

SP 42-1

Shear Transfer By Aggregate Interlock

By

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Synopsis : In a test program, comprising 44 specimens, the nature of shear transfer across prepared cracks by aggregate interlock mechanism was further explored. Within a narrow range aggregate shapes and sizes had no discernible effect upon the mechanism. Shear displacements under monotonic and cyclic loading were found to be approximately proportional to the preselected width of preformed cracks. Thus crack widths affected predominantly the stiffness of aggregate interlock mechanism. The restraining stresses required to prevent the unrestricted opening of a crack, while interface shear stresses were being transferred, were found to be independent of aggregate shape or size and of selected crack widths. The shear stress - restraining stress relationship₂ was approximately linear up to a shear stress of 1000 psi (70 kgf/cm²) and this corresponded with a mean coefficient of friction of 1.7.

Keywords: aggregate interlock; aggregate shape and texture; beams (supports); concretes; crack width and spacing; cracking (fracturing) cyclic loads; joints (junctions); shear stress; stiffness; stress transfer; stresses; tests.

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INTRODUCTION

When a crack is developed in a concrete mass the surfaces of the crack are usually rough and irregular. The majority of the coarse aggregate particles remain embedded in one or the other of the crack faces. When this crack forms along a continuous plane a parallel displacement in this plane is possible and, therefore, projecting particles from one face of the crack come into contact with the matrix of the other face. Further movement is then restricted by the bearing and friction of the aggregate particles on the crack surface. Provided that restraint is available, to prevent large increases in the crack width, substantial shear forces can be transmitted across the crack interface. This is aggregate interlock action. It is usually defined in terms of the average shear stress, the shear displacement in the plane of the interface and the width of the crack.

In a number of situations aggregate interlock has only recently been recognised as a viable shear transfer mechanism. Contraction joints in concrete pavements were probably the first ones in which aggregate interlock in combination with dowel bars were relied upon for interface shear load. (1, 2) Cracks can also easily form at the interface of precast and cast in situ components where monolithic action is expected. Direct shear load must be considered in such a situation. (3, 4, 5, 6) Similar conditions can exist at construction joints in cast in situ concrete, in particular in horizontal construction joints across shear walls which have to resist large lateral forces. (7) Shear displacement along diagonal tension cracks, caused by shear forces in reinforced concrete beams, have also been observed. The resulting interface shear transfer by aggregate interlock has been found (8, 9) to account for a major share of the shear resistance in beams without web reinforcement. Design rules for shear friction, largely based on the work of Mast (10) and Mattock (11) have now been incorporated in the ACI Building Code (12).

The scope of this study was to continue the investigation into the fundamental aspects of shear transfer by aggregate interlock.

A total of 44 specimens were tested to examine the effect of aggregate shape, size and the crack width, using monotonic and cyclic static loading.

TEST SPECIMENS AND LOADING

The dimensions of the test specimen, similar to that used by Mattock et al., (11) are shown in Fig.1. The contact area, shown shaded, over which the shear stresses were transmitted was 33.7 in^2 (217 cm^2). Fig.2 shows the formwork used for the simultaneous casting of two specimens. Some reinforcement was provided to control possible cracking away from the test surface. The bolt inserts, seen in Fig.2, were used to introduce a controlled tension force at right angles to the shear plane. They did not pass through the shear plane but attached the two concrete blocks to the stiff frame surrounding the specimen.

The load, acting across the shear plane, was introduced to one half of the specimen which was free to move vertically, while the other half was rigidly mounted in the test frame. The mounting arrangement for the moving part was such that the crack width could be controlled within 2% of the required value. Shear displacements and crack widths were measured at both faces of the specimen with demountable mechanical strain gauges. These gave the change of distance between stainless steel locating discs attached to the concrete surface at 2 in (5.08 cm) centres. Details of the test rig and test procedure are reported elsewhere (14),

Ordinary Portland cement and good quality alluvial gravel and sand from the Canterbury district of New Zealand were used for making the concrete, which had a four inch (10cm) cube strength of approximately 5300 psi (370 kgf/cm²) at the time of testing. Only three types of coarse aggregate were used, 3/8 in (9.5mm) and 3/4 in (19mm) round maximum size and 3/4 in (19mm) crushed maximum size. To speed up testing sometimes calcium chloride was added in making the concrete so that testing could be carried out 48 hours after casting. The more important properties of the specimens are summarised in Table 1. The load was introduced in approximately 20 equal increments and a test was terminated when the specimen could carry no further load or when shear displacements could no longer be measured or when the crack width exceeded the desired value by 50%. During the test crack widths were adjusted, before a load increment, but only when the deviation from the specified value exceeded 5%.

The specified nominal crack widths used in the tests were 0.005 in, 0.01 in or 0.02 in (0.13, 0.25 and 0.51 mm). In a few specimens the width of the preformed crack was varied, the applied shear stress to crack width ratio being kept constant.

TESTS WITH CONSTANT CRACK WIDTH

In 27 tests the load was monotonically increased till failure or the termination of the test because of other reasons. For three types of aggregate and three different crack widths, the load displacement relationships were established. Typical observations are shown in Fig.3, which records the response of three specimens with 3/4 inch

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(19 mm) round maximum size aggregate for three different crack widths.

The first significant trend which emerged from the testing was the effect of sedimentation and water gain under the coarse aggregate particles, situated in the top of the concrete, when the specimen was cast, see Fig.2. The shear displacement readings were significantly and consistently larger at the "top" side of the specimen when compared with measurements on the "bottom" side. This is clearly evident in Fig.3, where the shear stress - shear displacement relationship has been approximated by a bilinear one.

From a comparison of all responses it becomes evident that the aggregate size or shape used in this study had no noticeable effect. Specimens with 3/8 in (9.5 mm) maximum size aggregate failed suddenly when the shear stress reached approximately 1100 psi (80 kgf/cm²) as opposed to a more gradual failure when 3/4 in (19 mm) size aggregate was used. Crack widths could not be controlled when the load was in excess of 1000 psi (70 kgf/cm²). Specimens with small crack widths usually failed more suddenly than those with 0.02 in (0.51 mm) crack width.

The trends shown in Fig.3 are representative of all other tests. Up to a shear stress of approximately 300 psi (21 kgf/cm²) the stiffness of the specimens increased to a maximum value which was then maintained to shear stress level of approximately 900 psi (63 kgf/cm²). Further loading caused a degradation in stiffness.

A bilinear regression analysis was used to express the two phase behaviour of all specimens tested. This is shown in Fig.4. The shaded area includes 75% of all the individual results. The limits are expressed as a percentage of the mean shear stress. The change of stiffness at low load intensity is due to the "initial looseness of fit". (2) A certain displacement must occur before the aggregate particles, projecting across the crack, can firmly bear against the matrix of the opposite face. Once this bearing is established, a linear elastic response, represented by the steeper line, was encountered. The origin of the first line does not coincide with the origin of coordinate system because first a "no load displacement" must occur. (13) After the formation of a tension crack, particles are not in contact with each other and therefore only a very small shear load is required to establish the first contact and hence resistance against further shear displacement is small. The larger the initial crack width, the larger the "no load shear displacement".

CYCLIC LOAD TESTS

In most structural elements the load is of a fluctuating nature. Therefore it is relevant that the response of the aggregate interlock mechanism under cyclic loading without reversals should also be examined.

The fatigue strength of interface shear transfer in concrete pavements has been studied. (12) The effectiveness of shear transfer for low intensity, approximately 30 to 40 psi (2 to 3 kgf/cm²), loading of up to a million cycles was found to be drastically affected by the width of the crack and to a lesser degree by the size, shape

and hardness of the aggregates.

In this study, load intensities more likely to be encountered during severe seismic disturbances were applied to the specimens. Therefore shear stresses a little over 800 psi (56 kgf/cm²) were applied 33 times with time intervals after certain cycles to allow detailed measurements to be made. Concrete with 3/4 in (19 mm) or 3/8 in (9.5 mm) maximum round aggregate size was used in nine tests in which only the crack width was varied, as in the previous tests. Three to four hours were required to complete the testing of one specimen.

At no stage was a sudden breakdown of aggregate interlock action observed. The failure was similar to that encountered in the tests with monotonic loading. However, as could be expected, there was an accumulation of residual shear displacements with progressing loading. These increased proportionally with the crack width. In specimens with 0.02 in (0.51 mm) crack width, the testing became impractical after the 17th cycle. In each test the load in the last cycle was increased till failure occurred as in the previous tests with monotonic loading.

Crack surfaces after the testing were, as expected, heavily striated. A large quantity of crushed matrix and smaller aggregate particles could be seen. The surface irregularities were worn down and edges of aggregate indentations were rounded off. Again no difference in behaviour of the specimens with different coarse aggregate sizes were observed.

For a number of fully instrumented load cycles the bilinear shear stress - shear displacement relationship was determined as described earlier. Where more than one test was carried out for a given constant crack width, these bilinear responses were averaged. This enabled at least a qualitative comparison of the responses to cyclic loading to be made for the three selected crack widths. The results, which are self-explanatory, are presented in Fig.5, Fig.6 and Fig.7. Between the 2nd and 17th cycles and similarly between the 18th and 32nd cycles, no shear displacements were measured for individual load cycles.

As the number of load cycles increased the lower portions of curves become less linear. The predominant feature of these bilinear curves is the increased stiffness of the mechanism as cyclic loading progressed. From a comparison of the three responses it is evident that, with respect to the first loading, 210%, 280% and 320% increase of stiffness occurred in specimens with a 0.005 in, 0.010 in and 0.020 in crack width respectively. When comparing the residual shear displacements for the three cases, shown in Fig.5, Fig.6 and Fig.7, the different horizontal scales used in each case should be noted.

The loading, 830 psi (59 kgf/cm²), was extremely severe. For the concrete strength used shear stresses of this magnitude, approximately $13\sqrt{f'_c}$ psi, are not likely to be encountered because the structure would probably show distress due to other causes. Moreover in real structures the constant crack width could not be

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maintained over such a large range of shear stresses. It is also to be noted that the load was applied in one direction only.

There are situations where the cyclic loading may be associated with load reversals. It is likely that a very much larger strength degradation and accumulation of residual shear displacements would occur. In tests of construction joints, in which the cracks of the shear plane were neither preformed nor controlled with respect to width, such phenomena were observed. (7)

TESTS WITH VARIABLE CRACK WIDTH

Tests with monotonic or cyclic loading and constant crack widths provide useful information with respect to behaviour but they do not represent well situations encountered in practice. It is more likely that as the load intensity increases on the aggregate interlock mechanism the crack width will also increase. Indeed, in beams the cracks increase as the tensile steel stresses increase. It was, therefore, decided to conduct a few tests in which the ratio of load to crack width was to be kept constant at the rate of 100 psi/0.002 in (13.8 kgf/cm² / 0.1 mm).

In these tests the load was increased at 50 psi (3.5 kgf/cm²) increments, but the crack widths were adjusted only at every second increment because a crack, set before an increment, always increased only slightly during the application of the incremental load. At higher loads little adjustment was required as the crack width increased, of its own accord, approximately at the desired rate.

The majority of specimens failed when a sudden increase of crack width and a breakdown of aggregate interlock shear transfer, as manifested by a large increase in shear displacement, was observed. An examination of the crack surface showed more damage than in specimens with constant crack widths. A larger amount of ground material from the matrix and dislodged small aggregate particles were found. In spite of difficulties encountered with the adjustment of crack widths, while the load was maintained, there was little observed scatter in the shear stress - shear displacement relationships both individually and between tests.

Fig.8 shows the result obtained from a regression analysis carried out for all but one of the specimens. The limits enclose 75% of all test results. A comparison of individual tests showed that neither strength variation nor aggregate size used had a discernible effect upon the response. Upon this curve the results from the previous "constant crack width tests" (Fig.4) have also been transposed to enable a comparison to be made. The heavy broken line connects the appropriate stress values for the three distinct crack widths used in the tests. It reveals the same form as the relationship obtained from the "variable crack width tests". The latter, however, show considerably larger shear displacements at any given shear stress level. It is believed that this is mainly due to the test procedure and in particular the technique of crack width adjustments during the tests, which could not be controlled with complete satisfaction. The tests

demonstrate once more the drastic loss of stiffness in interface shear transfer when the crack width is allowed to increase.

TESTS WITH RESTRAINING FORCES

From the first series of tests and the work of others (11) it became evident that considerable restraining forces, acting at right angles to the shear plane, are required to maintain a crack width at its chosen magnitude, while interface shear load is being applied. To measure this, a load cell was designed and built into the modified test rig and a new series of tests was carried out. The system was not stable enough to carry out tests with cyclic load or variable crack width and so monotonic load was applied to these restrained specimens, while the crack width was held constant.

As in previous tests the width of the cracks tended to increase with increased load and so after every two or three load increments adjustments had to be made to reduce the crack width to the specified magnitude. The restraining force did not change during small load increment, but it increased suddenly whenever crack width adjustments were made. Typical results are shown in Fig.9. It is seen that for an approximately constant restraining force several different shear stress intensities were obtained. For all individual specimens and for selected groups of specimens a logarithmic regression analysis was performed. (14) Results of this are also shown in Fig.9.

From a comparison of different groups of specimens in which either the aggregate size (3/4 in or 3/8 in) or the crack width (0.005, 0.010 and 0.020 in) was kept constant, it became evident that none of these parameters has a distinct effect upon the restraining force required to maintain a given shear stress transfer by aggregate interlock. This is demonstrated by Fig.10, which shows the curves, obtained by regression analyses, for each group and the magnitude of the particular parameter which was the same for all specimens in that group.

