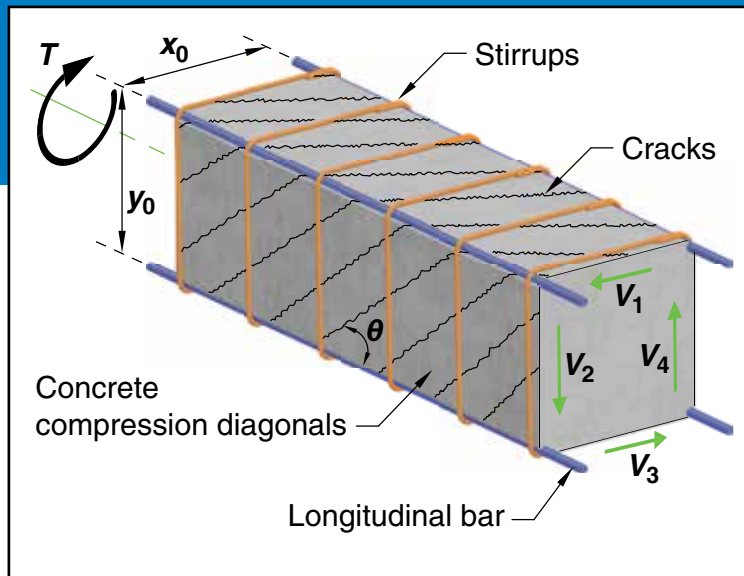


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Examples for The Design of Reinforced and Prestressed Concrete Members Under Torsion

Editors:
Eva Lantsoght, Gary Greene, and Abdeldjelil Belarbi

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Examples for the Design of Reinforced and Prestressed Concrete Members Under Torsion

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Editors:
Eva Lantsoght,
Gary Greene, and
Abdeldjelil Belarbi



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PREFACE

Examples for the Design of Reinforced and Prestressed Concrete Members Under Torsion

The design and analysis of structural concrete elements is a topic of practical interest. While sometimes the effect of torsion is only addressed based on simple examples, practicing engineers are faced with the need to include the effects of torsion in their designs of a variety of structures and load arrangements.

This Special Publication (SP) contains papers about the design of reinforced and prestressed concrete elements for torsion. The focus of the SP is on practical design examples according to different concrete bridge and building codes. In addition to the design examples, papers dealing with the current state of the art on torsion in structural concrete, as well as recent advances in the analysis and design of concrete elements failing in torsion, are added.

The objectives of this SP are to provide practicing engineers with the tools necessary to better understand and design concrete elements for torsion. The need for this SP arose after the development of the State-of-the-Art Report on Torsion of Joint ACI-ASCE Committee 445 “Shear and Torsion” and Subcommittee 445-E “Torsion”. Usually, the attention that is paid to torsion in engineering education is limited to simplified textbook examples. The examples in this SP show applications in bridges and buildings, where the torsion design is combined with the design for flexure and shear. Additionally, the examples in this SP give insight on the different outcomes when using different bridge and building codes. Finally, the papers that include theoretical considerations give practicing engineers a deeper understanding and background on torsion in structural concrete.

The views from an international group of authors are included in this SP, subsequently representing a variety of building and bridge codes the engineer may encounter in practice. In particular, authors from the United States, Canada, Ecuador, the Netherlands, Italy, Greece, and the Czech Republic contributed to the papers in this SP. Views from academia and the industry are included.

To exchange experience in the design of torsion-critical structures as well as new research insights on torsion, Joint ACI-ASCE Committee 445 and Subcommittee 445-E organized two sessions titled “Examples for the Design of Reinforced and Prestressed Concrete Members under Torsion” at the ACI Fall Convention 2020. This SP contains several technical papers from experts who presented their work at these sessions, in addition to papers submitted for publication only.

In summary, this SP addresses numerous practical examples of structural elements under torsion in bridges and buildings, as well as insights from recent research applied to practical cases of elements under torsion. The co-editors of this SP are grateful for the contributions of the authors and sincerely value the time and effort they invested in preparing the papers in this volume, as well as the contributions of the reviewers of the manuscripts.

Eva Lantsoght, Gary Greene, and Abdeldjelil Belarbi
Co-editors

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OVERVIEW OF TORSION DESIGN METHODS

Camilo Granda Valencia and Eva Lantsoght

Synopsis: Large torsional moments, which need to be considered in a design, can result among others, in structures with an asymmetric layout or loading. To find the required longitudinal and transverse reinforcement to resist these torsional moments, the link between the three-dimensional action of the torsional moment and sectional analysis methods is necessary. This paper reviews the existing methods and code provisions for torsion. First, an overview of the principles of torsion from the mechanics perspective is given. Then, a survey of the available mechanical models for torsion is presented. Finally, the code provisions for torsion of ACI 318-19, CSA-A23.3-04, AASHTO-LRFD-17, EN 1992-1-1:2004, and the fib Model Code 2010 are summarized. Additionally, current research topics on torsion in structural concrete are summarized. It is expected that with this paper, engineers will have a useful overview and background knowledge for the design and assessment of torsion-critical elements.

Keywords: codes, concrete, reinforcement, shear, torsion

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INTRODUCTION

In general, concrete structures are subjected to four principal actions: axial force, shear, bending moment, and torsion. Engineers and researchers focused on the understanding of the first three phenomena in concrete structures because these usually control the design of a member, i.e. they control the resulting reinforcement layout. For example, beams are typically designed for sectional moment and shear. Columns work as flexion-compression elements, around both axes of the cross-section. Nevertheless, torsion is a special topic. It was left apart because generally its influence on the resulting design is limited. For this reason, building codes accounted for torsion's small influence in the safety factors¹. Throughout the 1960s, extensive research on torsion was made. As a result, the first design recommendations for torsion made by the American Concrete Institute (ACI) were formulated in 1969². These recommendations led to the inclusion of provisions for torsion in the 1971 edition of the ACI Building Code, ACI 318-71³. The research carried out over the past decades led to a better understanding of the behavior of concrete members subjected to a torsional moment. The Space Truss Analogy, the Skew-Bending Theory, and other theories provided mechanical models to predict the behavior of concrete structures under torsion after cracking.

Torsion can be defined as the moment that twists an element around its axis. This torsional moment causes shearing stresses at each point of the cross-section of an element. These stresses change according to the proximity to the member's axis⁴. In circular cross-sections, the stress caused by a torsional moment is zero at the neutral axis and reaches the maximum value on the outermost fiber, see Figure 1(a). For rectangular cross-sections, the shear stress is also zero at the neutral axis and at the corners. It increases towards its maximum value at the surface of the longest side, see Figure 1(b).

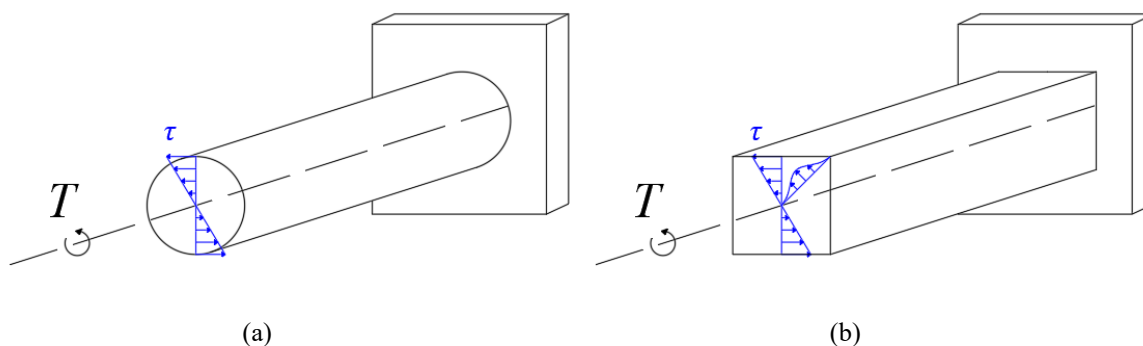


Figure 1—Shear stresses (τ) due to an applied torsional moment (T) on a circular (a) and rectangular solid (b) element

Torsion can be a result of primary or secondary actions. The primary action occurs when the member can only support the action of an external load by generating a torsional moment. This is also called equilibrium torsion and is common in statically determinate structures. Equilibrium torsion is important for the stability of the structure. This occurs, for example, when a load acts on a fixed-end beam, but it is applied eccentric with respect to the z-axis, like in Figure 2. As a result, a torsional moment is generated around this axis.

Torsion can also be found as a result of secondary actions in statically indeterminate structures. This happens because the structure needs to satisfy compatibility requirements. In this case a twist is required to maintain the compatibility, not a torsional moment⁵. Spandrel continuous beams supporting other secondary beams or slabs are often subjected to this phenomenon, as shown in Figure 3.

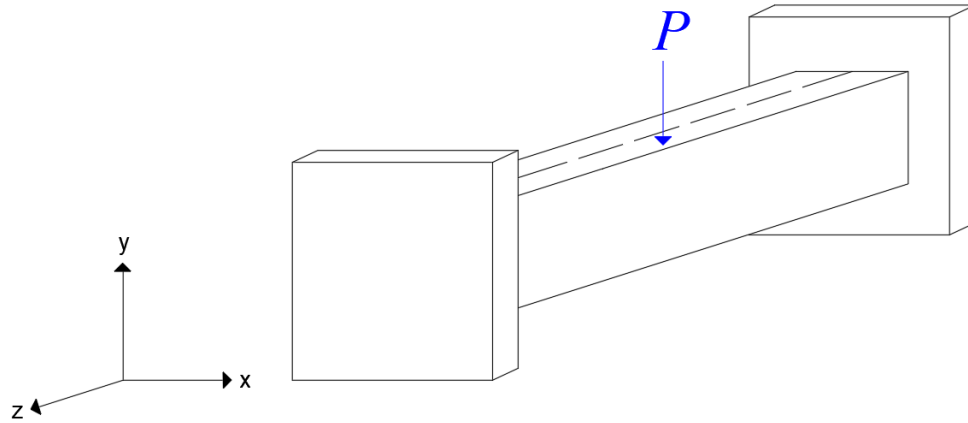


Figure 2—Equilibrium torsion at the ends of the beam, generated by the action of a point load

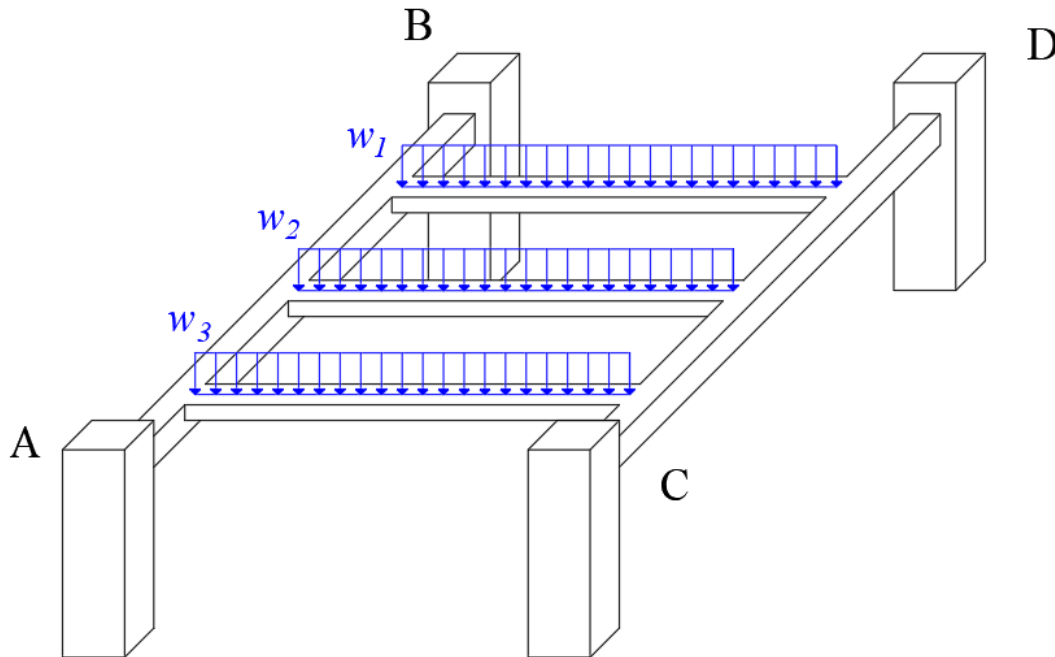


Figure 3—3D Frame system where spandrel beams AB and CD are subjected to compatibility torsion due to the load on the secondary beams joints

More complex and asymmetric concrete structures are designed every year around the world thanks to the reduction times in analysis and design when using structural software. As a result, the effect of torsion on concrete structures has become more important. For example, horizontally curved bridges and cantilever members should be designed for torsion. Standardization institutions like American Concrete Institute (ACI)⁶, Canada Standards Association (CSA)⁷, American Association of State Highway and Transportation Officials (AASHTO)⁸, the European Normalization Committee (CEN)⁹, and the International Federation for Structural Concrete (fib)¹⁰ have developed provisions for situations when torsion needs to be considered. The design philosophy that each code uses is:

- ACI 318-19⁶ uses a thin-tube and space truss analogy.
- CSA (CSA-A23.3-04)⁷ uses a General Design Method for torsion derived from the Modified Compression Field Theory (MCFT); it includes the tensile contribution of concrete.

- AASHTO-LRFD-17⁸ code provisions for torsion are obtained from the MCFT. The torsion equations on this code are similar to the CSA-A23.3-04 ones. A Strut and Tie Model can also be used as an alternative for design.
- Eurocode (EN 1992-1-1:2004)⁹ uses a spatial truss model with an equivalent thin-walled tube and wall thickness for the torsion design.
- The fib Model Code 2010¹⁰ uses a variable angle truss model, generalized stress field approach, or a simplified modified compression field theory, depending on the Level of Approximation.

The assumptions that lie at the basis of each of these models, and the resulting mechanics, will be discussed in the section about the mechanical models. The resulting code provisions will be given in the section with the code provisions.

BRIEF HISTORY OF TORSION RESEARCH

Mechanics of torsion

In this section, an overview of the history of torsion mechanics is given. Kurrer in “The History of the Theory of Structures” shows various important investigations on this topic¹¹. The first known person to study the effect of torsion on materials, as a consequence of his research on electric charges, was Coulomb. Using his torsion balance, he deduced that the torsional moment is proportional to the torsional angle¹². About 40 years later, Navier was the first to postulate a theoretical equation to compute the torsional moment on shafts with a circular cross-section. The two assumptions that he made were: 1) the shape of the cross-section cannot change after twist, and 2) plane sections must remain plane. The latter assumption implies that warping does not occur¹³.

Later, it was found that there are two possible ways in which a structural member can resist torsion: by circulatory torsion or by warping torsion. Saint-Venant developed in 1847 the first theory, in which he stated that the cross-section of an element counters the effect of torsion by producing a circulatory shear flow (torsional shear multiplied by the wall thickness) on its plane. This means that the shear stress resisting the external torsional moment is constant within the flow area, see Figure 4(a). This effect usually occurs in solid and hollow members, which are free to bend around their axis⁴. The second way in which structural members can withstand torsion is by warping torsion. It was first investigated by Timoshenko in 1905¹⁴ and further researched by Vlasov in 1940^{15,16}. Warping torsion produces different shear stresses along the same circumference, see Figure 4(b). Consequently, the planar sections do not remain plane due to the changing strain at points over a determined circumference. Longitudinal bending results from these strains. Warping torsion arises when the entire section or part of it is restrained, for example, by end conditions^{4,17}. This is usually expected in members formed by at least three connected walls, or with a fixed-end support.

Both resisting torsional moments need to be in equilibrium with the applied torsional moment (T) on the member. This means that $T = T_s + T_w$ where T_s is the Saint-Venant torsion and T_w is the warping torsion. Both happen at the same time, consequently; there is not a clear way to classify sections according to how they resist torsion. Some practical examples have demonstrated that the action of one of the resisting methods can be neglected compared to the effect of the other. Nevertheless, there are other cases where neither of them is predominant over the other; this case is called mixed torsion¹⁸. One example of mixed torsion is an I-shaped simply supported beam. If the torsional moment is applied at midspan, the cross-sections at the left and right of it experience warping torsion. Close to the ends, the beam can twist freely, therefore Saint-Venant torsion occurs.

In 1890 Bach, in his book, “Elasticität und Festigkeit” presented all the torsion cases proposed by Saint-Venant and interpreted them theoretically. Bach tested numerous cast-iron and hard lead bars under torsion. Using the results between the proposed theory and the experiment, Bach developed a simple proof equation to check the shear stress for the Saint-Venant torsion in bars, equilateral triangles, and regular hexagons¹¹. In 1896, Bredt offered a promising solution to the Saint-Venant torsion problem. His solution equation states that the sum of the tangential shear forces (τ) per unit area (ds) on a closed curve within the cross-section under the effect of an external torsional moment is equal to two times the area enclosed by the forces (A_m), shear modulus (G) and the product of rotation (θ_r)¹¹, i.e.:

$$\int \tau ds = 2A_m \theta_r G \quad (1)$$

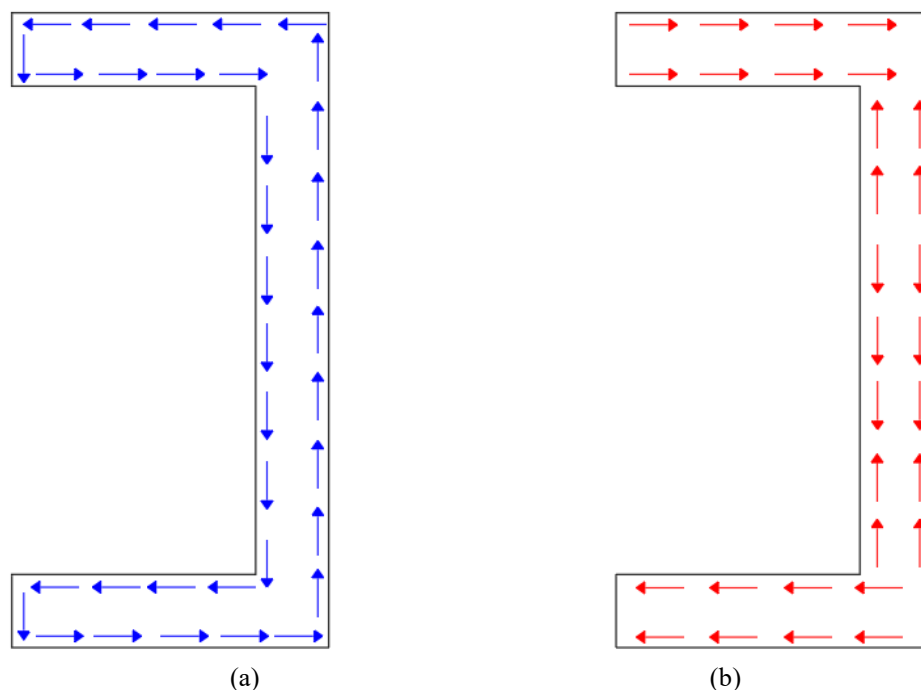


Figure 4—Circulatory torsion (a) and warping torsion (b) shear stresses on C-shaped members

Torsion in reinforced concrete

Graf and Mörsch were the first researchers to study torsion in plain and reinforced concrete. They tested different circular, square, and rectangular beams to study the effect of the reinforcement on the ultimate strength for elements under torsion¹⁷. In 1929, Rausch published his dissertation, in which he presented the 3D-truss analogy for torsion. Rausch provided an equation to predict the torsional resistance of reinforced concrete members based on the space truss model^{19,20}. This method lies at the basis for the current torsion design provisions.

