

SP 41-1

Strength And Deformation Characteristics of Plain Concrete Subjected to High Repeated and Sustained Loads

By

M.E. Award and H.K. Hilsdorf

Synopsis: 300 concrete prisms were subjected to high repeated or sustained compressive loads with maximum stresses ranging from 80 to 95 percent of their short time compressive strength. Stress range was varied between zero and 95 percent of the compressive strength. Speed of testing was varied between 600 and 60,000 psi/min (42-4200 kgf/cm²/min). Specimens were tested at an age of 3; 7 or 90 days. The results indicate that concrete response to high repeated loads is to a large extent controlled by the duration of time during which the concrete has to resist stresses higher than its sustained load strength. Therefore, the speed of testing has a substantial influence on the fatigue life of concrete: A decrease of rate of loading by one order of magnitude results in a decrease of the number of cycles to failure by almost one order of magnitude. For young concrete the continued hydration may partially or completely offset the damage caused by high loads. An analytical procedure was developed to predict the number of cycles to failure for concrete subjected to various stress ranges at various rates of loading.

Keywords: age; compression tests; compressive strength; concretes; cyclic loads; deformation; fatigue (materials); hydration; loading rate; loads (forces); strength; stress-strain relationships.

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INTRODUCTION

Under extreme conditions reinforced concrete structures may be exposed to a small number of high overloads approaching the ultimate load carrying capacity of a structural member. Such conditions may arise when a structure is exposed to earthquakes, high winds or blasts. They are also of significance for young concrete structures which may have to sustain heavy loads during construction, and for the application of limit analysis to the design of reinforced concrete structures. However, only limited information is available on the behavior of concrete subjected to repetitions of high stresses, often referred to as low cycle fatigue (1-3).

In the prediction and evaluation of the low cycle fatigue behavior of concrete particular attention has to be given to the fact that the repeatedly applied stresses may be larger than the sustained load strength of the concrete. Under such conditions the period of time during which concrete has to resist stresses larger than the sustained load strength may be as significant as the number of applied cycles. Then, concrete response is both cycle and time dependent, and the frequency at which the repeated loads are applied may become a major parameter affecting the fatigue behavior of concrete.

The objective of this study was to investigate the strength and deformation characteristics of plain concrete subjected to repeatedly applied high compressive stresses. Particular emphasis was placed upon studies of the role of time during which concrete is subjected to high stresses. Analytical procedures were developed to predict concrete response under such loading conditions.

EXPERIMENTAL PROGRAM

Approximately 300 prismatic specimens, 4 by 4 by 12 in, were subjected to concentric compression. The investigation was limited to one type of concrete with a cylinder strength of 4000 psi (280 kgf/cm²) after 28 days. The specimens were either loaded to failure at a constant strain rate or subjected to sustained or repeated loads. In the fatigue tests the specimens were loaded or unloaded with a constant

stress rate, $\pm \dot{\sigma}$, so that a saw-tooth loading pattern was obtained, as shown in the top right corner of Fig. 1.

To correlate the results obtained from all batches, the maximum and minimum stresses in a fatigue test are given as a ratio of the static strength of companion specimens, f_{cup} , hereafter referred to as the stress levels, σ_{max} and σ_{min} . The stress level in a sustained load test is referred to as σ_{sus} .

The test program was divided into three phases: In Phase One 133 specimens were subjected to high repeated or sustained loads. The maximum stress level, σ_{max} , and the stress range $R = \sigma_{max} - \sigma_{min}$ were the parameters in the repeated load tests. Four levels of maximum stress, $\sigma_{max} = 0.80; 0.85; 0.90$ or 0.95 were investigated. The stress range, R was varied between 0.05 and the level of the maximum stress ($\sigma_{min} = 0$). In the sustained load studies three levels of sustained stress, $\sigma_{sus} = 0.85; 0.90$ or 0.95 were studied. In both repeated and sustained load tests the concrete age at initial load application was 7, 28 or 90 days, respectively.

Phase Two was concerned with the effect of stress rate and frequency of testing on concrete response to high repeated loads. Stress rates of $\dot{\sigma} = \pm 600$ (42) and $\pm 60,000$ psi/min. ($4200 \text{ kgf/cm}^2/\text{min}$), were chosen to supplement the data obtained in Phase One where a stress rate of $\pm 6,000$ psi/min. ($420 \text{ kgf/cm}^2/\text{min}$) was used in all tests. A maximum stress level $\sigma_{max} = 0.90$ and stress ranges $R = 0.10; 0.50$ and 0.90 were investigated. Note that the frequency of testing (cycles/min) was different for the same maximum stress level but varying stress ranges whenever the stress rate (stress/min) was kept constant. The age of concrete at the time of loading was 28 days.

In Phase Three the extent of damage caused by high sustained or repeated loads which did not as yet cause failure was investigated. The specimens were loaded to 30; 60 or 90 percent of their predicted lives. Then the compressive strength of the preloaded specimens, hereafter referred to as the reloading strength, was determined and compared with the strength of companion specimens without previous load history. One level of sustained stress, $\sigma_{sus} = 0.90$, and two levels of repeatedly applied stresses, $\sigma_{max} = 0.90$ and 0.95 , ($\sigma_{min} = 0$) were chosen. The age of concrete at the time of loading was 28 days.

In all tests longitudinal and lateral concrete strains were observed. Further details of experimental procedures, tests results and their evaluation are given in (4).

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TESTS RESULTS

Effect of Maximum Stress Level and Range of Stress

Strength data--In Fig. 1 the relationship between the maximum stress level, $\sigma_{\max}$, and the number of cycles to failure, n_u , for a constant stress range, R , are given. Each data point corresponds to the logarithmic average of between three and seven individual tests. As is true for most fatigue data, the number of cycles to failure increases with decreasing maximum stress level and with decreasing stress range. The effect of stress range becomes even clearer in Fig. 2. There, the relation between the stress range, R , and the number of cycles to failure, n_u , is given for constant maximum stress levels. For maximum stress levels below the sustained load strength of concrete the relationship between R and n_u should approach the n_u -axis asymptotically giving a value of $n_u \rightarrow \infty$ for $R = 0$. According to Fig. 2, however, these relationships become almost vertical particularly for $\sigma_{\max} = 0.95$ and $R < 0.50$. At these stress levels the maximum stress is well above the sustained load strength so that the total time of testing or the time during which a specimen has to sustain high stresses becomes more significant than the number of applied cycles. Therefore, further reduction of the stress range R does not result in a significant increase of the number of cycles to failure, n_u .

A sustained load test may be considered a limiting case of fatigue loading where $R = 0$. Therefore, the fatigue tests were supplemented by tests on concrete under sustained loads. The results of these studies are summarized in Fig. 3. There, the relationship between the level of sustained stress, σ_{sus} , and the time to failure, t_u , is given for specimens loaded at an age of 7, 28 or 90 days respectively. Similar to the results from previous investigations (5) the time to failure increases with decreasing sustained stress level. For high stress levels causing failure after less than 500 min ($\sigma_{\text{sus}} > 0.90$), old concrete is more resistant to sustained loads than young concrete. At lower stress levels causing failure after more than 1000 min, this trend is reversed: The time to failure for concrete loaded at an age of 7 days is larger than it is for older concrete. It was shown in (5) that this is due to continued hydration occurring while the specimen is under load. Such hydration may partially or completely offset the damage caused by high sustained loads. Hydration under load is insignificant at high stresses when failure occurs already after a few hours. However, it becomes significant for low stress levels when the time to failure may be several days.

Strain data--In Fig. 4 the longitudinal and lateral strains observed on specimens which were subjected to repeated loads are given as a function of the number of cycles to failure. Similar to the behavior of concrete under sustained loads both longitudinal and lateral strains increase with increasing number of cycles. The strains at failure are the larger the lower the applied maximum stress level or the longer the ti-

me to failure. In Fig. 5 the failure strains obtained for the various loading conditions and stress ranges are summarized. In this diagram the dashed line gives the stress-strain diagram under static loading as determined in a short time test in which the specimens were loaded at a constant strain rate of 10^{-5} in./in./sec. The failure strains observed on specimens which were subjected to repeated loads fall between two limits: For a given maximum stress level the upper strain limit is given by the failure strains observed in sustained load tests ($R = 0$). The lower limit is given by the descending portion of the static stress strain relationship. As the stress range decreases also the influence of the time under load becomes more significant and the failure strains increase.

Effect of Stress Rate and Frequency of Testing

In Fig. 6 the relationship between the stress rate, in psi/min and the number of cycles to failure, n_u , is given for three different stress ranges, R . In the same diagram also the frequency of testing in cycles/min is given for each data point. According to Fig. 6 an increase of the stress rate by one order of magnitude leads to an increase of the number of cycles to failure by almost one order of magnitude. This effect is most pronounced for small stress ranges.

The effect of frequency of loading is likely to diminish with decreasing maximum stress level, and other investigators have, in deed, reported that the frequency of testing has little influence on the fatigue behavior of concrete when subjected to comparatively low stresses (6). This may be explained by the accumulation of both cycle and time dependent damage caused by fatigue loading at high stresses and will be discussed further in the evaluation of the test results.

Reloading Strength

For the evaluation of the experimental data an estimate of the accumulation of damage which occurs during a repeated or sustained load test was needed. It was assumed that the reloading strength as defined earlier would give such an estimate. The results of this investigation are given in Fig. 7. There, the reloading strength expressed as a fraction of the short-time static strength of companion specimens is given as a function of the life ratio which is the fraction of the total life to which a specimen was subjected. According to Fig. 7 repeated or sustained loads cause a reduction of the reloading strength below the static strength only after 30 to 70 percent of the total life of the specimen had been applied. For smaller life ratios even a slight strength increase was observed.

EVALUATION OF TEST RESULTS

The purpose of the evaluation of the experimental results was to develop an analytical procedure by which the fatigue behavior of concrete under high compressive loads can be predicted for various stress ranges, frequency of testing and other load history parameters. The

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derivations given in the following are based upon the principles of cumulative fatigue damage. Their application to the behavior of concrete is discussed e.g. in (7). In these derivations the following assumptions were made:

- a) Damage in concrete caused by repeated loads can be separated in a time dependent and a cycle dependent part.
- b) The cycle dependent as well as the time dependent damage are linear functions of the number of applied cycles or of the time under sustained load, respectively.
- c) Time dependent damage accumulates only above a certain threshold stress σ_{sh} . It is equal to the sustained load strength of concrete if the minimum stress level in a fatigue test is less than σ_{sh} . For minimum stress levels larger than the sustained load strength $\sigma_{sh} = \sigma_{min}$.
- d) The total damage in a fatigue test can be determined by arithmetic addition of the cycle and the time dependent damage.

In the following, damage, D , is defined such that failure of the specimen will occur if $D = 1.0$. Assuming linear damage accumulation, the cycle dependent damage, D_n , can be expressed by:

$$D_n = \frac{n}{n_0} \quad (1)$$

where

n = number of applied cycles
 n_0 = number of cycles to failure under pure fatigue conditions ($P \rightarrow \infty$) for a given σ_{max} and R

Assuming linear damage accumulation the time dependent damage in a sustained load test, can be expressed by:

$$D_t = \frac{t}{t_u} \quad (2)$$

where

t = duration of loading under a constant stress level, σ
 $t_u = f(\sigma)$ = time of failure under a constant stress level, σ

Using eq. (2) the time dependent damage caused by one load cycle may be expressed by:

$$D_{t1} = 2 \int_{\sigma_{sh}}^{\sigma_{max}} \frac{dt}{t_u}$$

Then, the total time dependent damage after n cycles is:

$$D_t = 2n \int_{\sigma_{sh}}^{\sigma_{max}} \frac{dt}{t_u} \quad (3)$$

Since the loads are applied at a constant stress rate $d\sigma/dt = \beta$ $P/f_{cup} = \beta$ we can transform eq. (3) to:

$$D_t = \frac{2n}{\beta} \int_{\sigma_{sh}}^{\sigma_{max}} \frac{d\sigma}{t_u} \quad (4)$$

From the experimental data of this investigation as well as the data given in (5) the following approximation for $t_u = f(\sigma)$ was derived by curve fitting:

$$t_u = 240 \left(\frac{1 - \sigma}{\sigma/\sigma_s - 1} \right)^2 \quad (5)$$

with

$$\sigma_s = 0.70 \text{ and } t_u \text{ in minutes.}$$

After substituting eq. (5) in eq. (4) and integrating, the time damage can be expressed as follows:

$$D_t = K_1 \cdot \frac{n}{\beta} \quad (6)$$

where

$$K_1 = \frac{1}{120\sigma_s^2} \left[\frac{(\sigma_{max} - \sigma_s)^2}{1 - \sigma_{max}} - \frac{(\sigma_{sh} - \sigma_s)^2}{1 - \sigma_{sh}} + 2(\sigma_{max} - \sigma_{sh}) + 2(1 - \sigma_s) \log_e \frac{1 - \sigma_{max}}{1 - \sigma_{sh}} \right] \quad (7)$$

Then, the total damage after n cycles, D_{tn} is:

$$D_{tn} = D_n + D_t = \frac{n}{n_o} + K_1 \cdot \frac{n}{\beta}$$

By definition, failure will occur if $D_{tn} = 1.0$ and $n = n_{uc}$. With these conditions we obtain for the calculated number of cycles to failure, n_{uc} :

$$n_{uc} = \frac{n_o}{1 - K_1 \cdot \frac{n_o}{\beta}} \quad (8)$$

To evaluate eq. (8) the number of cycles to failure for pure fatigue conditions, n_o , have to be known. Therefore, eq. (8) was rearranged to calculate values for n_o from the experimental data which had been obtained for the fastest stress rate. Then, values for n_{uc} for other stress rates were calculated from eq. (8).