These results are similar to those obtained by Mattock et al. (11), but a direct comparison cannot be made. In the latter tests the restraining force was provided by reinforcement which crossed the preformed crack. The width of the crack was not controlled, but it was permitted to increase till yielding in the reinforcement set in or failure of the interlock mechanism occurred.

It may be seen in Fig.10 that for design purposes the mean shear stress - restraining stress relationship could be approximated by a straight line corresponding with a coefficient of friction of $\mu = 1.7$. When the restraining force is provided by reinforcement the ACI Building Code (12) recommends $\mu = 1.4$. It is to be noted, however, that in the latter case shear forces can also be transferred by the dowelling action of the reinforcement.

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CONCLUSIONS

From an experimental study of the nature of shear transfer by aggregate interlock action, the following observations emerged :

1. The shear stress - shear displacement relationship does not seem to be affected by either the size or the shape of the coarse aggregate particles used in making the concrete. It should be noted, however, that the maximum aggregate size varied only between the narrow limits of $3/8$ in (9.5 mm) and $3/4$ in (19 mm).
2. The response of coarse aggregate particles is appreciably influenced by their position relative to casting of the concrete. Large shear displacements occur when the embedment of particles in the top of the concrete is weakened by water gain and sedimentation.
3. The largest single factor affecting shear displacement is the width of a crack across which shear stresses are to be transferred. Within the ranges considered, the shear displacement was found to be approximately proportional to the selected crack width.
4. For a constant crack width the response could be satisfactorily represented by a bilinear relationship. Initial looseness of fit of aggregate particles accounts for the relatively small stiffness of the mechanism at low loads.
5. The magnitude of the constant crack width was found to dominate the response of test specimens also under cyclic loading of relatively high intensity. The accumulation of residual shear displacement after each load cycle was proportional to the preselected crack width. The stiffness of aggregate interlock mechanism during repetitive loading was two to three times the stiffness observed during first loading.
6. Tests, in which the crack width increased proportionally with the load, verified that the stiffness of the mechanism gradually decreased as the shear stress across the interface increased. This is the situation most likely to be encountered in structures.
7. The restraining stress required to prevent the opening of a preformed crack, while interface shear is being transferred, was found to be independent of aggregate shape and size and also of crack width.
8. The mean ratio of shear stress to restraining stress, i.e. the coefficient of friction in these tests, was 1.7 .

ACKNOWLEDGMENTS

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TABLE 1 - PROPERTIES OF TEST SPECIMENS

Type and Size of Aggregate	Crack Width		Number * of tests	Average Cube ** Strength	
	inch	mm		psi	kgf/cm ²
3/8 R	0.005	0.13	1	5290	372
"	0.010	0.25	4 (1)	5340	375
"	0.020	0.51	1	5290	372
3/4 R	0.005	0.13	4 (1)	5070	356
"	0.010	0.25	9 (4)	5200	366
"	0.020	0.51	7 (3)	5220	367
3/4 CR	0.005	0.13	2	5600	394
"	0.010	0.25	5	5630	396
"	0.020	0.51	3	5530	389
3/4 R	variable		7	5410	380
3/8 R	variable		1	5450	383

R = Round Aggregate

CR = Crushed Aggregate

* The numbers in brackets indicate cyclic loading tests.

** Determined on three 4 in (10cm) cubes.

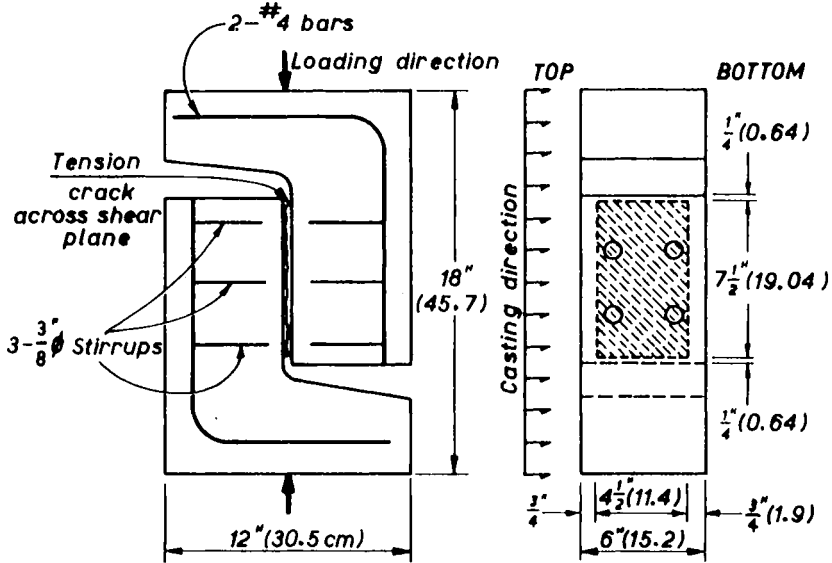


Fig. 1 - Dimensions and reinforcement of test specimens

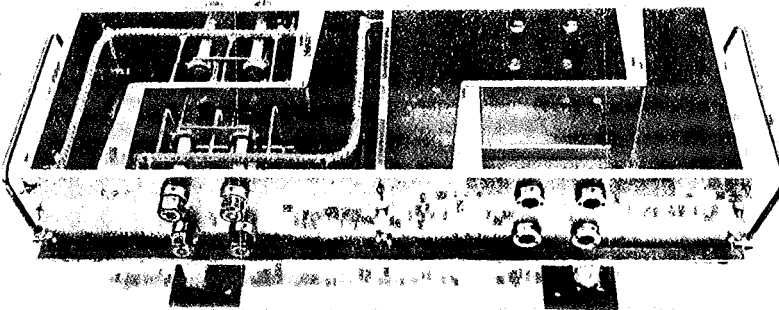


Fig. 2 - Formwork, reinforcement and tension attachments of test specimens

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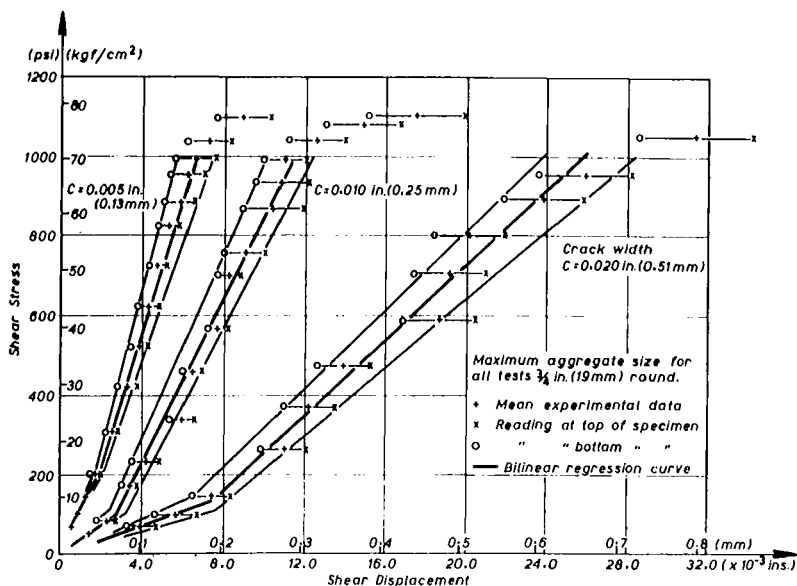


Fig. 3 - Typical shear stress - shear displacement relationships for different crack widths

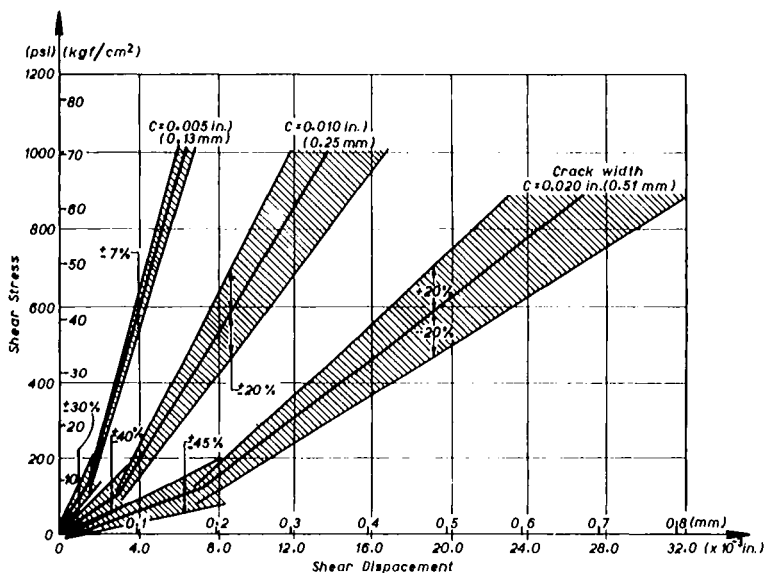


Fig. 4 - The mean shear stress - shear displacement relationships for aggregate interlock action

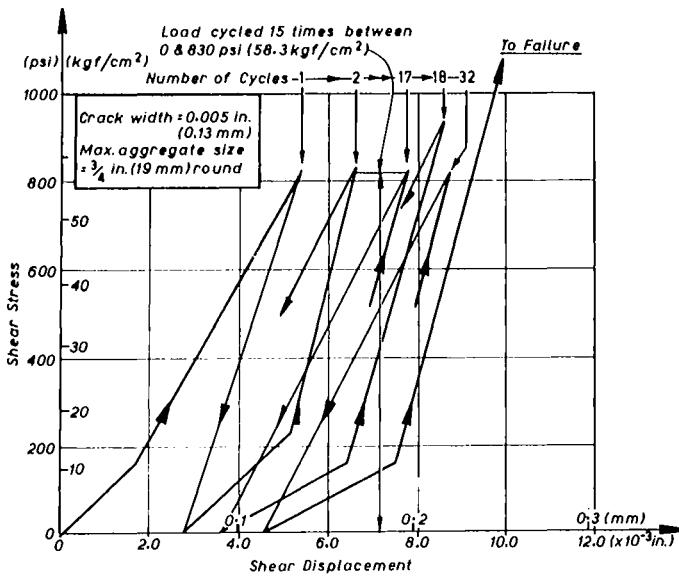


Fig. 5 - Response to cyclic loading with small crack width

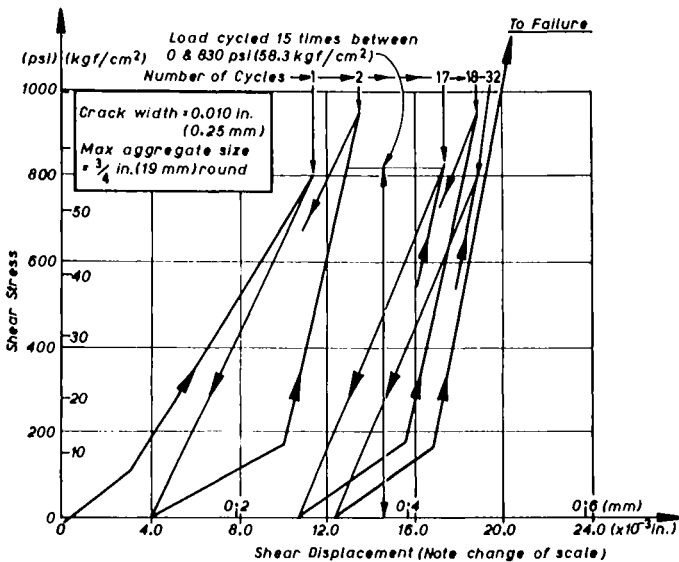


Fig. 6 - Response to cyclic loading with medium crack width

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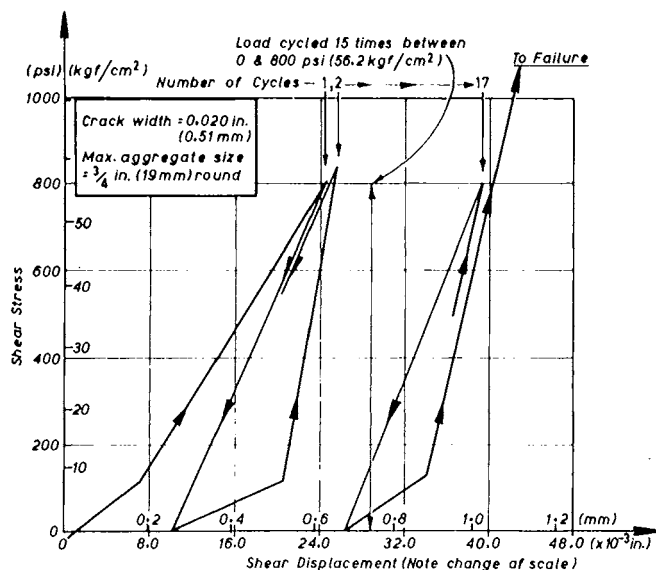