More researchers started to study torsion in structural concrete at the beginning of the second half of the twentieth century. In 1959, Lessig used equilibrium equations to propose a skew-bending theory for the failure mechanism of torsion²¹. This theory assumes that a beam under torsion will have a skewed failure surface. Lessig proposed two failure modes. The first one has a compression zone near the top face of the beam, while the second failure mode uses a compression zone along the side face. In 1962, Yudin²² realized that the skew-bending theory proposed by Lessig was not able to determine three unknowns: the longitudinal reinforcement area, the web steel area, and the depth of the compression zone. To solve this, Yudin proposed three equilibrium equations, while Lessig's analysis only used two: the equilibrium of moments about the neutral axis of the member, and the equilibrium of forces along the normal to the compression zone. Yudin's equations were: equilibrium of moments about an axis through the centroid of the compression zone and parallel to the longitudinal axis of the beam, equilibrium of moments about an axis through the centroid of the compression zone and perpendicular to the longitudinal axis, and equilibrium of forces along the normal to the compression zone. Nevertheless, this analysis is limited to only symmetrically reinforced elements.

Elfgren developed a method to determine the capacity of elements under combined shear, moment and torsion¹⁶. He used a truss analogy to predict the ultimate load carried by multiple sets of reinforced beams and tested these at Chalmers University of Technology. Elfgren established an interaction equation which can be used to plot an interaction surface. This model predicts accurately the strength of reinforced concrete beams subjected to torsional moment, shear force and bending moment.

Collins and Mitchell introduced another approach to study torsion in structural concrete in 1973²³. They presented the diagonal compression field theory for beams under pure torsion. They considered equilibrium equations, geometry of deformations, and stress-strain relationships of the concrete and steel to propose their theoretical model.

The basis of their approach is a truss analogy model, and their main assumption is that after cracking the concrete will not carry tension, therefore, the torsion will be resisted by a field of diagonal compression in the concrete. Afterwards, in 1985, Hsu and Mo developed a variation of the compression field theory. In this case, they softened the concrete stress-strain curve and called the new model the softened truss model (STM)¹⁹. In the STM equilibrium, compatibility and softened stress-strain relationships are combined to develop a theory that has shown good results in predicting the test results of reinforced concrete structures subjected to shear and torsion²⁴. Rahal and Collins have developed analytical computational models to calculate the response of concrete members subjected to combined torsion and bending²⁵ and to combined torsion and shear²⁶.

MECHANICAL MODELS FOR TORSION

Reinforced concrete before and after cracking under torsion

Prior to cracking, reinforced concrete members subjected to torsion can be analyzed as homogenous plain concrete sections. Therefore, their behavior can be predicted using Saint-Venant's theory²⁷. After the element cracks, the study of its behavior becomes more complicated. From now on, the structural member acts as a composite section, and Saint-Venant's theory can no longer be used because cracking violates the material homogeneity premise of the elastic theory. When the web of the beam cracks, its capacity to transmit diagonal tension forces is reduced. The load is then carried by diagonal compression members between the cracks and by the steel reinforcement resisting tension. Together, they form a truss-like mechanism⁴.

Shear truss analogy

The shear truss analogy was first proposed by Ritter at the end of the twentieth century²⁸. It is a strut-and-tie model and considers that a cracked reinforced concrete beam under shear will have diagonal cracks which separate the concrete into multiple struts. They modeled the beam as a plane truss consisting of longitudinal and transverse reinforcement to carry the load. In this assumption, the top and bottom longitudinal bars act as the top and bottom chords of the truss, while the transverse reinforcement and concrete struts work as the web members. To simplify this model, the strut's inclination is assumed to be 45°²⁹.

3D space truss analogy

To apply the concept of a truss model to members subjected to torsion, the truss model needs to be extended to a three-dimensional model, i.e. a space truss analogy. A member subjected to torsion is treated as a space truss formed by a series of joined planar trusses²⁰. The concrete member reinforced with longitudinal and transverse reinforcement resists torsion by producing a circulatory shear flow at the outermost part of the cross-section. Each straight segment of the tube walls behaves like a planar truss in which the shear stresses are resisted as in the shear truss analogy. Struts only carry axial compression; longitudinal and transverse reinforcement carries the tension forces, see Figure 5.

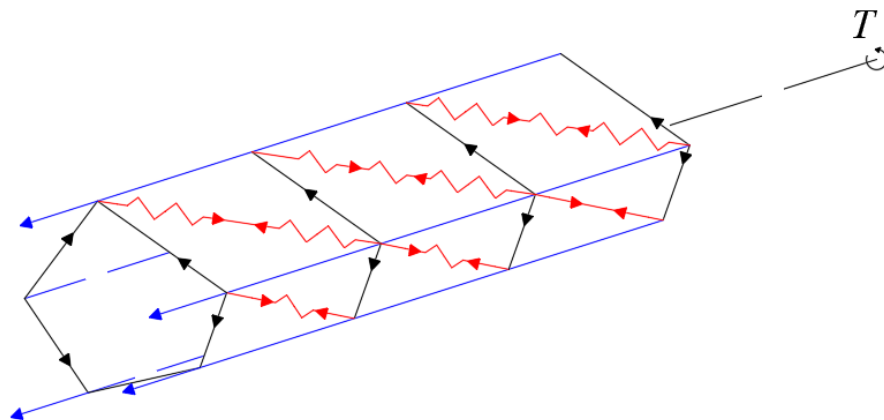


Figure 5—Space truss analogy for an asymmetrical beam under torsion. The tension forces are supported by the longitudinal and transverse reinforcement (black and blue) and the concrete struts resist compression (red)

Skew-bending theories

This theory is characterized by the assumption of a skewed failure surface. This surface is generated by a helically-shaped crack on three faces of a rectangular beam. On the fourth face, the helical crack is connected by a compression zone. The failure surface intersects the longitudinal and transverse reinforcement. The forces in the steel reinforcement generate the required internal forces and moments to carry external loads. Failure occurs when the steel starts to yield³⁰. At failure, the two parts of the member separated by the failure surface rotate against each other about a neutral axis on the inside edge of the compression zone. Then, the associated equilibrium equations at the ultimate limit state can be derived²⁷.

Thin-walled tube analogy

The most efficient cross-section to resist torsion is a thin tube. The thin-walled tube analogy states that the shear stresses and shear flow are constant around the cross-section of a member. This shear flow is enclosed by an area of pre-determined thickness. Therefore, solid and hollow sections can be calculated in the same way as tubes³¹, see Figure 6. Concrete members can be modelled as tubes because the concrete core does not contribute to the element's torsional strength²⁷. Within the walls of the tube, the external torsion is resisted by a shear flow, defined as the torsional shear multiplied by the thickness of the tube.

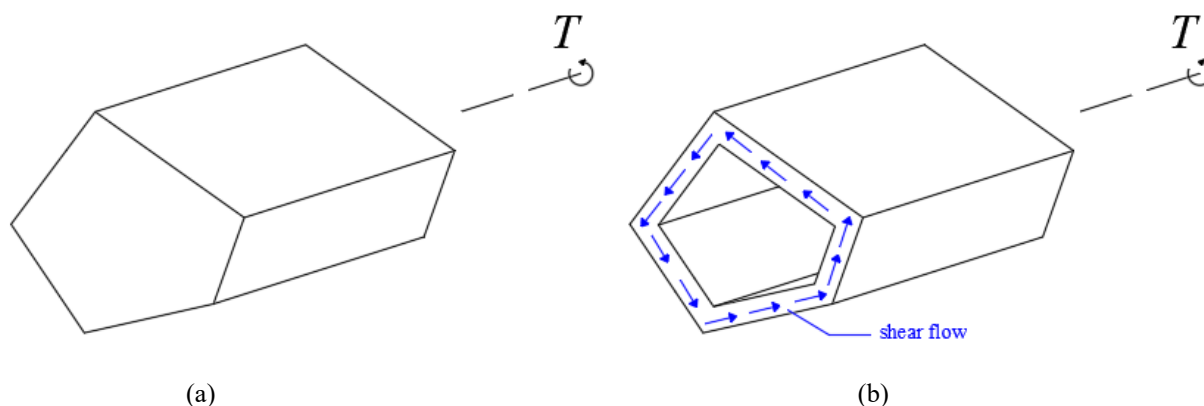


Figure 6—Original section (a) and the same member after the thin-walled tube analogy is applied (b)

Compression Field Theory (CFT)

The CFT is a model developed by Mitchell and Collins that considers equilibrium conditions, geometry of deformation and the strain-stress characteristics of the steel and concrete. This model predicts the shear strength of a reinforced concrete member after it cracks. This theory, based on the truss analogy, assumes that after cracking, the torsion shear stresses are carried by a field of diagonal compression in the concrete and balanced by the tension developed in the longitudinal and transverse reinforcement²³. In 1986, Vecchio and Collins expanded the CFT to the Modified Compression Field Theory (MCFT). The CFT assumed that the cracks of the diagonal field compression in the concrete were only able to withstand shear and compression. Nevertheless, between the concrete's cracks tension stresses exist. To have a more accurate answer of the reinforced concrete element's capacity under shear and torsion, the MCFT uses experimentally verified average stress-strain relationships instead of assuming them. Also, it considers the tension in the cracked concrete³². Although the MCFT can predict the shear and torsional strength with great precision, the process of solving the equations of this theory by hand is complex. For this reason, Bentz, Vecchio and Collins developed a simplified MCFT using the Membrane-2000 computer program to get more practical expressions. This method showed excellent predictions of the shear strength. The accuracy between the simplified MCFT and the full theory is almost the same³³.

CODE PROVISIONS FOR TORSION

All the equations in this section are expressed in SI units. The conversion factors are: 1 kN = 0.225 kip, 1 kN·m = 8.849 kip·in, 1 mm = 0.0394 in and 1 MPa = 145 psi.

ACI 318-19

ACI 318-19 first checks if torsion can be neglected. If the following expression from §9.5.4.1 is satisfied, torsional effects do not need to be considered:

$$T_u < \phi T_{th} \quad (2)$$

T_u is the factored torsional moment. ϕ , the reduction factor for the nominal capacity of torsion, is equal to 0.75. T_{th} is the threshold torsional moment given by §22.7.4. For solid sections it is:

$$T_{th} = \begin{cases} 0.083\lambda\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right) & \text{Non-prestressed member} \\ 0.083\lambda\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{f_{pc}}{0.33\lambda\sqrt{f'_c}}} & \text{Prestressed member} \\ 0.083\lambda\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{N_u}{0.33A_g\lambda\sqrt{f'_c}}} & \text{Non-prestressed member under axial load} \end{cases} \quad (3)$$

In statically indeterminate structures where $T_u \geq \phi T_{cr}$, it is permitted to reduce T_u to ϕT_{cr} due to redistribution of internal forces after cracking. This applies to typical and regular framing conditions. ϕT_{cr} is the cracking torsional moment and is defined in §22.7.5.1. Equation (3) is valid for solid cross-sections. For hollow cross-sections, all the A_{cp} terms in Equation (3) are substituted with A_g , the gross area of the concrete cross-section. f'_c [MPa] is the specified compressive strength of the concrete, A_{cp} is the area enclosed by the outside perimeter of the concrete cross-section, p_{cp} is the outside perimeter of concrete's cross-section, f_{pc} [MPa] is the compressive stress in the concrete, after allowance for all prestress losses, at the centroid of the cross-section resisting the externally applied loads or at the junction of the web and flange where the centroid lies within the flange. In a composite member, it is the resultant compressive stress at the centroid of the composite section, or at the junction of the web and flange, when the centroid lies within the flange, due to both prestress and moments resisted by the precast member acting alone, N_u is the factored axial force, taken as negative for tension and positive for compression, λ is a coefficient which accounts for the properties of lightweight concrete (see §19.2.4).

The shear strength provided by the concrete V_c according to §22.5.5.1 is determined as:

$$V_c = 0.17\lambda\sqrt{f'_c}b_wd \quad \text{with } f'_c \text{ in [MPa]} \quad (4)$$

b_w is the web width or diameter of a circular section and d is the effective depth. The last expression applies to reinforced concrete members without axial force and with $A_v \geq A_{v,min}$. A_v in Equation (4) is the required transverse reinforcement for shear and $A_{v,min}$ is the minimum transverse reinforcement for shear force. For other cases in reinforced concrete members, §22.5.5.1 through §22.5.5.1.3 are governing. For prestressed members, the shear strength provided in concrete is listed in §22.5.6 and §22.5.7.

The next expression from §22.7.7.1 checks if the dimensions of the member are large enough to avoid crushing of the concrete:

$$\begin{cases} \sqrt{\left(\frac{V_u}{b_wd}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \leq \phi \left(\frac{V_c}{b_wd} + 0.66\sqrt{f'_c}\right) & \text{for solid sections} \\ \left(\frac{V_u}{b_wd}\right) + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right) \leq \phi \left(\frac{V_c}{b_wd} + 0.66\sqrt{f'_c}\right) & \text{for hollow sections} \end{cases} \quad (5)$$

If Equation (5) is fulfilled, the reinforcement for torsion can be designed. For hollow sections with a variable wall thickness, the maximum value of the left side of Equation (5) should be evaluated, which is often at the point of the cross-section where shear and torsional stresses can be added. V_u is the factored shear force, p_h is the perimeter of the centerline of the outermost closed transverse torsional reinforcement, A_{oh} is the area enclosed by p_h . §22.7.7.1.1

mentions that for prestressed members the value of d in Equation (5) should be greater than $0.8h$, where h is the overall height of the element.

According to §22.7.6.1, θ , the angle between the struts and the tension chord, can be taken as any value between 30 and 60 degrees. §22.7.6.1.2 states that θ is usually 45° for reinforced concrete members with $A_{ps}f_{se} < 0.4(A_{ps}f_{pu} + A_s f_y)$ and 37.5° for prestressed elements with $A_{ps}f_{se} \geq 0.4(A_{ps}f_{pu} + A_s f_y)$. A_{ps} is the area of the prestressed longitudinal tension reinforcement, A_s is the area of the non-prestressed longitudinal tension reinforcement, f_{se} is the effective stress in prestressing reinforcement after allowance for all prestress losses, f_{pu} is the specified tensile strength of prestressing reinforcement, and f_y is the yield strength for non-prestressed longitudinal reinforcement. The required area of transverse reinforcement of one leg of a closed stirrup A_t for torsion is:

$$\frac{A_t}{s} \geq \frac{T_u}{1.7\phi A_{oh} f_{yt}} \tan \theta \quad (6)$$

s is the spacing between the stirrups, f_{yt} is the specified yield strength of the transverse reinforcement.

The next step is to calculate the required area of longitudinal steel for torsion A_l :

$$A_l \geq \frac{A_t}{s} \frac{f_{yt}}{f_y} p_h \cot^2 \theta \quad (7)$$

§9.5.4.3 mentions that the longitudinal and transverse reinforcement required for torsion need to be added to the reinforcement demanded by shear force, bending moment and axial force actions.

For the transverse reinforcement limit, §9.6.4.2 states that for members under torsion and shear, the stirrups for torsion and shear effects cannot be less than:

$$\frac{(A_v + 2A_t)_{\min}}{s} = \max \begin{cases} 0.062 \sqrt{f'_c} \frac{b_w}{f_{yt}} \\ 0.35 \frac{b_w}{f_{yt}} \end{cases} \quad (8)$$

In Equation (8), A_v is the required area of two legs of a closed stirrup for shear. If the analyzed element is only experiencing torsion, the value of the A_v term in Equation (8), is equal to zero. The minimum area of longitudinal steel reinforcement $A_{l,\min}$ for torsion can be calculated with §9.6.4.3 as:

$$A_{l,\min} = \min \begin{cases} \frac{0.42 \sqrt{f'_c} A_{cp}}{f_y} - \left(\frac{A_t}{s} \right) p_h \frac{f_{yt}}{f_y} \\ \frac{0.42 \sqrt{f'_c} A_{cp}}{f_y} - \left(\frac{0.175 b_w}{f_{yt}} \right) p_h \frac{f_{yt}}{f_y} \end{cases} \quad (9)$$

According to §9.7.6.3.3 the limits to the stirrup spacing are:

$$s \leq \min \begin{cases} \frac{p_h}{8} \\ 300 \text{ mm} \end{cases} \quad (10)$$

§9.5.4.3 states that the final amount of longitudinal and transverse reinforcement needs to be added to the required reinforcement for shear force, bending moment and axial effects.