Since the agreement between calculated and experimental values was not satisfactory, eq. (8) had to be further modified. Two additional assumptions were made:

a) Damage starts to accumulate only after an initial life ratio n , has been applied. This assumption is in agreement with the experimental

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data shown in Fig. 7 which indicate, that the strength of concrete drops below its initial value only for $0.30 < \eta < 0.70$.

b) If the strength of the concrete falls below its initial value, then time dependent damage may also be caused at stresses below the sustained load strength. This is of particular significance when R is large. Therefore, it was assumed that the threshold stress σ_{sh} decreases with increasing stress range, R . With these assumptions eq. (8) was modified to

$$n_{uc} = \frac{n_o}{1 + K_2(R) \cdot (1 - \eta) \cdot \frac{n_o}{\beta}} \quad (9)$$

There, $K_2(R)$ is equal to K_1 (eq. 7) except that σ_{sh} is not constant but stress range dependent. Eq. (8) was evaluated using the following values for σ_{sh}

$R = 0.10$	$\sigma_{sh} = \sigma_{min} = 0.80$
$R = 0.50$	$\sigma_{sh} = 0.65$
$R = 0.90$	$\sigma_{sh} = 0.55$

In Fig. 8 experimental and calculated values for n_u as well as a relation for the pure fatigue failure cycles n_o are given. The agreement between experimental and calculated results is satisfactory. Fig. 8 clearly shows the expected trends:

- The pure fatigue failure cycles n_o approach a value of $n_o \rightarrow \infty$ for $R = 0$.
- The higher the stress range the larger the cycle dependent damage. Therefore, at high stress rates the observed failure cycles are close to the estimated values for pure fatigue conditions.
- For small stress ranges and low stress rates the time dependent damage dominates. This is evidenced by the small increase in failure cycles with a reduction of the stress range.

The values for n_o are also dependent on the maximum stress level, however, no reliable values for n_o at stress levels $\sigma_{max} = 0.95$ and 0.85 can be given since for these stress levels only one stress rate was investigated.

CONCLUSIONS

The results from this investigation may be summarized as follows:

- When concrete is subjected to high repeated compressive stresses, a decrease in either the maximum stress level or the stress range results in an increase of the number of cycles to failure. The increase in failure cycles with decreasing stress range becomes insignificant at high maximum stress levels and small stress ranges.
- At high stresses, a reduction in the speed of testing results in a significant reduction of the fatigue life of concrete.
- Under repeated loads the failure strains of concrete increase with decreasing stress level or decreasing range of loading.
- High repeated or sustained loads may cause a significant strength decrease only after 30 to 70 percent of the number of cycles

causing failure have been applied.

e) Damage caused by high repeated loads depends both on the number of applied cycles and the total time concrete has to sustain high stresses. Based upon this observation an analytical procedure was developed which allows the prediction of the fatigue properties of concrete for various stress ranges and stress rates.

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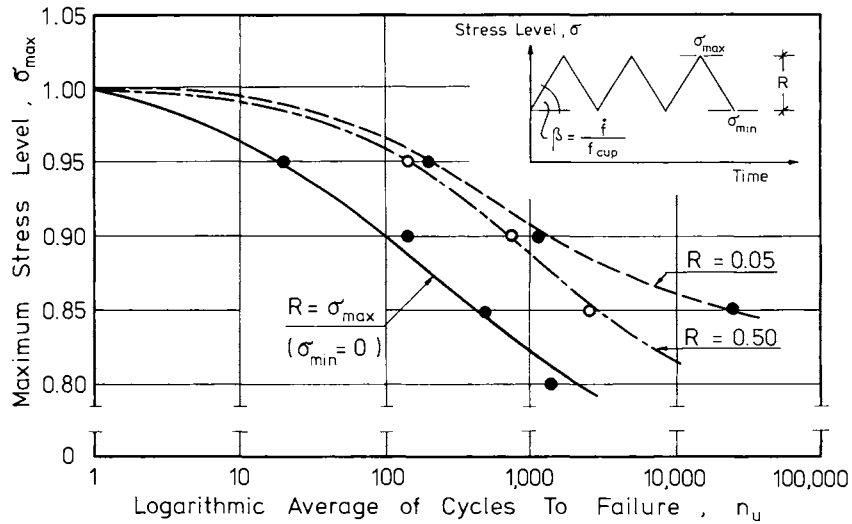


Fig. 1-Relation between maximum stress level and number of cycles to failure for different stress ranges

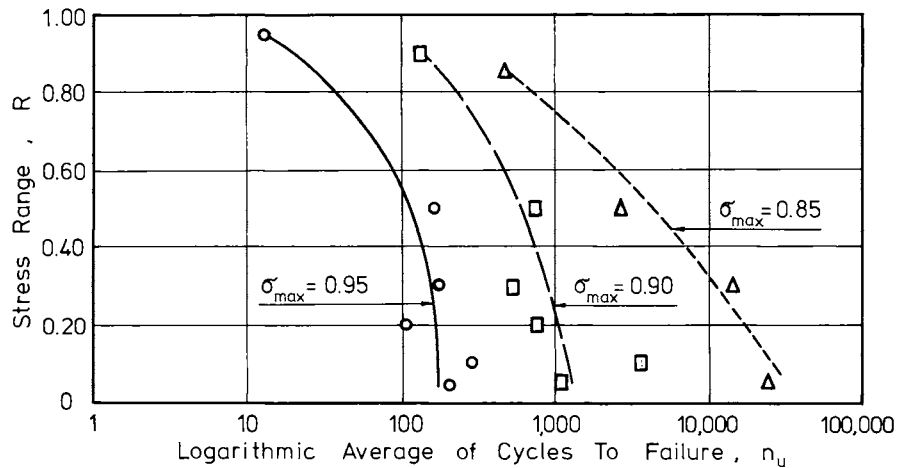


Fig. 2-Relation between stress range and number of cycles to failure for different maximum stress levels

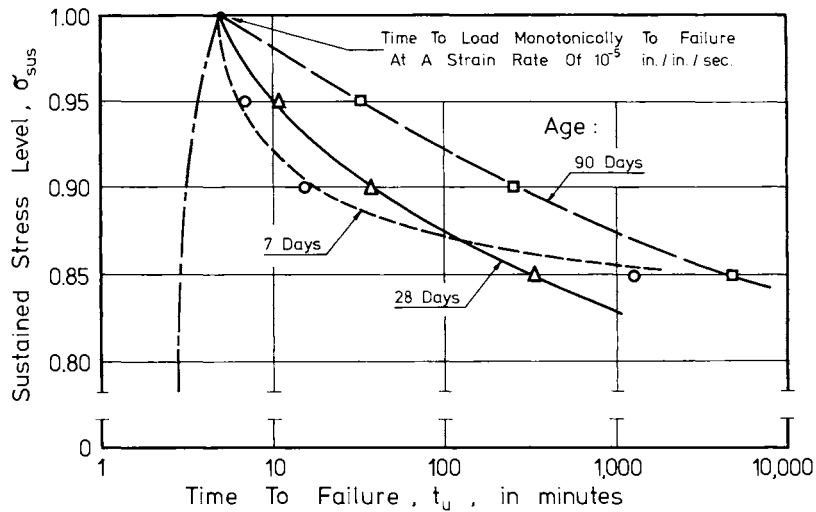


Fig. 3-Development of longitudinal and lateral strains at maximum stress during fatigue tests with various maximum stress levels

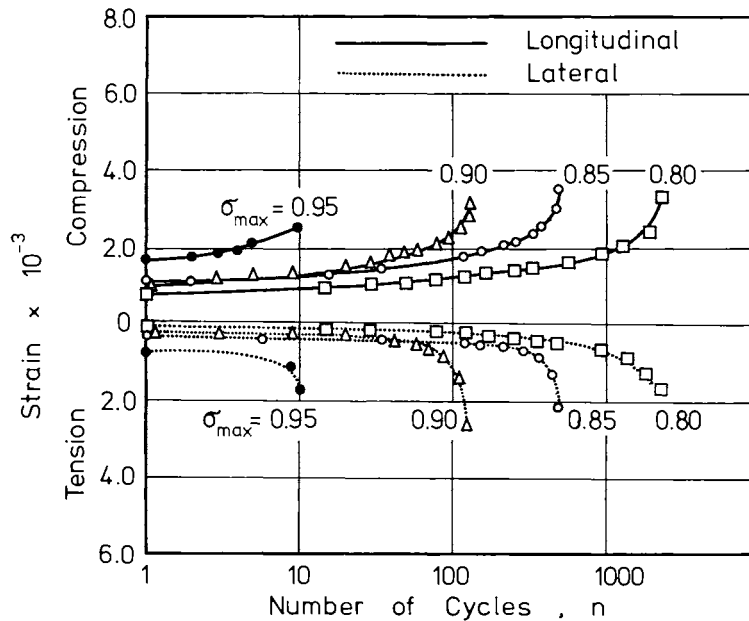


Fig. 4-The effect of age at loading on the time to failure of concrete subjected to different levels of sustained stress

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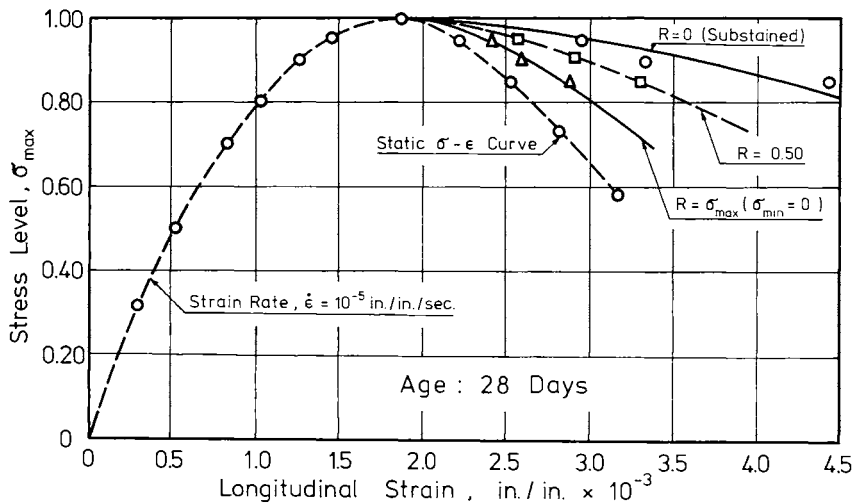


Fig. 5-Failure strain of concrete subjected to sustained or repeated loads with different stress ranges; age at loading: 28 days

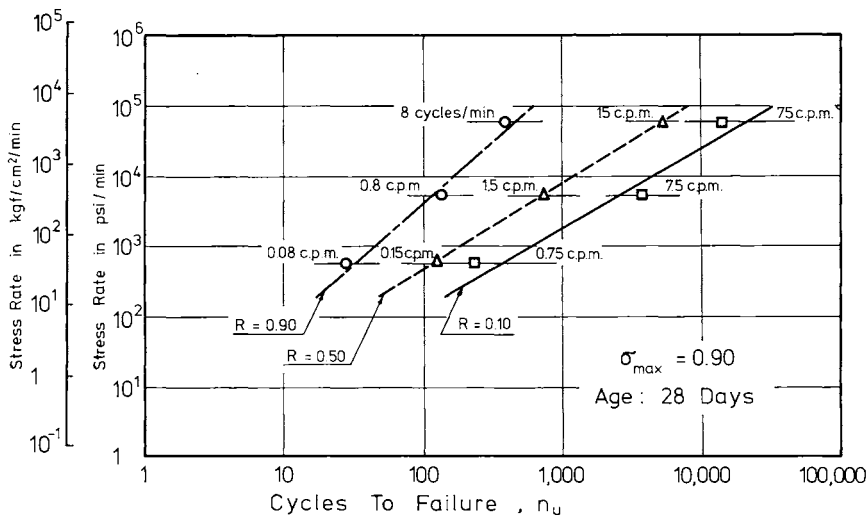


Fig. 6-Influence of stress rate and frequency of testing on the fatigue life of concrete; $\sigma_{\max} = 0.90$; age at loading: 28 days