Fig. 7 - Response to cyclic loading with large crack width

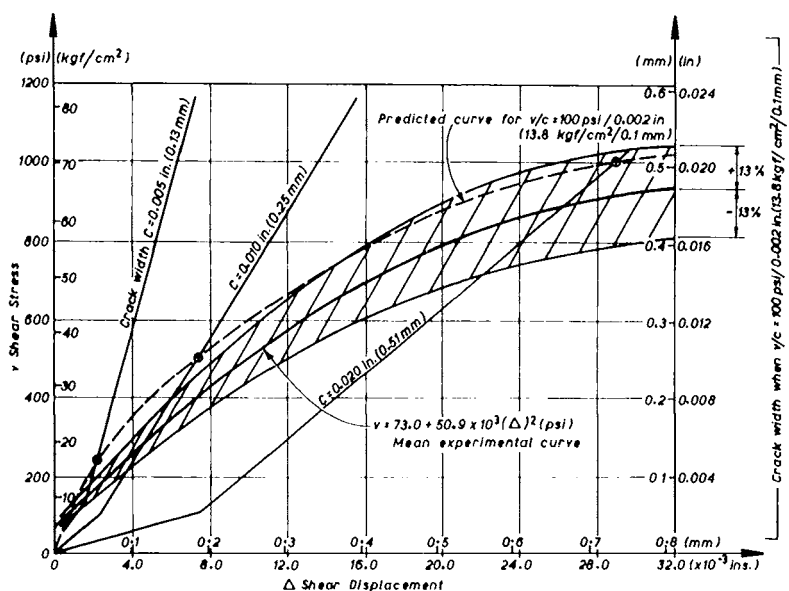


Fig. 8 - Mean regression curve for shear stress - shear displacement relationship with constant shear stress to crack width ratio

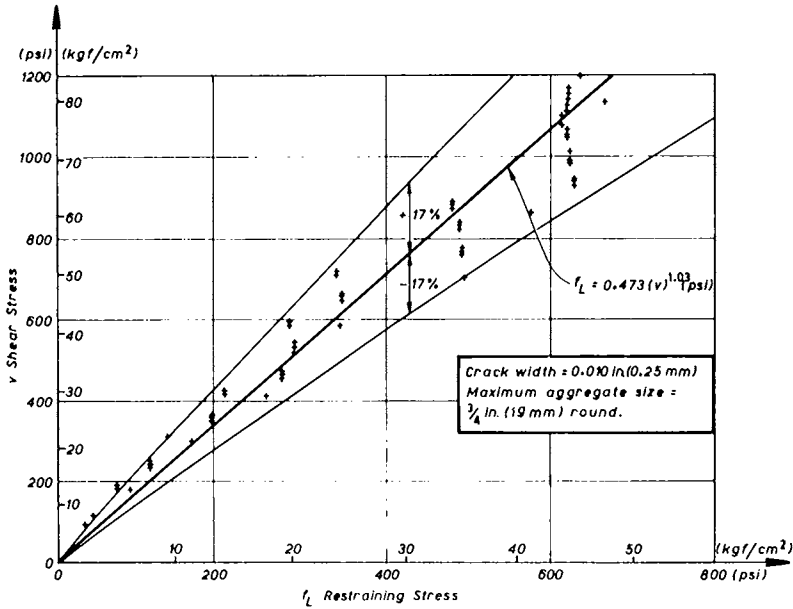


Fig. 9 - Typical test results for shear stress - restraining stress relationship

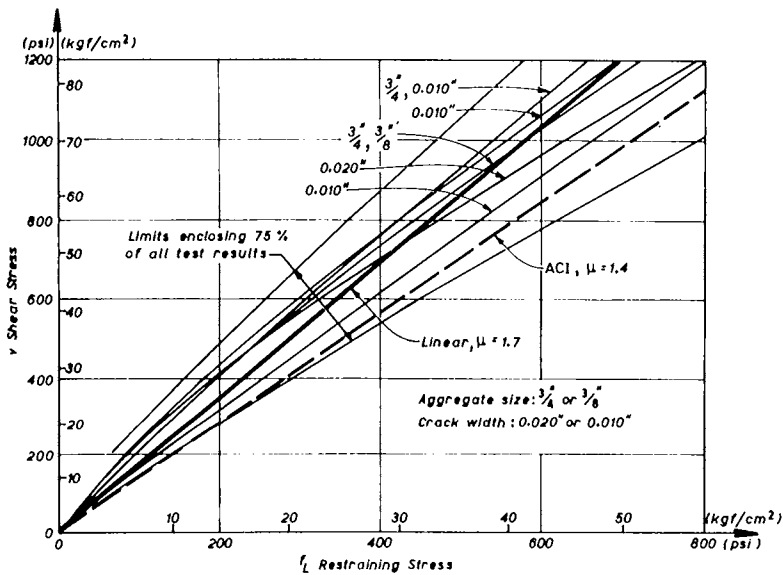


Fig. 10 - Mean shear stress - restraining stress relationships for grouped test specimens

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Shear Transfer In Concrete Having Reinforcement At An Angle To The Shear Plane

By
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Synopsis: This paper presents an experimental and analytical study of shear transfer across a plane inclined at an arbitrary angle to a parallel or orthogonal array of reinforcement. Twenty three push-off type specimens were tested, having either orthogonal or parallel arrays of reinforcement arranged at various angles to the shear plane. About two thirds of the specimens were cracked along the shear plane before being subject to shear. Hypotheses previously developed (4) for shear transfer in concrete having reinforcement at right angles to the shear plane, were extended to the cases under study. The results of the tests indicated that these hypotheses for shear transfer behavior are applicable to the case of concrete having reinforcement at an angle to the shear plane, provided the component of the bar forces parallel to the shear plane is taken into account in calculating shear resistance.

Keywords: cracking (fracturing); loads (forces); precast concrete; reinforced concrete; reinforcing steels; research; shear properties; stress transfer; structural analysis; tests.

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INTRODUCTION

Situations can arise in design where the transfer of shear across a specific plane must be considered. Examples of such situations in precast concrete construction have been discussed by Birkeland and Birkeland (1) and by Mast (2), who also pointed out the need to consider the case of a crack existing in the shear plane before shear is applied. Such cracks can occur for a variety of reasons unrelated to shear, such as tension forces caused by restraint of shrinkage or temperature deformations, etcetera.

In the design of prestressed concrete reactor containment vessels for lateral forces, it is necessary to calculate the in-plane shear which can be transferred across cracks arbitrarily inclined to the direction of the longitudinal and circumferential reinforcement usually present near the inner and outer faces of the shell. In the general case, the shears are accompanied by in-plane tension forces. Lacking data on actual behavior, designers faced with the problem have produced conservative behavior models for design purposes. As a step toward solution of the general problem, this paper presents an experimental and analytical study of shear transfer across a plane inclined at an arbitrary angle to the reinforcement, a situation which may also occur in precast concrete connection design. Both parallel and orthogonal arrays of reinforcement were considered in the study.

EXPERIMENTAL STUDY

Test Specimens

The test specimens were designed to simulate the conditions existing in a shear plane crossing a large array of equally spaced reinforcing bars at an arbitrary angle θ as in Fig. 1. They took the form of push-off specimens reinforced with closed #3 bar stirrups arranged parallel to one another or orthogonally at various angles to the shear plane as shown in Fig. 2. When loaded axially as shown by the arrows V in Fig. 2, shear without moment is produced on the shear plane indicated. If adequate longitudinal and end reinforcement is provided, failure of this type of specimen occurs along the shear plane. The length of the shear plane varied with the angle θ between the stirrups and the shear plane and was equal to the length of the shear plane contained within the 10 inch square superposed on the reinforcement array in Fig. 1. This was done so that the area of concrete involved along with each reinforcing bar should be the same as when a large array of reinforcing bars is crossed by an arbitrarily inclined

shear plane. Closed stirrups were used to ensure that the full yield strength of the reinforcement could be developed in tension where it crossed the shear plane.

Three series of specimens with orthogonal reinforcement and two with parallel reinforcement were tested. The spacing of the orthogonal reinforcement was the same in both directions and was 5 inches for Series 1 and 2 and 2-1/2 inches for Series 3. The spacing of the parallel stirrups in Series 4 and 5 was 5 inches. The details of the specimens are given in Tables 1 and 2. All the specimens were 7 inches thick and 14 inches wide. The specimens of Series 1, 3 and 4 were cracked along the shear plane by the application of a transverse line load before being tested. The specimens of Series 2 and 5 were uncracked at the commencement of the shear test. The concrete had a nominal compressive strength of 4000 psi at the time of test. It was made using Type III Portland cement, 3/4 inch maximum size glacial outwash gravel and natural sand. The mix proportions were 1.0: 2.87: 3.54 (cement: sand: gravel) with water added to produce a 3-inch slump. The stirrups were made from #3 rebar which conformed to ASTM Specification A615 and had a yield strength of 50 ksi.

Test Procedures

The specimens were loaded monotonically to failure in about 35 increments, using a hydraulic testing machine as shown in Fig. 3. The load was applied through the plates and rollers in order to prevent lateral confinement of the specimen by the testing machine structure. After each increment of load was applied, measurements were made of slip along the shear plane, separation across the shear plane, and of average strain in 10-inch gage lengths aligned with the stirrup reinforcement. The measurements were made using mechanical gages attached to reference points epoxied to the face of the specimen as may be seen in Fig. 3.

Test Results

Typical shear stress-slip curves for specimens with and without cracks in the shear plane before testing are shown in Figs. 4 and 5. There was slip from the beginning of the test in the case of the initially cracked specimens. In the initially uncracked specimens there was no relative movement between the concrete on opposite sides of the shear plane until diagonal tension cracks occurred at shear stresses of between 400 and 600 psi. These short diagonal tension cracks crossed the shear plane at intervals of a few inches and inclined at from 25 to 45 degrees. After diagonal tension cracking, relative movement (subsequently referred to as "slip") occurred as a result of rotation of the short concrete struts formed by the diagonal tension cracks. No diagonal tension cracks occurred in the initially cracked specimens.

The load at which the slip increased continuously without increase in load was taken to be the ultimate load. At ultimate, some compression spalling was observed in the region of diagonal tension cracking. The ultimate strengths attained in the tests are given in

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Table 3, both as the total shear on the shear plane, V_u , and as an average ultimate shear stress, v_u , obtained by dividing V_u by the area of the shear plane.

Although the mechanical extensometers could not measure the actual local strain in the reinforcement where it crossed the shear plane, the readings obtained indicated the following trends in behavior:

(a) For initially cracked specimens - the bars inclined at both 26.5° and 45° were in substantial compression. The bars at 63.5° were only lightly strained, some in compression, some in tension. Bars inclined at 90° or more to the shear plane were subject to considerable tension strains.

(b) For initially uncracked specimens - the bars inclined at 26.5° were subject to small amounts of compression. The bars at 45° were subject to a very small tension strain. Those at 63.5° were subject to about three times as much tension as those at 45° , and bars at greater angles to the shear plane were subject to very substantial tension strains at failure.

HYPOTHESES FOR BEHAVIOR

Shear Transfer Behavior of Initially Uncracked Concrete With Orthogonal Reinforcement at Angle 0 to the Shear Plane

External loads are assumed to cause a shear stress v along the shear plane and direct stresses σ_{Ny} and σ_{Nx} parallel to and normal to the shear plane, respectively. Under increasing shear, the principal tensile stress in the concrete becomes equal to the tensile strength of the concrete and several short diagonal tension cracks occur along the length of the shear plane and inclined to it at an angle α . This situation is shown in Fig. 6. The angle α will depend upon the particular combination of v , σ_{Nx} and σ_{Ny} existing at the time of cracking.

When the shear is further increased, a truss action develops as shown in Fig. 7(a). Diagonal struts of concrete are formed by the short, parallel diagonal tension cracks. When shear acts on the truss, the struts tend to rotate, causing a displacement of the concrete on one side of the shear plane relative to that on the other. This relative displacement produces stresses in the reinforcement crossing the shear plane. The resultant of the reinforcing bar forces provides the forces T and T_v at the ends of each strut normal to and parallel to the shear plane respectively. Because the diagonal struts are continuous with the concrete on both sides of the shear plane, there will be both compression and transverse shear in the struts. The applied shear is therefore resisted by the component of the reinforcing bar forces parallel to the shear plane, plus the components of the strut compression and shear forces acting parallel

to the shear plane as shown in Fig. 7(b).

When reinforcement is provided normal to the shear plane, it has been found (3) to develop its yield strength at ultimate, provided a failure of the concrete does not occur first. It is therefore conservative to assume that at ultimate the strain in reinforcement normal to the shear plane is equal to its yield strain, ϵ_y . The following additional assumptions are also made to enable the stress in bars inclined to the shear plane to be calculated:

1. The stress in the reinforcement is proportional to the component of the relative displacement of concrete on the two sides of the shear plane in the direction of the reinforcement.

2. The relative displacement at ultimate, δ_u , is constant and equal to that necessary to produce a strain ϵ_y in reinforcement normal to the shear plane.

Since the relative displacement occurs as a result of rotation of the inclined concrete struts, it will occur in a direction at right angles to the struts. This is borne out by observation. The direction of the relative displacement is therefore at $(90 + \alpha)^\circ$ to the shear plane. Referring to Fig. 6(b) and using the foregoing assumptions, the strain at ultimate ϵ_s in reinforcement at angle θ to the shear plane is given by:

$$\epsilon_s = C_1 \delta_u \cos(90 + \alpha - \theta)$$

where C_1 is a constant

when $\theta = 90^\circ$, $\epsilon_s = \epsilon_y$

$$\therefore \epsilon_y = C_1 \delta_u \cos \alpha$$

Hence we may write

$$\epsilon_s = \epsilon_y \sec \alpha \cos(90 + \alpha - \theta) \quad \dots\dots\dots (1)$$

(+ tension, - compression)

Equating ϵ_s to $-\epsilon_y$ in Equation (1), we find that the reinforcement yields in compression when θ is equal to or less than $(2\alpha - 90)^\circ$. The following expressions can now be written for the stress at ultimate f_s in reinforcement at an angle θ to the shear plane:

$$\left. \begin{array}{ll} 0 < \theta < (2\alpha - 90)^\circ & , \quad f_s = -f_y \\ (2\alpha - 90)^\circ \leq \theta \leq 90^\circ & , \quad f_s = f_y \sec \alpha \cos(90 + \alpha - \theta) \\ 90^\circ \leq \theta \leq 180^\circ & , \quad f_s = f_y \end{array} \right\} \quad \dots\dots (2)$$

Shear transfer failure will finally occur when the concrete struts fail under the combined action of the compression and shear in the struts, while the reinforcement is stressed as indicated above.