When the cross-section and the reinforcement of the member are designed, the ACI 318-19 §22.7.6.1 gives two equations to analyze the torsional strength T_n . Once the member cracks under the effect of a torsional moment, the strength is provided primarily by the transverse and longitudinal reinforcement. The concrete contribution to the torsional strength is neglected:

$$T_n = \min \left\{ \begin{array}{l} \frac{1.7 A_{oh} A_t f_{yt}}{s} \cot \theta \\ \frac{1.7 A_{oh} A_l f_y}{p_h} \tan \theta \end{array} \right. \quad (11)$$

Finally, the torsional strength should be greater than or equal to the factored applied torsional moment.

$$\phi T_n \geq T_u \quad (12)$$

CSA-A23.3-04

According to §11.2.9.1, reinforcement for torsion should be provided when the factored torsional moment T_f exceeds $1/4$ of the pure torsional cracking resistance T_{cr} , given in CSA-A23.3-04 Eq. 11-2 as:

$$T_{cr} = 0.38 \lambda \phi_c \left(\frac{A_c^2}{p_c} \right) \sqrt{f'_c} \sqrt{1 + \frac{\phi_p f_{cp}}{0.38 \lambda \phi_c \sqrt{f'_c}}} \quad (13)$$

In statically indeterminate structures, where redistribution of torsional moments can occur, §11.2.9.2 specifies that T_f can be reduced to $0.67 T_{cr}$ at the face of the support. For hollow sections, A_c is the area enclosed by the outside perimeter of the concrete cross-section, including the area of holes. In Equation (13), for hollow cross-sections, A_c can be replaced with $1.5 A_g$ (gross concrete area) if the wall thickness is less than $0.75 A_c / p_c$. p_c is the outside perimeter of the cross-section, f_{cp} [MPa] is the average compressive stress in the concrete due to the effective prestress force only (after allowance for all prestress losses), f'_c [MPa] is the specified compressive strength of concrete, $\phi_c = 0.65$ is the material factor for concrete, $\phi_p = 0.9$ is the material factor for prestressing tendons, and λ is the factor that accounts for lightweight concrete, see §8.6.5.

Equations 11-18 and 11-19 in the CSA-A23.3-04 code give minimum dimensions to avoid concrete crushing:

$$\left\{ \begin{array}{l} \left(\frac{V_f - V_p}{b_w d_v} + \frac{T_f p_h}{1.7 A_{oh}^2} \right) \leq 0.25 \phi_c f'_c \quad \text{for hollow sections} \\ \left(\sqrt{\left(\frac{V_f - V_p}{b_w d_v} \right)^2} + \left(\frac{T_f p_h}{1.7 A_{oh}^2} \right)^2 \right) \leq 0.25 \phi_c f'_c \quad \text{for other sections} \end{array} \right. \quad (14)$$

V_f is the factored shear, V_p is the component in the direction of the applied shear of the effective prestressing force factored by ϕ_p , d_v is the effective shear depth, taken as the greater of $0.9d$ or $0.72h$, where h is the overall height of the member, and d is the effective depth (d cannot be less than $0.8h$ for prestressed members and circular sections), b_w is the minimum web width within d or the diameter of a circular cross-section, A_{oh} is the area enclosed by the centerline of the exterior closed transverse torsion reinforcement, including the area of holes, p_h is the perimeter of the centerline of the closed transverse torsion reinforcement. If the wall thickness of the box section is less than A_{oh} / p_h , the second term on the left side of Equation (14) should be replaced by $T_f / (1.7 A_{oh} t)$. Where t is the wall thickness at the location where the stresses are being checked.

Next, it is needed to compute the longitudinal strain ϵ_x at mid-depth of the member due to the factored loads. With this variable, the angle of the diagonal compression field can be obtained to calculate the required transverse reinforcement. The longitudinal strain is computed substituting Equation 11-20 on Equation 11-13 of §11.3.6.4, which leads to:

$$\epsilon_x = \frac{\frac{M_f}{d_v} + \sqrt{(V_f - V_p)^2 + \left(\frac{0.9 p_h T_f}{2 A_o}\right)^2} + 0.5 N_f - A_p f_{po}}{2(E_s A_s + E_p A_p)} \quad (15)$$

If the value of Equation (15) is negative, ϵ_x can be taken as zero. M_f is the factored moment and cannot be less than $(V_f - V_p)d_v$, A_o is the area enclosed by the shear flow path including the area of holes, which can be taken as $0.85A_{oh}$ according to §11.3.10.3, N_f is the factored axial load, positive for tension and negative for compression. A_p is the area of prestressing tendons, f_{po} is the stress in the prestressing tendons when the strain in the surrounding concrete is zero (may be taken as $0.7f_{pu,CSA}$ for bonded tendons outside the transfer length and f_{pe} for unbonded tendons), $f_{pu,CSA}$ [MPa] is the specified tensile strength of the prestressing tendons, f_{pe} is the effective stress in the prestressing tendons after allowance for all prestress losses, E_s is the modulus of elasticity of non-prestressed reinforcement, E_p is the modulus of elasticity of prestressing tendons, A_s is the area of non-prestressed tension reinforcement, and A_p is the area of tendons on the flexural tension side of the member. The bending moment and shear force on Equation (15) are absolute values. The axial load is positive for tension and negative for compression. Once the longitudinal strain is computed, the angle of inclination of the diagonal compressive stresses, θ_{CSA} , is defined in §11.3.6.4, Eq. 11-12 as:

$$\theta_{CSA} = 29^\circ + 7000\epsilon_x \quad (16)$$

For special members like slabs or footings with an overall thickness less than 0.35 m, footings in which the distance from the point of zero shear to the face of the column, pedestal, or wall is less than three times the effective shear depth of the footing, beams with an overall thickness less than 0.25 m, beams cast integrally with slabs where the depth of the beam below the slab is not greater than one-half the width of web or 0.35 m, and concrete joist construction defined in §10.4, the angle of the struts can be taken as 42° . §11.3.6.3 also mentions that if the yield strength of the longitudinal reinforcement does not exceed 400 MPa and f'_c is smaller than 60 MPa, θ_{CSA} can be taken as 35° .

The next step is to get the required area of transverse reinforcement for torsion, A_t using Equation (17) :

$$\frac{A_t}{s} \geq \frac{T_f}{1.7\phi_s A_{oh} f_{yt}} \tan \theta_{CSA} \quad (17)$$

s is the spacing between stirrups for torsion, f_{yt} is the specified yield strength of transverse reinforcement, and $\phi_s = 0.85$ is the resistance factor for non-prestressed reinforcement. Combining Equations 11-14 and 11-21, the longitudinal reinforcement area A_{st} needed to withstand a torsional moment is given by:

$$A_{st} = \frac{\frac{M_f}{d_v} + 0.5 N_f + \left[\sqrt{(V_f - 0.5 V_s - V_p)^2 - \left(\frac{0.45 p_h T_f}{1.7 A_{oh}}\right)^2} \right] \cot \theta_{CSA}}{f_y} - A_s \quad (18)$$

f_y is the specified yield strength of non-prestressed longitudinal reinforcement for torsion.

The maximum spacing s between stirrups follows the expression described in §11.3.8.1:

$$s \leq \min \begin{cases} 0.7d_v \\ 600 \text{ mm} \end{cases} \quad (19)$$

The CSA-A23.3-04 code provisions for torsion do not specify any minimum longitudinal or transverse reinforcement for torsion. §11.2.7 indicates that a longitudinal reinforcing bar or bonded prestressing tendon shall be

placed in each corner of closed transverse reinforcement required for torsion. The nominal diameter of the bar or tendon shall be not less than $s/16$.

The torsional moment resistance T_r is only provided by the transverse reinforcement and is defined in §11.3.10.3 as:

$$T_r = 1.7\phi_s A_{oh} f_{yt} \frac{A_t}{s} \cot \theta_{CSA} \quad (20)$$

The torsional strength T_r should be greater than or equal to the applied torsional moment:

$$T_r \geq T_f \quad (21)$$

AASHTO-LRFD-2017

Torsion must be considered, according to §5.7.2.1 Eq. 5.7.2.1-3, if:

$$T_u > 0.25\phi T_{cr} \quad (22)$$

T_u is the applied factored torsional moment over the analyzed member. ϕ , the resistance factor is given in §5.5.4.2 and is equal to 0.90 for normal and lightweight concrete. To determine the torsional cracking moment, T_{cr} , first K , the effective length factor for compression members, must be computed according to Eq. 5.7.2.1-6:

$$K = \sqrt{1 + \frac{f_{pc,AAS}}{0.335\lambda\sqrt{f'_c}}} \leq 2.0 \quad (23)$$

$f_{pc,AAS}$ [MPa] is the unfactored compressive stress in concrete after prestress losses have occurred, taken either at the centroid of the cross-section resisting transient loads or at the junction of the web and flange where the centroid lies in the flange. f'_c [MPa] is the design compressive strength of concrete, λ_{AAS} is the concrete density modification factor given in §5.4.2.8. T_{cr} is provided in Eq. 5.7.2.1-4 and 5.7.2.1-5 as:

$$T_{cr} = \begin{cases} 0.329K\lambda\sqrt{f'_c} \frac{A_{cp}^2}{p_c} & \text{for solid shapes} \\ 0.329K\lambda\sqrt{f'_c} 2A_o b_e & \text{for hollow shapes} \end{cases} \quad (24)$$

A_o is the area enclosed by the shear flow path, including inner holes, A_{cp} is the area enclosed by the outside perimeter of the concrete cross-section, p_c is the outside perimeter of the concrete cross-section, b_e is the effective width of the shear flow path taken as the minimum thickness of the exterior webs or flanges comprising the closed box section. b_e should account for the presence of ducts, the diameters of ungrouted ducts or one-half the diameters of grouted ducts need to be subtracted from the web or flange thickness at the location of these ducts. b_e cannot exceed A_{cp} / p_c unless a more refined analysis is used.

To compute the required transverse reinforcement, ε_s needs to be determined, which is the net longitudinal tensile strain in the section at the centroid of the tension reinforcement. It can be obtained from §5.7.3.4.2, Eqs. 5.7.3.4.2-4, 5.7.3.4.2-5 and 5.7.3.4.2-6

$$\varepsilon_s = \begin{cases} \frac{\left(\left(\frac{|M_u|}{d_v} \right) + 0.5N_u + \left| \sqrt{V_u^2 + \left(\frac{0.9p_{h,AAS}T_u}{2A_o} \right)^2} - V_{p,AAS} \right| - A_{ps,AAS}f_{po,AAS}}{E_s A_s + E_p A_{ps}} & \text{for solid sections} \\ \frac{\left(\left(\frac{|M_u|}{d_v} \right) + 0.5N_u + \left| V_u + \frac{T_u d_s}{2A_o} - V_{p,AAS} \right| - A_{ps,AAS}f_{po,AAS}}{E_s A_s + E_p A_{ps}} & \text{for hollow sections} \end{cases} \quad (25)$$

§5.7.2.1 mentions that in statically indeterminate structures where redistribution of torsional moment can occur over a determined element, T_u can be taken as ϕT_{cr} . M_u is the factored applied moment and cannot be less than $|V_u - V_p|d_v$, N_u is the factored applied axial load, taken as positive for tension and negative for compression, V_u is the factored sectional shear, $V_{p,AAS}$ is the component of prestressing force in the direction of the shear force, $A_{ps,AAS}$ is the area of prestressing steel on the flexural tension side of the member, A_s is the area of non-prestressed tension reinforcement, d_s is the distance from the extreme compression fiber to the centroid of the non-prestressed tensile reinforcement measured along the centerline of the web, d_v is the effective shear depth taken as the distance between the resultants of the tensile and compressive forces due to flexure, $p_{h,AAS}$ is the perimeter of the centerline of the closed transverse torsion reinforcement for solid members, or the perimeter of the centroid of the transverse torsion reinforcement in the exterior webs and flanges for hollow members, and $f_{po,AAS}$ is a parameter taken as the modulus of elasticity of prestressing steel multiplied by the locked-in difference in strain between the prestressing steel and the surrounding concrete. For usual levels of prestressing, f_{po} can be taken as $0.7f_{pu}$ for both pretensioned and post-tensioned members. f_{pu} is the specified tensile stress of prestressing steel. E_s is the modulus of elasticity for steel reinforcement and E_p is the modulus of elasticity of prestressing steel. Now, with the longitudinal tensile strain ϵ_s , the angle of inclination of diagonal compressive stresses, θ_{AAS} is defined by §5.7.3.4.2, Eq. 5.7.3.4.2-2:

$$\theta_{AAS} = 29^\circ + 3500\epsilon_s \quad (26)$$

According to §5.7.3.4.1 θ_{AAS} can be taken as 45° for the following cases: concrete footings with a distance less than $3d_v$ from the point of zero shear to the face of the column, piers or walls with or without transverse reinforcement, and other non-prestressed concrete sections not subjected to axial tension, containing at least the minimum transverse reinforcement specified in §5.7.2.5 or having an overall depth of less than 0.40 m.

To design the transverse reinforcement for torsion A_t Eq. 5.7.3.6.2-1 is used:

$$\frac{A_t}{s} \geq \frac{T_u}{2\phi A_o f_{yt}} \tan \theta_{AAS} \quad (27)$$

The required stirrups for torsion should be added to those needed for shear. The total transverse provided transverse reinforcement should not be less than the sum of the required transverse reinforcement for shear and torsion. s is the spacing between stirrups and f_{yt} is the yield strength of transverse reinforcement.

The required area of longitudinal reinforcement for torsion A_l is given by Eq. 5.7.3.6.3-1 and 5.7.3.6.3-2. The longitudinal reinforcement for torsion should be added to the required reinforcement for bending moment:

$$A_l = \begin{cases} \frac{\left| \frac{M_u}{\phi d_v} + \frac{0.5N_u}{\phi} + \cot \theta_{AAS} \sqrt{\left(\left| \frac{V_u}{\phi} - V_p \right| - 0.5V_s \right)^2 + \left(\frac{0.45 p_h T_u}{2A_o \phi} \right)^2} - A_{ps,AAS} f_{ps}}{f_y} - A_s & \text{for solid sections} \\ \frac{A_t}{s} \frac{f_{yt}}{f_y} p_h \cot \theta_{AAS} & \text{for hollow sections} \end{cases} \quad (28)$$

V_s is the shear resistance provided by the transverse reinforcement, f_y is the yield strength for the longitudinal reinforcement, f_{ps} is the average stress in the prestressing steel at the time for which the nominal resistance of the member is required. The longitudinal steel reinforcement for solid sections should be distributed uniformly around the perimeter. For box sections, interior webs should not be considered in the calculation of the longitudinal torsional reinforcement. The values of $p_{h,AAS}$ and A_l should be for the box shape defined by the outermost webs and the top and bottom slabs of the box girder. Also, A_l needs to be distributed around the outermost webs and top and bottom slabs of the box girder.

(29)

To compute the maximum stirrup spacing, the shear stress v_u stated in §5.7.2.8, Eq. 5.7.2.8-1 is required:

$$v_u = \frac{|V_u - \phi V_p|}{\phi b_v d_v} \quad (30)$$

b_v is the web width adjusted for the presence of ducts as specified in §5.7.2.8. For circular cross-sections, b_v is the diameter of the cross-section, modified for the presence of ducts where applicable. The maximum spacing of the stirrups is provided in §5.7.2.6, Eq. 5.7.2.6-1 and 5.7.2.6-2:

$$s \leq \begin{cases} 0.8d_v \leq 600 \text{ mm} & \text{if } v_u < 0.125f'_c \\ 0.4d_v \leq 300 \text{ mm} & \text{if } v_u \geq 0.125f'_c \end{cases} \quad (31)$$

The AASHTO-LRFD-2017 code does not give any equation to compute the minimum longitudinal or transverse reinforcement for torsion.

T_n is the nominal torsional resistance, specified in Eq. 5.7.3.6.2-1 as:

$$T_n = \frac{2A_o A_t f_{yt} \cot \theta_{AAS}}{s} \quad (32)$$

The factored capacity of the element, ϕT_n , should be greater or equal than the factored demand T_u :

$$\phi T_n \geq T_u \quad (33)$$

EN 1992-1-1:2004

If the static equilibrium of the structure depends on the torsional resistance of some members of the structure, a full torsion design that fulfills both ultimate and serviceability provisions is necessary. However, if torsion is acting in statically indeterminate structures and it is present as a secondary effect, i.e. due to compatibility requirements, it may be neglected if the structure does not depend on the torsional resistance for its stability. In the last case, a minimum amount of longitudinal and transverse torsion reinforcement is needed to control excessive cracking. The minimum amounts are given in EN 1992-1-1:2004, §7.3.2, 9.2.1 and §9.2.2

The first step of a torsion design is to check if the cross-sectional dimensions are adequate. For this, the effective wall thickness t_{ef} expressed in §6.3.2 is required:

$$t_{ef} = \frac{A}{u} \geq 2c \quad (34)$$

If the analyzed member has a hollow cross-section, the effective wall thickness should be less than the actual wall thickness. A is the total area of the cross-section, including inner hollow areas, u is the perimeter of the cross-section, and c is the distance between the edge of the member and the centroid of the longitudinal reinforcement. The next step is to determine the torsional shear stress within the equivalent thin-walled tube, τ_t , according to Eq. 6.3.2 (6.26):

$$\tau_t = \frac{T_{Ed}}{2A_k t_{ef}} \quad (35)$$

T_{Ed} is the design torsional moment and A_k is the area enclosed by the centerlines of the connecting walls, including inner hollow areas. The applied shear force, V_{Ed} , caused by the design torsional moment T_{Ed} obtained from Eq. 6.3.2 (6.27) is:

$$V_{Ed} = \tau_t t_{ef} z \quad (36)$$

z is the distance along the centerline between the intersection points of the adjacent walls of the equivalent thin-walled tube, usually taken as the height of the element. The strength reduction factor for cracked concrete in shear, v , is provided in EN 1992-1-1:2004, §6.2.2 (6), Eq. (6.6N):

$$v = 0.6 \left[1 - \frac{f_{ck}}{250} \right] \quad (37)$$

f_{ck} [MPa] is the characteristic compressive cylinder strength of concrete at 28 days. Subsequently, α_{cw} , which is a coefficient that takes account the state of the stress in the compression chord, is computed using EN 1992-1-1:2004, §6.2.3 (3), Eq. (6.11.aN), (6.11.bN), and (6.11.cN):

$$\alpha_{cw} = \begin{cases} 1 & \text{for non-prestressed structures} \\ 1 + \frac{\sigma_{cp}}{f_{cd}} & \text{for } 0 < \sigma_{cp} \leq 0.25f_{cd} \\ 1.25 & \text{for } 0.25f_{cd} < \sigma_{cp} \leq 0.5f_{cd} \\ 2.5 \left(1 - \frac{\sigma_{cp}}{f_{cd}} \right) & \text{for } 0.5f_{cd} < \sigma_{cp} \leq 1.0f_{cd} \end{cases} \quad (38)$$

σ_{cp} is the mean compressive stress in the concrete due to the design axial force or prestressing. The value of σ_{cp} does not need to be calculated at a distance less than $0.5\cot\theta_{EN}$ from the edge of the support. f_{cd} is the design value of the concrete compressive strength. θ_{EN} is the angle of the strut inclination given in EN 1992-1-1:2004, §6.2.3 (4), Eq. (6.7N). The effects of torsion and shear may be added if the angle of the strut inclination is the same. The limits of the angle are $21.8^\circ \leq \theta_{EN} \leq 45^\circ$. The upper limit of the torsional strength, $T_{Rd,max}$ is given in Eq. (6.30) as:

$$T_{Rd,max} = 2v_1\alpha_{cw}f_{cd}A_k t_{ef} \sin\theta_{EN} \cos\theta_{EN} \quad (39)$$

The recommended value of v_l is v , see Equation (37). If the design yield strength of the transverse reinforcement, f_{ywd} , is below 80% of the characteristic yield strength of reinforcement f_{yk} , v_l can be taken according to Eq. 6.2.3 (6.10.aN) and 6.2.3 (6.10.bN) as:

$$v_l = \begin{cases} 0.6 & \text{for } f_{ck} \leq 60\text{MPa} \\ 0.9 - \frac{f_{ck}}{200} > 0.5 & \text{for } f_{ck} \geq 60\text{MPa} \end{cases} \quad (40)$$

$V_{Rd,max}$, the upper limit of the shear strength, is calculated using Eq. (6.9) of EN 1992-1-1:2004, §6.2.3 (2):

$$V_{Rd,max} = \frac{\alpha_{cw}b_{w,EN}z v_1 f_{cd}}{\cot\theta_{EN} + \tan\theta_{EN}} \quad (41)$$

$b_{w,EN}$ is the width of the cross-section and for T, I or L beams it is the width of the web. If the web width contains ducts, the web width should be calculated according to §6.2.3 (6). Once $T_{Rd,max}$ and $V_{Rd,max}$ are obtained, the maximum combined shear and torsion capacity should be checked according to Eq. (6.29) to check if crushing of the concrete occurs:

$$\frac{T_{Ed}}{T_{Rd,max}} + \frac{V_{Ed}}{V_{Rd,max}} \leq 1.0 \quad (42)$$

If the inequality is not satisfied, f_{cd} or A need to be modified.

