wall. The tangential slab moments decrease less rapidly and flexural cracking develops parallel to, and away from, the column face, and thus in the direction and close to the location of the inclined crack in the slab.

9. Existence of slab-column connection side faces and their role in moment transfer:

Exterior and many interior slab-column connections transfer moments from the slab to the column. Conditions on the side faces, as well as on the front face and any back face, become extremely important in the overall performance of the slab-column connection in combined shear and moment transfer; the sides of beams can have no such roles. Even in the absence of an edge beam, a portion of the slab entering the column side faces functions as a torsion member, which combined with the pattern of vertical shear forces along the column sides, assists the shear stresses and moment capacities along the column front face and any back face in providing moment resistance.

10. Typical slab versus beam depths – size/depth effects: Floor and roof slabs typically have a thickness of 5 to 10 inches (127 to 254 mm), less than the typical depth of beams. As a result, the magnitude of any size effect for slabs is generally less than for a typical beam, although size effects for footings and other heavily loaded thick slabs can be expected similar to those for an equally deep beam.

11. Typical slab versus beam depths – shear reinforcement: The typically smaller member depth for slabs than for beams presents construction and other practical challenges. The use of traditional stirrups involves many small and closely spaced stirrups, challenges in stirrup anchorage, intrusion of the hooks and bends of stirrups into the usual cover dimensions over slab flexural reinforcement, etc. The result is a need for shear reinforcement systems specifically developed for slabs.

12. Member versus connection area design statement: Several of the previously stated differences between beams in shear and slabs in shear support the overall observation that beam shear can almost always be viewed as a member design consideration, while slab shear (unless the result of a localized load away from a column area) involves the behavior and design of a connection area. For slab structures, the slab punching shear problem is effectively synonymous to the slab-column connection problem.

3 Slab-column connections – Strength performance level

3.1 Development of basic strength design requirements and associated studies

The studies of the performance and strength of slabs in punching shear that led to the development of the familiar design provisions in the ACI 318 Code, provisions not greatly changed since 1963, were mostly conducted to determine the shear strength of slabs which failed in shear before the slab flexural capacity was reached. Several of the earliest test series addressed the strength in shear of isolated spread footings loaded by a column, for which punching is a major design concern. A similar isolated slab plus column specimen became the standard test specimen for study of the punching shear strength of elevated slabs. The slab segment was sized to include the negative moment area around the column, with vertical supports located near the perimeter of the square, rectangular, circular, or octagonal slab and load applied via a column stub. Although these isolated slab-column models are much more