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Consider an element of concrete lying in the shear plane at the middle of the thickness of a strut. With reference to coordinates x' and y' , the stresses acting on the element will be as shown in Fig. 7(c). They comprise a compression stress σ_y , acting parallel to the direction of the diagonal tension cracks and shear stresses $\tau_{x'y'}$, oriented as shown. Because the faces of the strut formed by the diagonal tension cracks are unloaded free surfaces, $\sigma_{x'}$ is zero. It has been shown elsewhere (4) that pairs of values of σ_y , and $\tau_{x'y'}$, at failure of the concrete can be obtained from the concrete failure envelope using a simple geometric construction.

The state of stress in the element on the shear plane can also be expressed as σ_x , σ_y and τ_{xy} with respect to the axes x and y , normal and parallel to the shear plane respectively. These stresses can be stated in terms of σ_y , and $\tau_{x'y'}$, as follows:

$$\sigma_x = \sigma_y \sin^2 \alpha - 2\tau_{x'y'} \sin \alpha \cos \alpha \quad \dots\dots\dots (3)$$

$$\sigma_y = \sigma_y \cos^2 \alpha + 2\tau_{x'y'} \sin \alpha \cos \alpha \quad \dots\dots\dots (4)$$

$$\tau_{xy} = \sigma_y \sin \alpha \cos \alpha - \tau_{x'y'} (\cos^2 \alpha - \sin^2 \alpha) \quad \dots\dots\dots (5)$$

When there is no externally applied direct stress σ_{Nx} , α is usually about 45° . In this case:

$$\sigma_x = \sigma_y / 2 - \tau_{x'y'} \quad \dots\dots\dots (3a)$$

$$\sigma_y = \sigma_y / 2 + \tau_{x'y'} \quad \dots\dots\dots (4a)$$

$$\tau_{xy} = \sigma_y / 2 \quad \dots\dots\dots (5a)$$

Now at failure, σ_x is the direct stress acting across the shear plane as a result of the stresses in the reinforcement, plus any externally applied direct stress σ_{Nx} acting across the shear plane at failure, i.e.:

$$\sigma_x = \frac{F}{lw} + \sigma_{Nx} \quad \dots\dots\dots (6)$$

where l and w are the length and width of the shear plane and F is the total reinforcement force acting across the shear plane.

$$F = \left(\frac{l}{s_a} \sin \theta\right) A_{sa} f_{sa} \sin \theta + \left(\frac{l}{s_b} \cos \theta\right) A_{sb} f_{sb} \cos \theta \quad \dots\dots (7)$$

$$\therefore \frac{F}{lw} = \frac{A_{sa} f_{sa}}{s_a w} (\sin^2 \theta) + \frac{A_{sb} f_{sb}}{s_b w} (\cos^2 \theta) \quad \dots\dots\dots (8)$$

where f_{sa} and f_{sb} are the stresses in, and s_a and s_b are the spacing of the reinforcing bars crossing the shear plane at angles θ and

(90 + 0) respectively. f_{sa} is calculated using Equation (2), f_{sb} will be equal to f_y . Hence Equation (6) may be restated as:

$$\sigma_x = \frac{A_{sa} f_{sa}}{s_a w} (\sin^2 \theta) + \frac{A_{sb} f_y}{s_b w} (\cos^2 \theta) + \sigma_{Nx} \quad \dots\dots\dots (6a)$$

At failure τ_{xy} is the shear stress in the shear plane at the center of the strut. We may then write

$$\frac{V_u - F_v}{l_w} = K \tau_{xy} \quad \dots\dots\dots (9)$$

$$\text{or } v_u = \frac{V_u}{l_w} = \frac{F_v}{l_w} + K \tau_{xy} \quad \dots\dots\dots (10)$$

where F_v is the component of the bar forces parallel to the shear plane, opposing shear, and K is a coefficient.

$$F_v = -\left(\frac{l}{s_a} \sin \theta\right) A_{sa} f_{sa} \cos \theta + \left(\frac{l}{s_b} \cos \theta\right) A_{sb} f_{sb} \sin \theta \quad \dots\dots (11)$$

$$\frac{F_v}{l_w} = \sin \theta \cos \theta \left[-\frac{A_{sa} f_{sa}}{s_a w} + \frac{A_{sb} f_{sb}}{s_b w} \right] \quad \dots\dots\dots (12)$$

Hence Equation (10) may be restated as

$$v_u = \frac{\sin 2\theta}{2} \left[\frac{A_{sb} f_y}{s_b w} - \frac{A_{sa} f_{sa}}{s_a w} \right] + K \tau_{xy} \quad \dots\dots\dots (10a)$$

The coefficient K relates the true maximum shear stress in the concrete to the nominal average shear stress in the concrete. Factors influencing the value of K have been discussed previously (4) and it was found that in push-off specimens with reinforcement normal to the shear plane an appropriate value of K was 0.84.

Using Equations (3) and (5) it is possible to calculate pairs of values of σ_x and τ_{xy} corresponding to shear transfer failure, and a plot can be made of the relationship between σ_x and τ_{xy} for a particular strength of concrete. The shear transfer strength corresponding to a particular arrangement of reinforcement is then obtained as follows. First, calculate the value of σ_x corresponding to the reinforcement provided using Equation (6a). From the plot relating σ_x and τ_{xy} for the concrete, read off τ_{xy} . Substitute τ_{xy} in Equation (10a) to obtain the shear transfer strength v_u .

For the isotropic orthogonal reinforcement used in the tests reported here Equations (6a) and (10a) respectively reduce to

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$$\sigma_x = \frac{A_s}{sw} [f_{sa} \sin^2 \theta + f_y \cos^2 \theta] + \sigma_{Nx} \dots\dots\dots (6b)$$

$$\text{and } v_u = \frac{A_s}{2sw} \sin 2\theta [f_y - f_{sa}] + K \tau_{xy} \dots\dots\dots (10b)$$

For the case where σ_{Nx} is zero, α may be taken as 45° and f_{sa} is then given by

$$\begin{aligned} 0 < \theta < 90^\circ \quad f_{sa} &= f_y \sec 45^\circ \cos(135 - \theta) \\ &= 1.4 f_y \cos(135 - \theta) \dots\dots\dots (13) \end{aligned}$$

Equations (6a) and (10a) now become

$$\sigma_x = \frac{A_s f_y}{sw} [1.4 \cos(135 - \theta) \sin^2 \theta + \cos^2 \theta] \dots\dots\dots (6c)$$

$$\text{and } v_u = \frac{A_s f_y}{2sw} \sin 2\theta [1 - 1.4 \cos(135 - \theta)] + K \tau_{xy} \dots\dots\dots (10c)$$

In Fig. 8 a comparison is made between the shear transfer strengths measured in the tests of Series 2 and the calculated values obtained using Equations (3), (5), (6) and (10) as described above and using a value of 0.84 for K. The intrinsic shape of the failure envelope used to obtain the values of σ_y and $\tau_{x'y}$ substituted in

Equations (3) and (5) was obtained from biaxial tests of concrete reported by Kupfer, Hilsdorf and R sch (6). Use was made of data reported for concrete having a ratio of tensile to compressive strength corresponding to that of the concrete used in the push-off specimens. Also plotted in Fig. 8 as an open point at $\theta = 0$ and 90° is the average shear transfer strength for the case of reinforcement normal to the shear plane, based on several previous tests (4). It can be seen that although there is a fair amount of experimental scatter, the proposed method of calculation yields a reasonably close estimate of the ultimate shear transfer strength. The average value of $v_u(\text{test})/v_u(\text{calc.})$ for the tests of Series 2 is 1.07. It can be seen that the maximum shear transfer stress is developed when θ is 20° . At this angle the contribution to shear transfer strength of the rebar force component parallel to the shear plane has already reached about 90 percent of its maximum value, and the truss action component of the shear resistance is only reduced a small amount from that corresponding to $\theta = 0^\circ$. As θ increases beyond 20° , the truss action resistance decreases faster than the rebar force component parallel to the shear plane increases, so that the net shear transfer strength decreases to a minimum at about $\theta = 68^\circ$. Beyond this value of θ , the truss action resistance increases significantly so that the net shear transfer strength increases once more.

Shear Transfer Behavior of Initially Uncracked Concrete With Reinforcement at Angle θ to the Shear Plane

The shear transfer behavior of initially uncracked concrete in which the shear plane is crossed at angle θ by a parallel array of bars will be exactly similar to the case of orthogonally reinforced concrete. Equations (6) and (10) for σ_x and v_u will apply to this case providing the appropriate values of F and F_v are substituted in the equations. For bars A_s at spacing s ,

$$\frac{F}{l_w} = \frac{A_s f_s}{sw} \sin^2 \theta \quad \dots\dots\dots (14)$$

and

$$\frac{F_v}{l_w} = -\sin \theta \cdot \cos \theta \frac{A_s f_s}{sw} \quad \dots\dots\dots (15)$$

The stress in the bars f_s will be calculated using Equation (2). For the case where $\sigma_{Nx} = 0$ and taking α as 45° .

$$0 < \theta \leq 90^\circ \quad f_s = 1.4 f_y \cos(135-\theta) \quad (\text{as before})$$

$$90^\circ \leq \theta \leq 180^\circ \quad f_s = f_y$$

Then

$$\sigma_x = \frac{A_s f_s}{sw} \sin^2 \theta \quad \dots\dots\dots (6d)$$

and

$$v_u = -\frac{A_s f_s}{2sw} \sin 2\theta + K \tau_{xy} \quad \dots\dots\dots (10d)$$

In Fig. 9 a comparison is made between the shear transfer strengths measured in the tests of Series 5 and the calculated values obtained using Equations (6d) and (10d) as described for the case of orthogonally reinforced concrete, and using a value of 0.84 for K . Also plotted as open points at $\theta = 90^\circ$ and 180° are measured shear transfer strengths obtained in earlier tests (4). It can be seen that the method of calculation proposed correctly predicts the trends in behavior and yields a reasonably close estimate of the ultimate shear transfer strength. In this case, the maximum shear stress is developed when θ is 113° . The average value of $v_u(\text{test})/v_u(\text{calc.})$ for the tests of Series 5 is 1.0.

Shear Transfer Behavior of Initially Cracked Concrete With Orthogonal Reinforcement at Angle θ to the Shear Plane

When initially cracked concrete is subject to shear along the crack, slip will occur. The faces of the crack are rough, and hence when slip occurs, the crack faces are forced to separate. The relative displacement of the concrete on the two sides of the crack produces strains in the reinforcement crossing the crack. The

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resultant of the forces induced in the reinforcement will have a component parallel to the crack resisting the applied shear, and a component normal to the crack which produces a compression force across the crack. This compression force produces a frictional resistance to sliding between the faces of the crack, thus opposing the applied shear. The relative movement of the concrete on opposite sides of the crack also subjects the individual reinforcing bars to a shearing action. The resistance of the bars to this shearing action, sometimes referred to as dowel action, also contributes to the shearing resistance. A further contribution to shearing resistance is made by the resistance to the shearing off of asperities projecting from the faces of the crack.

In an under-reinforced shear plane with reinforcement normal to the crack, it has been found (3) that the amount of separation of the crack faces is sufficient to cause the reinforcement to yield. It will therefore be conservative to assume that the strain at ultimate in reinforcement normal to the crack is equal to its yield strain, ϵ_y . The following additional assumptions will also be made to enable the stress in bars inclined to the crack to be calculated:

1. The strain in the reinforcement is proportional to the component of the relative displacement of the faces of the crack in the direction of the reinforcement.
2. The relative displacement at ultimate, δ_u , is constant and equal to that necessary to produce a strain ϵ_y in reinforcement normal to the crack.
3. The relative displacement δ_u takes place in a direction at angle ϕ to the crack. ϕ is equal to $\tan^{-1}\mu$, where μ is the coefficient of friction between the crack faces. $\mu = 0.8$, hence $\phi = 38.7^\circ$. This is based on the simplified "saw tooth" model of frictional resistance in which the slope of the saw teeth is $\tan^{-1}\mu$, and relative displacement takes place parallel to the sloping face of the teeth. The directions of the actual measured relative displacements were in reasonable agreement with this assumption.

Referring to Fig. 10(b) and using the foregoing assumptions, the strain at ultimate ϵ_s in reinforcement at angle θ to the crack is given by:

$$\epsilon_s = C_2 \delta_u \cos(\theta + \phi) \quad \dots\dots\dots (16)$$

where C_2 is a constant.