The first step for the torsion design is to compute the required amount of transverse reinforcement A_{sw} . The stirrups for torsion must be added to the calculated reinforcement for shear. The code does not specify an equation to

calculate the required number of stirrups for torsion. EN 1992-1-1:2004 mentions that the required area of transverse torsion reinforcement A_{sw} should be obtained using the same method as for shear stirrups, therefore:

$$\frac{A_{sw}}{s} \geq \frac{T_{Ed}}{2A_k f_{ywd}} \tan \theta_{EN} \quad (43)$$

s is the spacing of the stirrups. The longitudinal reinforcement, A_{sl} , for torsion needs to be added to the computed reinforcement required for flexure. The longitudinal reinforcement should generally be distributed over the length of the side, z . EN 1992-1-1:2004, §9.2.3 (3) states that the longitudinal reinforcement bars required for torsion A_{sl} need to be organized to have at least one bar at each corner of the stirrups. The remaining steel bars can be distributed uniformly around the stirrups' perimeter inner face. Using Eq. 6.3.2 (6.28) the required longitudinal reinforcement for torsion A_{sl} is:

$$A_{sl} \geq \frac{T_{Ed} u_k}{2A_k f_{yd}} \cot \theta_{EN} \quad (44)$$

u_k is the perimeter of the A_k area and f_{yd} is the design yield strength of the longitudinal reinforcement.

The transverse reinforcement ratio to compute the minimum transverse reinforcement for torsion is given in Eq. 9.2.2 (9.5N) as:

$$\rho_w \geq 0.08 \frac{\sqrt{f_{ck}}}{f_{yk}} = \rho_{w,min} \quad (45)$$

If torsion arises from compatibility in statically indeterminate structures, then it is unnecessary to consider torsion as an ultimate limit state. In this case, minimum longitudinal and transverse reinforcement should be provided to prevent excessive cracking. According to Eq. 9.2.2 (9.4) and the value found in Equation (45), The minimum area of transverse reinforcement $A_{sw,min}$ is calculated as:

$$\frac{A_{sw,min}}{s} = \rho_{w,min} b_{w,EN} \sin \alpha \quad (46)$$

α is the angle between the transverse reinforcement and the longitudinal axis. The maximum spacing of the stirrups is defined by EN 1992-1-1:2004, §9.2.3 (3) as:

$$s \leq \min \begin{cases} \frac{u}{8} \\ 0.75d(1 + \cot \alpha) \\ \min(\text{all dimensions}) \end{cases} \quad (47)$$

d is the effective depth of the cross-section. To find the minimum amount of longitudinal reinforcement, which is based on a requirement to control excessive cracking, first h^* , which is the overall height of the cross-section within the tensile zone, needs to be determined according to §7.3.2 as:

$$h^* = \begin{cases} h & \text{for } h < 1000 \text{ mm} \\ 1000 \text{ mm} & \text{for } h \geq 1000 \text{ mm} \end{cases} \quad (48)$$

h is the overall depth of the cross-section. The next step is to calculate k_l , which is a coefficient given in §7.3.2 that considers the effects of axial forces on the stress distribution:

$$k_1 = \begin{cases} 1.5 & \text{if } N_{Ed} \text{ is a compressive force} \\ \frac{2h^*}{3h} & \text{if } N_{Ed} \text{ is a tensile force} \end{cases} \quad (49)$$

N_{Ed} is the axial force at the serviceability limit state acting on the part of the cross-section under consideration (compressive force positive), resulting from the characteristic values of prestress and axial forces under the relevant load combination.

k_c is a coefficient which takes into account the stress distribution within the section immediately prior to cracking and of the change of the lever arm. It is defined by Eq. 7.3.2 (7.2) and (7.3) as:

$$k_c = \begin{cases} 1.0 & \text{for pure tension, any cross section} \\ 0.4 \left[1 - \frac{\frac{N_{Ed}}{bh}}{k_1 \left(\frac{h}{h^*} \right) f_{ct,eff}} \right] \leq 1.0 & \text{only for rectangular sections, webs of box sections and T-sections} \\ 0.9 \frac{F_{cr}}{A_{ct} f_{ct,eff}} \geq 0.5 & \text{only for flanges of box and T-sections} \end{cases} \quad (50)$$

A_{ct} is the area of concrete within the tension zone before the formation of the first crack, b is the overall width of the cross-section, or the actual flange width in a T- or L-shaped beam. F_{cr} is the absolute value of the tensile force within the flange immediately prior to cracking due to the cracking moment calculated with $f_{ct,eff}$. $f_{ct,eff}$ is the mean value of the tensile strength of the concrete, effective at the time when the cracks are first expected to occur.

k , given in §7.3.2 is a coefficient which accounts for the effect of non-uniform self-equilibrating stresses, which lead to a reduction of restraint forces.

$$k = \begin{cases} 1.0 & \text{for webs with } h \leq 0.3 \text{ m or flanges with } b < 0.3 \text{ m} \\ 0.65 & \text{for webs with } h \geq 0.8 \text{ m or flanges with } b > 0.8 \text{ m} \end{cases} \quad (51)$$

For other intermediate values of height and width, interpolation is allowed. With these parameters, the minimum area of longitudinal steel for torsion $A_{sl,min}$ is given by Eq. 7.3.2 (7.1) and 9.2.1.1 (9.1N):

$$A_{sl,min} = \max \left\{ \begin{array}{l} \frac{k_c k f_{ct,eff} A_{ct}}{\sigma_s} \\ 0.26 \frac{f_{ctm}}{f_{yk}} b_t d \geq 0.0013 b_t d \end{array} \right. \leq 0.04 A_{c,EN} \quad (52)$$

σ_s is the absolute value of the maximum stress permitted in the reinforcement immediately after formation of the crack. σ_s is often taken as the yield strength of the reinforcement, f_{yk} . A lower value may however, be needed to satisfy the crack width limits according to the maximum bar size or spacing, see §7.3.3 (2). b_t denotes the mean width of the tension zone, for a T-beam with the flange in compression, only the width of the web is considered for calculating the value of b_t , f_{ctm} is the mean value of the axial tensile strength of the concrete, and $A_{c,EN}$ is the gross area of the concrete.

Next, the torsional capacity T_{Rd} of the element is computed:

$$T_{Rd} = \min \begin{cases} \frac{2A_k A_{sw} f_{ywd}}{s} \cot \theta_{EN} \\ \frac{2A_k A_{sl} f_{yd}}{u_k} \tan \theta_{EN} \end{cases} \quad (53)$$

Finally, the torsional capacity needs to be larger than or equal to T_{Ed}

$$T_{Rd} \geq T_{Ed} \quad (54)$$

When the cross-section of the shape is irregular, like a T-section, it can be divided into rectangular subsections. Each of these needs to be modeled using the space truss, thin-walled tube analogy to obtain the torsional resistance. The overall resistance of the irregular section will be the sum of the subdivisions. The external torsional moment applied on each individual subsection is proportional to each uncracked torsional stiffness. The maximum resistance of an element under torsion is limited by the capacity of the concrete struts [9].

If torsion is not as important as other actions, a minimum longitudinal and transverse reinforcement for torsion must be provided. Warping torsion can be neglected in hollow thin-walled and solid sections. In open thin-walled shapes (like T-, I- or L-shapes) the calculation of the effect of warping torsion should be made for every slender cross-section using a beam-grid model. For other cases, the analysis can be carried out by a truss model.

MC2010

This code establishes that if static equilibrium depends on the torsional resistance of the elements of the structure, a full torsional design must be provided. On the other hand, if torsion arises due to compatibility, generally a torsion design is not needed. In cases where compatibility torsion occurs, minimum longitudinal and transverse reinforcement for torsion should be provided.

The first step is to check if the dimensions of the cross-section are adequate. For this, the longitudinal strain $\varepsilon_{x,MC}$ at mid-depth of the effective shear depth, needs to be computed. It is defined in §7.3.3.1, Eq. 7.3-14 and 7.3-16 as:

$$\varepsilon_{x,MC} = \begin{cases} \frac{1}{2E_s A_s} \left[\frac{M_{Ed}}{z_{MC}} + V_{Ed} + N_{Ed,MC} \left(\frac{1}{2} \pm \frac{\Delta_e}{z_{MC}} \right) \right] & \text{for non-prestressed members} \\ \frac{M_{Ed0} \pm F_p \cos \delta_p e_p + M_{p,ind} + V_{Ed0} - F_p \sin \delta_p + (N_{Ed0} - F_p \cos \delta_p) \frac{z_p - e_p}{z_{MC}}}{2 \left(\frac{z_s}{z_{MC}} E_s A_s + \frac{z_p}{z_{MC}} E_p A_{p,MC} \right)} & \text{for prestressed members} \end{cases} \quad (55)$$

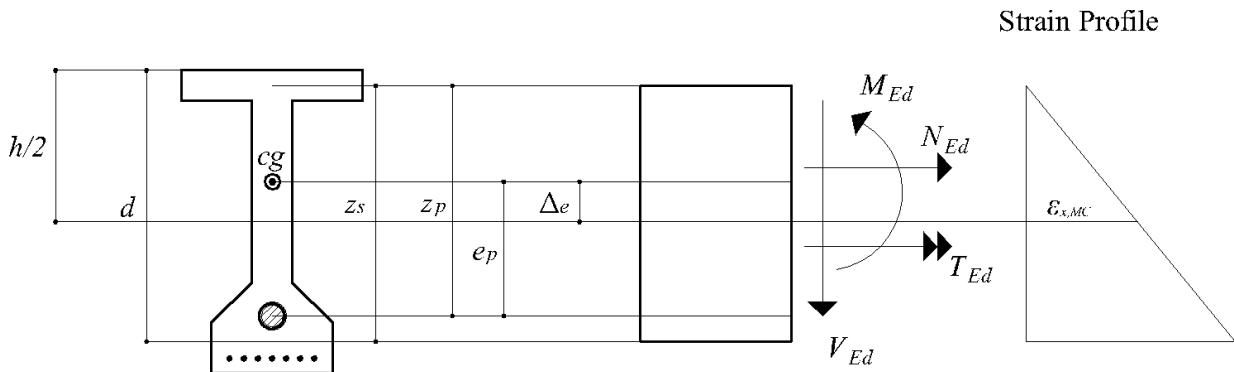


Figure 7—Definition of the variables used in Equation (55)

M_{Ed} is the design bending moment, V_{Ed} is the design shear force (M_{Ed} and V_{Ed} are positive), $N_{Ed,MC}$ is the applied axial force (positive for tension and negative for compression), z_{MC} is the effective shear depth which cannot be less than $0.9d$ for non-prestressed members, and d is the effective depth. In case of a support that penetrates the beam or slab, z_{MC} is replaced with $d_{v,MC}$, which is the distance from the centroid of the reinforcement layers to the supported area. Δ_e is the difference between the position of the applied axial load and the centroid of the cross-section, E_s is the modulus of elasticity of the reinforcing steel, E_p is the modulus of elasticity of the prestressing steel, A_s is the area of longitudinal reinforcement, and $A_{p,MC}$ is the area of prestressing reinforcement. M_{Ed0} , V_{Ed0} , and N_{Ed0} are the bending moment, shear and normal force without the effect of prestressing. $M_{p,ind}$ is the secondary moment caused by prestressing, F_p is the prestressing force, e_p is the eccentricity of prestressing, δ_p is the tendon angle, z_s is the distance between the centerline of the compressive chord and the reinforcement, and z_p is the distance between the tendon axis and the compressive chord.

The design approach of the MC2010 code is by Levels of Approximation. Level I represents the simplest and quickest approach, valid for standard design cases. The use of higher Levels of Approximation means more computational effort and time but will result in a more accurate solution. For the value of the minimum compressive stress field inclination θ_{min} , four levels of approximation can be used. The angle θ_{MC} selected to make the calculations can be chosen according to §7.3.3.3, Eq. 7.3-35 between:

$$\theta_{min} \leq \theta_{MC} \leq 45^\circ \quad (56)$$

The Level of Approximation I for θ_{min} , using a variable angle truss model approach, states:

$$\theta_{min} = \begin{cases} 25^\circ & \text{for members with significant axial compression or prestress} \\ 30^\circ & \text{for reinforced concrete members} \\ 40^\circ & \text{for members with significant axial tension} \end{cases} \quad (57)$$

§7.3.3.3, Eq. 7.3-39 in the MC2010 gives a definition for the minimum angle for a Level of Approximation II (based on a generalized stress field approach) and III (represents a general form of sectional shear equations and is based on the simplified modified compression field theory), defined as:

$$\theta_{min} = 20^\circ + 10,000\varepsilon_{x,MC} \quad (58)$$

The Level of Approximation IV states that the angle can be determined using a finite element method. Appropriate stress-strain models for the steel and for diagonally cracked concrete should be used.

Now, the parameter ε_l is required to calculate the strength reduction factor, which will be used later to check if the cross-sectional dimensions are adequate. It is defined in §7.3.3.3, Eq. 7.3-41 as:

$$\varepsilon_l = \varepsilon_{x,MC} + (\varepsilon_{x,MC} + 0.002) \cot^2 \theta_{MC} \quad (59)$$

Consequently, k_e , a factor that considers the influence of the state of strain in the web, is computed according to Eq. 7.3-37 or 7.3-40:

$$k_e = \begin{cases} 0.55 & \text{for Level I or when } \varepsilon_x < 0.0001 \\ \frac{1}{1.2 + 55\varepsilon_l} \leq 0.65 & \text{for Level II and III} \end{cases} \quad (60)$$

Eq. (7.3-28) defines η_{fc} as:

$$\eta_{fc} = \left(\frac{30}{f_{ck}} \right)^{\frac{1}{3}} \leq 1.0 \quad (61)$$

f_{ck} [MPa] is the characteristic value of the compressive strength of concrete. The strength reduction factor, $k_{c,MC}$ is calculated according to Eq. 7.3-27:

$$k_{c,MC} = k_e \eta_{fc} \quad (62)$$

The next step is to compute the maximum shear resistance $V_{Rd,max}$, using θ_{min} found in Equation (57) or (58). $V_{Rd,max}$ is defined by Eq. 7.3-26 as:

$$V_{Rd,max} = \frac{k_c f_{ck} b_w z}{\gamma_c} \sin \theta_{min} \cos \theta_{min} \quad (63)$$

§7.2.3.1.4 specifies that γ_c is the partial safety factor for concrete. $\gamma_c = 1.5$ for standard loading and 1.2 for incidental loading. b_w is the width of the web. The effective panel thickness t_{ef} is according to §7.3.4.1, Eq. 7.3-54:

$$t_{ef} \leq \frac{d_k}{8} \quad (64)$$

d_k is the diameter of the circle that can be inscribed at the narrowest part of the cross-section. The effective panel thickness should have at least a value of twice the distance between the concrete surface and the center of the closest layer of longitudinal reinforcement. In the case of box-girders, the effective panel thickness corresponds to the wall thickness, if the wall is reinforced on all sides. The upper bound of the torsional resistance $T_{Rd,max}$ can be obtained from §7.3.4.1, Eq. 7.3-56:

$$T_{Rd,max} = 2k_c \frac{f_{ck}}{\gamma_c} t_{ef} A_k \sin \theta_{MC} \cos \theta_{MC} \quad (65)$$

A_k is the area enclosed by the centerlines of the connecting walls, including inner hollow areas. With $V_{Rd,max}$ and $T_{Rd,max}$ known, the dimensions of the cross-sections can be checked according to §7.3.4.1, Eq. (7.3-55)

$$\left(\frac{T_{Ed}}{T_{Rd,max}} \right)^2 + \left(\frac{V_{Ed}}{V_{Rd,max}} \right)^2 \leq 1.0 \quad (66)$$

Where T_{Ed} is the applied torsional moment.

The required amount of transverse reinforcement for torsion A_{sw} is obtained by assuming that the torsional moment will be resisted only by the stirrups. For this, Eq. 7.3-53 was substituted into Eq. 7.3-29:

$$\frac{A_{sw}}{s_w} \geq \frac{T_{Ed}}{2A_k f_{ywd}} \tan \theta_{MC} \quad (67)$$

f_{ywd} is the yield strength of the transverse reinforcement and s_w is the spacing of the stirrups.

