

SP 43-1

Code Criteria For Deflection Limitation In Reinforced Concrete Structures

By

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Synopsis: Code criteria for deflection limitation in reinforced concrete structures, from a number of countries, are reviewed and the two major methods used for deflection limitation are discussed. These methods are: limiting deflections to a fraction of the span length, and limiting the span-to-depth ratio.

Keywords: building codes; creep properties; deflection; flexural strength; loads (forces); reinforced concrete; shrinkage; span depth ratio; standards; stiffness.

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INTRODUCTION

The fact that so many prominent authors have contributed to this symposium indicates that deflection has become an important consideration in the design of reinforced concrete structures. Further evidence of its importance is that all of the codes of practice, either existing or proposed, and committee recommendations that were reviewed include some sort of provision for deflection limitation. In general, two methods are used to limit deflections; specifying maximum deflection as a fraction of the span length, and specifying the maximum span-to-depth ratio. Both methods will be discussed in this paper by citing examples of codes using each method. The examples will be taken from the documents reviewed and listed in the Bibliography.

ALLOWABLE DEFLECTIONS

If deflection is limited by the method of specifying allowable deflections, the designer is faced with two problems, determining the value of the limiting deflection and determining the method to be used for evaluation of the deflection of the structure. Before discussing these problems, it should be noted that analysis and design for deflection should be significantly different from analysis and design for strength. When a concrete member is designed for strength, inherent in the procedure are a number of assumptions, factors of safety for stress, factors of safety for load, and, in general, conservative estimates of all quantities involved. If the same approach is used to evaluate deflections or allowable span-to-depth ratios, unnecessarily conservative structures will result. Such computation should predict the true deformation of the structure as accurately as possible, with all factors of safety set equal to one.

Consider first the problem of the limiting deflection. There are a number of reasons, other than structural

integrity, for setting limits on deflections. Some of these are: aesthetic, physiological, psychological, and the effect on non-structural elements. In addition, it is important to consider what component of the deflection should be limited. For example, if the effect on non-load bearing walls is being considered, the limitation should be placed on the deflection that will occur after the wall is constructed.

In 1968, ACI 435 (1) proposed the allowable or limiting deflection shown in Table 1. Table 2, taken from the Czech code Design of Concrete Structures (2) provides a second example. Both tables indicate the importance of considerations other than structural integrity, and the importance of incremental deflections.

Consider now the problem of calculating the deflection to be compared with the limiting deflection taken from, for example, Table 1 or Table 2. As 12 of the papers in this volume are concerned with this subject, a detailed discussion is not appropriate. However, a brief discussion of the important parameters involved in such calculation might be helpful. In a recent meeting of C. E. B., Dr. J. Brakel presented a comprehensive report on the calculation and limitation of deflections. The following is a summary of Dr. Brakel's comments on the parameters involved in deflection calculation.

1. Type of loading and support conditions

This can generally be taken into account by writing the deflection equation as

$$\delta = \beta \frac{M \ell^2}{EI}$$

where β is a correction factor for loading and support conditions which can be calculated from basic principles.

2. Variation in stiffness

Branson (3) and the ACI (4) code use an effective moment of inertia (I_e) to include the effect of stiffness variation caused by various amount of cracking along the length of the beam. I_e is a function of the cracking moment, maximum moment, the uncracked moment of inertia, and the cracked moment of inertia. Other methods utilize a simple deflection correction factor that depends on the moment distribution and cracking moment.

3. Creep curvature

The increase of curvature due to creep may be taken into account by multiplying the instantaneous curvature by a factor K , or by replacing the instantaneous modulus of elasticity by a reduced creep modulus.

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4. Shrinkage curvature

Restrained shrinkage causes force in the steel which results in an additional bending moment, usually in the direction of the applied loads, thereby increasing deflection.

5. Other factors

Depending on the accuracy required, it is also possible to consider the contribution of other variables such as the uncracked concrete under the neutral axis, and compressive reinforcement.