When $\theta = 90^\circ$, $\epsilon_s = \epsilon_y$

$$\therefore \epsilon_y = C_2 \delta_u \cos(90 + \phi) = -C_2 \delta_u \sin \phi$$

Hence we may write

$$\epsilon_s = -\epsilon_y \operatorname{cosec} \phi \cos(\theta + \phi) \quad \dots\dots\dots (17)$$

(+ tension, - compression)

Substituting $\phi = 38.7^\circ$ in Eq. (17), we find that when θ is equal to or less than 12.7° , the reinforcement yields in compression, and when θ is equal to or greater than 90° the reinforcement yields in tension. We may now write the following expressions for the stress at ultimate f_s , in reinforcement at angle θ to the crack,

$$\left. \begin{aligned} 0 < \theta \leq 12.7^\circ, & \quad f_s = -f_y \\ 12.7^\circ \leq \theta \leq 90^\circ, & \quad f_s = -f_y \operatorname{cosec} 38.7 \cos(\theta + 38.7) \\ & \quad = -1.6 f_y \cos(\theta + 38.7) \\ 90^\circ \leq \theta \leq 180^\circ, & \quad f_s = f_y \end{aligned} \right\} \quad (18)$$

It has been proposed (4) that for the case of reinforcement crossing a crack at right angles the ultimate shear transfer strength may, for purposes of design, be taken as:

$$v_u = 200 + 0.8\rho f_y, \text{ but } \nless 0.3f'_c \quad \dots\dots\dots (19)$$

providing ρf_y is ≥ 200 psi.

This equation was shown to be a lower bound to all available push-off and pull-off test data for initially cracked specimens. The following modifications of Equation (19) predict mean values for shear transfer strength when the reinforcement is at 90° to the crack:

$$v_u = 400 + 0.8\rho f_y, \text{ but } \nless 0.3f'_c \quad (\text{psi}) \quad \dots\dots\dots (20)$$

$$\text{or } v_u = 400lw + 0.8F, \text{ but } \nless 0.3f'_c lw \quad (\text{lbs}) \quad \dots\dots\dots (21)$$

where l and w are the length and width of the shear plane, and F is the total reinforcement force acting across the shear plane.

It is proposed to adapt Equation (21) to the case of a crack crossing an orthogonal array of reinforcement as shown in Fig. 10(a) by adding a term F_v equal to the component of the bar forces parallel to the crack, opposing the shear:

$$v_u = 400lw + 0.8F + F_v \quad (\text{lbs}) \quad \dots\dots\dots (22)$$

$$\begin{aligned} \text{where } F &= \left(\frac{l}{s} \sin\theta\right) A_s f_{sa} \sin\theta + \left(\frac{l}{s} \cos\theta\right) A_s f_{sb} \cos\theta \\ &= \frac{A_s l}{s} [f_{sa} \sin^2\theta + f_{sb} \cos^2\theta] \quad \dots\dots\dots (23) \end{aligned}$$

$$\begin{aligned} \text{and } F_v &= -\left(\frac{l}{s} \sin\theta\right) A_s f_{sa} \cos\theta + \left(\frac{l}{s} \cos\theta\right) A_s f_{sb} \sin\theta \\ &= \frac{A_s l}{2s} \sin^2\theta [-f_{sa} + f_{sb}] \quad \dots\dots\dots (24) \end{aligned}$$

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f_{sa} and f_{sb} are the stresses in the reinforcement crossing the crack at angles θ and $(90+\theta)$ respectively. f_{sa} is calculated using Eq. (18), f_{sb} will be equal to f_y .

The first term of Equations (21) and (22) primarily represents the contribution to shear transfer strength of dowel action of the reinforcement. The use of the same value in Equation (22) as in Equation (21) is to some extent arbitrary, but is also based on the following qualitative reasoning. The dowel force normal to the rebar will be small when θ is small, but it will increase as θ increases to 90° . As θ becomes larger than 90° , the displacement at right angles to the bar which causes dowel action gradually reduces to zero as the bar finally becomes aligned with the direction of relative displacement, and hence the dowel force developed decreases to zero. Shear along the crack is resisted by the component of the dowel force parallel to the crack, which will be $\sin \theta$ times the dowel force normal to the bar. The exact way in which the dowel force normal to the bar varies with θ is not known but could perhaps be approximated as being proportional to $\sin \theta$. The component parallel to the crack would then be proportional to $\sin^2 \theta$, say equal to $D \sin^2 \theta$. Since one set of bars is at angle θ and the other at $(90+\theta)$ in any instance, the component parallel to the crack of the dowel forces in both sets of reinforcement may be written $D[\sin^2 \theta + \sin^2(90+\theta)] = D$, i.e., the dowel force contribution is independent of θ according to the foregoing argument. The variation of the dowel force component parallel to the shear plane with angle θ , reported by Dulacska, (5) can be approximated reasonably well by an expression of the form $D \sin^2 \theta$ for the range of values of θ (50° to 80°) included in her tests. Additional support for the use of the constant $400/l_w$ as the first term in Equation (22) is provided by the correlation obtained between Equation (22) and the test data reported here.

Equation (22) may also be expressed in the form of stresses:

$$\begin{aligned} v_u &= V_u / l_w \\ &= 400 + 0.8 \frac{A_s}{s w} [f_{sa} \sin^2 \theta + f_{sb} \cos^2 \theta] \\ &\quad + \frac{A_s}{2 s w} \sin 2\theta [-f_{sa} + f_{sb}] \dots\dots\dots (25) \end{aligned}$$

where f_{sa} and f_{sb} are calculated using Equations (18).

In Fig. 11 a comparison is made between the shear transfer strengths measured in the tests of Series I and the calculated values obtained using Equation (25). It can be seen that Equation (25) correctly predicts the way in which the ultimate shear transfer strength changes as angle θ changes, and for this series ($s=5$ in.) predicts the ultimate strengths very closely. When applied to Series 3 ($s=2-1/2$ in) the results were not quite so consistent, the ratios of $v_u(\text{test})/v_u(\text{calc})$ for specimens 3.1 through 3.4 being 0.84, 1.13, 0.99 and 1.16, for an average value of $v_u(\text{test})/v_u(\text{calc})$ of 1.03. The maximum and minimum values of shear stress occur at $\theta = 22^\circ$ and 72° respectively.

It is interesting to note that the difference between the calculated shear transfer strength of initially uncracked concrete and that of initially cracked concrete as θ varies from 0 to 90° is almost a constant value of 200 psi, the same value as previously reported (3) for concrete with reinforcement normal to the shear plane.

Shear Transfer Behavior of Initially Cracked Concrete With Parallel Reinforcement at Angle θ to the Shear Plane

The behavior of initially cracked concrete with parallel reinforcement crossing the crack at angle θ is similar to that of orthogonally reinforced concrete, providing the angle θ is large enough so that tension is developed in the reinforcement. It was observed that for small values of θ the relative displacement of the concrete on opposite sides of the crack was in a direction approximately at right angles to the bars, i.e. at $(90+\theta)$ to the crack. It appears as if this displacement corresponds to a rotation of the bars as they tear themselves out of the opposite faces of the crack. Under these conditions the shear transfer resistance will be developed primarily by the dowel action of the reinforcing bars, since there is little or no component of displacement in the direction of the reinforcing bars to cause a significant compressive strain in those bars.

The same assumptions will be made in order to calculate the ultimate shear transfer strength of initially cracked concrete reinforced with a parallel array of bars at angle θ to the crack, as were made in the case of orthogonally reinforced, initially cracked concrete, except as follows:

1. The relative displacement δ_u is assumed to take place in a direction at angle $(90+\theta)$ to the crack when θ is less than $(90-\phi)$ and in a direction at angle ϕ to the crack when θ is equal to or greater than $(90-\phi)$.

2. The resistance to shear along the crack due to dowel action is assumed to vary directly with $\sin^2\theta$, for reasons already cited in the discussion of the contribution of dowel action to the shear transfer strength of orthogonally reinforced, initially cracked concrete.

The stress in the reinforcement f_s is then given by:

$$\begin{array}{lll} 0 < \theta < (90-\phi) & f_s = 0 & \vdots \\ 90-\phi \leq \theta < 90 & f_s = -1.60f_y \cos(\theta+38.7) & \sim \\ 90 \leq \theta \leq 180 & f_s = f_y & \vdots \end{array} \quad (26)$$

$$(90-\phi) = 51.3^\circ$$

The use of $f_s = 0$ when θ is less than 51.3° will be conservative since there will be a small indeterminate amount of compression in the bars. Making use of Equations (26) and assumption (2) we may write:

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$$V_u = 400Z_w \sin^2 \theta + 0.8F + F_v \quad \dots\dots\dots (27)$$

$$\text{where } F = \frac{A_s l}{s} f_s \sin^2 \theta \quad \dots\dots\dots (28)$$

$$\text{and } F_v = - \frac{A_s l}{2s} f_s \sin 2\theta \quad \dots\dots\dots (29)$$

$$\begin{aligned} \therefore V_u &= V_u / l_w \\ &= 400 \sin^2 \theta + 0.8 \frac{A_s f_s}{s w} \sin^2 \theta - \frac{A_s f_s}{2 s w} \sin 2\theta \\ \text{or } V_u &= 400 \sin^2 \theta + \frac{A_s f_s}{s w} [0.8 \sin^2 \theta - 0.5 \sin 2\theta] \quad \dots\dots (30) \end{aligned}$$

In Fig. 12 a comparison is made between the shear transfer strengths measured in the tests of Series 4 and the calculated values obtained using Equations (30) and (26). It can be seen that the trend of the experimental data is predicted reasonably closely by the calculated relationship. The average value of $v_u(\text{test})/v_u(\text{calc.})$ for the tests of Series 4 is 1.18. The conservatism in the calculated values for small values of θ probably reflects the neglect of the bar force component parallel to the crack implicit in the use of Equations (26). The conservatism when θ is greater than 90° may be due to the assumption of a too rapid reduction in dowel force effects as θ increases beyond about 100° , but it is difficult at present to be sure of the reason.

CONCLUDING REMARKS

It has been shown that the previously developed (4) hypotheses for shear transfer behavior in both initially cracked and initially uncracked concrete reinforced with bars normal to the shear plane can be extended to the case of concrete reinforced with orthogonal reinforcement or parallel reinforcement inclined at an arbitrary angle to the shear plane, provided the component of the bar forces parallel to the shear plane is also taken into account in calculating shear resistance-

The tests reported did not include a normal stress acting across the shear plane and therefore they strictly only validate the hypotheses for that situation. However, it has previously been shown (4) that for the case of reinforcement crossing the shear plane at right angles, an externally applied compressive stress acting transversely to the shear plane is additive to ρf_y in calculations of the ultimate shear transfer strength of both initially cracked and uncracked concrete. It is felt that the same will be true in the case of orthogonal reinforcement and inclined parallel reinforcement. The validity of the hypotheses when tension acts across the shear plane is not proved and experimental studies of this combination of stresses should be made.

For a given *normal* spacing of the reinforcing bars, the maximum value of ultimate shear transfer stress will be developed when the.

bars (or one set of bars in the case of orthogonal reinforcement) are inclined at approximately 110° to the shear plane. This is true for both initially cracked and initially uncracked concrete. It should be noted, however, that if the bar spacing along the shear plane is constant and θ is varied, calculations indicate that the maximum ultimate shear stress will be developed when θ is 135° .

As indicated earlier, this study is presented as a step toward solution of the general problem of shear transfer across a crack encountered in the design of concrete reactor containment vessels for lateral forces. It should be noted that the experimental work reported involved only single direction loading and relatively small diameter reinforcing bars. Further work is necessary to determine whether cyclic reversal of shear results in a degradation of shear transfer strength. It is also necessary that studies should be made using large size reinforcing bars to determine whether their effectiveness in dowel action resistance to shear is limited by local splitting of the concrete, and whether the slip displacements necessary to mobilize the dowel resistance of very large bars become excessive. Studies of these problems are currently under way or planned at Cornell University and the University of Washington.

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APPENDIX 1

NOTATION

- A_s = Cross sectional area of one rebar, in.²
 A_{sa} = Cross sectional area of one of the rebars A, in.²
 A_{sb} = Cross sectional area of one of the rebars B, in.²
 C = Compression force in concrete strut, lb.
 C_1 } = Coefficients relating strain and displacement.
 C_2
 D = Dowel action shear transfer resistance when $\theta = 90^\circ$, lb.
 F = Component normal to the shear plane of the total rebar force at ultimate, lb.
 F_v = Component parallel to the shear plane of the total rebar force at ultimate, lb.
 f'_c = Concrete compression strength measured on 6x12-inch cylinders, psi.
 f_s = Stress in rebar at ultimate, psi.
 f_{sa} = Stress in rebars A at ultimate, psi.
 f_{sb} = Stress in rebars B at ultimate, psi.
 f_t = Concrete split cylinder tensile strength measured on 6x12-inch cylinders, psi.
 f_y = Yield strength of rebar, psi.
 K = Coefficient relating maximum and average shear stresses.
 l = Length of shear plane, in.
 N = Externally applied force acting normal to the shear plane, lb.
 s_a = Space between rebars, in.
 s_a = Space between rebars A inclined at angle θ to the shear plane, in.
 s_b = Space between rebars B inclined at angle $90+\theta$ to the shear plane, in.
 T = Component normal to shear plane of rebar force acting at one end of a strut, lb.
 T_v = Component parallel to shear plane of rebar force acting at one end of a strut, lb.
 V = Applied shear acting along shear plane, lb.
 V' = Shear in strut, lb.
 V_u = Ultimate shear force, lb.
 v_u = Ultimate shear stress, psi. ($= V_u/lw$).
 w = Width of shear plane, in.