The required longitudinal reinforcement for torsion, A_{st} results from substituting Eq 7.3-53 into 7.3-34:

$$A_{st} = \frac{\left(V_{Ed} + \frac{T_{Ed} z}{2A_k} \right) \cot \theta_{MC}}{f_{yk}} \quad (68)$$

f_{yk} is the characteristic value of the yield strength of reinforcing steel in tension.

The minimum area of transverse reinforcement $A_{sw,min}$ required for torsion should also fulfill §7.13.5.2, Eq. 7.13-9.

$$\frac{A_{sw,min}}{s_w} = 0.08 \sqrt{f_{ck}} \frac{b_w}{f_{ywd}} \quad \text{with } f_{ck} \text{ in [MPa]} \quad (69)$$

The maximum spacing between stirrups s_w is defined in §7.13.5.2 as:

$$s_w \leq \min \begin{cases} 0.75d \\ 500 \text{ mm} \end{cases} \quad (70)$$

d is the effective depth. The minimum longitudinal reinforcement for torsion $A_{st,min}$ according to §7.13.5.2, Eq. 7.13-8 is:

$$A_{st,min} = 0.26 \frac{f_{ctm}}{f_{yk}} b_{t,MC} d \quad (71)$$

$b_{t,MC}$ is the width of the tension zone and f_{ctm} is the mean value of the axial tensile strength of concrete.

The torsional capacity T_{Rd} is computed as:

$$T_{Rd} = \min \begin{cases} \frac{2A_k A_{sw} f_{ywd}}{s_w} \cot \theta \\ \frac{4A_k A_{st} f_{yd}}{z} \tan \theta \end{cases} \quad (72)$$

Finally, the torsional capacity T_{Rd} needs to be larger than the applied torsional moment T_{Ed} .

$$T_{Rd} \geq T_{Ed} \quad (73)$$

DISCUSSION

The previous sections showed that there are two major philosophies for determining the torsional capacity of structural concrete members: 1) skewed-bending analysis, and 2) truss analogy (with or without the consideration of the concrete's contribution). However, all the building codes presented in this document use a 3D-truss model and the thin-walled tube analogy to predict the failure of the members. According to Hsu²⁷, the advantages of this theory are: the interaction of shear and torsion with bending and axial load is well-described, the effect of prestress can be included in a logical way, it provides a reasonable accuracy between the model and the experimental tests, and the distinct advantage over the skewed-bending theory is that the truss analogy can predict the deformation of a member throughout the loading history. Within the space truss model, the codes presented here use either a variable angle truss or a MCFT method to predict the behavior of concrete members under torsion. One of the differences between them is how each one obtains the angle of inclination of the concrete struts or compressive field. The variable angle truss method fixes an assumed angle for the inclination of the struts, while the MCFT considers compatibility and equilibrium conditions to determine the angle of the compression field. The other difference is that the first method does not contemplate the tensile contribution of the concrete to the torsional strength, whereas the MCFT does. Nevertheless, other models have shown to predict the behavior of structural members with good accuracy. One of them is the Softened Membrane Model for Torsion³⁴ which is an extension of the Softened Membrane Model for Shear³⁵. Another new model³⁶ that follows the skew-bending theory has shown better prediction results on the shear strength of hollow circular structural concrete cross-sections compared to the methods used in EN 1992-1-1:2004 and fib Model Code 2010. This statement is based on the experimental testing of 45 specimens³⁷. However, this model still needs to be extended to other types of cross-sections and to the torsion design problem.

Several subjects of discussion remain concerning torsion in structural concrete. The first topic is the capacity of the members resisting loads by warping torsion. All five codes listed here assume that the external torsional moment will be resisted by circulatory torsion. Nevertheless, box-, T-, or I-shaped concrete beams tend to produce differential shear stresses on their cross-sectional planes to resist torsion, due to the characteristic restriction of their connected flanges and webs. None of the codes give clear provisions on how to deal with members resisting torsional moments by warping torsion. A second important subject is the torsion effect on slabs. Point loads on slabs close to the edges produce large torsional moments³⁸. None of the codes presented in this document give clear provisions on how to address the effect of torsion on the shear capacity of concrete slabs at the edges.

One topic of recent research is the torsional behavior of structural concrete members under different physical and geometric conditions. Examples include the analysis of limitations of torsional reinforcement to prevent a brittle failure³⁹. Based on the experiments of 15 beams with the maximum torsional reinforcement ratio as the main parameter and 99 existing tests obtained from the literature, it was observed that the ACI 318-19 and JSCE-07⁴⁰ codes predicted the torsion failure with good accuracy when having the maximum ratio of torsional reinforcement. On the other hand, EC2-04 and CSA-14 building codes overestimated the limit between a brittle and ductile failure. Another research topic is the torsional performance of beams subjected to pure torsion with low levels of torsional reinforcement⁴¹. In this research it was found that high strength concrete beams (HSC) with a total torsional reinforcement ratio of less than 0.95% presented a brittle failure. On the other hand, HSC and normal strength concrete (NSC) specimens with a total torsional reinforcement larger than 0.95% and 0.87%, respectively, showed a ductile torsional failure. Moreover, an experimental study⁴² on the comparison of HSC and NSC beams under torsion with the same amount of reinforcement concluded that HSC elements provided a higher torsional strength than NSC. The uncracked torsional stiffness and the cracked stiffness of HSC beams was approximately 2 times and 1.4 times, respectively, compared to the NSC elements. Another example is the torsional behavior of concrete elements using CTR (continuous transverse reinforcement)⁴³. In summary, it was demonstrated that the pure torsional resistance using CTR sometimes exceeds the strength obtained with conventional stirrups. Nevertheless, if the cracks due to torsion have the same direction as the CTR, the strength is decreased. Experimental tests of the torsional behavior of high-strength reinforced concrete under-reinforced beams showed that torsional strength of these elements is independent of the concrete strength as long as the beam is under-reinforced⁴⁴.

A second topic of research is the use of innovative materials. An example includes beams with glass fiber-reinforced polymer (GFRP) bars and stirrups. The advantage of such bars is the superior performance from a durability point of view. These bars cost less than carbon fiber-reinforced polymer bars and offer a different solution to the corrosion problem. Investigation on this topic concluded that the GFRP-reinforced concrete beams under torsion exhibited a similar strength and cracking behavior compared to the counterpart steel reinforced concrete (RC) beams⁴⁵. Waste materials like oil palm shell have been tested as a substitute to granite aggregate to produce a lightweight concrete. Experimental analysis⁴⁶ on the torsional behavior of oil palm shell concrete (OPSC) compared to normal weight concretes (NWC) demonstrated that the OPSC had a 280% larger twist at failure than the NWC and a better torsional ductility. Another application is the use of steel fiber reinforced concrete (SFRC). Abundant research has been carried out on rectangular SFRC beams⁴⁷⁻⁵². However, most beams in real structures have T- or L-shaped cross-sections. Therefore, it is important to understand how steel fibers influence the torsional behavior of non-rectangular beams. Experimental investigation⁵³ on this topic showed that steel fibers can increase the torsional strength after cracking and are very effective in preventing a sudden brittle failure in flanged beams that presented a steel fiber volume of 3%.

A third topic of current research on torsion is the strengthening of structures that are subjected to torsional moments. The various types of wrapping using carbon fiber-reinforced polymer (CFRP) fabrics⁵⁴ showed that the full-wrapping technique enhances the torsional behavior. However its practical application is limited because the access to the sides of the beam is restricted. On the other side, the U-jacket technique is the most achievable and practical wrapping. Nevertheless, it showed less effectiveness in strengthening for torsion compared to the extended U-jacket and the full wrapping technique. An analytical model⁵⁵ which uses a smeared crack analysis for plain concrete in torsion for the pre-cracking behavior and a softened truss theory for the post-cracking performance has shown good prediction of the torsional capacity of beams retrofitted with CFRP. CFRP sheets are also used to repair damaged RC elements under torsion. After tests were made⁵⁶, it was shown that the torsional capacity of columns was larger than the original torsional strength, after they were repaired with CFRP.

SUMMARY OF CONCLUSIONS

This study summarizes the provisions for torsion design in structural concrete. All the required provisions given in ACI 318-19, CSA-A23.3-04, AASHTO-LRFD-17, EN 1992-1-1:2004, and the fib Model Code 2010 are listed. A short literature review on the early stages of torsion research plus brief descriptions of the mechanical models used to describe the torsional behavior are given. The two major philosophies, 1) space truss analogy and 2) skew-bending theory are summarized. All the codes listed here use a 3D truss analogy. The ACI 318-19, EN 1992-1-1:2004 and Level I of Approximation of the fib Model Code 2010 do not consider the contribution of the concrete to the torsional strength. On the contrary, the CSA-A23.3-04, AASHTO-LRFD-17 and Levels II and III of approximation in the fib Model Code 2010 include the concrete contribution in the determination of the torsional capacity.

Furthermore, topics outside the scope of current provisions such as how to design structural concrete elements under warping torsion or the effect of torsion on the shear capacity of concrete slabs at the edges are discussed. Finally, an overview of recent topics in torsion research was presented.

LIST OF NOTATIONS

b	= overall width of the cross-section, or the actual flange width in a T-or L-shaped beam,
b_e	= effective width of the shear flow path taken as the minimum thickness of the exterior webs or flanges comprising the closed box section or flanges comprising the closed box section,
b_t	= mean width of the tension zone; for a T-beam with the flange in compression, only the width of the web is considered for calculating the value of b_t ,
$b_{t,MC}$	= width of the tension zone,
b_v	= web width adjusted for the presence of ducts,
b_w	= web width or diameter of a circular section,
$b_{w,EN}$	= web width of the cross-section for T, I or L beams,
c	= distance between the edge of the member and the centroid of the longitudinal reinforcement,
d	= effective depth,
ds	= unit area,
d_s	= distance from extreme compression fiber to the centroid of the non-prestressed tensile reinforcement measured along the centerline of the web,
d_k	= diameter of the circle that can be inscribed at the narrowest part of the cross-section,
d_v	= effective shear depth,
$d_{v,MC}$	= in case of a support that penetrates the beam or slab, it is the distance from the centroid of the reinforcement layers to the supported area,
e_p	= eccentricity of prestressing,
f'_c	= specified compressive strength of concrete,
f_{cd}	= the design value of concrete compressive strength,
f_{ck}	= characteristic compressive cylinder strength of concrete at 28 days,
f_{cp}	= average compressive stress in concrete due to the effective prestress force only, after allowance for all prestress losses,
$f_{ct,eff}$	= mean value of the tensile strength of concrete, effective at the time when the cracks may first be expected to occur,
f_{ctm}	= mean value of axial tensile strength of concrete,
f_{pc}	= compressive stress in concrete, after allowance for all prestress losses, at centroid of cross-section resisting externally applied loads or at junction of web and flange where the centroid lies within the flange. In a composite member, it is the resultant compressive stress at centroid of composite section, or at junction of web and flange where the centroid lies within the flange, due to both prestress and moments resisted by precast member acting alone,
$f_{pc,AAS}$	= unfactored compressive stress in concrete after prestress losses have occurred either at the centroid of the cross-section resisting transient loads or at the junction of the web and flange where the centroid lies in the flange,
f_{pe}	= effective stress in prestressing tendons after allowance for all prestress losses,
f_{po}	= stress in prestressing tendons when the strain in the surrounding concrete is zero,
$f_{po,AAS}$	= parameter taken as modulus of elasticity of prestressing steel multiplied by the locked-in difference in strain between the prestressing steel and the surrounding concrete,
f_{ps}	= average stress in prestressing steel at the time for which the nominal resistance of member is required
f_{pu}	= specified tensile strength of prestressing reinforcement,

$f_{pu,CSA}$	= specified tensile strength of prestressing tendons,
f_{se}	= effective stress in prestressing reinforcement, after allowance for all prestress losses,
f_y	= specified yield strength for non-prestressed longitudinal reinforcement,
f_{yd}	= design yield strength of the longitudinal reinforcement,
f_{yk}	= characteristic yield strength of reinforcement,
f_{yt}	= specified yield strength of transverse reinforcement,
f_{ywd}	= design yield strength of the transverse reinforcement,
h	= overall height of the member,
h^*	= overall height of the cross-section to calculate the minimum longitudinal reinforcement to control excessive cracking, within the tensile zone,
k	= coefficient which accounts for the effect of non-uniform self-equilibrating stresses,
k_c	= coefficient which accounts the stress distribution within the section immediately prior to cracking and of the change of the lever arm,
k_c	= strength reduction factor,
k_e	= factor which considers the influence of the state of strain in the web,
k_l	= coefficient that considers the effects of axial forces on the stress distribution,
n_{fc}	= coefficient needed to compute k_c ,
p_c	= outside perimeter of the cross-section,
p_{cp}	= outside perimeter of concrete cross-section,
p_h	= perimeter of the centerline of outermost closed transverse torsion reinforcement,
$p_{h,AAS}$	= perimeter of the centerline of the closed transverse torsion reinforcement for solid members, or the perimeter of the centroid of the transverse torsion reinforcement in the exterior webs and flanges for hollow members,
s	= center-to-center spacing of stirrups,
s_w	= spacing of the stirrups for torsion,
t	= wall thickness at the location where the stresses are being checked,
t_{ef}	= effective wall thickness,
u	= perimeter of the cross-section,
u_k	= perimeter of the A_k area,
z	= distance along the centerline, between the intersection points of the adjacent walls of the equivalent thin-walled tube, usually taken as the height of the element,
z_{MC}	= effective shear depth,
z_p	= distance between the tendon axes and the compressive chord,
z_s	= distance between the centerline of the compressive chord and the reinforcement,
A	= total area of the cross-section, including inner hollow areas,
A_c	= area enclosed by outside perimeter of concrete cross section, including area holes,
A_{cp}	= area enclosed by the outside perimeter of concrete cross-section,
A_{ct}	= area of concrete within tensile zone before the formation of the first crack,
$A_{c,EN}$	= cross-sectional area of the concrete,
A_g	= gross area of the concrete cross-section,
A_k	= area enclosed by the centerlines of the connecting walls, including inner hollow areas,
A_l	= required area of longitudinal reinforcement to resist torsion,
$A_{l,min}$	= minimum area of longitudinal reinforcement to resist torsion,
A_m	= area enclosed by the shear forces,
A_o	= area enclosed by the shear flow path including area of holes,
A_{oh}	= area enclosed by centerline of the outermost closed transverse torsional reinforcement, including area holes,
A_p	= area of tendons on the flexural tension side of the member,
A_{ps}	= area of prestressed longitudinal tension reinforcement,
$A_{ps,AAS}$	= area of prestressing steel on the flexural tension side of the member,
$A_{p,MC}$	= area of prestressing reinforcement,
A_s	= area of non-prestressed longitudinal tension reinforcement,
A_{sl}	= area of longitudinal reinforcement bars required for torsion,
$A_{sl,min}$	= minimum area of longitudinal reinforcement bars required for torsion,
A_{st}	= area of longitudinal reinforcement for torsion,
A_{sw}	= area of transverse torsion reinforcement,

$A_{sw,min}$	= minimum area of transverse torsion reinforcement,
A_t	= area of one leg of a closed stirrup resisting torsion,
$A_{t,min}$	= minimum area of transverse reinforcement for torsion,
A_v	= required area of shear reinforcement. Required area of two legs of a closed stirrup for shear,
$A_{v,min}$	= minimum transverse reinforcement for shear force,
E_p	= modulus of elasticity of prestressing tendons,
E_s	= modulus of elasticity of non-prestressed reinforcement,
F_{cr}	= absolute value of the tensile force within the flange immediately prior to cracking due to the cracking moment calculated with $f_{ct,eff}$,
F_p	= the prestressing force,
G	= shear modulus,
K	= the effective length factor for compression members,
M_f	= moment due to factored loads,
$M_{p,ind}$	= secondary moment caused by prestressing,
M_u	= factored applied moment,
M_{Ed}	= design bending moment,
M_{Ed0}	= bending moment without the effect of prestressing,
N_{Ed}	= axial force at the serviceability limit state acting on the part of the cross-section under consideration (compressive force positive), resulting from the characteristic values of prestress and axial forces under the relevant load combinations,
$N_{Ed,MC}$	= applied axial force, positive for tension and negative for compression,
N_{Ed0}	= normal force without the effect of prestressing,
N_f	= factored axial load normal to the cross-section. Taken as positive for tension, negative for compression
N_u	= factored axial force, taken as negative for tension and positive for compression,
T_{cr}	= cracking torsional moment,
T_f	= factored torsional moment,
T_n	= nominal torsional resistance,
T_r	= factored torsional resistance,
T_{th}	= threshold torsional moment,
T_u	= applied factored torsional moment,
T_{Ed}	= applied torsional moment,
T_{Rd}	= factored torsional strength,
$T_{Rd,max}$	= upper limit of the torsion strength,
V_c	= shear strength provided by concrete,
V_f	= factored shear force,
V_p	= component in the direction of the applied shear of the effective prestressing force factored by ϕ_p ,
$V_{p,AAS}$	= component of prestressing force in the direction of the shear force,
V_s	= shear resistance provided by transverse reinforcement,
V_u	= factored shear force,
V_{Ed}	= applied shear force,
$V_{Rd,max}$	= maximum shear resistance,
α	= angle between the transverse reinforcement and the longitudinal axis,
α_{cw}	= coefficient that takes into account the state of the stress in the compression chord,
γ_c	= partial safety factor for concrete,
δ_p	= tendon angle,
ϵ_s	= net longitudinal tensile strain in the section at the centroid of the tension reinforcement,
ϵ_x	= longitudinal strain at mid-depth of the member due to factored loads,
$\epsilon_{x,MC}$	= longitudinal strain at mid-depth of the effective shear depth,
ϵ_l	= coefficient required to compute k_c ,
θ	= angle between the struts and the tension chord,
θ_{check}	= actual angle of the strut inclination,
$\theta_{check,MC}$	= actual angle of the compression field,
θ_{min}	= minimum value of the compressive stress field inclination,
θ_t	= angular rotation,
θ_{AAS}	= angle of inclination of diagonal compressive stresses,
θ_{CSA}	= angle of inclination of diagonal compressive stresses, measured from the longitudinal axis of the member,