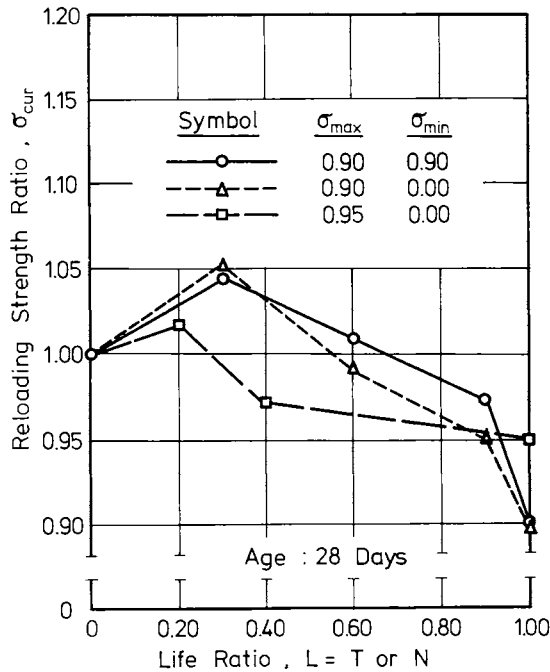


Fig. 7-Change of static strength of concrete during repeated or sustained load tests

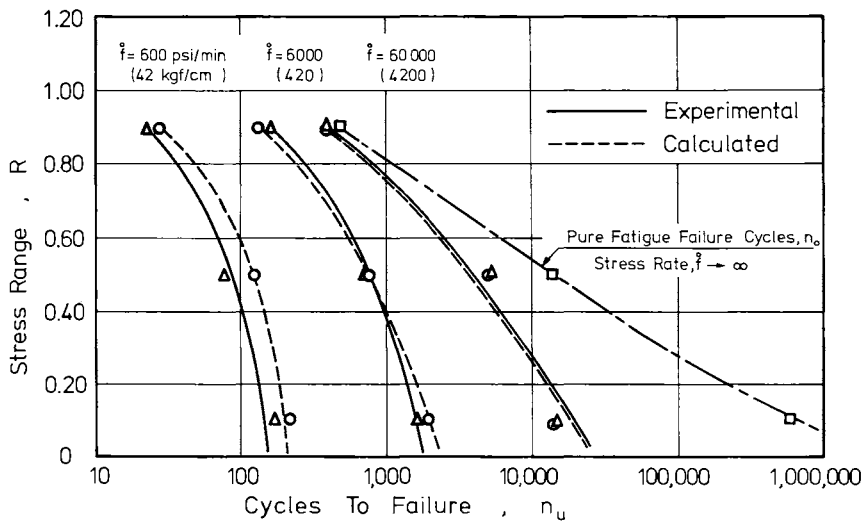


Fig. 8-Comparison of experimental and calculated relationships between stress range and number of cycles to failure for various stress rates; $\sigma_{max} = 0.90$; age at loading: 28 days

SP 41-2

Effects Of Moisture Condition, Age, And Rate Of Loading On Fatigue Of Plain Concrete

By
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Synopsis: Results are given for flexural fatigue tests on small concrete beams to investigate the effects of moisture state, age and rate of loading on the fatigue performance of three types of concrete as used in typical highway construction. The fatigue performance is strongly dependent on the age and moisture state of the concrete and can be related to flexural strength under similar conditions. It is suggested that a reasonably good prediction of long-term performance might be obtained by extrapolation from the results of short-term tests under appropriate conditions. The importance of moisture condition is stressed and some indications of further research are given.

Keywords: age; beams (supports); concrete construction; concretes; fatigue (materials); fatigue tests; flexural strength; loading rate; moisture; strength; stress-strain relationships.

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J W Galloway is a Senior Scientific Officer at the TRRL and over a period of 20 years has had experience of research into the structural performance of road pavements and the behaviour of road materials. Mr Galloway was responsible for performing the tests described.

INTRODUCTION

A major objective of research at the Transport and Road Research Laboratory of the Department of the Environment is the development of improved methods of structural design for highway pavements based on an assessment of probable traffic loading throughout the life of the pavement and the ability of the constituent materials to withstand the repeated stresses and strains resulting from such loads for the range of environmental conditions encountered in service. In defining allowable strength values for materials used in highway pavements, more attention is now being paid to their behaviour under repeated loading.

There are four main applications for which fatigue strength data are needed:

- (i) To help provide a rational explanation for the observed behaviour of full-scale experiments.
- (ii) To provide design data for new road construction.
- (iii) To assess the remaining life of existing roads and hence to decide when overlaying or reconstruction becomes necessary.
- (iv) To form a basis for improved mix design and for assessing the potential uses of new materials, such as fibre-reinforced concrete.

Although aimed mainly at flexible pavements the current programme of fatigue tests at TRRL covers tests on concrete as well as on soils and bituminous mixtures. In each case exploratory research programmes are being pursued firstly to establish major factors having a significant effect on fatigue performance under conditions in which the material is stressed in the road structure and then to use such information to define simple

accelerated laboratory tests that can be used for comparison of different materials and for the development of new materials.

This paper describes tests on unreinforced concrete beams to study the effects of moisture condition, age and, to a limited extent, rate of loading, on the fatigue performance of two types of concrete as used for pavement slabs and on one type of lean concrete which may be used for road bases in flexible road construction.

All results are given in S.I. Units: (1 mm = 0.0394 in., 1N=0.2248 lbf, 1 MN/m² = 145.038 psi).

SELECTION OF FACTORS TO BE STUDIED

A review of previous work on the flexural fatigue strength of plain concrete (1) led to the conclusion that there were a number of factors relevant to the long-term behaviour of the material in the road which had received insufficient attention. Among these were the effect of age on fatigue performance and the effect of rate of loading and rest periods between loading pulses (related to vehicle speed and axle spacing).

Kesler (2) has indicated that rate of loading is not important within the range 1 to 7 Hz but this finding seemed inconsistent with the fact that quasi-static flexural strength is affected quite markedly by rate of loading (3). It was decided therefore to explore rate of loading effects over a wider range both under "static" and fatigue loading. Rest period tests were included in the exploratory programme in view of the intermittent nature of traffic loading and because it has been shown that with rolled asphalt, rest periods between successive loading cycles have a very marked effect on fatigue performance (4).

Little work appears to have been done hitherto on the effects of age on flexural fatigue performance of concrete and so a main objective of the work described was to study the effects of curing time on the flexural strength and fatigue performance of small concrete beams. At an early stage in the test programme the surface moisture condition of the test beams was found to have a predominant effect on fatigue performance, so this was studied in some detail and formed another main part of the investigation.

TEST CONDITIONS

All tests were carried out on 102 x 102 x 510 mm beams, simply supported on a span of 406 mm and loaded at the middle third points. The test specimens were cured under water before being tested at various ages, ranging from 28 days to 3 years.

To study the effects of moisture state, flexural strength and fatigue tests were made under each of the following conditions:

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- (i) Test specimens saturated throughout the duration of the test.
- (ii) Specimens allowed to dry for one week in the laboratory atmosphere before being tested.
- (iii) Specimens removed from the curing tank then oven-dried at 105°C for one week before being tested.
- (iv) Specimens oven-dried, then resoaked in water for 3 weeks before being tested in a wet condition.

The fatigue tests were performed under constant amplitude sinusoidal repeated loading to give bending stresses which varied from approximately zero to a maximum in each cycle. The minimum load in the cycle, although nominally zero, was kept at 25N to avoid backlash in the loading system. The maximum load varied from about 55 to 95 percent of the "static" failing load, to give lives to failure up to about 10^7 cycles. Most of the tests were carried out at a frequency of 20 Hz but some tests were run at 4 Hz. In a further limited series of tests a rest period of constant duration was introduced after each loading cycle, with rests of up to 2s.

In some tests central deflections of the beam were measured relative to the supported ends and in some strain gauges were used to measure strains at the top and bottom surfaces throughout the test.

As part of the investigation into the effects of rate of loading "single shot" tests were made to determine the flexural strength at various rates of load application, ranging from that equivalent to an increase in flexural stress of $0.017 \text{ MN/m}^2/\text{s}$ (slightly less than the British Standard rate of loading for flexural strength tests (5) to $170 \text{ MN/m}^2/\text{s}$, which is of the order of the rate of load application in the sinusoidal repeated load tests at 20 Hz.

METHOD OF TEST

All flexural tests were carried out in two identical closed-loop electro-hydraulic testing machines, one of which is shown in Fig 1. The beams were simply-supported on steel rollers, which were mounted on a stiff cross beam of the reaction frame and were free to rotate in two planes to avoid any restraint on the test specimen. Load was applied in a downward direction to the top surface of the test specimen (upper surface as cast) through similar steel rollers mounted on an aluminium alloy load-spreading beam which was connected through a swivel joint to a load cell attached to the end of the actuator rod. For the tests on saturated beams and on oven-dried beams, each specimen was sealed in a polyethylene bag before being placed in the testing machine, in order to maintain the appropriate surface condition throughout the test.

In the fatigue tests constant amplitude sinusoidal cyclic loading was applied through the electro-hydraulic loading system and was controlled by

feedback from the load cell. For the tests in which rest periods were included the command signal was interrupted after each load cycle to give a series of sinusoidal pulses with a rest of constant duration at the minimum load of each cycle.

Vertical deflections were measured relative to the loading frame at the centre of the beam and over the end supports, using linear variable displacement transformers (LVDT's). In the tests in which strains were measured electrical resistance strain gauges of 102 mm gauge length were attached to the top and bottom surfaces of the test specimen. A successful technique was developed for bonding the foil gauges to wet concrete, using a commercial epoxy resin adhesive that had been specially developed for such applications. Outputs from the LVDT's and the strain gauges were displayed either on an ultra violet oscillograph recorder or on a cathode ray oscilloscope. A multi-channel semi-automatic digital data recording system was used to record peak values of deflection, strain and load throughout the tests in which they were measured.

For the quasi-static tests at various rates of loading a "ramp" function was used to apply load at a constant rate to failure, over the range of rates of loading given in the preceding section.

MIX DESIGN AND PREPARATION OF TEST SPECIMENS

The majority of the tests were made on a medium-strength concrete having an uncrushed flint gravel aggregate from the Thames Valley, this being a material for which a great deal of strength and performance data already exist. This mix is referred to as PQ1. For comparison a more limited amount of testing was performed on two other mixes - one (PQ2) a pavement quality concrete having the same workability and nominal 28-day indirect tensile strength as PQ1 but made with crushed limestone coarse aggregate; the other (LC1) a lean concrete, with flint gravel aggregate similar to that used in PQ1. The mix proportions and strengths of the trial mixes for each material are given in Table 1. No air-entraining agents or other admixtures were used.

To maintain a sufficient supply of concrete specimens for testing at particular ages one or two batches were made under carefully controlled conditions every week for over two years. The same batch of blended Portland cement was used for all test specimens and the aggregates were taken from stockpiles, dried, graded and recombined with excess water before mixing, in accordance with the TRRL's standard method of mixing for laboratory trials (6). Mixing was performed in a 50-litre open pan mixer and from each batch of concrete six beams and five 102 mm diameter x 152 mm long control cylinders were made. The mixes were compacted in moulds either on a vibrating table or, in the case of the lean concrete, with a vibrating hammer. Demoulding took place after curing for one day (or in the case

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of LC1 two days) under damp matting and polyethylene sheet and the specimens were then cured under water at 20 C until required for test.

As a means of checking the quality of the concrete throughout the two years of specimen preparation, indirect tensile tests were made on four control cylinders from each batch after 28 days. The results, summarised in Table 2, show that the scatter in strength was acceptably low and probably represents the minimum that can be expected for carefully controlled laboratory conditions over a long period of time.

EFFECTS OF MOISTURE CONDITION ON FATIGUE STRENGTH

Tests were made on Type PQ1 specimens only.

Apart from the saturated specimens which had been cured under water throughout their life it is difficult to define precisely the moisture state of the beam specimens at the time of test. Even for specimens which have been subjected to controlled drying, there are moisture gradients which affect the local surface strains and therefore may be expected to have an effect on both flexural strength and fatigue performance. It was beyond the scope of the current investigation to attempt to determine the moisture distribution within the test specimens and its effect on surface strains and crack initiation but some measurements were taken on control beams to determine the average evaporable moisture content under various conditions. Those beams which were allowed to dry in a controlled laboratory atmosphere (20 C) and 65 percent relative humidity) showed a continuing loss of weight after three years, by which time the loss of moisture was about 2.8 percent of the original saturated weight. For the partially dry beams which were tested 7 days after removal from the curing tank the average moisture loss was about 0.7 percent. In the case of the oven-dried specimens evaporation appeared to be virtually complete after about 8 days, with a moisture loss of about 5.5 percent of the initial weight. For oven-dried specimens which were then re-soaked in water for 3 weeks, the final weight was approximately 0.8 percent less than for the fully saturated specimens.