- α = Angle between diagonal tension cracks and the shear plane, degrees.
- δ_u = Relative displacement at ultimate of the concrete on opposite sides of the shear plane, in.
- ϵ_s = Strain in rebar at ultimate, in./in.
- ϵ_y = Yield strain of rebar, in./in.
- θ = Angle of inclination of rebars to the shear plane, degrees.
- μ = Coefficient of friction.
- ρ = Shear transfer reinforcement ratio ($= A_s/sw$)
- σ = Direct stress, psi.
- τ = Shear stress, psi.
- ϕ = $\tan^{-1}\mu$

TABLE 1 - DETAILS OF SPECIMENS WITH ORTHOGONAL REINFORCEMENT

Specimen No.	Angle θ (degrees)	Stirrup spacing, s (in.)	Number of stirrups @ θ° to the shear plane	Number of stirrups @ $(90+\theta)^\circ$ to the shear plane	Length of shear plane (in.)	$f_c^{(1)}$ (psi)	$f_t^{(2)}$ (psi)
1.1*	0	5	0	2	10.00	4370	420
1.2*	26.5	5	1	2	11.17	4370	438
1.3*	45.0	5	2	2	14.14	4015	367
1.4*	63.5	5	2	1	11.17	4120	439
2.1	0	5	0	2	10.00	4175	425
2.2	26.5	5	1	2	11.17	4245	407
2.3	45.0	5	2	2	14.14	3990	362
2.4	63.5	5	2	1	11.17	4270	395
3.1*	0	2.5	0	4	10.00	3920	326
3.2*	26.5	2.5	2	4	11.17	4300	436
3.3*	45.0	2.5	4	4	14.14	4010	383
3.4*	63.5	2.5	4	2	11.17	4470	-

* Cracked along shear plane before test.

(1) Concrete compressive strength measured on 6 x 12-inch cylinders.

(2) Concrete splitting tensile strength measured on 6 x 12-inch cylinders.

To convert inches (in.) to centimeters (cm.) multiply by 2.540

To convert psi to kilogram-force/square centimeter (kgf/cm^2) multiply by 0.0703.

TABLE 2 - DETAILS OF SPECIMENS WITH PARALLEL REINFORCEMENT

Specimen No.	Angle θ (degrees)	Stirrup spacing s (in.)	Number of stirrups at θ° to the shear plane	Length of shear plane (in.)	f'_c (1) (psi)	f_t (2) (psi)
4.1*	26.5	-	1	11.17	3975	406
4.2*	45.0	5	2	14.14	4275	381
4.3*	63.5	5	2	11.17	3880	419
4.4*	90.0	5	2	10.00	4370	420
4.5*	116.5	5	2	11.17	4050	314
4.6*	135.0	5	2	14.14	4040	360
4.7*	153.5	-	1	11.17	4090	419
5.1	90.0	5	2	10.00	4175	425
5.2	116.5	5	2	11.17	3885	358
5.3	135.0	5	2	14.14	3805	381
5.4	153.5	-	1	11.17	3825	405

* Cracked along shear plane before test

(1) Concrete compressive strength measured on 6 x 12-inch cylinders

(2) Concrete splitting tensile strength measured on 6 x 12-inch cylinders

To convert inches (in.) to centimeters (cm.) multiply by 2.540

To convert psi to kilogram-force/square centimeter (kgf/cm^2) multiply by 0.0703

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TABLE 3 - TEST RESULTS

Specimen No.	V_u , Ultimate Shear (kips)	v_u^* Ultimate Shear Stress (psi)	Specimen No.	V_u Ultimate Shear (kips)	v_u^* Ultimate Shear Stress (psi)
1.1	45.8	654	4.1	7.4	95
1.2	62.5	799	4.2	23.75	240
1.3	65.0	657	4.3	28.15	360
1.4	47.5	607	4.4	45.8	654
			4.5	60.0	767
2.1	70.0	1000	4.6	67.5	682
2.2	79.0	1010	4.7	25.1	321
2.3	80.0	808			
2.4	69.5	889	5.1	70.0	1000
			5.2	75.0	959
3.1	52.75	754	5.3	81.0	818
3.2	101.75	1301	5.4	49.0	627
3.3	96.0	970			
3.4	72.5	927			

* Nominal ultimate shear stress obtained by dividing the ultimate shear by the area of the shear plane.

To convert kips to kilogram-force (kgf) multiply by 453.6.

To convert psi to kilogram-force/square centimeter (kgf/cm²) multiply by 0.0703.

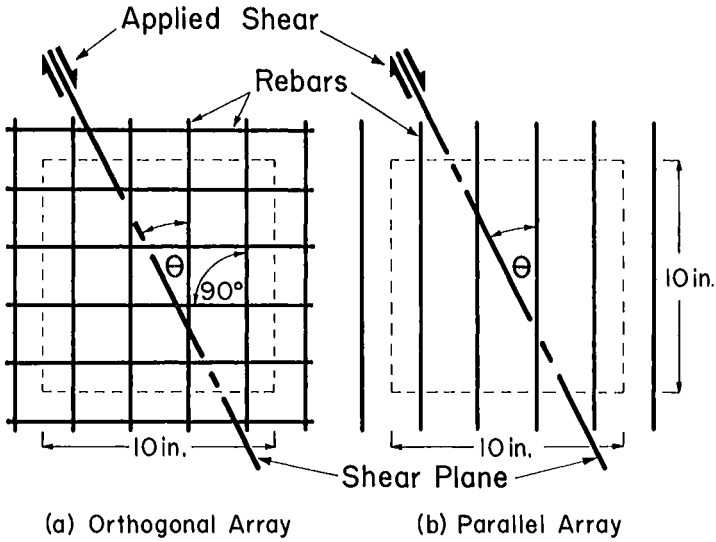


Fig. 1—Shear plane crossing reinforcement array at an arbitrary angle θ

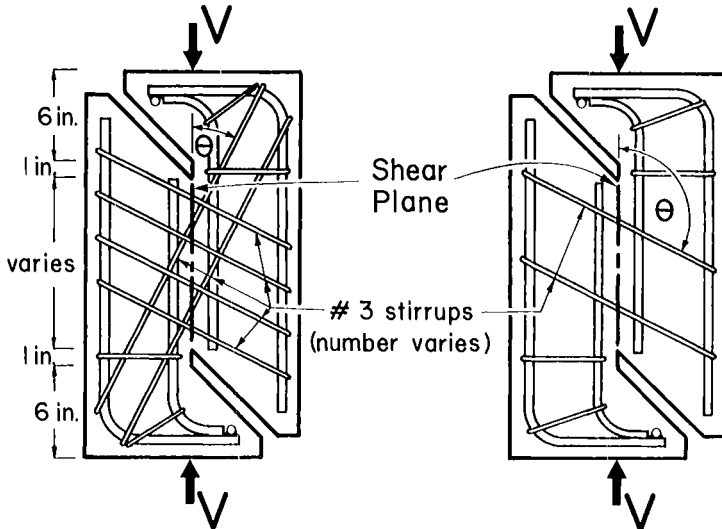


Fig. 2—Typical push-off specimens with orthogonal and parallel reinforcement

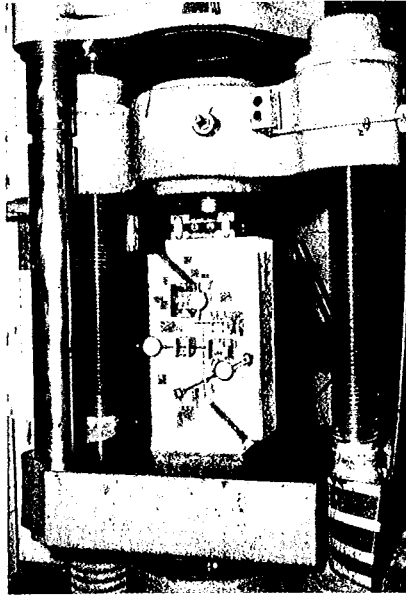


Fig. 3—Arrangement of push-off test

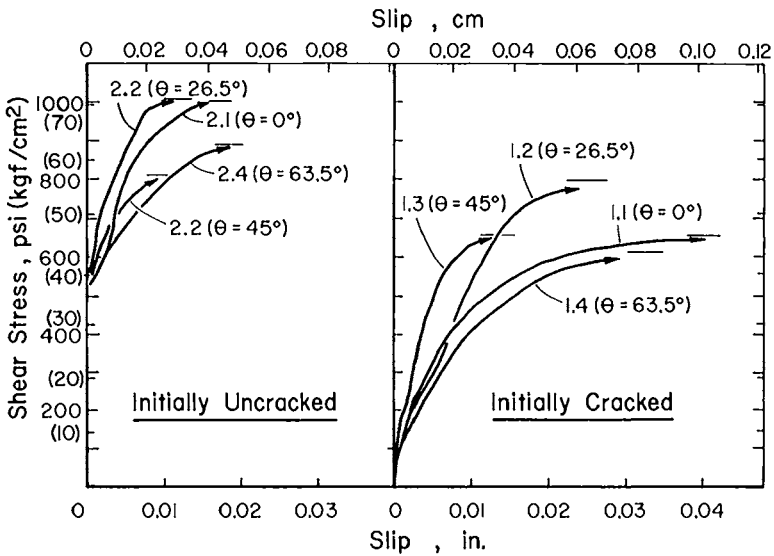


Fig. 4—Typical shear stress-slip curves for specimens with orthogonal reinforcement ($s = 5$ in.)

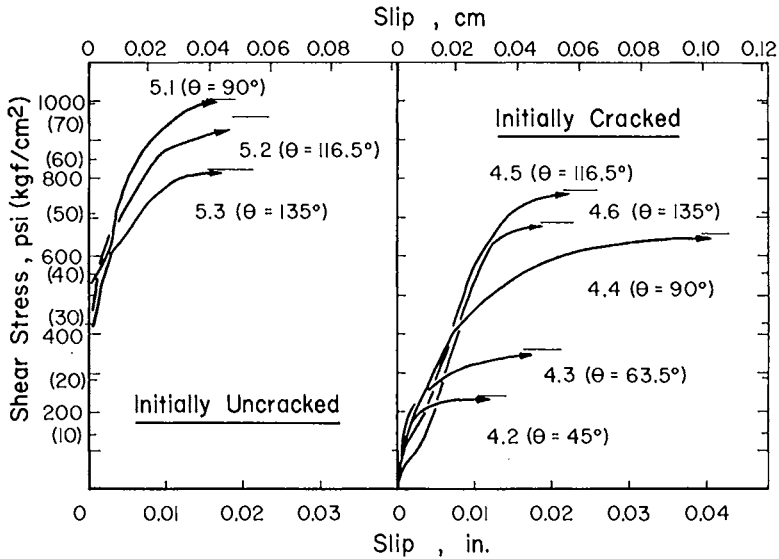


Fig. 5—Typical shear stress-slip curves for specimens with two parallel stirrups as reinforcement

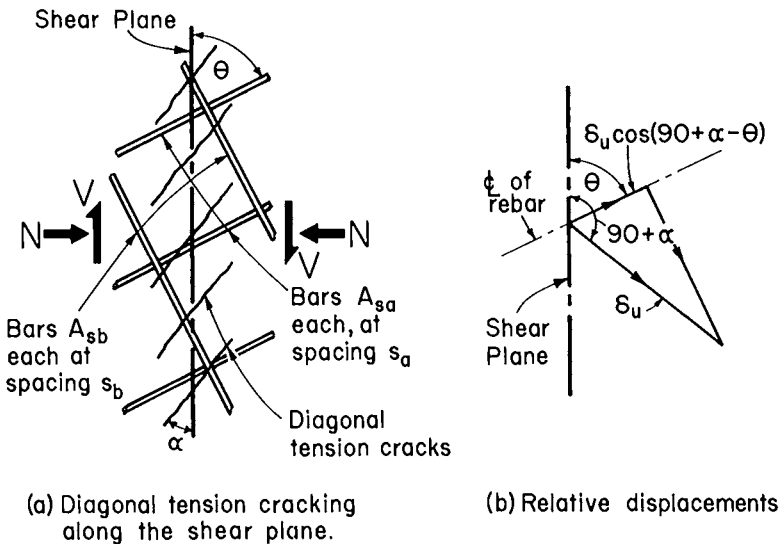


Fig. 6—Shear transfer in initially uncracked concrete with orthogonal reinforcement

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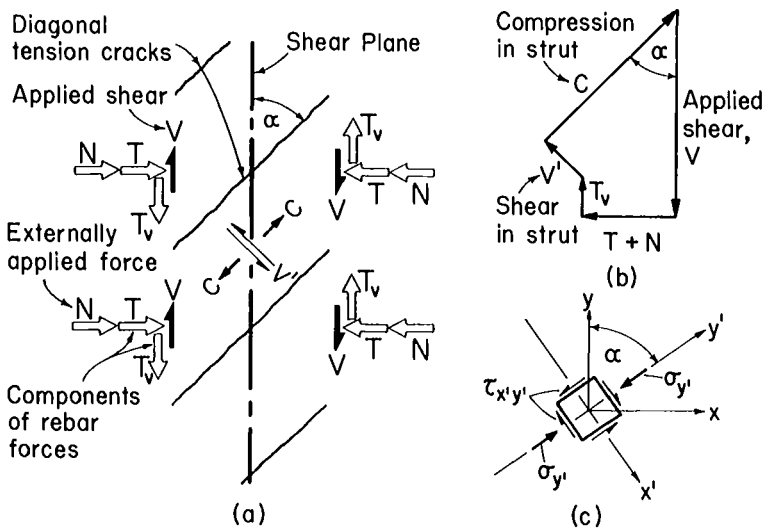