θ_{EN}	= assumed angle of the strut inclination,
θ_{MC}	= assumed angle of the compression field,
λ	= modification factor which accounts for the properties of lightweight concrete,
λ_{AAS}	= concrete density modification factor,
v	= strength reduction factor for cracked concrete in shear,
v_l	= strength reduction factor for cracked concrete in shear when the design yield strength of the transverse reinforcement, is below 80% of the characteristic yield strength of reinforcement,
v_u	= shear stress,
ρ_w	= transverse reinforcement ratio,
$\rho_{w,min}$	= minimum transverse reinforcement ratio,
σ_{cp}	= mean compressive stress in the concrete, due to the design axial force or prestressing,
σ_s	= absolute value of the maximum stress permitted in the reinforcement immediately after formation of the crack,
τ	= tangential shear forces,
τ_t	= torsional shear stress,
ϕ	= resistance factor,
ϕ_c	= resistance factor for concrete,
ϕ_p	= resistance factor for prestressing tendons,
ϕ_s	= resistance factor for non-prestressed reinforcement,
Δ_e	= difference between the position of the applied axial load and the centroid of the cross-section,

REFERENCES

- ¹Nilson, A., Winter, G. & Urquhart, L. 1991. Design of concrete structures, McGraw-Hill., New York, NY.
- ²ACI Committee 438. 1969. Tentative Recommendations for the Design of Reinforced Concrete Members to Resist Torsion, Journal of the American Concrete Institute, Proc., 66(1), 1-8.
- ³ACI Committee 318. 1971. Building Code Requirements for Reinforced Concrete (ACI 318-71), Detroit, MI
- ⁴Wight, J. 2016. Reinforced concrete: mechanics and design 7ed., Pearson Education, Inc., New Jersey, NJ
- ⁵Collins, M.P., and Lampert, P. 1973. Redistribution of Moments at Cracking—The Key to Simpler Torsion Design?, American Concrete Institute (ACI) Special Publication (SP-35), 343-384.
- ⁶ACI Committee 318. 2019. Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary, Farmington Hills, MI
- ⁷Canada Standards Association. 2004. CSA-A23.3-04 Design of concrete structures, Ontario, Canada
- ⁸American Association of Highway and Transportation Officials. 2017. AASHTO LRFD Bridge Design Specifications, 8th Edition, Washington, D.C.
- ⁹European Committee for Standardization. 2004. Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings, Brussels, Belgium
- ¹⁰The International Federation for Structural Concrete. 2010. fib Model Code for Concrete Structures 2010, Lausanne, Switzerland
- ¹¹Kurrer, K. 2008. The history of the theory of structures: from arch analysis to computational mechanics, Ernst & Sohn, Berlin
- ¹²Elfgren, L. 2009. Development of Models for Torsion of Concrete Structures in Northern Europe, American Concrete Institute (ACI) Special Publication (SP-265), 309-326.

¹³Navier, C. L. 1826, “Résumé des leçons données à l'École des ponts et chaussées sur l'application de la mécanique à l'établissement des constructions et des machines”, Première partie, “continant les leçons sur la résistance des matériaux et sur l'établissement des constructions en terre, en maçonnerie et en charpente”, Article V, “de la résistance d'un corps prismatique à la torsion”, Article VI, “de la résistance d'un corps prismatique à la rupture causée par la torsion”, pp. 71-76, pp. 76-80, Firmin Didet, Paris.

¹⁴Timoshenko, S. P. 1905. Lateral buckling of I-beams under the influence of forces acting in the plane of largest rigidity. In Russian. Proceedings of the St. Petersburg Polytechnic Institute, vol 4, No 3-4, 1905, pp 151-219 and vol 5, No 1-2, 1906, pp 3-34. (The writer has not seen the original paper. The reference is quoted from Timoshenko's autobiography “As I Remember”, translated from the Russian by Robert Addis, D vanNostrand, Princeton, New Jersey 1968, 430 pp.)

¹⁵Vlasov, V. Z. 1940. Thin-walled elastic beams. English translation of the 2nd Russian edition by US Department of Commerce, Office of Technical Services, OTS 61-11400, Israel Program for Scientific Translations, Jerusalem 1961, 493 pp.

¹⁶Elfgren, L. 1972. Reinforced Concrete Beams Loaded in Combined Torsion, Bending and Shear. A Study of the Ultimate Load-Carrying Capacity. PhD thesis, Division of Concrete Structures, Chalmers University of Technology, second edition, Göteborg, Sweden.

¹⁷Kollbrunner, C. and Basler, K. 1969. Torsion in Structures, An Engineering Approach, Springer-Verlag Berlin Heidelberg, Berlin, Germany

¹⁸Graf, O. and Mörsch, E, 1922, “Verdrehungsversuche zur Klärung der Schubfestigkeit von Eisenbeton. Forschungsarbeiten auf dem Gebiete des Ingenieurwesens, Heft 258, Verein Deutscher Ingenieure (VDI)”, Berlin, 52 pp.

¹⁹Hsu, T.T.C., and Mo, Y. 1985. Softening of Concrete in Torsional Members - Theory and Tests, Journal of the American Concrete Institute, Proceedings, 82(3), 290-303.

²⁰Rausch, E. Design of reinforced concrete in torsion (Berechnung des Eisenbetons gegen Verdrehung). Ph.D. thesis. Berlin: Technische Hochschule; 1929. p. 53

²¹Lessig, N. N. 1959. Determination of the Load Carrying Capacity of Reinforced Concrete Elements with Rectangular Cross-Section Subjected to Flexure with Torsion, Work 5, Institute Betona i Zhelezobetona, Moscow, pp. 4-28; also available as Foreign Literature Study 371, PCA Research and Development Labs, Skokie, IL.

²²Yudin, V. K. 1962, “Determination of the Load-Carrying Capacity of Rectangular Reinforced Concrete Elements Subjected to Combined Torsion and Bending”, Beton i Zhelezobeton (Concrete and Reinforced Concrete), No. 6, pp. 265-269 (In Russian). Translated by the Portland Cement Association, Foreign Literature Study No. 377

²³Mitchell, D., and Collins, M.P. 1974. Diagonal Compression Field Theory—A Rational Model for Structural Concrete in Pure Torsion, ACI Journal, 71(8), 396-408.

²⁴Hsu, T.T.C. 1988. Softening Truss Model Theory for Shear and Torsion, ACI Structural Journal, 85(6), 624-635

²⁵Rahal, K.N., and Collins, M. P. 2003. Combined Torsion and Bending in Reinforced and Prestressed Concrete Beams, ACI Structural Journal, 100(2), 157-165.

²⁶Rahal, K.N., and Collins, M.P. 1995. Analysis of Sections Subjected to Combined Shear and Torsion-A Theoretical Model, ACI Structural Journal, 92(4), 459-469.

²⁷Hsu, T.T.C. 1984. Torsion of reinforced concrete, Van Nostrand Reinhold Company Inc., New York, NY.

²⁸Ritter, W. 1899. Die Bauweise Hennebique, Schweiz Bauzeitung, 33(7), 59-61

- ²⁹Collins, M.P., and Mitchell, D. 1980. Shear and Torsion Design of Prestressed and Non-Prestressed Concrete Beams, *PCI Journal*, 25(4), 32-100.
- ³⁰Collins, M.P., Walsh, P.F., Archer, F.E., and Hall A.S. 1968. Ultimate Strength of Reinforced Concrete Beams Subjected to Combined Torsion and Bending, American Concrete Institute (ACI) Special Publication (SP-18), 379-402.
- ³¹Hsu, T.T.C. 1990. Shear flow zone in torsion of reinforced concrete, *Journal of Structural Engineering ASCE*, 116(11), 3206-3226.
- ³²Vecchio, F.J., and Collins, M.P. 1986. The Modified Compression Field Theory for Reinforced Concrete Elements Subjected to Shear. *ACI Journal, Proceedings*. 82(2), 219-231.
- ³³Bentz, E.C., Vecchio, F.J., and Collins, M.P. 2006. Simplified Modified Compression Field Theory for Calculating Shear Strength of Reinforced Concrete Elements. *ACI Structural Journal*, 103(4), 614-629.
- ³⁴Jeng, C.-H., and Hsu, T.T.C. 2009. A softened membrane model for torsion in reinforced concrete members, *Engineering Structures*, 31(9), 1944-1954.
- ³⁵Hsu, T.T.C., and Zhu, R.R.H. 2002. Softened membrane model for reinforced concrete elements in shear, *ACI Structural Journal*, 99(4), 460-469.
- ³⁶Völgyi, I., and Windisch, A. 2014. Resistance of reinforced concrete members with hollow circular cross-section under combined bending and shear - Part II: New calculation model. *Structural Concrete*, 15(1), 21-29.
- ³⁷Völgyi, I., Windisch, A., and Farkas, G. 2014. Resistance of reinforced concrete members with hollow circular cross-sections under combined bending and shear - Part I: experimental investigation, *Structural Concrete*, 15(1), 13-20.
- ³⁸Valdivieso, D.A., Lantsoght, E.O.L., and Sánchez, T.A. 2016. Effect of torsion on shear capacity of slabs, 6th International Conference on Structural Engineering, Mechanics and Computation: Proceedings of the 6th International Conference on Structural Engineering, Mechanics and Computation, Cape Town, South Africa, Conference Paper, 1-6
- ³⁹Lee, J.-Y., Kim, K.-H., Lee, S.H., Kim, C., and Kim, M.-H. 2018. Maximum Torsional Reinforcement of Reinforced Concrete Beams Subjected to Pure Torsion, *ACI Structural Journal*, 115(3), 749-756.
- ⁴⁰Japan Society of Civil Engineering. 2007. Standard Specifications for Concrete Structures, Japan Society of Civil Engineering, Tokyo, Japan
- ⁴¹Chiu, H.-J., Fang, I.-K., Young, W.-T., and Shiau, J.-K. 2007. Behavior of reinforced concrete beams with minimum torsional reinforcement, *Engineering Structures*, 29(9), 2193-2205.
- ⁴²Fang, I.-K., and Shiau, J.-K. 2004. Torsional behavior of normal- and high-strength concrete beams, *ACI Structural Journal*, 101(3), 304-313.
- ⁴³Shahrooz, M.B., Forry, L.M., Anderson, S.N., Bill, L.H., and Doellman M.A. 2016. Continuous Transverse Reinforcement—Behavior and Design Implications, *ACI Structural Journal*, 113(5), 1085-1094.
- ⁴⁴Koutchoukali, N., and Belarbi, A. 2001. Torsion of High-Strength Reinforced Concrete Beams and Minimum Reinforcement Requirement, *ACI Structural Journal*, 98(4), 462-469.
- ⁴⁵Mohamed, H., and Benmokrane, B. 2015. Torsion Behavior of Concrete Beams Reinforced with Glass Fiber-Reinforced Polymer Bars and Stirrups, *ACI Structural Journal*, 112(5), 543-552.

- ⁴⁶Yap, S.P., Khaw, K.R., Alengaram, U.J., and Jumaat, M.Z. 2015. Effect of fibre aspect ratio on the torsional behaviour of steel fibre-reinforced normal weight concrete and lightweight concrete, *Engineering Structures*, 101, 24-33.
- ⁴⁷Wafa, F.F., Hasnat, A., and Tarabolsi, O.F. 1992. Prestressed fiber-reinforced concrete beams subjected to torsion. *ACI Structural Journal*, 89(3), 272–83.
- ⁴⁸Mansur, M.A., and Lim, T.Y. 1985. Torsional behaviour of reinforced fibre concrete beams, *International Journal of Cement Composites and Lightweight Concrete*, 7(4), 261–267.
- ⁴⁹Narayanan, R., and Kareem-Palanjian, A.S. 1986. Torsion in beams reinforced with bars and fibers, *Journal of Structural Engineering ASCE*, 112(1), 53–66.
- ⁵⁰Mansur, M.A., Nagataki, S., Lee, S.H., and Oosumimoto, Y. 1989. Torsional response of reinforced fibrous concrete beams. *ACI Structural Journal*, 86(1), 36–44.
- ⁵¹El-Niema, E.I. 1993. Fiber reinforced concrete beams under torsion. *ACI Structural Journal*, 90(5), 489–95.
- ⁵²Gunneswara Rao, T. D., and Rama Seshu, D. 2003. Torsion of steel fiber reinforced concrete members. *Cement and Concrete Research*, 33(11), 1783–1788.
- ⁵³Chalioris, C. E., & Karayannis, C.G. 2009. Effectiveness of the use of steel fibres on the torsional behaviour of flanged concrete beams, *Cement and Concrete Composites*, 31(5), 331-341.
- ⁵⁴Deifalla, A., and Ghobarah, A. 2010. Strengthening RC T-beams subjected to combined torsion and shear using FRP fabrics: Experimental study, *Journal of Composites for Construction*, 14(3), 301-311.
- ⁵⁵Chalioris, C. E. 2007. Analytical model for the torsional behaviour of reinforced concrete beams retrofitted with FRP materials, *Engineering Structures*, 29(12), 3263-3276.
- ⁵⁶He, R., Sneed, L., & Belarbi, A. 2014. Torsional Repair of Severely Damaged Column Using Carbon Fiber-Reinforced Polymer, *ACI Structural Journal*, 111(3), 705-716.

NEGLECTING COMPATIBILITY TORSIONS IN THE DESIGN OF CONCRETE STRUCTURES

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Synopsis: The traditional approach in the design of reinforced and prestressed concrete building structures has been to design each of the two orthogonal directions independently. In calculating the distribution of moments in a structure, this two-dimensional approach neglects the effects of the intersecting members. That is, in the case of compatibility torsions, the torsional stiffness is neglected. This paper provides a summary of the progression of the ACI code and commentary pertaining to the zero torsional stiffness assumption and its origins. The paper then introduces a recently developed nonlinear finite element analysis tool, VAST II, capable of predicting the response of reinforced and prestressed concrete structures in three-dimensions. The tool, based on the Modified Compression Field Theory, is capable of modelling entire structures or large portions of structures in order to assess their performance in a manner that accounts for three-dimensional effects, such as compatibility torsions. VAST II is then used to model a case study transit center. The transit center is a post-tensioned concrete structure that was designed using the traditional approach of neglecting the effects of compatibility torsions. The results indicate that the traditional approach recommended by the ACI code and commentary, to neglect compatibility torsions, is appropriate and gives robust designs. The paper concludes by providing recommendations for future studies that could be conducted using three-dimensional nonlinear tools such as VAST II.

Keywords: assessment, compatibility torsion, design, finite element, prestressed concrete, shear

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INTRODUCTION

Prior to the advent of computers, the design of indeterminate reinforced and prestressed concrete structures relied on hand calculations to determine how moments would be distributed in a frame structure. Traditionally, such structures would be designed in each of the two orthogonal directions independently. It was very computationally difficult to calculate torsion in highly statically indeterminate reinforced concrete structures. This was one of the main reasons that, except for some special structures such as free-standing spiral staircases, torsion was ignored.

As computers became readily available, structural engineers began using computer methods in place of hand calculations to determine the distribution of moments in a structure. As was the case for hand calculations, engineers would design structures in each of the two orthogonal directions independently, thereby ignoring three-dimensional effects that occur as a result of intersecting members in orthogonal directions. This practice is still a commonly used and acceptable method for the design of many structures. With improved computational power, it has become more common to develop three-dimensional models to determine the elastic distribution of forces in a structure. In conducting such three-dimensional analyses, the effects of torsions in three-dimensions are no longer ignored. Torsions that arise from the compatibility of the connected members, compatibility torsions, will be dependent on the choice of the relative stiffness of the flexural members and the members subjected to torsion. After cracking, the flexural stiffness of reinforced concrete members decreases by about half of its elastic value, whereas after cracking the torsional stiffness of the member will reduce by much more than half (about 80-90%) of its elastic stiffness. Thus, after cracking, the ratio of torsional stiffness to the flexural stiffness is lower than the ratio of the elastic torsional stiffness to the elastic flexural stiffness. In the limit when torsional stiffness is neglected, the three-dimensional analysis will provide the same results as the two-dimensional analyses conducted in each of the two orthogonal directions. Conversely, using the elastic stiffnesses may significantly overestimate the torsions and underestimate the flexural demand after cracking. Since, elastic analysis programs are used for the purposes of determining the distribution of forces in a structure, a single assumption must be made for the relative stiffnesses used. In situations where torsions arise in order to satisfy equilibrium, equilibrium torsion, non-zero torsional stiffness must be used to determine the stress resultants.

This paper examines a case study structure to investigate the performance of an indeterminate transit center designed using the assumption of zero torsional stiffness. While a three-dimensional elastic analysis of such a structure using the elastic torsional and flexural stiffnesses would give rise to large compatibility torsions, this paper examines the performance of the structure when these torsions are ignored in design. The paper first describes the progression of the ACI code and commentary with respect to compatibility torsion and the zero torsional stiffness assumption. The paper then introduces a recently developed three-dimensional macro element nonlinear finite element program, VAST II, and uses the program to model a portion of the transit center. The performance of the structure is then discussed in the context of the design loads and assumptions.

DESIGNING FOR COMPATIBILITY TORSION, THE ACI CODE AND COMMENTARY

This section provides an overview of the progression of the ACI clauses and commentary pertaining to compatibility torsion and stiffness assumptions permitted in design.

The first ACI code to mention torsion was the 1963 edition, ACI 318-63, which gave torsional stiffness advice in Clause 905—Frame analysis—Details, and gave some simple torsional detailing rules in Clause 921-Torsion. These clauses are reproduced in Fig. 1 (ACI, 1963).