The results of the flexural strength and fatigue tests for different moisture states are given in Table 3 and the mean values are plotted in the form of endurance curves in Fig 2. The points shown plotted at one cycle are the flexural strengths determined from the standard quasi-static test procedure expressed in terms of the modulus of rupture. It should be noted, however, that the rate of loading in these tests was considerably less than in the fatigue tests. The significance of this is discussed later. Some tests were terminated before failure, particularly at the lower stress levels, and for these the number of cycles applied when the test was stopped was used as a conservative estimate of the life of the specimen.

The results of both the flexural strength tests and the fatigue tests showed a consistent pattern, the highest strengths (and longest lives) being

obtained with the oven-dried specimens and the lowest with the specimens which had partially dried in the laboratory before test; the fully saturated specimens gave intermediate strength and endurance values. The specimens which had been oven-dried and then soaked for 3 weeks before being tested in a wet condition had a "static" strength midway between the oven-dried and the fully saturated specimens but the fatigue performance of two specimens was not significantly different from that of the saturated beams.

There is no direct evidence from the above tests to explain the differences in performance between the various moisture states but it seems likely that the explanation will lie in differential strains generated by moisture gradients within the beams. This would explain why the partially-dried specimens were weaker than the fully saturated specimens. It is unlikely that the higher strengths of the oven-dried specimens can be attributed to accelerated ageing during the one-week drying period, since the specimens had already been cured for 26 weeks before being dried. It may be that the higher strength results from increased internal friction due to shrinkage during drying.

As the moisture state of the specimens turned out to have a critical influence on fatigue performance it was decided for most of the subsequent tests to use fully saturated beams, as their moisture state could be defined with more certainty. There was an added advantage that the scatter in fatigue test results appeared to be rather less with saturated beams. Any interpretation of the results of such tests in terms of conditions in the road would have to take into account the local moisture conditions at critical locations in the road structure.

EFFECTS OF RATE OF LOADING AND REST PERIODS

It is well established that the rate of loading has a significant effect on the flexural strength of concrete beams loaded monotonically to failure but most of such studies have been made at rates of loading considerably less than the rate at which load is applied in fatigue tests. In the present investigation flexural strength tests were made on saturated beams of both PQ1 and PQ2 at various rates of loading up to the rate at which flexural stress was imposed in the fatigue tests at 20 Hz. The results are given in Fig 3, in which the modulus of rupture (mean of ten tests at each rate of loading) is plotted against the rate of increase of extreme fibre stress. At rates of loading comparable to the fatigue tests the flexural strength is about 50 percent greater than that measured at the standard rate of loading. The rate of increase of strength with rate of loading agrees broadly with indications from earlier tests over a more limited range of loading rates by Wright, Klettke, Kesler and others on various types of concrete at different ages (3).

To see whether loading rate had a similar effect on fatigue strength, comparative fatigue tests were carried out at 4 Hz and 20 Hz on both sat-

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urated and on surface-dry beams of PQ1 (Fig 4). There is an apparent slight increase in fatigue strength at 20 Hz for the surface dry beams but the difference is not statistically significant. The tests on saturated beams showed no frequency effect. These findings confirm the observation by Kesler (2) that loading frequency is unimportant in fatigue, even though there is a substantial difference in flexural strength under single load application over this range.

For convenience in testing, in all subsequent fatigue tests the loading frequency was maintained at 20 Hz.

The results of the rest period tests, which were restricted to one stress level and were carried out only on concrete PQ2, are shown in Table 4. Although rests between successive loading cycles appeared to give a slight reduction in life, the differences are not statistically significant at the 5 percent level. There is therefore no evidence of any recovery of damage during short rests such as occurs with rolled asphalt, where increases in mean life of up to 25 have been recorded with rests of 1s duration (4). Kesler has reported some improvement in the fatigue strength of concrete as a result of rests (7) but in his tests rest periods of 1 to 27 minutes were applied at the end of blocks of several thousand load cycles.

EFFECTS OF AGE ON FATIGUE STRENGTH

Having established the basic endurance curves for each of the three types of concrete in the saturated condition at 26 weeks (Fig 5), the effect of age on fatigue life was studied by testing specimens of each type at particular stress levels for various curing times up to 3 years. The modulus of rupture was also determined for each concrete by standard flexural tests over the same range of curing times.

The results of the fatigue tests (Fig 6) indicate a substantial increase in fatigue endurance with age, the actual gain being dependent on the type of concrete and on the stress level. For concrete PQ1 at the lower stress level (3.20 MN/m^2) the mean life to failure of 2-year-old beams was 2000 times the life at 4 weeks. At the higher stress level (3.53 MN/m^2) the rate of increase appeared to be slightly less but the life was still increasing after 3 years. At each stress level the age-endurance curves tend towards an infinite life at particular ages, which may range from about one year to three or four years, depending on the stress level and the type of concrete. Based on the limited results available, the lean concrete (LC1) appeared to reach such a limit rather earlier than the two pavement quality concretes. Further tests are to be made on PQ2 and LC1 at 2 years to confirm the behaviour of these mixes.

If the cyclic stresses of Fig 6 are expressed in terms of the values of modulus of rupture at the appropriate ages (Fig 7), the results may be plotted as points on an endurance curve with non-dimensional ordinates.

This has been done in Fig 8 for comparison with non-dimensional endurance curves derived from Fig 5 for the three concrete mixes at 26 weeks. It will be seen that when plotted in this way the curves for the three mixes come very close together and the results of the tests at different ages fall close to these curves. This suggests that it may be possible to derive characteristic strength values for any particular concrete from which the fatigue performance may be predicted with acceptable accuracy at any age, at least up to 3 years. The characteristic values needed are (i) the modulus of rupture, preferably at several different ages to check on the relationships of Fig 7, and (ii) fatigue test data at one stress level, say 80 percent of the modulus of rupture at a particular age, say 4 weeks. Further work is needed to determine the range of conditions over which relations such as those shown in Fig 8 hold.

STRESS-STRAIN RELATIONSHIPS AND THE EFFECTS OF CYCLIC LOADING

Analysis is not yet complete but some typical results are shown in Figs 9 and 10. Fig 9 shows both central deflection and maximum tensile cyclic strain plotted against the percentage of fatigue life for a typical beam of series PQ2 (limestone aggregate). Both curves show the same characteristic of little change until, at about 80 percent of the life there is a marked and rapid increase in the strains and deflections, presumably associated with the development of small surface cracks. In Fig 10 dynamic stress-strain curves at 20 Hz are plotted for two specimens of different ages at various percentages of the life to failure. The values were derived from an oscillograph record of load and strain taken while the fatigue test was in progress. The shape of the curves does not appear to change very much during the course of a test but towards the end of the test there is usually a marked change of slope together with an increase in permanent strains. The stress-strain curves are fairly linear and are somewhat steeper for the older beam, indicating a higher modulus of elasticity.

Values of secant modulus calculated from the peak dynamic strains have generally shown good agreement with those derived from central deflection measurement up to about 80 percent of the life to failure, typical values being about $34,000 \text{ MN/m}^2$. Thereafter the deflection measurements tend to indicate higher values of modulus, as may be expected as small cracks develop in the tension surface.

MODE OF FAILURE AND EFFECT OF AGGREGATE TYPE

All specimens failed between the two loading points i.e. in the region of constant bending moment and the failures were fairly uniformly distributed over this section. In no case were there any visible cracks before failure. On a few specimens high speed photography (400 frames/s) was used in an attempt to detect the onset of cracking but nothing was visible on the photographs until the final loading cycle which led to failure.

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In the case of the specimens with flint gravel aggregate (PQ1 and LC1) the appearance of the failure was very similar in fatigue, modulus of rupture and indirect tensile tests. Cracks had progressed around the aggregate particles and there had been considerable loss of adhesion between the mortar and aggregate, with very few of the stones fractured. With the limestone aggregate (PQ2) however there was no evidence of bond failure and many of the stones had failed in tension in all three types of test. The flexural and fatigue strength of the limestone concrete was appreciably higher at all ages than the flint gravel concrete, although the 28-day indirect tensile strength was the same.

CONCLUSIONS

Flexural fatigue tests on two mix designs of pavement quality concrete and on a lean concrete similar to road base material have shown the importance of moisture condition on fatigue performance and have indicated the way in which fatigue strength increases with age. Changes in fatigue strength follow closely changes in quasi-static flexural strength under these conditions and it appears reasonable to expect to be able to predict long-term fatigue performance from a few short-term tests under appropriate conditions.

In view of the importance of moisture condition and concrete age at the time of test there is an obvious need to study the effects of interactions between these factors by using different curing methods; this is being done in the continuing programme of research at the Transport and Road Research Laboratory.

A limited study of the effects of frequency of loading on fatigue performance has confirmed the conclusions of other research workers that at least up to 20 Hz the rate of loading is unimportant. This conclusion seems to be at variance with the fact that the flexural strength is affected strongly by rate of loading. Further research is needed to explain this apparent discrepancy.

ACKNOWLEDGEMENTS

The work described forms part of a more general research programme on the structural behaviour of road materials, under the direction of Mr D F Cornelius, Head of the Structural Properties Division of the Structures Department, TRRL. The assistance of Mr H M Harding in performing much of the test work is gratefully acknowledged. This paper is contributed by permission of the Director, Transport and Road Research Laboratory, Crown copyright 1973, and is reproduced by permission of the Controller of Her Britannic Majesty's Stationery Office.

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TABLE 1
Details of Concrete Mixes

Mix			PQ1		PQ2		LC1	
Aggregate	Type		Uncrushed Flint		Crushed Limestone		Uncrushed Flint	
	Particle Shape		Irregular		Angular		Irregular	
	Surface Texture		Smooth		Fairly Rough		Smooth	
	Water Absorption % and mix proportions %	19 - 9.5 mm	1.9	41	0.8	17	1.9	35
		9.5 - 4.8 mm	3.3	20	0.7	39	3.3	25
		Passing 4.8 mm	1.1	39	0.5	44	1.1	40
BS 882 Grading of Fines		ZONE 2		ZONE 1		ZONE 2		
Mix Proportions	Aggregate/cement ratio		6.7		5.2		18.0	
	Free water/cement ratio		0.5		0.62		61% of total oven dry weight of aggregate and cement	
Trial Mix Details	Workability		Compacting Factor 0.84		'V-B' Test 7 secs. (equivalent to C.F. of 0.84)		Compacting Factor 0.79	
	28 day indirect tensile strength (152 mm x 102 mm diameter cylinders)		3.50 MN/m ²		3.48 MN/m ²		2.00 MN/m ²	
	28 day compressive strength (102 mm cubes)		44.80 MN/m ²		33.30 MN/m ²		21.00 MN/m ²	

TABLE 2
Indirect Tensile Tests on Control Cylinders

	CONCRETE MIX	PQ1	PQ2	LC1
ALL	Number of Results	608	136	96
INDIVIDUAL	Mean Tensile Strength MN/m ²	3.47	3.33	1.70
RESULTS	Standard Deviation MN/m ²	0.187	0.222	0.127
	Coefficient of Variation %	5.4	6.7	7.4
AVERAGE*	Number of Batches	152	34	24
WITHIN-	Tensile Strength MN/m ²	3.47	3.33	1.70
BATCH	Standard Deviation MN/m ²	0.152	0.154	0.084
VALUES	Coefficient of Variation %	4.4	4.6	4.9
	Number of Batches	152	34	24
BATCH	Mean Tensile Strength MN/m ²	3.47	3.33	1.70
VALUES	Standard Deviation of Batch Strengths MN/m ²	0.132	0.181	0.105
	Coefficient of Variation of Batch Strengths %	3.8	5.4	6.2
	Mean Sample Range MN/m ²	0.306	0.313	0.172

* Average within-batch standard deviation, σ_M , computed from:

$$\sigma_M = \sqrt{\frac{\sigma_1^2 + \sigma_2^2 + \dots + \sigma_N^2}{N}}$$

where N = number of batches

σ_1, σ_2 etc = Standard Deviations within Batches

TABLE 3
Test Results, effects of moisture condition, Concrete PQ1

Moisture State	Mean * loss of weight percent	Modulus of Rupture			Fatigue Performance			
		Number of Tests	Mean Value MN/m ²	Coefficient of Variation %	Maximum stress MN/m ²	Number of Tests	Geometric + Mean Endurance Cycles	Coefficient of variation of log endurance %
Saturated	0	10	4.21	7.8	3.16	10	70,660	14.3
					2.69	9	> 700,000	11.1
					2.53	11	> 1,480,000	11.2
					2.35	5	> 15,580,000	8.3
Air-dried for 1 week	0.73	5	3.31	4.8	3.22	6	421	24.7
					2.70	10	77,460	17.0
					2.53	13	> 153,530	19.2
Oven-dried for 1 week	5.43	9	6.35	7.7	5.64	10	2,340	16.9
					4.82	9	24,900	17.3
					4.69	10	> 239,200	21.1
Oven-dried then re- soaked for 3 weeks	0.78	10	5.24	4.5	3.68	2	1,350	23.7