Fig. 7—The mechanics of shear transfer in initially uncracked concrete with orthogonal reinforcement

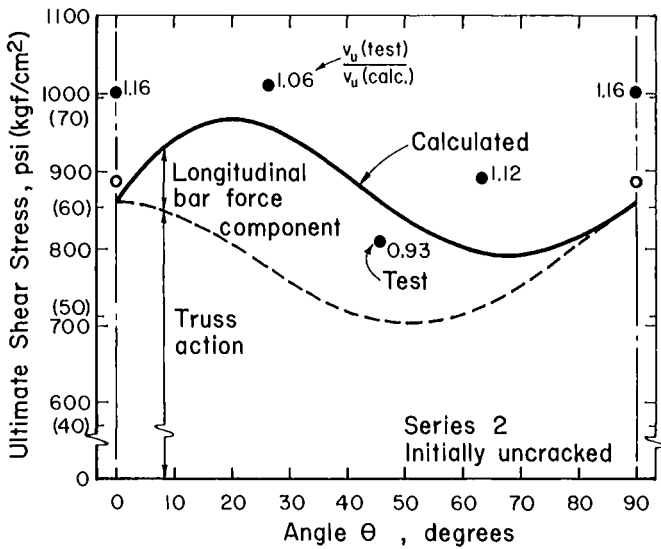


Fig. 8—Comparison of calculated and test shear transfer strengths of initially uncracked concrete with orthogonal reinforcement

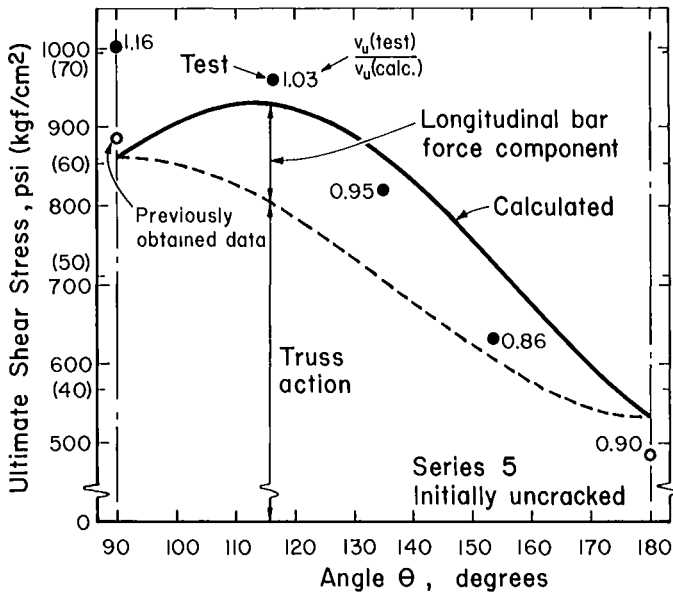


Fig. 9—Comparison of calculated and test shear transfer strengths of initially uncracked concrete with parallel reinforcement

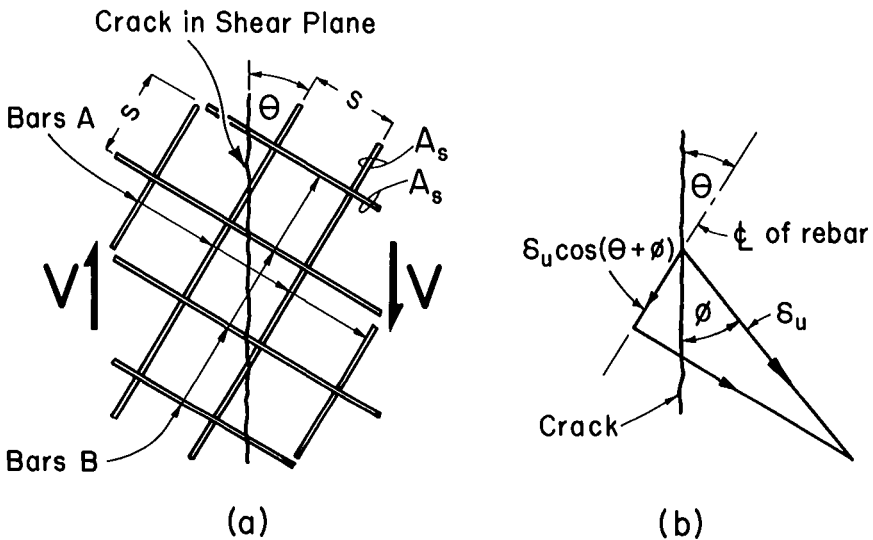


Fig.10—Shear transfer in initially cracked concrete with orthogonal reinforcement

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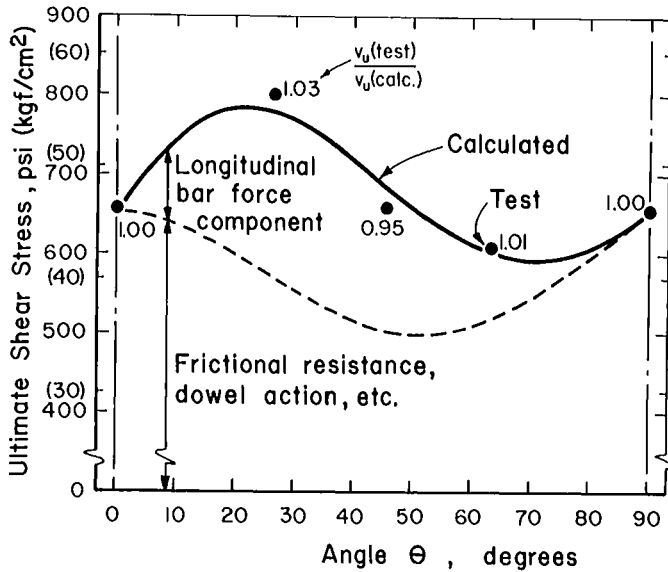


Fig. 11— Comparison of calculated and test shear transfer strengths of initially cracked concrete with orthogonal reinforcement

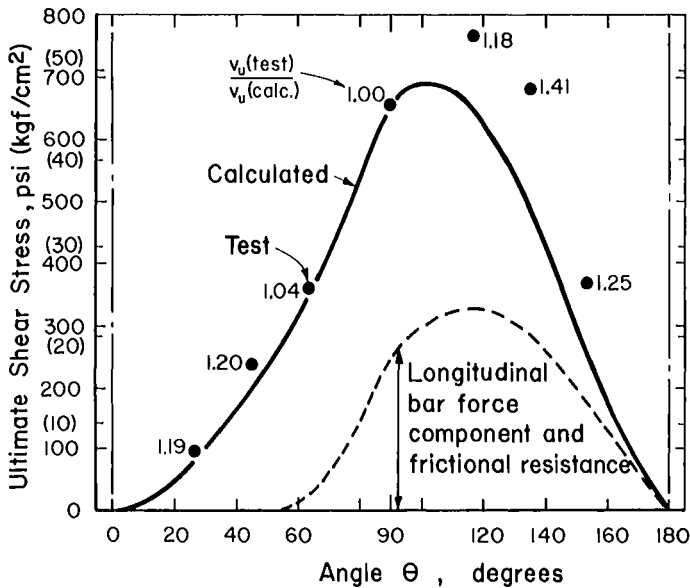


Fig. 12— Comparison of calculated and test shear transfer strengths of initially cracked concrete with parallel reinforcement

SP 42-3

The Fundamental Behavior Of Reinforced Concrete Beams In Bending And Shear

By
H. P. J. Taylor

Synopsis:

Research on the shear strength of reinforced concrete beams in bending and shear indicates that although reliable empirical conclusions can be made about beam behaviour, little is known about the way in which reinforced concrete beams carry shear.

Tests are described in which the distribution of shear force is studied in beams subject to bending and shear. In the first of the tests, the shear force carried by beam compression zones was investigated from measurements of surface strains and these tests showed that only a small proportion of the total shear force is carried by the compression zone. Two separate investigations are described in which the shear forces carried across the cracked tensile zone of the beam were studied. Firstly the force carried across the cracks by the main steel acting as a dowel was measured in a series of model and prototype tests. Secondly, the forces carried across cracks by interlock of the aggregate were estimated in two separate tests, one on cracked blocks of concrete and one on beams specially designed to eliminate other methods of shear transfer.

The strains and displacements from elastic analysis of the concrete cantilevers between cracks loaded by the forces measured in the experiments described previously are shown to be consistent with the observed crack patterns and displacements in beams.

Keywords: aggregate interlock; beams (supports); bending; compressive strength; cracking (fracturing); dowels; elastic analysis; modulus of elasticity; reinforced concrete; reinforcing steels; shear properties; shear strength; stress transfer.

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Howard P.J. Taylor joined the Cement and Concrete Association in 1964, his previous experience being in the design and site supervision of power station works. On joining the Association he has carried out research into various aspects of the structural behaviour of reinforced and prestressed concrete particularly on shear, cracking, reinforcement detailing and model testing. He is a member of the ACI, a committee member of ACI-ASCE 426, a member of the Institutions of Civil and Structural Engineers in England and participates with the work of the CEB shear committee.

INTRODUCTION

Throughout the history of the structural use of reinforced concrete, research has been carried out into the strength of members subject to bending and shear. Because of the abrupt nature of shear failures and the difficulty of formulating reliable mathematical analyses of the behaviour of beams in shear, research has tended to concentrate on predicting the collapse load of such members, usually on an empirical basis. The trend has been accentuated by the increasing use of new materials and changes in structural form.

It is unfortunate that, because of the need for the large amount of test work to be carried out, studies of basic behaviour of beams and the way in which they carry shear forces have been neglected. If this understanding were available, the extension of design rules to cover future changes in practice, for example the increasing use of high-strength steel and concrete leading to members of greater slenderness, would be much simpler and would not involve as much test verification.

The intention of this paper is to describe experimental and theoretical work that has been carried out at the Cement and Concrete Association in recent years in which basic behaviour was studied.

The most convenient way to define the various kinds of beam failure is to relate them to the shape of beam in which they occur and the crack pattern that is exhibited. Two very distinct failure crack patterns of beams without stirrups are shown in Figure 1. The shorter of the two shear spans has a crack pattern in which, although some flexural cracking had started, the diagonal failure crack formed in a straight line between load and support point and did not develop from the flexural action. This type of failure is common in beams with an a_v/d ratio of less than 2.5. The shear span with a higher a_v/d ratio shows a typical crack pattern in which the failure crack developed from a previously formed flexural crack. It is this, the predominant shear behaviour, that was studied in experimental work described in this paper.

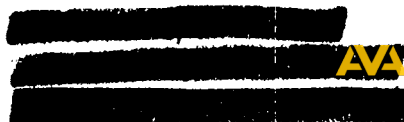
In the early days of reinforced concrete design, the shear stress distribution in beams was calculated from the assumption that the

section could be analysed elastically with the concrete being incapable of carrying tension. This led to a distribution of stress that was parabolic in the compression zone and rectangular below the neutral axis. The average shear force on the section was V/bz , it being assumed that the centroid of the compression block was at one-third of the neutral axis depth. This method of calculating the nominal shear stress on a section, or the simplified assumption of V/bd , has remained in use to this day.

After some years, this distribution of shear stress was challenged, mainly on the grounds that "shear forces obviously cannot be transmitted across cracks" even though no experiments had been carried out to prove the point. The corollary of this, that all the shear force was therefore carried in the compression zone, was also very popular with theorists and this led the way to a number of shear failure theories in which the stress conditions in the compression zone were compared with failure criteria in order to predict member shear failure.

In the last two decades, this theory has in turn been challenged by people who realized that the internal force distributions that were implied required parts of a beam to carry forces that they obviously could not sustain. In particular, Acharya and Kemp¹ estimated the shear force that could be carried by a beam compression zone when the concrete fails under the biaxial stress conditions present. They showed that, if a beam were to fail at the level of load measured in tests, it must be carrying less than 40% of the total shear force in the compression zone.

The implication of the assumption on the beam tension zone was that, as all the shear force was carried in the compression zone, the bond force moment on the concrete teeth between cracks was resisted entirely by flexure at the tooth root (Figure 2a). A number of workers, among them Moe² and Fenwick & Paulay³, pointed out that this mechanism is not possible as the flexural stresses in the tooth root were much too high to be resisted by the concrete. In order that the tooth should not break off prematurely, some of the bending moment produced by the bond forces must be resisted by shears carried across the crack, as shown in Figure 2b. This means that a diagonal tensile stress field forms across the tooth, leading to the cracking shown in Figure 2b. This form of cracking exists in practice. Further evidence of the importance of shear transfer in the tension zone in providing stability came from the work of Kani⁴ who postulated shear failure as the ripping of the concrete teeth from the rest of the beam. Without the strengthening effect of the shear transferred across the cracks, Kani found that his theory would give good results only if very short teeth, of the length shown shaded in Figure 2a, were used. Finally, it was shown experimentally^{3, 5} that beams constructed with preformed smooth-sided cracks, conforming with Kani's simple model, failed by the teeth tearing off but at a lower load than would be expected from a beam without preformed cracks. Figure 3 shows such a beam at failure and it can be seen that the crack pattern is very unlike the failure crack pattern of a normal beam.