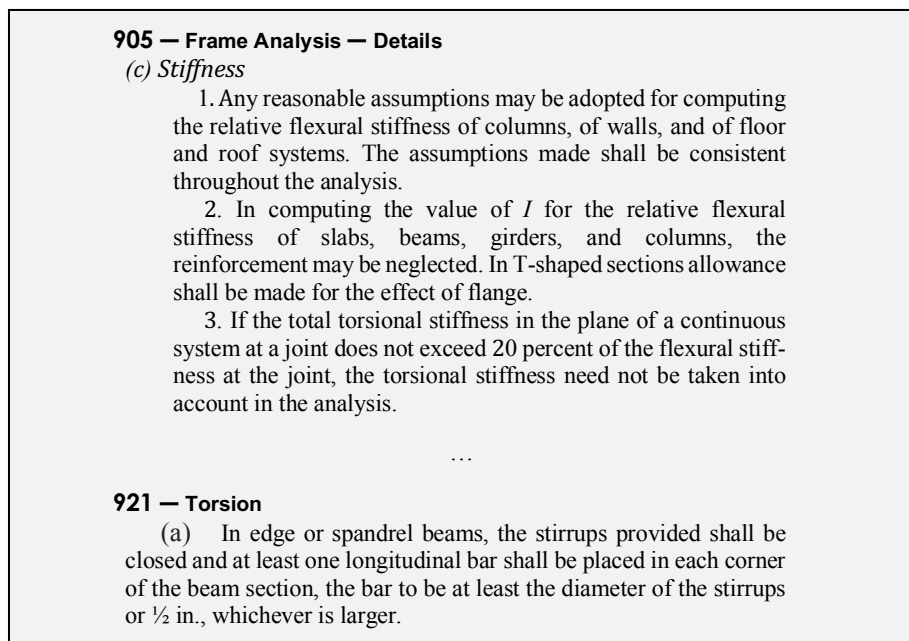


Figure 1 – ACI 318-63 Code excerpt (ACI, 1963)

In many practical situations the requirements of Clause 905.(c).3 are met and thus the torsions in the members could be assumed to be zero. The very simple detailing rules in Clause 921 for edge or spandrel beams were the only torsion provisions which most designers needed to satisfy. By the time of the 1971 ACI code, ACI 318-71, provisions for calculating the torsional strength of reinforced concrete members were included, but no provisions were given for prestressed concrete members. The section describing assumptions about stiffness, Section 8.5.3, then explicitly included torsional stiffness. The commentary for this section specifically notes that neglecting torsional stiffness “is allowed by the Code” (ACI, 1971). An excerpt is shown in Fig 2.

8.5.3 Stiffness

8.5.3.1 Any reasonable assumptions may be adopted for computing the relative flexural and torsional stiffnesses of columns, walls, floors, and roof systems. The assumptions made shall be consistent throughout the analysis.

...

The provision in the 1963 Code which allows the torsional stiffness to be neglected in frame analysis under certain conditions has been removed because of misinterpretations, and the Code is now silent on this matter. It has been customary to neglect torsional stiffness in frame analysis when torsional stiffness is small compared to flexural stiffness and, except for the provisions of Chapter 13, this practice is allowed by the Code. Because of the approximations used in design, the neglect of this small factor will have no important effect on the flexural moments. In members subjected to significant torsional stresses, torsional stiffness should be considered in determining torsional moments.

Figure 2 – ACI 318-71 Code and commentary excerpt (ACI, 1971)

The concept of distinguishing between equilibrium torsion (where the torsion is required to satisfy equilibrium) and compatibility torsion (where the torsion arises because the member must twist to satisfy compatibility) was introduced in a paper by M. P. Collins and P. Lampert and presented at an ACI symposium in Denver in March 1971. The paper was titled: “Redistribution of Moments at Cracking—the Key to Simpler Torsion Design?” (Collins and Lampert, 1971). Their experiments showed that for the case of compatibility torsion, satisfactory designs resulted when the torsional stiffness was assumed to be zero resulting in a calculated torsional load of zero in the spandrel members. The experiments demonstrated that floor beam - spandrel beam specimens designed assuming zero torsional stiffness in the spandrels had similar crack control and similar ultimate strengths to the companion specimens designed assuming uncracked torsional stiffness values, see Fig. 3. The specimens designed assuming zero torsional stiffness had arrangements of flexural reinforcement that more closely matched the actual flexural demands caused by the loading. The ACI Special Publication, SP-35, which published the paper along with 11 other papers, was devoted to explaining how structural systems could be analyzed to determine the magnitude of the design torsions. Fig. 12-5 from the Collins and Lampert paper was the cover photograph for the volume, see Fig. 4.

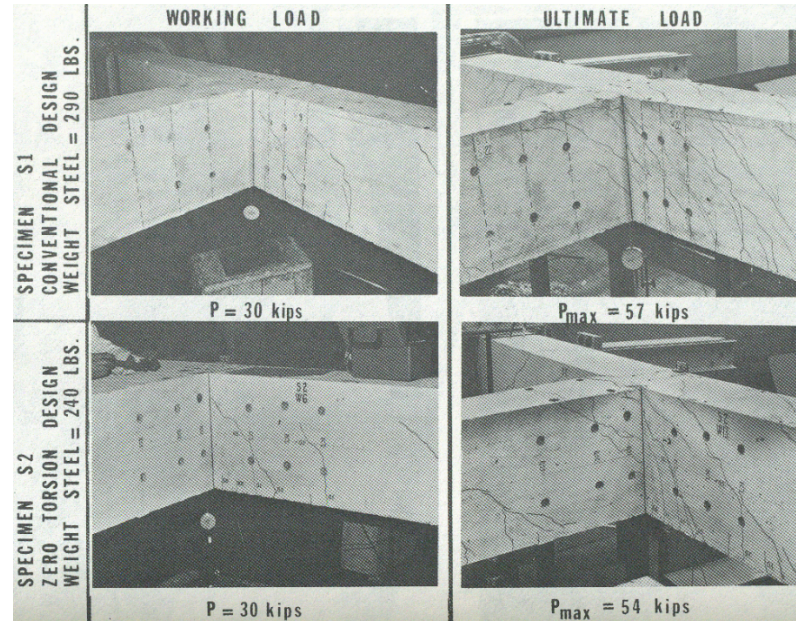
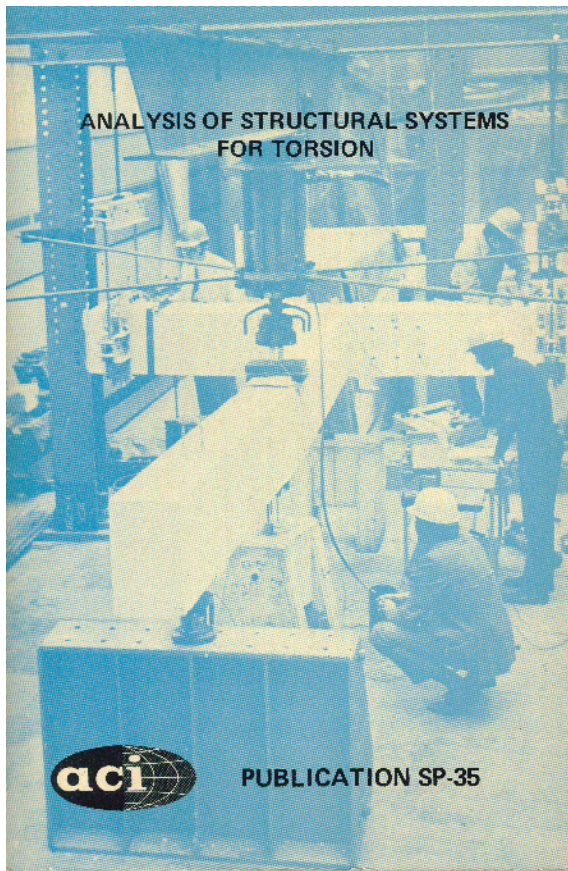


Figure 3 – Comparison of full torsion design and zero torsion design, reproduced from Collins and Lampert, 1971 [Note: 1 kip = 4.45 kN, 1 lb = 0.454 kg]

The concept that zero torsional stiffness could be used for compatibility torsion was introduced in the commentary to the stiffness clause in the ACI 1977 code, see Fig. 4. The commentary outlines that any reasonable assumption for the relative stiffnesses can be made. It also describes the difference between compatibility torsion and equilibrium torsion and indicates that while it is necessary to consider torsions arising from equilibrium, in most cases compatibility torsions can be neglected.

8.6 – Stiffness

8.6.1 – Any reasonable assumption may be adopted for computing relative flexural and torsional stiffnesses of columns, walls, floors, and roof systems. Assumptions shall be consistent throughout analysis.



8.6– Stiffness

8.6.1 – This section allows the use of any reasonable assumption when computing the stiffnesses for use in a frame analysis provided the assumptions made are consistent throughout the analysis. Ideally, the member stiffnesses EI and GJ should reflect the degree of cracking and inelastic action which has occurred along each member immediately prior to the onset of yielding. However, the complexities involved in selecting the ideal values of EI for all members of a frame would make frame analyses uneconomical in design offices. Hence, simpler assumptions are required to define the flexural and torsional stiffnesses for practical analyses.

A provision in the 1963 code which allowed the torsional stiffnesses of transverse beams to be neglected if they did not exceed 20 percent of the flexural stiffnesses at the joint was deleted in the 1971 code, and the current code only requires that reasonable assumptions of the relative flexural and torsional stiffnesses be made. Two conditions determine whether it is necessary to include the torsional stiffness in the analysis of a given structure: the relative magnitude of the torsional and flexural stiffnesses and, whether the torsion is required for equilibrium of the structure (equilibrium torsion) or simply arises due to members twisting to maintain compatibility of deformations (compatibility torsion). In the case of compatibility torsion, the torsional stiffness can be frequently be neglected, while it generally must be included in the analysis if equilibrium torsion exists. Prior to torsional cracking, the torsional stiffness of a rectangular beam will be in the order of 10 to 20 percent of the flexural stiffness. Torsional cracking will reduce the torsional stiffness of such a beam by 80 to 90 percent so that the torsional stiffness drops to a small fraction of the corresponding flexural stiffness. Because of this high reduction, the torsional stiffness of beams transverse to the frame being analyzed is usually not considered in the analysis. It is, however, necessary to consider the torsional stiffnesses of edge beams in Chapter 13.

Figure 4 – Cover of ACI Special Publication 35, 1973 (ACI, 1973) and ACI clause 8.6 code and commentary, 1977 (ACI, 1977)

By the 1989 ACI code, the commentary on stiffness had been further simplified and noted: “In the case of compatibility torsion, the torsional stiffness may be neglected” and this version of the commentary has remained in all ACI codes up to the current 2019 standard. In the ACI 2019 Code and Commentary, the commentary continues to recognize that compatibility torsions can be neglected, the commentary to express this has changed to say, “In the case of compatibility torsion, torsional stiffness usually is not included in the analysis. This is because the cracked torsional stiffness of a beam is a small fraction of the flexural stiffness of the members framing into it.” (ACI, 2019). See excerpt in Fig. 5.

CODE	COMMENTARY
6.3 — Modeling Assumptions 6.3.1 — General 6.3.1.1 Relative stiffnesses of members within structural systems shall be selected based on a reasonable set of assumptions. The assumptions shall be consistent throughout each analysis.	R6.3 — Modeling Assumptions R6.3.1 — General 6.3.1.1 Separate analyses with different stiffness assumptions may be performed for different objectives such as to check serviceability and strength criteria or to bound the demands on elements where stiffness assumptions are critical. Ideally, the member stiffnesses $E\mathbf{I}$ and $G\mathbf{J}$ should reflect the degree of cracking and inelastic action that has occurred along each member before yielding. However, the complexities involved in selecting different stiffnesses for all members of a frame would make frame analyses inefficient in the design process. Simpler assumptions are required to define flexural and torsional stiffnesses. For braced frames, relative values of stiffness are important. A common assumption is to use $0.5\mathbf{I}_g$ for beams and $\mathbf{I}_g$ for columns. For sway frames, a realistic estimate of $\mathbf{I}$ is desirable and should be used if second-order analyses are performed. Guidance for the choice of $\mathbf{I}$ for this case is given in 6.6.3.1. Two conditions determine whether it is necessary to consider torsional stiffness in the analysis of a given structure: 1) the relative magnitude of the torsional and flexural stiffnesses; and 2) whether torsion is required for equilibrium of the structure (equilibrium torsion) or is due to members twisting to maintain deformation compatibility (compatibility torsion). In the case of equilibrium torsion, torsional stiffness should be included in the analysis. It is, for example, necessary to consider the torsional stiffnesses of edge beams. In the case of compatibility torsion, torsional stiffness usually is not included in the analysis. This is because the cracked torsional stiffness of a beam is a small fraction of the flexural stiffness of the members framing into it. Torsion should be considered in design as required in Chapter 9.

Figure 5 – ACI 318-19 Code and commentary excerpt (ACI, 2019)

It is also relevant to note that the 92 page 2013 ACI state-of-the-art “Report on Torsion in Structural Concrete” continues to support the practice of “disregarding structural torsion” in cases of compatibility torsion (ACI-ASCE, 2013). Additionally, provisions for calculating torsional strength of prestressed concrete members were first introduced into the 1995 ACI code, ACI 318-95 (ACI, 1995). These provisions were influenced by the torsional design procedures developed by Collins and Mitchell and summarized in their 1991 textbook “Prestressed Concrete Structures” published by Prentice Hall (Collins and Mitchell, 1991).

The case study transit center that will be discussed in subsequent sections was built in the United States and was designed using the ACI 318-02 code. The structure was designed using the assumption of zero torsional stiffness. Much like the 1989 code and the 2019 code, the 2002 ACI code and commentary indicates that, “In the case of compatibility torsion, the torsional stiffness may be neglected” (ACI, 2002). See excerpt shown in Fig. 6.

CODE	COMMENTARY
8.6 — Stiffness 8.6.1 — Use of any set of reasonable assumptions shall be permitted for computing relative flexural and torsional stiffnesses of columns, walls, floors, and roof systems. The assumptions adopted shall be consistent throughout analysis.	R8.6 — Stiffness R8.6.1 — Ideally, the member stiffnesses EI and GJ should reflect the degree of cracking and inelastic action that has occurred along each member before yielding. However, the complexities involved in selecting different stiffnesses for all members of a frame would make frame analyses inefficient in design offices. Simpler assumptions are required to define flexural and torsional stiffnesses. For braced frames, relative values of stiffness are important. Two usual assumptions are to use gross EI values for all members or, to use half the gross EI of the beam stem for beams and the gross EI for the columns. For frames that are free to sway, a realistic estimate of EI is desirable and should be used if second-order analyses are carried out. Guidance for the choice of EI for this case is given in R10.11.1. Two conditions determine whether it is necessary to consider torsional stiffness in the analysis of a given structure: (1) the relative magnitude of the torsional and flexural stiffnesses, and (2) whether torsion is required for equilibrium of the structure (equilibrium torsion) or is due to members twisting to maintain deformation compatibility (compatibility torsion). In the case of compatibility torsion, the torsional stiffness may be neglected. For cases involving equilibrium torsion, torsional stiffness should be considered.

Figure 6 – ACI 318-02 Code and commentary excerpt (ACI, 2002)

The transit center was comprised of prestressed slabs, beams and girders and was designed to withstand substantial traffic and pedestrian loads. The structure was designed in each of the two orthogonal directions independently and the elastic distribution of flexural moments were calculated neglecting the torsional stiffness of intersecting members. The remainder of this paper introduces a nonlinear finite element modelling tool, VAST II, capable of predicting the full nonlinear three-dimensional response of members (Proestos, 2018). While the initial structural design was conducted in two dimensions, the tool is used to model a portion of the transit center in three-dimensions. Additionally, while in the design of the structure the compatibility torsions were neglected, in a three-dimensional model those compatibility torsions can be calculated. The structural response of the transit center is then evaluated for three critical load cases to assess the performance of the beams and girders. Since the VAST II model is capable of predicting the full three-dimensional nonlinear response of the structure, the effects of compatibility torsions can be evaluated to assess their influence on the global structural response throughout loading.

A NONLINEAR FINITE ELEMENT PROGRAM FOR MEMBERS IN THREE-DIMENSIONS, VAST II

In 1974, 1977 and 1978 Rabbat and Collins published works on the design and analysis of members subjected to all six of the stress resultants including members subjected to combinations of shear, moment and torsions (Collins and Rabbat, 1974; Rabbat and Collins, 1977; Rabbat and Collins, 1978). In addition to several series of experiments, this work included the development of a program called Design by the Variable Angle Space Truss (DEVAST) which was based on a sectional analysis program for variable angle space trusses. The program was eventually used in the design of a guideway girder system in Toronto, Canada (Collins and Rabbat, 1974). Figure 7 shows a guideway girder and the six sectional resultants acting on the member.

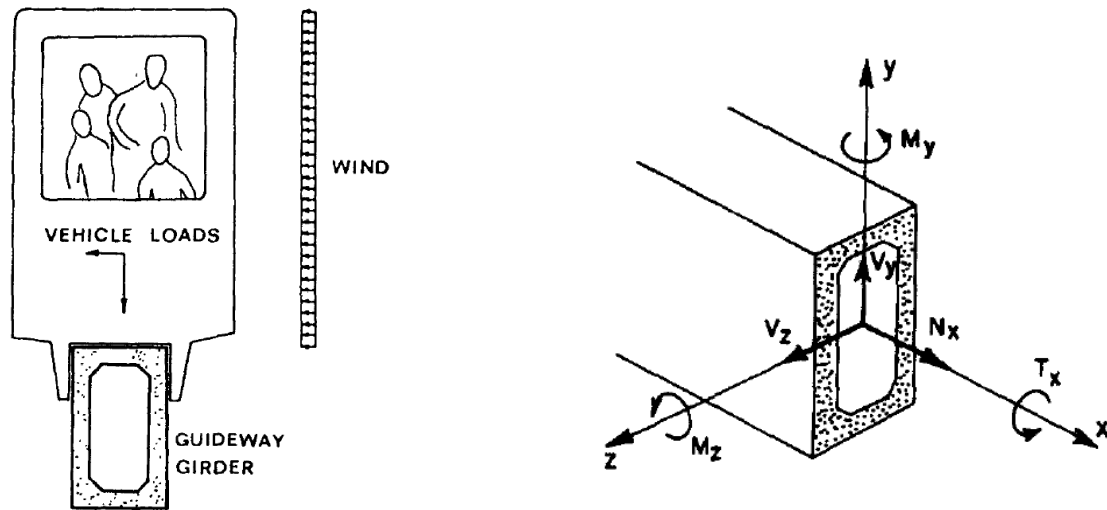


Figure 7 – Guideway girder (left) and DEVAST section subjected six sectional resultants (right), reproduced from Rabbat and Collins, 1977

DEVAST is based on the concept of a ‘space truss’ that is: the axial loads and two-way bending moments are carried through axial stresses in truss elements along the x -direction while panel elements connecting the truss elements are capable of carrying two-way shears and torsion. The ‘variable angle’ refers to the idea that the cracks arising from the shear flow (shear and torsions) are able to rotate and are not fixed. The two vertical webs carry the vertical shear, the two horizontal webs carry the horizontal shear and the four webs together carry the shear flow that make up the torsion. The model is a sectional model and therefore the concept that plane sections remain plane was also used. The Compression Field Theory (CFT) was used to calculate the strength and response of the webs while conventional techniques accounting for the stress-strain and compatibility were used for the trusses. A visualization of the truss model is summarized in Fig. 8. The model is based on the behavior of a hollow box, but extensive work was conducted to demonstrate that the model provides accurate results for hollow or solid sections for both reinforced and prestressed concrete members (Rabbat and Collins, 1978).