* From saturated condition

+ > Signifies some specimens unbroken

All specimens cured for 26 weeks before test

TABLE 4
Fatigue Test Results - Effects of Rest Periods - Concrete PQ2

Loading Condition	Number of Tests	Geometric Mean Endurance Cycles	Coefficient of variation of log endurance %
Continuous cycling at 20 Hz	10	8,590	25.2
0.5 sec rest after each cycle	10	3,380	17.0
2.0 sec rest after each cycle	10	2,550	16.5

All specimens tested wet at a nominal maximum cyclic stress of
4.59 MN/m² after 13 weeks curing

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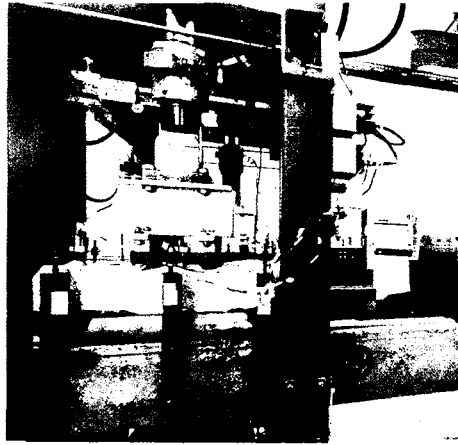


Fig.1— Flexural fatigue tests on concrete beams

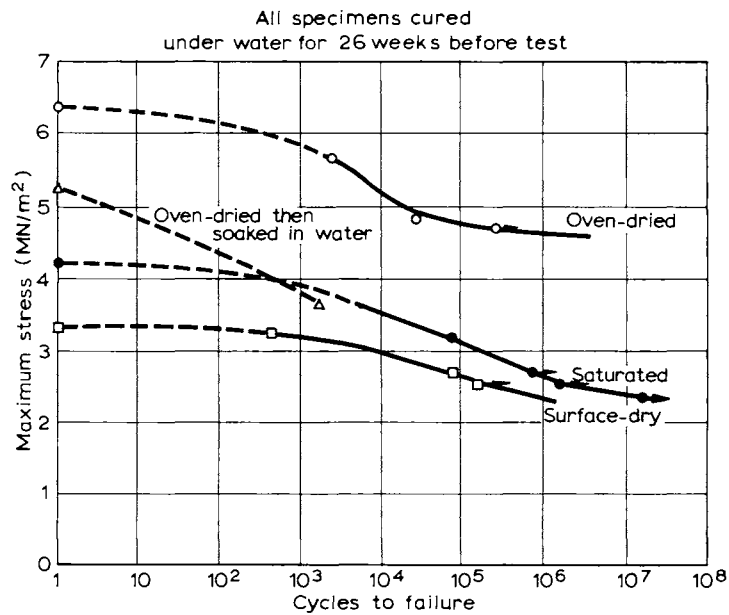


Fig.2— Effect of moisture condition on fatigue performance of concrete PQ1

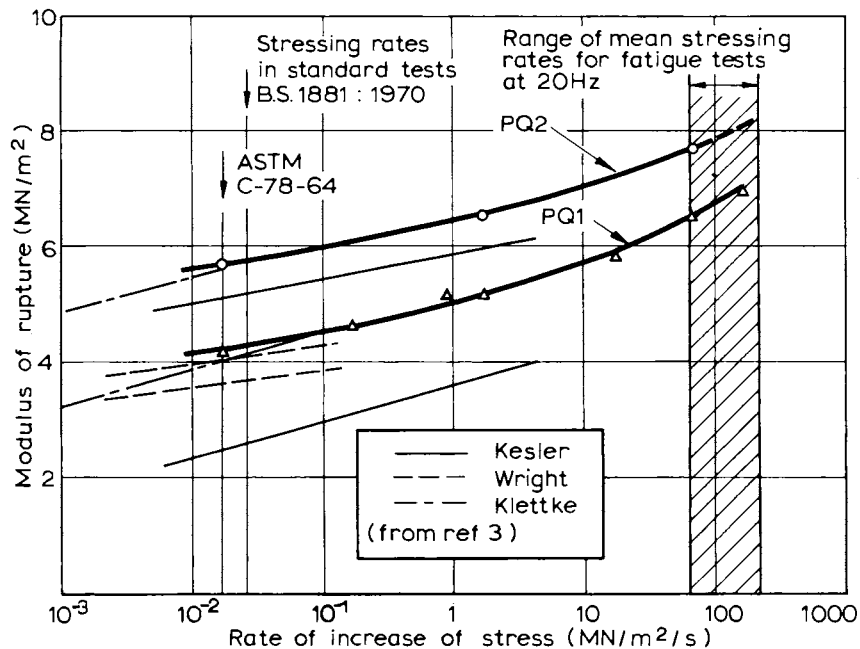


Fig.3 — Effect of rate of loading on flexural strength

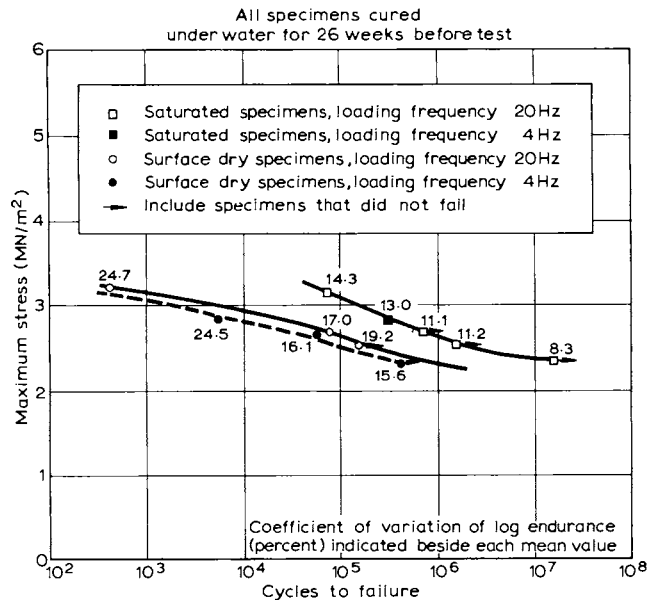


Fig. 4 — Effect of rate of loading on fatigue performance - concrete

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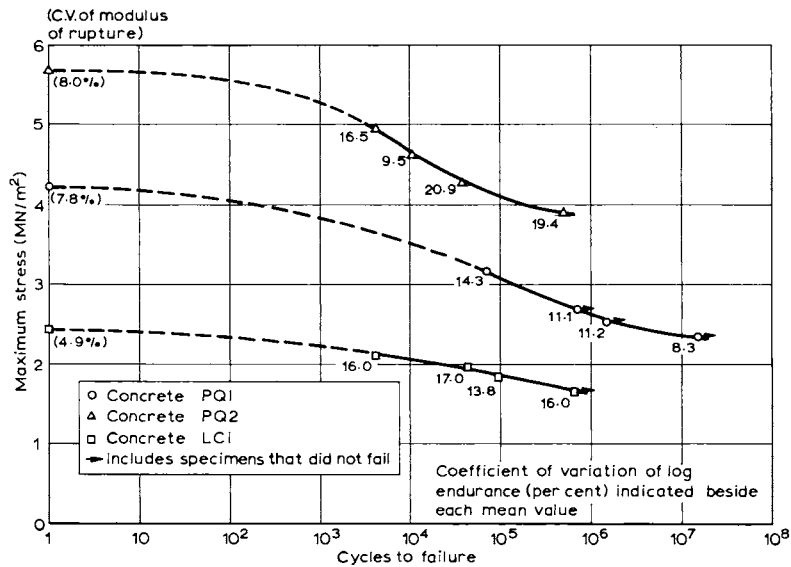


Fig.5 — Endurance curves for concretes PQ1, PQ2 and LC1 (saturated specimens cured for 26 weeks)

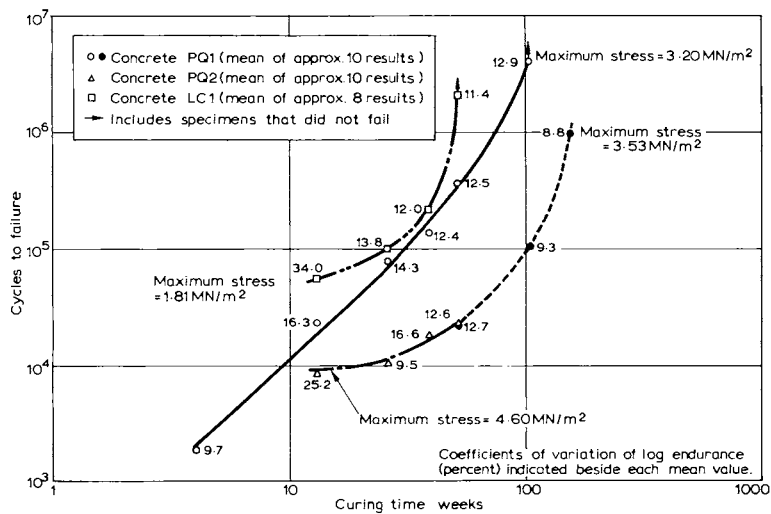


Fig.6 — Effect of age on fatigue endurance

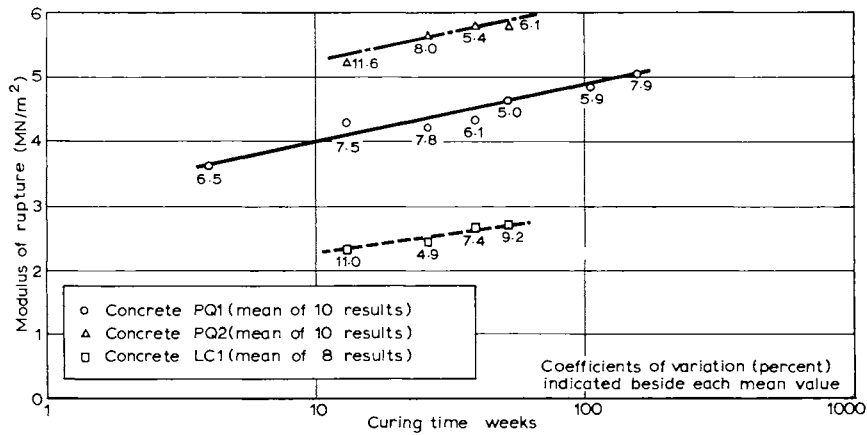


Fig. 7 — Effect of age on modulus of rupture (standard loading rate)

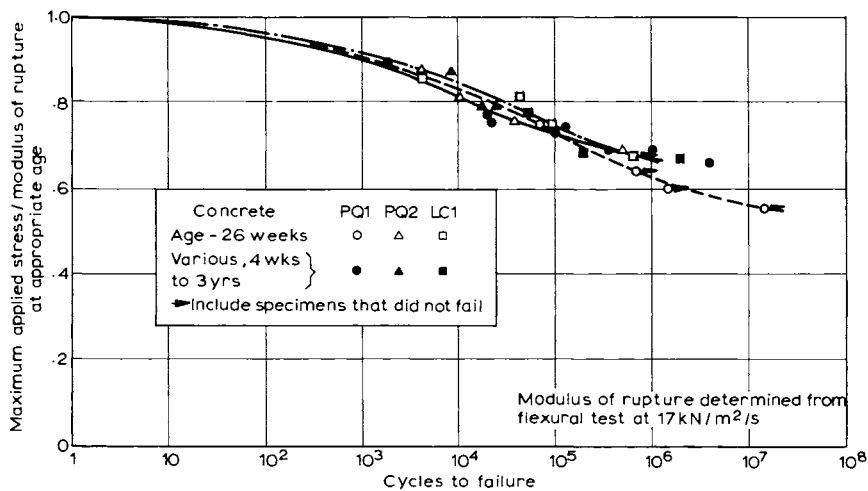


Fig. 8 — Fatigue performance related to modulus of rupture (saturated beams)

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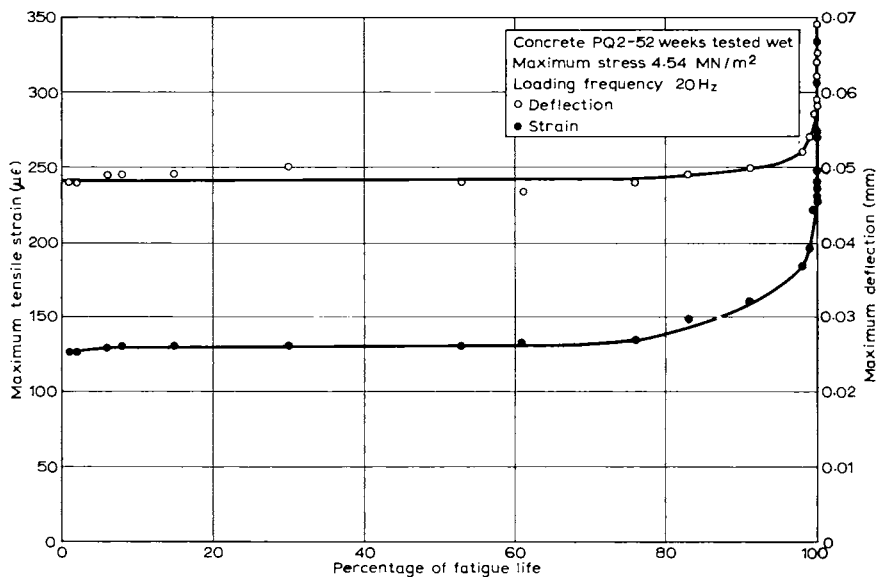