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The forces carried across the shear crack in a beam may be considered in the form of a free body diagram (Figure 4). From this Figure, three simple equilibrium equations may be stated:

$$P_c + P_{ih} - P_s = 0 \quad \dots (1)$$

$$V - V_1 - V_2 - V_3 = 0 \quad \dots (2)$$

$$V a_h - V_2 s - P_s z - V_3 n + P_{ih} (z - t) = 0 \quad \dots (3)$$

At this point, the stage was set for investigators to carry out experiments to quantify the relative contributions to the vertical equilibrium, equation 2. Before this work is discussed and the recent work carried out at the Cement and Concrete Association is presented, it is worth considering the ways of designing experiments to investigate methods of force transfer within beams. Two basic forms of experiment are possible: either the particular form of force transfer is isolated by designing the beam with the other forms either omitted or replaced by instrumented links; or, basic tests are carried out, not necessarily on a beam but in a rig in which the same force-deformation behaviour recorded in beam tests can be imposed. Preferably both forms of experiment should be carried out and the results compared. The first method has the advantage that it is direct and the disadvantage that the test specimen may not behave in exactly the same way that a complete beam would. The second method, on the other hand, has the advantage that the test results are related to real beam displacements and stiffnesses and the disadvantage that the test is indirect. An example of the use of the two methods is shown in Figure 5 for dowel tests. Here the upper test is of the direct type and the lower test is indirect. In this case, the direct test gives only the ultimate dowel strength; the indirect test gives the dowel force in a beam at all load stages up to and including failure.

EXPERIMENTAL PROGRAMME

Displacements

The first stage in the work carried out at the Cement and Concrete Association was to test complete beams with instrumentation capable of measuring the relative displacements across cracks both at the steel level and higher up the beam. Crack width measurements were not sufficient; a knowledge of the normal and shear displacements across the cracks was also required. Nine beams were tested^{5, 6} of which three were fully instrumented. Displacements were measured across the cracks by using a delta rosette of gauge points for a demountable mechanical strain gauge and taking readings at all load stages. These rosettes do not show the complete picture from the formation of cracks onwards, as they were only stuck onto the beam after the cracks had formed. Figure 6 shows the crack pattern and rosette layout for one of the beams. A typical set of rosette results is also given. A linear relationship was found in nearly all cases between Δv , the vertical displacement between two points initially on opposite sides of a crack, and ΔH , the horizontal displacement. The slope of the graphs was assumed to be a linear function of the crack geometry, expressed by $\frac{\delta V}{\delta H}$. A plot of $\frac{\Delta H}{\Delta V}$ against $\frac{\delta V}{\delta H}$ is shown in Figure 7.

It seems reasonable that $\frac{\Delta V}{\Delta H}$ should be proportional to $\frac{\delta V}{\delta H}$ and

Figure 7 supports this view. Displacement measurements of this type are of great importance to the fundamental study of shear and much remains to be done in this area in the future. It is necessary to know how the shear displacements are related to the important parameters of beam shape, steel percentage and degree of shear reinforcement before the results of basic aggregate interlock and dowel tests can be generally applied.

Dowel Forces

A number of studies of dowel forces in beams has been carried out, notably by Krefeld & Thurston⁷, Fenwick³ and Baumann⁸. In these, a variety of test specimens was conceived, having the basic factor in common that a bar was cast projecting from a concrete face, to simulate conditions at a crack; this bar was then pulled transversely and sometimes longitudinally until it split out of the concrete. From a study of the previous test arrangements, it was decided that the specimens tested by Krefeld & Thurston⁷ approximated closest to conditions in a beam and this design was therefore adopted both for specimens of the size used by Krefeld & Thurston and for ²/₇ scale models of them. One significant difference between these new tests and the previous ones was that the complete load-displacement behaviour of the dowels was measured, including the falling branch. The test rig that was developed for the model specimens is shown in Figure 8 and diagrammatically in Figure 9.

The test specimens were cast with a centre section separated from the rest of the beam by a preformed crack bridged only by the dowel bars. In the test rig the centre section of the beam was connected to a lever by a threaded rod which passed through the lever and had a load cell and thrust washer below it. Load was applied by tightening the screw which lifted the lever up against an upper stop. The lever was then balanced by adding lead shot to a bucket at its outer end. If the dowel split, the bucket dropped onto a lower stop, restraining the splitting. In order to follow the falling branch, a fixed displacement could be given to the dowel and the bucket load could be adjusted until it floated between the stops. Load and displacement measurements were taken only when the lever was in its floating position. This rig did not prove completely successful in following falling branches, particularly immediately after cracking, but falling branches were observed and these were found to be significant when the beam displacements were considered.

Altogether 34 model tests and 12 prototype tests were carried out to study the parameters of concrete strength, number and layout of reinforcing bars, shear span, cover and preformed crack width. The dowel load-displacement curve was found to approximate to that shown in Figure 10. After an initial steep section the curve flattens out at the dowel splitting load (the load at which a horizontal split formed on the outside of the beam at the steel level) and then follows a steep falling branch before levelling out at approximately half of the splitting load. The dowel splitting force was found to fit the following function

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$$P_{sp} = 9.1 + 0.0001 \left[\sum (c_s + c_i) \right]^2 f_t \text{ kN}$$

where f_t is the splitting tensile strength of the beam concrete (N/mm^2).

$\sum (c_s + c_i)$ is the width of the concrete failure surface (mm).

c_s side cover to bars (mm).

c_i distance between bars (mm).

In Imperial units, the above equation is

$$P_{sp} = 2040 + 0.1 \left[\sum (c_s + c_i) \right]^2 f_t \text{ lbf}$$

where c_s and c_i are in inches and f_t is in lbf/in^2 .

The curve up to P_{sp} is defined by the expression

$$P_s = 1.55 P_{sp} \Delta^{0.25} \text{ kN}$$

or

$$P_s = 3.48 P_{sp} \Delta^{0.25} \text{ lbf in Imperial units}$$

It is worth recognizing at this stage that the bar diameter was not considered as a variable in the test series because the dowel study was carried out primarily to estimate the dowel forces in the beams from the displacement tests. Baumann⁸ showed in his work that dowel strength increased with bar diameter.

The test series that has been described briefly here has been reported fully in a report produced by the Cement and Concrete Association⁹. The significance of the dowel forces estimated from these tests is discussed later in this paper.

Aggregate Interlock Tests

Two previous investigations on this subject had shown that aggregate interlock forces were significant but at the same time had highlighted that it was very difficult to devise a perfect test method.

Gergely¹⁰ tested a beam in which one end had a preformed smooth-sided diagonal crack while the other end was completely sound. The end with the preformed crack failed at a considerably lower load than the sound end and, from this, Gergely estimated that the contribution of the aggregate interlock force to the shear capacity was 45%. Fenwick & Paulay³ in their pioneering work carried out interlock tests indirectly by applying a shear force parallel to a preformed crack in a piece of plain concrete. The crack was held at a fixed width throughout each test and Fenwick was therefore able to produce a series of plots of shear stress against shear displacement for various crack widths.

Although this latter method had the advantage that it was indirect, the displacements that were imposed on the test specimen did not match the displacements measured in beams, as shown in Figure 6. Cracks open and shear simultaneously; they do not open to their final width and then shear. The test rig that was used in the Cement and Concrete Association tests, shown diagrammatically in Figure 11 and with a specimen installed in Figure 12, was designed to overcome this problem. The ratio of normal to shear displacement could be changed between tests but was constant during the test, simulating the measured behaviour shown in Figures 6 and 7. The test specimen, an unreinforced concrete block with a pre-cracked section 127 mm long and 140 mm wide (5 in. x 5 1/2 in.), was bolted to a pair of linked cross-heads. The lower crosshead was bolted to the floor and the upper crosshead was pulled horizontally. The linkages were made in the form of a parallel ruler system so that vertical and horizontal displacements were induced simultaneously. By casting test blocks with cross-head fixing grooves at various spacings, it was possible to set up the specimen with varying values of the angle α_i ; each value, for the small displacements used in the test, defined a different ratio of normal to shear displacement.

Displacements across cracks in previous tests on beams were measured in the vertical and horizontal directions, giving the ratio $\Delta V/\Delta H$. These displacements may be converted to normal and shear displacements, ΔN and ΔS , once the slope of the crack at that point is known. The ratio of $\Delta N/\Delta S$, $\cot\alpha$, was defined in the block tests by the slope of the links.

The results of the block tests showed that the ultimate aggregate interlock strength and the interlock stress-displacement curve depended on the concrete strength, aggregate type and displacement ratios imposed by the test specimen.

The ultimate aggregate interlock stresses, obtained with a $\Delta N/\Delta S$ value of 1.0, are shown plotted against concrete strength in Figure 13. It can be seen that an essentially linear relationship was obtained, except for three specimens which had a high-strength concrete made with limestone aggregate. In these cases, the aggregate did not tear out of the matrix to give a rough crack but tended to split. The crack surface was therefore relatively smooth, giving low interlock strengths. Tests using the same aggregate and a weaker concrete gave a higher ultimate strength. The ultimate interlock stress, f_{su} , was found to be related inversely to $\Delta N/\Delta S$, the ratio of normal displacement to shear displacement, as shown in Figure 14. In the absence of a complete series of tests for all concrete strengths and displacement ratios, the relationship shown in Figure 13 was used to modify the relationship in Figure 14 which was constructed for a concrete strength of 32 N/mm².

The shape of the aggregate interlock-displacement curves was also found to be dependent on the ratio of normal displacement to shear displacement. A series of such curves for sensible values of $\Delta N/\Delta S$, with the strength plotted in the non-dimensional form f_s/f_{su} , is shown in Figure 15. The curves were all trilinear with the coordinates of turning points

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f_s/f_{su}	ΔN (mm)
0.55	0.06 (0.0024 in.)
0.7	0.15 (0.0059 in.)
1.0	ΔN_u

The ultimate normal displacement was found to be a function of the displacement ratio, in the form shown in Figure 16.

Summarizing the test results, it is now possible to show an interlock stress-displacement curve for comparison with a beam test.

Firstly, the ultimate interlock strength can be obtained from Figure 13, which has been drawn from the results of tests in which $\Delta N/\Delta S = 1.0$.

Secondly, knowledge of the $\Delta N/\Delta S$ relation for the beam crack allows a value of f_{su} to be read from Figure 14. The Figure 13 stress may then be modified to suit the correct displacement ratio and concrete strength.

Finally, the appropriate value of ΔN_u can be obtained from Figure 16 and, with the rest of the coordinates given before, the interlock stress-displacement curve of the form shown in Figure 15 may be constructed.

The test method enables the normal forces across the crack to be measured, as well as the shear forces, and these had similar relationships to the ones already described. The ultimate normal stress across the crack was a function of $\Delta N/\Delta S$ but was about one-quarter of the shear stress. These tests are described more fully in reference 11.

In the direct aggregate interlock tests, a beam was cast with cut-outs both in the tension zone around the steel and in the compression zone. The beam was instrumented so that all the forces apart from the interlock forces could be measured.

The beam was cast, in the form shown in Figure 17, without the inclined crack which was formed during the test. The main steel in the cut-out was instrumented with points for a demountable mechanical strain gauge with which the steel strains would be measured. The dowel force carried by the steel was assumed to be negligible. When the beam was set up to be tested, the compression zone notch was fitted with a steel packer, a jack and a load cell so that the load carried by it could be varied and measured accurately during the test. The compression zone jack reacted against a plate which was separated by a roller system from the rest of the beam. The plate was clamped to the rest of the beam via two pairs of pre-loaded load cells. The shear force carried by the compression zone was therefore measured as the change in the load cell readings. A total of six tests was carried out, of which four were exploratory and used to develop the test method.

In the final, successful tests, the beam was loaded as shown in Figure 17 until the inclined crack formed. Once the crack formed, it was possible to adjust the compression zone force until the crack passed through the beam with the trajectory shown in the Figure, without causing a shear failure. The beam was finally loaded until it failed in shear, at which point the ultimate interlock shear stress was 1.27 N/mm^2 (184 lbf/in^2) and the ultimate normal stress was 0.39 N/mm^2 (56 lbf/in^2). These two test results from the final successful beam tests are shown on Figure 13 and good agreement can be seen with the results of the block tests.

The two test methods, direct and indirect, gave similar results and both pointed to the fact that aggregate interlock forces are likely to provide a significant contribution to the shear capacity of a member.

Compression Zone Shears

The last remaining section of the free body shown in Figure 2 which can carry shear force is the compression zone. This was the subject of the experimental investigations reported in references 5 and 6.

The shear stresses were estimated by using a grid of electrical resistance gauges located as shown in Figure 18. Shear stresses may be calculated from the expression

$$\tau_{xy} = \int_0^y \frac{\partial \sigma_x}{\partial x} dy \quad \dots (4)$$

Thus, once the rate of change of stress along the beam is measured, functions for σ_x against distance along the beam may be derived for various levels and the slope of these at a section may be integrated numerically to give the shear stress. A simple transform of this equation enables these calculations to be carried out with only one line of gauges, read at different loads:

$$\tau_{xy} = \int_0^y \frac{\partial \sigma_x}{\partial M} \frac{\partial M}{\partial x} dy \quad \dots (5)$$

It should be recognized that these calculations can give only an idea of the average level of shear stress in the compression zone, as the functions of σ_x against distance along the beam were not found to be strongly influenced by the position of cracks. It is doubtful whether there are large differences in the distribution of shear stress in compression zones above and between cracks. The transform used in equation 5 also assumes that local variations in stress around the heads of cracks are insignificant, as longitudinal stress is taken to be a unique function of moment.

Measurements of the type mentioned above showed that up to 40% of the beam shear could be traced to the compression zone, in a roughly parabolic distribution.