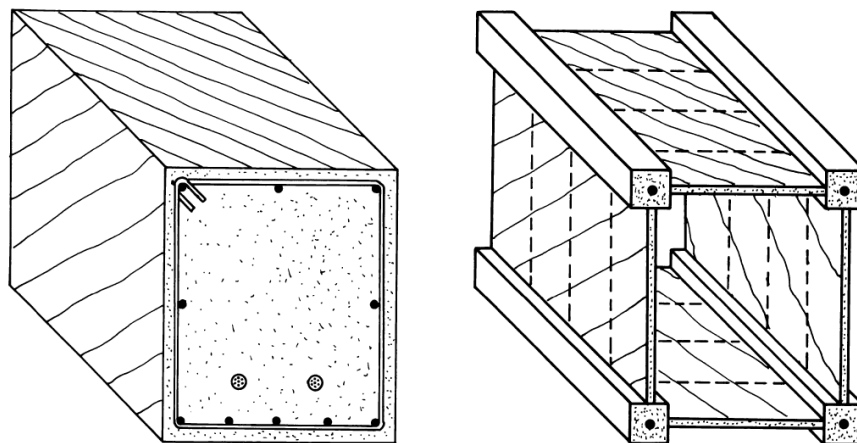


Figure 8 – Actual beam (left) and space truss model (right), reproduced from Rabbat and Collins, 1978

Since the development of DEVAST and the corresponding sectional analysis program, improvements have been made in the field of shear and torsion research. One of the major improvements was the development of the Modified Compression Field Theory (MCFT) and its corresponding constitutive relationships (Vecchio and Collins, 1986). VAST II was developed as the successor to DEVAST and uses refined constitutive models and assumptions that are consistent with international codes such as AASHTO and CSA A23.3 (AASHTO LRFD, 2017; CSA A23.3-14, 2014).

VAST II also has the benefit of being implemented as a finite element, thus allowing more complex situations to be considered.

VAST II CAPABILITIES

VAST II is a three-dimensional nonlinear finite element program capable of modeling reinforced and prestressed concrete beam, girder, slab, column, truss and membrane elements. There are two types of *beam* elements that can be used in the program, 2-vertical web elements and 3-vertical web elements. The 2-vertical web elements have 8 nodes and 24 degrees of freedom while the 3-vertical web elements have 12 nodes and 36 degrees of freedom. Figure 9 shows a visual representation of the 2 and 3-web elements. Both of the elements in Fig. 9 are capable of transmitting the six stress resultants on beams in three-dimensions: axial load, moment in two directions, shears in two directions and torsions. The orange and green elements represent the truss elements while the turquoise and blue elements represent the membrane elements that carry the shear flow. Thus, the 2-vertical web element has a total of 4 membrane and 4 trusses whereas the 3-vertical web element has 7 membrane elements and 6 truss elements.

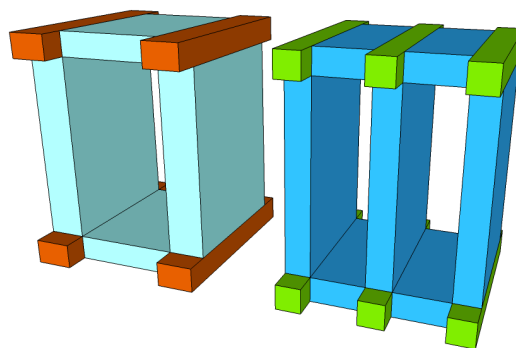


Figure 9 – VAST II beam elements: 2-vertical web element (left) and 3-vertical web (right) (Proestos, 2018)

Since the elements inherently capture flexure and can account for the shear stresses in the member, only one 2-vertical web or 3-vertical web element is needed through the height of the member. This makes the program computationally efficient for modelling the nonlinear behavior of complex structures. The constitutive models have been formulated so that additional considerations such as straight or inclined tendons and shrinkage strains can be accounted for in the analysis. The model is based on first principles and does not require assumptions regarding member stiffnesses following cracking. Rather, stiffnesses are calculated and used in the analysis at every load level. For example, the torsional stiffness of the members need not be assumed. Rather, it can be calculated based on the geometry of the members, material properties and compatibility with the rest of the structure. The program is thus able to automatically account for situations of compatibility torsion, as the stiffness reduction following cracking will lead to a different distribution of forces in the structure. Outputs from the program include parameters such as displacements, crack widths, strains and stresses in the steel and concrete, which are determined for every load level.

Consistent with the truss model, VAST II assumes that all the concrete area above the centroid is distributed in the top chord members and all the concrete area below the centroid is distributed in the bottom truss members. The web areas and reinforcement are also proportioned in this manner. The chord elements are modelled as nonlinear truss elements that utilize the strain compatibility approach for reinforced and prestressed concrete members. The constitutive models proposed by Collins and Mitchell are used. That is, the concrete compressive response is modelled using the Modified Popovics relationship, the passive steel is modelled using elastic-plastic response, the prestressing steel uses the Modified Ramberg-Osgood relationship, and the tension stiffening relationship proposed by Collins and Mitchell is used (Collins and Mitchell, 1991). For the web elements, the equations of the MCFT are used (Vecchio and Collins, 1986). The truss elements are assumed to be spaced a distance $d_v = 0.9d$ apart from each other and the depth of the web elements are assumed to be $d_v = 0.9d$ in height. The behavior of these membrane and truss elements is determined and a three-dimensional stiffness matrix is built by using finite element principles. These component stiffnesses matrices are then added to the global stiffness matrix and iterations are conducted to determine the structural response. In order not to double count the axial stiffness from the contribution of the webs and the axial elements, the axial stiffness of the webs is not added in the element stiffness matrix. This stiffness decoupling approach has been summarized elsewhere (Proestos, 2018). The 3-vertical web element was developed to better represent the response of members subjected to combined shear and torsion. That is, whereas the 2-vertical web element must carry the entire

shear flow in only one web, in the 3-vertical web element, part of the shear can be distributed to the central web. This internal redistribution is more representative of members that are subjected to large vertical gravity loads and some torsions, rather than members subjected to pure torsional response, which are rare in real structures.

To demonstrate the capabilities of VAST II in being able to predict the response of members subjected to combined loads, the tool was used to model several experimental series. Figure 10 shows the Mardukhi series of experiments that investigated the response of members subjected to combined torsion and flexure. The blue line in Fig. 10 indicates the experimental interaction diagram and the green line shows the response as predicted by VAST II. As can be seen, the program predicts well the interaction and the transition in behavior from flexurally governed members to torsionally governed members that occurs near experiment TB2. For this series of experiments the average test-to-predicted ratio is 1.02 with a coefficient of variation of 9.3% (Proestos, 2018).

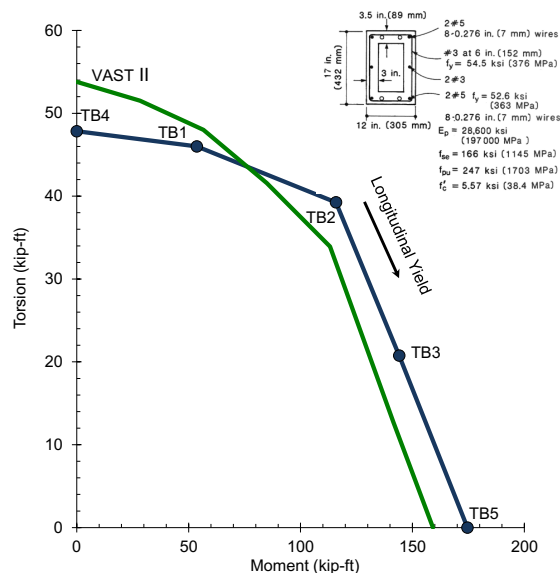


Figure 10 – Mardukhi series of experiments, inset from Mardukhi, 1973 [Note: 1 kip = 4.45 kN, 1 in. = 25.4 mm, 1 ft = 304.8 mm, 1 ksi = 6.90 MPa]

Additionally, to investigate the performance of VAST II for prestressed members with spalling cover, the program was used to predict the response of two tests conducted by Mitchell and Collins, PT5 and PT6 (Mitchell and Collins, 1974). The members were tested in pure torsion and were nominally identical except PT6 included a cover around the stirrups whereas PT5 did not. Figure 11 shows the experimental response of these specimens and also shows the prediction from VAST II. Since these two specimens are modelled in the same manner, VAST II predicts the same response for both PT5 and PT6. With an average test-to-predicted ratio for the two tests of 1.16, the results are somewhat conservative but the pre-cracking and post-cracking stiffnesses are predicted well. For this series of experiments, the VAST II model does not capture the portion of the curve near the ultimate loads when the stiffnesses rapidly reduce. However, the nonlinear stiffness, that is, the response after first cracking, is predicted well. Capturing this stiffness is important in being able to determine the influence of compatibility torsions after cracking. Additional verification and validation of VAST II can be found elsewhere (Proestos, 2018).

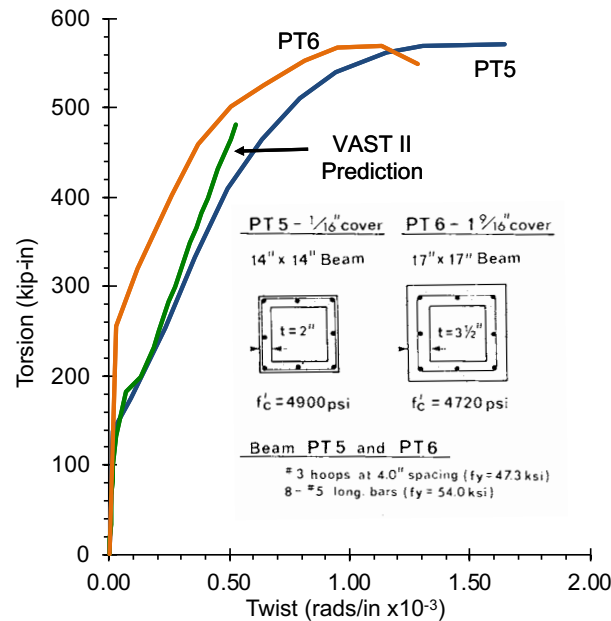


Figure 11 – VAST II predictions of pure torsion tests, adapted from Proestos, 2018 and inset adapted from Mitchell and Collins, 1974 [Note: 1 kip = 4.45 kN, 1 in. = 25.4 mm, 1 ksi = 6.90 MPa]

VAST II MODELLING OF TRANSIT CENTER

In the design of the case study transit center, the calculations of the member stress resultants were determined assuming zero torsional stiffness. That is, each of the two orthogonal directions were designed independently. Another reason this structure was selected for the case study, is because it is a very stiff, post-tensioned concrete structure. Thus, it is a good candidate to identify if problems may arise at ultimate loads when the zero torsional stiffness assumption is made in the initial design. To investigate the three-dimensional performance of the structure, designed in two dimensions, it is of interest to examine how it performs when subjected to multiples of the design loads. To this end, VAST II was used to model a representative three-dimensional portion of the structure (see shaded area of Fig. 12) consisting of three girders and 14 beams. The deck slab was assumed to act as a series of one-way members with slab strips being placed under chosen truck axle locations for the selected torsionally critical loading scenarios. Because this analytical model accounts for the reductions in stiffness caused by cracking, both flexural or torsional, it is capable of capturing the observed torsional redistribution that occurs after cracking.

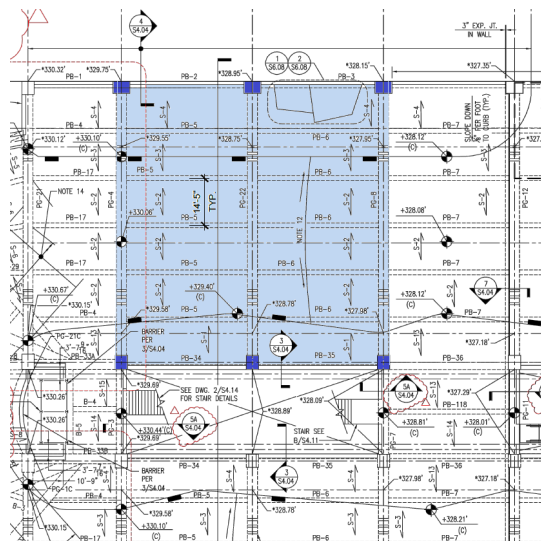


Figure 12 – Region of modeled transit center, three girders and 14 beams

The girders were each modeled with 43 VAST II elements along their length and the beams were modelled with 20 VAST II elements along their length, for each span. The VAST II elements used to model both the beams and girders were of the type that utilized 3-vertical webs (as shown in the right diagram of Fig. 9, above). The slab was not modelled. Thus, at the location of wheel loads where trucks would be placed on the structure, one-way slab strips were used. For these strips, VAST II, 2-vertical web beam elements were used. These 2-vertical web beam elements are shown on the left diagram of Fig. 9, above. As the analysis is intended to investigate the behavior of the beams and girders, rather than the slab, the slab strips were only included as a means to introduce point loads between the beams. The surface loads were applied tributary to the appropriate beams and girders. The girders were 'simply supported' at their ends, which conservatively assumes that no moment is transferred to the adjacent girders and supporting columns. The beams were modelled integral with the girders. Since only three girders of the structure are being modelled, to capture the influence of the adjacent beams, the nodes on the end beams were restrained from translating along their longitudinal axis and thereby also enforcing zero rotation at their ends. That is, the beams are assumed to be continuous over the girders. As is shown later, Fig. 14, below, the deformed shapes of the beams over the girders are consistent with the continuous connection imposed.

The three 36 in. (914 mm) wide by 72 in. (1,829 mm) tall girders that were considered in the analysis were 87 ft. (26.5 m) long. The girders nominally contained 0.144% of shear reinforcement near the center of the girders and 0.517% reinforcement near the girder ends. The longitudinal reinforcement in the girders was provided using 93- 0.6 in. (15.24 mm) strands and 2- 0.5 in. (12.70 mm) strands as prestressing, in addition to small quantities of passive reinforcement. A prestrain was input to achieve the 3400 kips (15,130 kN) of prestress force. The tendons were singly harped; over the column supports, the height of the tendon was 31 in. (787 mm) from the top of the section (near the centroid) and at the midspan of the girders the centroid of the tendon was 4 in. (101.6 mm) from the bottom of the girder. The beams which spanned the girders were 18 in. (457 mm) wide and 36 in. (914 mm) tall and contained 0.139% shear reinforcement along their 40 ft. (12.19 m) length. The longitudinal reinforcement consisted of parabolic tendon profiles consisting of 10- 0.6 in. (15.24 mm) strands in addition to small quantities of passive reinforcement. These tendons were used to develop the 450 kips (2,003 kN) of prestress force. Over the girder supports, the tendons in the beams have a centroid 4 in. (101.6 mm) from the top surface and at midspan they have a centroid 4 in. (101.6 mm) from the bottom surface. For the passive reinforcement, 60 ksi (414 MPa) steel was used. For the prestressing steel 7-wire low relaxation 270 ksi (1,862 MPa) strand was used. The concrete was modeled using 8,000 psi (55.2 MPa) concrete. Figure 13 provides a summary of the typical girder and beam details used in the analysis.

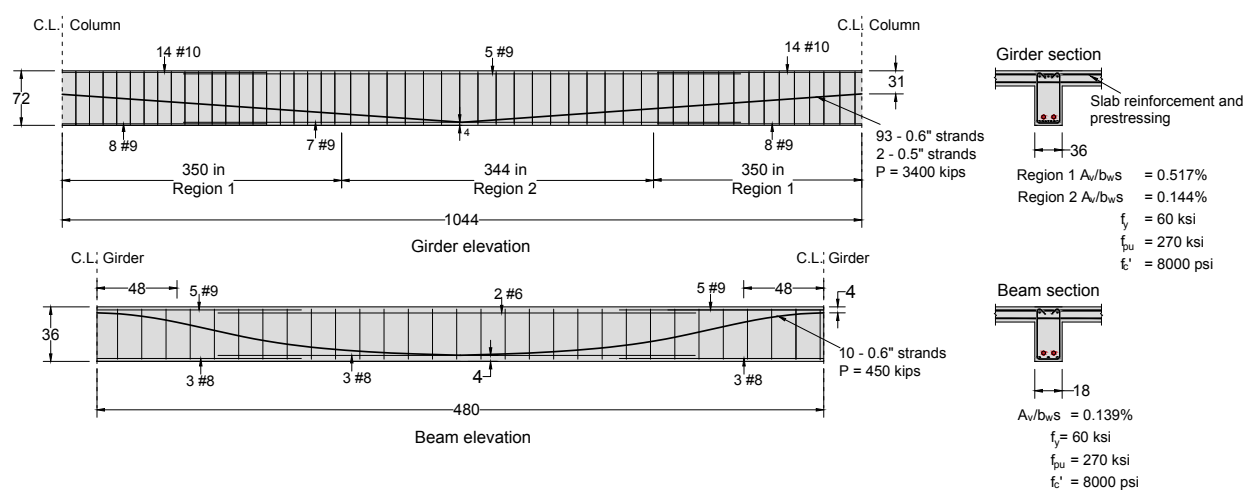


Figure 13 – Typical girder and beams used in analyses (dimensions in inches) [Note: 1 kip = 4.45 kN, 1 in. = 25.4 mm, 1 ksi = 6.90 MPa]

Three loading cases were assessed: Case 1: a situation where all 4 drive lanes are occupied by an HS-25-44 truck and positioned over the center girder, Case 2: a situation where all 4 drive lanes are occupied between the girders and Case 3: a single drive lane is occupied between the girders and beams. In Cases 1 and 2 since all 4 lanes are occupied, the truck loads are reduced by 25%. The 33% impact factor was applied to the truck loading in all cases. A truck axle