Fig.9—Variation of maximum tensile strain and central deflection with repeated loading

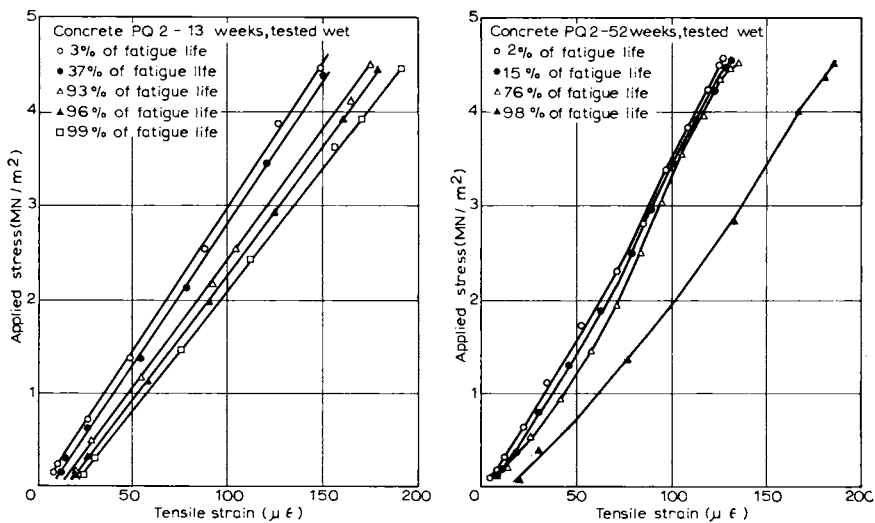


Fig.10—Dynamic stress-strain curves at 20 Hz

SP 41-3

Contribution To The Fatigue Strength of Reinforced Concrete

By
S. Soretz

Synopsis: The present report deals with the results of fatigue tests carried out on 249 reinforced-concrete beams reinforced with high-quality ribbed steel bars with a yield point of at least 40 kg/mm². The beams differed in size, shape of the cross-section and longitudinal section, type of reinforcement and type of loading. All these parameters seem to have a more or less strong influence on the fatigue strength of reinforced concrete, which in individual cases could already be proved by means of statistical methods. The beams used for the testing of the fatigue influence on reinforced concrete and reinforcing steel have to meet the rules established for the design of reinforced-concrete construction. The fatigue limit of the pulsating stress range decreases with an increasing lower limit of the pulsating stress. A well-rounded merging of the flanges of the lugs into the circumference of the core resulted in a higher fatigue strength than sharp-edged transitions.

Keywords: beams (supports); concrete construction; cracking (fracturing); fatigue (materials); fatigue tests; flexural strength; loads (forces); reinforced concrete; reinforcing steels; research; stresses.

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INTRODUCTION

The usually underreinforced reinforced-concrete beams fail when subjected to static bending by excessive steel elongations, which with a decreasing percentage of reinforcement may increase until tearing of the steel, but which under certain circumstances, e.g. with a weak bond, are not attained. Under fatigue load these beams fail due to fracture of the reinforcing bars. In the same way as the testing of the steel alone is not sufficient to determine the carrying capacity and the fracture behaviour of the reinforced-concrete beams reinforced with this steel, the testing of the fatigue strength of the steel in naked bars, too, does not seem sufficient to judge the fatigue strength of the reinforced-concrete beams reinforced with these bars.

For the testing of the fatigue strength of embedded reinforcing bars a beam was suggested in (1), which is shown here in Fig. 1b. For the extensive tests described in (2) additional beams were used, as shown here in Fig. 1c. For the tests described in (2) and (3), furthermore, rectangular beams were used, which had a cross-section of the 30 x 36 cm and were reinforced with one single bar No. 8 (ϕ 25 mm). In the cases mentioned above tin plate strips were inserted to initiate cracks in the area of the largest bending moments ranging from the tensile edge to and beyond the bar.

All these beams show deviations in various respects from the acknowledged rules established for the design of reinforced-concrete beams. They are:

- 1) One bar in a 30 cm wide beam leads to excessive crack widths at the level of the reinforcement, as is well known. For this reason at least 2 bars should always be placed in the corners.
- 2) One bar No. 8 in a 10 cm wide, short beam leads to an excessive stress of the bond when subjected to fatigue and results in a damage of the bond under working load conditions which can no longer be tolerated. Thus, the fatigue strength is decreased, as shown in (4).
- 3) The crack initiators have a stronger effect than natural cracks of the same width on the outer surface, because they extend up to the bar with the same width. The resulting crack width on the surface and even more on the bar itself is ten times the size generally permissible under working load. In addition to the artificial crack, furthermore, there is the excessive width of the natural crack mentioned under (1) which occurs under load close to the crack initiator. These

cracks, therefore, correspond to a state that is very close to failure. A fatigue stress, however, must occur only in the range of the permissible load, since millionfold overloads must not occur or will only occur in faulty designed structures.

- 4) Reinforced-concrete building unit subjected to fatigue in which all bars are bent with a relatively small radius at the site of the largest bending moment are not used. To prevent a crushing of the concrete in the curvature far bigger bending radii must be used, which, however, do not have any adverse effect on the fatigue strength, as was illustrated in (5).

For this reason a comprehensive investigation was started to determine the influence of the shape of the beam on the fatigue strength of reinforced concrete. The following is a summary report on the results obtained until now. The investigations were carried out upon commission of the TOR-ISTEG STEEL Corporation, Luxembourg, at the research institute for reinforcing steel and reinforced concrete of the SCHMIDTSTAHL-WERKE AG, Vienna, under the direction of the author.

TEST SPECIMENS AND PROCEDURE

- 1) This report is based on the results of 249 reinforced-concrete beams tested in fatigue tests. The beams (types A - F) used in these tests are shown in Fig. 1. With each type the number of tested beams (n) is indicated. In the case of beams A a crack initiator was inserted in midspan in the tensile zone and the compression zone was replaced by a steel articulation. Thus, the lever arm of the inner forces and also the stress in the bar to be tested are clearly indicated. All other beams were cast without a crack initiator.
- 2) The reinforcement of the beams consisted of ribbed steel with a diameter of 8 - 26 mm and a yield point of 40 - 65 kg/mm². The reinforcing steel was supplied by 12 steel mills from 6 European countries. Each mill furnished one or more lots. All steels were commercial steels. All bars had sickle-shaped lugs (see Fig. 2a). 96 reinforced-concrete beams were reinforced with hot-rolled ribbed steel (see Fig. 2b), 153 with cold-worked RIBBED-TORSTEEL (see Fig. 2c).

With beams A and B bending of the reinforcing bar around a mandrel 15 times the size of the bar diameter was carried out in such a way that the mandrel was lying on a lug during bending.

- 3) All beams were cast in steel moulds. The maximum size of the aggregate was adapted to the size of the beams, i.e. 32 mm for beams A, B and D and 16 mm for beams C, E and F. The concrete mix was adapted to a homogeneous strength so that after 28 days of curing the cube-crushing strength of the concrete corresponded to 320 kg/cm² on the average with a standard deviation of 38 kg/cm².
- 4) Beams A, B and C were tested in a fully automatic hydraulic pulsator with 440 load cycles/minute, in which up to 4 beams can be simultaneously tested. Beams D and F were individually tested in a fully automatic hydraulic pulsator with 280 load cycles/minute. Beams E were tested in 2 semi-automatic mechanical pulsators, in each of which up to 2 beams can be simultaneously tested with 280 or 440 load cycles/minute.

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- 5) All fatigue tests on reinforced-concrete beams were carried out as described below. With a first, pre-selected lower and upper load limit the test run until failure of the beam occurred after less than 2×10^6 load cycles or up to 2×10^6 load cycles were sustained. In the latter case the upper load limit or the pulsating stress range of the beam was increased by approximately 10 % and the procedure was carried out as above. This procedure was continued until failure of the beam occurred at a load stage after less than 2×10^6 load cycles. Each beam endured until failure an average of 2.15 load stages.
- 6) Of the two limits of the pulsating stress range one limit was kept constant for larger groups of beam tests as described in the following. The load limits are expressed here in terms of steel stresses, which were calculated with $E_s/E_c = 10$ from the applied loads and with due consideration of the dead weight for the zone of the highest bending moment. The only exception were beams A, as has been mentioned above.

With 123 beams the upper limit was kept constant at approximately 30 kg/mm^2 and the lower limit was decreased at each consecutive load stage.

With 91 beams the lower limit was kept constant at approximately 3 kg/mm^2 and the upper limit was increased at each consecutive load stage.

With 35 beams the lower limit was kept constant at approximately 20 kg/mm^2 and the upper limit was increased at each consecutive load stage.

- 7) Testing of the beams was carried out after 28 days of curing at the earliest. After failure all beams were destroyed in the area of the fracture and both halves of the fractured bar were removed to determine the cause and the origin of the fatigue failure in the bar.
- 8) One sample of all steel rods, from which bars for the test beams were taken, was subjected to a tensile test.

TEST RESULTS

- 1) In all reinforced-concrete beams cracks occurred already during the first load application. In all beams failure occurred in the bar or bars of the main reinforcement. The site of failure always coincided with a crack in the concrete. It was located between the 2 single loads or beyond them. In the latter case, however, the crack pointed at a load site, so that the same stress was present in the tested bar as under the point of or between the load.
- 2) Beams A and B deviate somewhat from this rule. Beams A always failed in the articulation. The failure originated from a generatrix located nearer to the compression edge of the beam, marked with an arrow in Fig. 1. The site of fracture of beams B was scattered over a wide range, as shown by the position of the small arrows in Fig. 3. The failure originating at the bottom of the bars of 51 % of the beams were beyond the bent area of the bars. The fractures starting from the top of the bars of 49 % of the beams were close to the end of the

inner side of the curvature or in the transition zone to the straight part of the bar. In the area of the curvature no crack or failure occurred.

- 3) The fractures of beams C and E started in 84 % of the cases at the bottom of the bars, close to the tensile edge of the beams and in 16 % of the cases at the top, close to the compression edge of the beams.
- 4) The fractures of beams D and F always started at a generatrix close to the tensile edge of the beams.
- 5) In 93.5 % of all beams the fracture originated at the groove of a lug or longitudinal rib and in the remaining 6.5 % of the beams from the circumference of the core. 4 % of the bars showed a rolling defect at the site of fracture and 1.6 % of the bars had a pipe there.
- 6) 2/3 of all C-beams showed a longitudinal crack in the concrete, which extends along the bar axis across the site of fracture.
- 7) In slabs F the site of fracture was under a point load, in the break of bending lines. It was always the two external bars that failed. In beams E with the same depth, however, failure always occurred between the two point loads in the area of the constant bending moment.

The limits of the pulsating stress ranges in the individual load stages and the load cycles endured as well as the strength characteristics of the steels used for the 249 beams are summarized in Table 2.

DISCUSSION OF THE TEST RESULTS

For the evaluation of the various influences the fatigue strength for 2×10^6 load cycles is used. In the individual tests fewer load cycles were reached until failure occurred in the last load stage. The fatigue strength of the individual beams for 2×10^6 load cycles was, therefore, calculated as follows. The highest pulsating stress range endured through 2×10^6 load cycles was increased in the next load stage by that percentage which resulted from the ratio between the number of actually endured load cycles and 2×10^6 load cycles. For beams fractured in the first load stage it was assumed that a 10 % lower pulsating stress would have been endured without failure through 2×10^6 load cycles; the same procedure was applied. The possible error in the assumption of the fatigue strength of the individual beams in this procedure is less than 1 kg/mm^2 and will probably not exceed 0.5 kg/mm^2 .

The test results prepared in this way were further evaluated in groups of related tests by means of statistical methods, which included: Arithmetic mean values (M); standard deviation (S); 5 % fractile (F5); 95 % area of reliability (V95) in which 95 % of all values are found with 95 % probability; testing of the reliability of the difference between mean values, the criterium being the threefold error of the difference; linear regression analyses.

The results of these calculations are compiled in Table 1 and below.

- 1) Beams A, B and C were tested with an upper limit of the pulsating stress of 30 kg/mm^2 on the average and are, therefore, directly comparable. The fatigue limit of the pulsating stress range is lowest

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for beams A with an articulation. The smallest single value amounted to approximately 13.5 kg/mm^2 . The sharp discontinuity in the range of the articulation leads to high notch stresses and secondary bending stresses which strongly reduce the fatigue limit of the pulsating stress range. The shape of these beams differs widely from the acknowledged rules established for the reinforced-concrete structures and is, therefore, not suited for the testing of the behaviour of bars in concrete reinforced in the usual way.

- 2) Beams B showed neither a crack nor a fracture in the range of the strongest curvature. There is no statistically significant difference between the beams in which failure occurred outside the curvature on the outer side of the bar and those beams in which failure occurred in the transition zone from the curvature to the straight part, at the inner side of the bar.

The fatigue limit of the pulsating stress range shows the largest scatter for these beams in this group. These beams are not suited for the testing of the fatigue limit of bent bars. Owing to the strong deviations from the acknowledged rules established for reinforced concrete these beams are not suited for the testing of straight bars either.

- 3) For pairs of beams A and B with a reinforcement taken from the same steel rod, the following results were obtained with a more than 99 % probability: The fatigue limit of beams B is on the average 2.3 kg/mm^2 or 14 % higher than for beams A.
- 4) Beams C with straight bars have the highest fatigue limit of the pulsating stress range of this group. The smallest single value can be estimated to amount to 17 kg/mm^2 .

For pairs of beams B and C with the reinforcement taken from the same steel rod no statistically significant difference in the fatigue limit was obtained. Owing to the smaller scatter of the tests with beams C the smallest single value is 1.7 kg/mm^2 or 11 % higher than that for beams B.

No statistically significant differences in the ranking of different steels on the basis of tests with beams A, B and C could be found.

- 5) Some of the beams D, E and F were tested with a lower limit of the pulsating stress of 3.4 kg/mm^2 on the average and are, therefore, directly comparable. No statistically significant difference between the 3 shapes of beams could be found. Further investigations are being carried out to determine whether it was only the number of tests that was not yet sufficient for a differentiation or whether beams corresponding to the acknowledged rules established for reinforced concrete really do not differ, i.e. whether they show the same behaviour.
- 6) A comparison of beams D, E and F with beams B and C yielded the following results:

For beams D, E and F the upper limit of the pulsating stress amounted to 31.7 kg/mm^2 on the average as compared to an average 30 kg/mm^2 of beams B and C. When neglecting this difference, beams D, E and F, which correspond to the acknowledged rules established for reinforced

concrete, on the average lead to a 7.6 kg/mm^2 or 37 % higher fatigue limit than beams B and C, which are far away from it. Beyond that, no significant difference in the ranking of different steels could be found with these 2 groups of beams.

- 7) Beams D, E and F furthermore permit an evaluation of the influence of the lower limit of the pulsating stress on the fatigue limit. The fatigue limit of the pulsating stress range endured over a lower limit of 3.4 kg/mm^2 is on the average 4.8 kg/mm^2 or 20 % higher than that over a lower limit of 20 kg/mm^2 , a fact which well corresponds to what is generally known about the fatigue strength.
- 8) 108 beams B, C, D, E and F, in which the fatigue failure started at the groove of a lug or longitudinal rib, were studied with regard to the influence of the merging of the rib flanks into the circumference of the core. Bars with an obviously well-rounded merger (see Fig. 4a) had a fatigue limit which was significantly higher by 4.3 kg/mm^2 or 20 % than the fatigue limit of bars with a sharp merger (see Fig. 4b). Owing to the rough surface of hot rolled bars it was not possible to measure the radius of the merger. It is often thought that the most favourable bond is achieved with rib flanks which run almost vertical to the bar axis and merge sharply into the circumference of the core. These bars, however, show the worst fatigue behaviour. For this reason it seems advisable to reconsider and to study once more the influence of the rounded mergers on the bond by taking into consideration the retrograde effects on the fatigue proven here. The rounding always recommended by us seems to continue to be the best compromise, since it helps to achieve a better bendability of the bars at the same time, which is very important for the processing of the bars (6).
- 9) Until now it was not possible to prove a statistically significant influence of the type of the melting of steels nor of their yield-points. The calculated correlation coefficients were always lower than 0.2. This may be partly due to the fact that 220 of the 249 beams were reinforced with ribbed steels with a yield-point of at least 40 kg/mm^2 and that the reinforcement of only beams consisted of ribbed steels with a guaranteed yield-point of 50 and 60 kg/mm^2 . Furthermore, it might be the case that the influence of the yield-point of the steels was covered by the influence of the shape of the groove of the lugs.

Besides, the observed rolling defects and pipes had no statistically significant influence on the fatigue limit of the beams.
- 10) Beams C partly showed longitudinal cracks. The statistical investigation showed that with a 99 % probability beams with longitudinal cracks have a 1 kg/mm^2 smaller fatigue limit on the average than beams without longitudinal cracks. This corresponds to the influence of the bond on the fatigue strength of reinforced concrete, illustrated under (4).
- 11) For the fatigue tests with beams A, B and C the fatigue strength was preselected as a constant upper limit of the load and the fatigue limit of the pulsating stress range was found for a lower limit of the pulsating stress variable from 2 to 16 kg/mm^2 . Owing to the decrease of the fatigue limit of the pulsating stress range with an

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increasing lower limit of the pulsating stress shown above, the already low values of the fatigue limit of the pulsating stress range are further reduced and the evaluation criterion is constantly changed. This type of testing is contradictory to the actual use of the steels in the reinforced concrete. The carrying structure is constantly under the permanent load, which in the test is replaced by the lower limit of the pulsating stress. It, therefore, constitutes a constant for the carrying structure. The pulsating stress based on it represents the additional stress through the live load on the carrying structure, and is adapted to the permanent load. The influence of the fatigue on the carrying capacity of reinforced concrete should, therefore, be investigated for a given lower limit of the load. The type of load used with beams A, B and C is not conclusive.

- 12) The range of scatter of the test results amounts to 6 - 13 kg/mm² and seems to increase with the number of tests and steel suppliers. To determine a significant lower limit of the fatigue strength or the fatigue limit of the pulsating stress range it is necessary, therefore, to carry out a large number of tests.

CONCLUSIONS

249 beams with 6 different shapes were tested with regard to their fatigue strength in 550 load stages. The results were evaluated by means of the fatigue limit of the pulsating stress range endured through 2×10^6 load cycles. From this the following conclusions were drawn:

- 1) Beams D, E and F correspond to the acknowledged rules established for reinforced concrete. For a lower limit of the pulsating stress of 3.4 kg/mm² the fatigue limit amounts to 28.3 kg/mm² on the average with a smallest single value of 23 kg/mm². These results confirm the results of tests on T-beams with 7.5 m span, which simulated a railway bridge for the heaviest class and for which a fatigue limit of 29 kg/mm² was found, as was reported under (5).
- 2) For the same beams the fatigue limit endured over a lower limit of the pulsating stress of 20 kg/mm² amounts to 23.5 kg/mm² on the average with a smallest single value of 18 kg/mm². This confirms the experience that the fatigue limit of the pulsating stress range decreases with an increasing lower limit of the pulsating stress.
- 3) Beams A, B and C with a shape differing widely from that admitted in the acknowledged rules for reinforced concrete show a considerably lower fatigue limit of the pulsating stress range. The fatigue limit values may decrease to 17.2 kg/mm² on the average and for the smallest single value to 13 kg/mm², i.e. by 40 % on the average. These shapes are, therefore, not suited for the testing of the fatigue influence on reinforced concrete or reinforcing steel.
- 4) Beams A and B with bent bars did not lead to a different ranking of the individual steels than beams with straight bars. If the bars are bent around a mandrel with a diameter that is at least 25 times that of the bar, in accordance with the rules established for reinforced concrete, this curvature has no adverse effect on the fatigue strength and fatigue limit, as was shown in (5). Sharper curvatures, e.g. around mandrels with 15 fold diameters, are unrealistic owing to the excessive stresses in the concrete in the curvature, and such tests are, therefore, no conclusive.

- 5) Well rounded mergers of the sides of the lugs into the circumference of the core, as shown with the bars tested in (3) lead to a fatigue limit which on the average is 4.3 kg/mm^2 or 20 % higher than in the case of sharp mergers.

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Table 1

Characteristics of Fatigue Limit in Reinforced Concrete Beam Tests

B e a m	A	B	C	D, E, F	
Bar Diameter mm	26	26	26	8, 12, 25, 26	
Number of Beams	22	53	48	91	35
min f_s ¹⁾	var ³⁾	var	var	3.4	2o
max f_s ²⁾	3o	3o	3o	var	var
Stress Range ⁴⁾					
V95	13.5	15.5	17.2	22.8	16.7
min	13.9	15.8	17.5	23.1	18.4
F5	14.3	16.3	17.9	23.8	17.9
average	17.2	2o.4	21.o	28.3	23.5
F5	2o.1	24.5	24.2	32.8	29.1
max	2o.o	27.9	25.9	35.8	29.3
V95	2o.9	25.3	24.9	33.8	3o.3
S	1.76	2.47	1.91	2.74	3.39
max-min	6.1	12.1	8.4	12.7	1o.9

¹⁾ Minimum Stress in a given Cycle kept constant

²⁾ Maximum Stress in a given Cycle kept constant

³⁾ Variable

⁴⁾ Computed Stress Range for 2×10^6 Cycles endured; all Stresses are Tensile Steel Stress in kg/mm^2 computed with $E_s/E_c = 1o$

Table 2

Steel: Type H = Hot-rolled

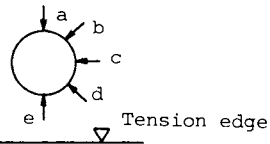
Type C = Cold-worked

 f_y = Yield Point or Proof Stress (kg/mm^2) $\min f_s$ = Minimum Stress (kg/mm^2) Δf_s = Stress Range (kg/mm^2)

N = Number of Load Cycles sustained

Failure Position:

"1" = Point of Failure at the Circumference of the Bar:



"2" = Origin of Failure: K = Groove of Oblique Rib

L = Groove of Longitudinal Rib

M = Surface of the Bar Core

W = Surface Crack at the Origin of Failure

"3" = Kind of Groove according to Fig. 4: R = Round

S = Sharp

- = Not observed

Beam Nr.	Steel		minf _y	Δf _s	10 ³ N	Failure Position		
	Type	f _y				1	2	3
Beam Shape A according to Fig. 1								
I	C	52.4	14.6	19.5	809	b	K	-
II	C	52.4	17.5	16.7	2000			
			14.6	19.5	955	b	L	-
1	C	46.2	16.2	15.3	2000			
2	C	46.2	13.5	18.0	1404	a	K	-
			16.2	15.3	2000			
			13.5	18.0	2000			
			11.7	19.8	205	b	K	-
3	C	47.4	16.2	15.3	1993	a	K	-
4	C	45.4	16.2	15.3	2000			
			13.5	18.0	2000			
			11.7	19.8	1271	a	K	-
5	C	44.5	10.1	18.9	767	a	K	-
6	C	45.8	10.1	18.9	912	a	K	-
7	C	46.6	14.1	14.5	2000			
			11.8	16.8	2000			
			10.0	18.6	2000			
			8.2	20.4	1095	b	K	-
8	C	45.7	14.1	14.5	2000			
			11.8	16.8	2000			
			10.0	18.6	2000			
			8.2	20.4	1535	a	K	-
9	C	46.1	10.0	18.6	1374	b	W	-
10	C	44.6	10.0	18.6	2000			
			8.2	20.4	652	b	K	-

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Table 2

Beam Nr.	Steel		minf _s	Δf _s	10 ³ N	Failure Position		
	Type	f _y				1	2	3
Beam Shape A according to Fig. 1								
11	H	43.7	10.1	18.9	581	a	K	-
12	H	43.9	10.1	18.9	436	a	K	-
13	H	40.6	12.1	17.2	619	a	K	-
14	H	40.2	11.9	17.0	874	a	K	-
15	H	43.9	14.5	14.9	1164	a	K	-
16	H	43.3	14.5	14.9	2000			
			12.2	17.2	2000			
			10.3	19.1	609	b	K	-
17	H	43.8	12.0	17.1	463	a	K	-
18	H	40.7	12.0	17.1	923	b	K	-
19	H	43.6	14.3	14.8	1768	b	K	-
20	H	44.0	14.3	14.8	807	b	K	-
Beam Shape B according to Fig. 1								
I	C	52.4	17.1	16.2	2000			
			14.3	19.0	661	e	W	-
II	C	52.4	17.5	16.5	2000			
			14.5	19.5	2279			
			12.6	21.4	2000			
			9.7	24.3	634	d	K	-
1	C	46.2	12.0	20.3	1316	d	K	-
3	C	47.4	16.6	15.7	2000			
			13.9	18.4	2000			
			12.0	20.3	2000			
			9.2	23.1	1305	b	K	-
4	C	45.4	17.1	16.2	2000			
			14.3	18.4	2000			
			12.4	20.9	8	a	K	-
5	C	44.5	10.4	19.4	2000			
			8.5	21.3	2000			
			6.6	23.2	52	a	K	-
6	C	45.8	10.7	20.0	2000			
			8.8	21.9	66	a	K	S
7	C	46.6	14.7	15.1	2000			
			12.3	17.5	2000			
			10.4	19.4	2000			
			8.5	21.3	2000			
			6.6	23.2	288	b	L	-
8	C	45.7	14.2	14.7	2000			
			11.9	17.0	2000			
			10.1	18.8	2000			
			8.3	20.6	2000			
			6.4	22.5	891	a	L	-
9	C	46.1	10.2	19.0	1847	a	W	-
10	C	44.6	10.2	19.0	2000			
			8.3	20.9	2000			
			6.5	22.7	609	a	W	-
11	H	43.7	10.0	18.7	1181	e	K	-

Table 2

Beam Nr.	Steel		minf _s	Δf _s	10 ³ N	Failure Position		
	Type	f _y				1	2	3
Beam Shape B according to Fig. 1								
12	H	43.9	10.0	18.6	2000	e	K	-
			8.1	20.5	2000			
			6.3	22.3	912			
13	H	40.6	11.9	17.0	2000	d	K	-
			10.1	18.8	72			
14	H	40.2	11.9	17.0	1087	e	K	-
15	H	43.9	14.0	14.9	2000			
16	H	43.3	12.1	17.2	2000	d	K	-
			10.2	19.1	2000			
			8.4	20.9	845			
			14.0	14.9	2000			
			12.1	17.2	2000			
17	H	43.8	10.2	19.1	2000	d	K	-
			8.4	20.9	669			
			12.8	18.2	1215			
18	H	40.7	12.4	17.6	1223	d	K	-
19	H	43.6	15.1	15.8	1946			
20	H	44.0	15.8	15.3	2000	b	K	-
			12.9	18.3	2000			
			10.9	20.3	2000			
			8.9	22.2	610			
			10.2	19.0	2000			
21	C	44.1	8.3	20.9	354	d	K	S
			10.9	17.7	1178			
22	C	44.5	10.9	17.7	1178	b	K	S
23	C	43.1	10.2	19.1	2000			
24	C	43.9	8.4	20.9	2000	b	K	R
			6.5	22.8	2000			
			4.2	25.0	464			
			10.0	18.6	1229			
			10.6	19.9	2000			
25	H	44.1	8.7	21.8	929	a	K	R
			10.5	19.6	2000			
26	H	44.0	8.6	21.5	2046	a	K	R
			10.6	19.9	2000			
27	H	43.8	8.7	21.8	1543	a	K	R
			8.9	16.6	2000			
28	H	47.0	7.3	18.2	2046	d	W	-
			5.7	19.8	1390			
			10.3	19.3	129			
29	C	44.7	10.4	19.4	2000	b	K	S
			8.5	21.3	715			
30	C	41.8	10.4	19.5	2000	a	K	R
			8.5	21.4	102			
31	C	44.3	10.7	20.0	2000	a	K	R
			8.8	21.9	429			
32	C	44.4	8.6	21.4	1283	b	K	R
33	H	43.9				a	K	R

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Table 2

Beam Nr.	Steel		minf _s	Δf _s	10 ³ N	Failure Position		
	Type	f _y				1	2	3
Beam Shape B according to Fig. 1								
34	H	45.2	8.4	21.1	2003	a	K	R
			6.6	22.9	2000			
			4.2	25.3	2000			
			2.3	27.2	2000			
35	H	43.4	2.3	29.1	709	e	K	R
			8.4	20.8	2003			
			6.5	22.7	2000			
36	H	44.0	4.2	25.0	1197	d	K	R
			5.1	12.9	2003			
			4.0	14.0	2000			
			2.6	15.4	2000			
37	C	45.3	1.4	16.6	1051	e	K	R
			10.4	19.4	1500			
38	C	44.1	10.4	19.4	2000	a	K	S
			8.5	21.3	1147			
39	C	45.3	10.2	19.1	2000	d	K	S
			8.4	20.9	2000			
			6.5	22.8	1116			
40	C	44.9	10.3	19.3	2000	d	K	R
			8.4	21.2	1738			
41	C	44.7	8.5	21.3	2000	d	K	S
			6.6	23.2	1225			
42	C	47.0	8.7	21.7	1608	e	K	R
43	C	44.7	8.5	21.1	1232	e	K	R
44	C	47.4	8.6	21.4	2000	a	W	R
			4.3	25.7	18			
			6.4	22.4	671			
			6.3	22.1	2000			
45	H	45.7	4.1	24.3	2000	d	K	R
			2.3	26.1	2000			
			2.3	27.9	497			
			6.6	23.2	1178			
46	H	43.7	6.7	23.5	1638	a	K	R
47	H	43.7	6.7	23.5	1638	e	K	R
			10.2	19.1	2000			
			8.4	20.9	157			
48	C	44.0	10.2	19.0	2000	d	K	S
			8.3	20.9	936			
			e	K	R			
49	C	42.5	10.2	19.0	2000	e	K	R
			8.3	20.9	936			
			10.2	19.0	2000			
			8.3	20.9	2000			
50	C	45.6	6.5	22.7	2000	d	K	R
			4.2	25.0	755			
			10.2	19.0	2000			
			8.3	20.9	2000			
51	C	42.1	4.2	25.0	755	a	K	S
			10.3	19.3	2000			
			8.5	21.1	530			
Beam Shape C according to Fig. 1								
1	C	47.1	6.8	22.2	1878	e	K	S
2	C	46.0	6.7	22.1	2000	-	-	-
			2.9	25.9	1736			

Table 2

Beam Nr.	Steel		min f_s	Δf_s	10 ³ N	Failure Position		
	Type	f_y				1	2	3
Beam Shape C according to Fig. 1								
3	C	47.1	6.8	22.2	978	d	K	S
4	C	46.0	6.8	22.2	669	d	K	R
5	H	44.0	6.9	22.6	599	b	K	R
6	H	44.0	7.0	22.8	821	e	K	S
7	H	40.3	7.0	22.8	733	e	K	S
8	H	40.3	6.9	22.6	794	e	K	R
9	C	48.8	8.8	20.4	1419	a	K	S
10	C	47.4	8.7	20.3	1806	a	K	S
11	C	47.4	8.7	20.3	2000			
			5.8	23.2	1993			
			2.9	26.1	178	e	K	-
12	C	48.8	8.8	20.4	2000			
			5.9	23.3	1828	e	K	-
13	H	42.9	8.9	20.9	1956	e	K	-
14	H	42.9	9.0	21.0	655	e	K	-
15	H	40.5	8.9	20.6	1289	e	K	-
16	H	40.5	8.6	20.0	819	e	K	-
17	C	47.2	8.9	20.6	1661	e	K	-
18	C	46.4	8.9	20.6	1198	e	K	-
19	C	47.2	8.9	20.9	2000			
			6.0	23.8	1383	e	K	-
20	C	46.4	8.8	20.4	2000			
			5.9	23.3	463	e	K	-
21	H	41.3	11.7	17.5	2000			
			8.8	20.4	750	d	K	-
22	H	43.7	11.8	17.7	2000			
			8.9	20.6	709	a	K	-
23	H	41.3	11.8	17.7	2000			
			8.9	20.6	912	d	K	-
24	H	43.7	11.7	17.5	2000			
			8.8	20.4	1769	b	K	-
25	C	44.1	9.0	21.0	974	e	K	S
26	C	44.5	8.7	20.3	666	d	K	R
27	C	43.1	9.0	21.0	2000			
			6.0	24.0	1212	e	K	R
28	C	43.9	8.8	20.4	529	b	K	S
29	H	44.1	8.9	20.6	2000			
			5.9	23.6	1453	d	K	R
30	H	44.0	8.9	20.6	1228	d	K	R
31	H	43.8	8.9	20.9	1093	d	K	R
32	H	47.0	8.6	20.0	2000			
			5.7	22.9	505	e	K	R
33	C	44.7	11.8	17.7	2000			
			8.9	20.6	292	e	K	S
34	C	41.8	12.0	18.0	2000			
			9.0	21.0	2000			
			6.0	24.0	23	d	K	R

50 fatigue of concrete

Table 2

Beam Nr.	Steel		minf _s	Δf _s	10 ³ N	Failure Position		
	Type	f _y				1	2	3
Beam Shape C according to Fig. 1								
35	C	44.3	11.9	17.9	1503	d	K	S
36	C	44.4	11.6	17.4	2000			
			9,7	20,3	630	d	K	S
37	H	45.2	11.7	17.5	2000			
			8.8	20.4	2000			
			5.9	23.3	2000			
			2.9	26.3	421	e	L	R
38	H	45.2	11,7	17.5	2000			
			8.8	20.4	2000			
			5.9	23.3	1410	e	K	R
39	H	43.4	11.4	17.2	2000			
			8,6	20,0	2000			
			5.7	22.9	2000			
			2.9	25.7	1058	d	K	R
40	H	44,0	11.7	17,5	2000			
			8.8	20.4	2000			
			5.9	23.3	596	a	K	R
41	C	44,0	8,7	20.3	758	d	K	S
42	C	42.5	8.5	19.7	2000			
			5.7	22.5	552	e	K	R
43	C	45.6	8,7	20.3	1546	e	K	S
44	C	42,1	8,7	20.3	2000			
			5.8	23,2	885	d	K	R
45	H	45.7	8,6	20.2	2000			
			5.8	23,0	2000			
			2.9	25.9	533	e	K	R
46	H	45.3	8,5	19,7	2000			
			5.7	22.5	248	e	K	R
47	H	43.7	8.8	20.4	2000			
			5.9	23.3	1926	d	K	R
48	H	43,7	8,7	20.3	2000			
			5.8	23.2	2000			
			2.9	26.1	1171	d	K	R
Beam Shape D according to Fig. 1								
95	C	40,9	3,7	28.9	2128			
			3.7	32.1	342	-	M	-
96	C	40.9	3,7	28.9	1690	-	K	-
97	C	42.8	3,7	29.4	2000			
			3.7	32.6	1793			
			3.7	39.1	888	-	W	-
98	C	42.8	3,7	29.4	2546			
			3.7	32.6	659	-	W	-
99	C	41.9	3.8	30.2	2020			
			4.5	32.9	1662			
			8.2	35.8	9	-	W	-
100	C	41.9	3.8	30.2	1295	-	W	-
101	C	43.8	3.8	29.8	1798	-	L	S

Table 2

Beam Nr.	Steel		minf _s	Δf _s	10 ³ N	Failure Position		
	Type	f _y				1	2	3
Beam Shape D according to Fig. 1								
102	C	43.8	3.8	29.8	2000			
			3.8	33.1	2002			
			3.8	39.0	930	-	K	-
103	C	44.5	3.8	29.8	1980			
			3.8	33.1	1503			
			3.8	39.0	724	-	K	-
104	C	44.5	3.8	29.8	2059			
			3.8	33.1	430	-	K	-
111	C	53.8	21.0	17.0	1990			
			21.0	21.0	2010			
			21.0	25.5	1680	-	-	-
112	C	53.8	21.0	17.0	2000			
			21.0	21.0	1990			
			21.0	25.5	1740	-	-	-
113	C	50.0	21.0	17.0	2000			
			21.0	21.0	2000			
			21.0	25.5	810	-	-	-
114	C	50.0	21.0	17.0	2000			
			21.0	21.0	1670	-	-	-
135	C	54.1	3.2	28.8	1191	b	W	-
136	C	54.1	3.2	28.8	1376	c	W	-
137	C	54.4	3.2	28.8	2000			
			3.2	35.2	1367	d	K	-
138	C	54.4	1.9	30.1	1468	a	K	-
Beam Shape E according to Fig. 1								
1	C	52.4	20.0	20.0	2000			
			20.0	24.0	1200	e	K	-
2	C	52.6	20.0	20.0	2000			
			20.0	24.0	2150			
			20.0	28.0	200	e	K	-
3	C	53.2	20.0	20.0	2140			
			20.0	24.0	80	e	K	-
4	C	51.0	20.0	20.0	2210			
			20.0	24.0	2160			
			20.0	28.0	770	e	K	-
21	C	54.0	2.9	26.2	2000			
			2.9	29.2	2060			
			2.9	32.1	110	-	L	-
22	C	53.0	3.0	27.3	2010			
			3.0	30.4	450	-	W	-
23	C	52.6	3.1	30.5	1200	-	L	-
24	C	53.3	3.1	27.6	2000			
			3.1	30.7	630	-	K	-
51	C	50.0	3.1	27.4	601	d	K	-
52	C	49.9	3.2	28.2	601	-	-	-
53	C	49.9	3.2	25.9	993	e	K	-
54	C	48.7	3.2	25.4	993	e	K	-