LIMITING SPAN-TO-DEPTH RATIOS

From a design point of view, it is probably easier to satisfy a limiting span-to-depth ratio than it is to satisfy an allowable deflection. However, this approach will most likely result in a more conservative design because it is difficult to include all of the appropriate influences in a single span-to-depth ratio. Therefore such ratios tend to be set at conservative levels. Many of the codes reviewed use the concept of limiting depth or span-to-depth ratio but the most comprehensive treatment is that of the British Code of Practice for Reinforced Concrete (5). The allowable span-to-depth ratio is evaluated in terms of support conditions, percentage of reinforcement, the ratio of permanent to total load, and mix and environmental conditions. Thus it can be seen that this method attempts to take into account any of the important parameters involved in the calculation of deflection. The basic span-to-depth ratios are given for various span lengths and support conditions in Table 3. A member is assumed to be fully fixed if

$$\frac{\Sigma K_1 + \Sigma K_2}{K_\beta} > 4$$

and partially fixed if

$$\frac{\Sigma K_1 + \Sigma K_2}{K_\beta} > 2$$

Where K_β is the stiffness of the member considered and ΣK_1 and ΣK_2 are the sums of the stiffnesses of all other members framing into ends 1 and 2 of member β . These basic values (Table 3) are then modified by multipliers for reinforcement, long-term effects, and mix proportions and environment. The multipliers for reinforcement are given in Table 4 and Table 5 gives the multipliers for long-term effects. Note that the effect of compressive reinforcement has been included. Multipliers for mix proportions and environmental conditions are given in Table 6. Creep coefficient is defined as creep plus elastic strain divided by elastic strain.

Once the span-to-depth ratio obtained by the above method is satisfied by design it may be assumed that the structure is adequate to withstand both long-term and short-term deflections.

SUMMARY AND CONCLUSION

It can be seen from the above discussion that codes of practice not only consider deflection control of reinforced concrete structures, but, in some cases, do so in a rather comprehensive manner. Either method, maximum deflections or maximum span depth ratios, can be used effectively to limit deflections if the following general procedures are followed in the analysis:

1. Evaluate as realistically as possible material properties, loading history and environmental conditions.
2. Set all factors of safety equal to 1.
3. Set all load factors equal to 1.
4. Consider the effects on deflection due to the following characteristics:
 - a. Type of loading and support conditions
 - b. Variation in stiffness
 - c. Creep
 - d. Shrinkage
 - e. Other effects as necessary for the structure under consideration

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Table 1
Deflection Limitations from ACI 435

REASONS FOR LIMITING DEFLECTION	EXAMPLES	DEFLECTION LIMITATION	PORTION OF TOTAL DEFLECTION ON WHICH THE DEFLECTION LIMITATION IS BASED
1. Sensory acceptability			
1.1 Visual	Droopy cantilevers and sag in long span beams	By personal preference	Total deflection
1.2 Tactile	Vibrations of floors that can be felt Lateral building vibrations	L/360	Full live load
		No recommendation	Gust portion of wind
1.3 Auditory	Vibrations producing audible noise	Not permitted	
2. Serviceability of structure			
2.1 Surfaces which should drain water	Roofs, outdoor decks	L/240	Total deflection
2.2 Floors which should remain plane	Gymnasias and bowling alleys	L/360 + camber or L/600	Incremental deflections after floor is installed
2.3 Members supporting sensitive equipment	Printing presses and certain building mechanical equipment	Manufacturer's recommendations	Incremental deflections after equipment is leveled

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Table 1
Deflection Limitations from ACI 435 (Continued)

REASONS FOR LIMITING DEFLECTION	EXAMPLES	DEFLECTION LIMITATION	PORTION OF TOTAL DEFLECTION ON WHICH THE DEFLECTION LIMITATION IS BASED
3. Effect on nonstructural elements			
3.1 Walls	3.1.1 masonry and plaster	$L/600$ or 0.30 in. (7.6 mm) max or $\phi = 0.00167$ rad.	Incremental deflections after walls are constructed
	3.1.2 metal movable partitions and other temporary partitions	$L/240$ or 1 in. (25.4 mm) max	Incremental deflections after walls are constructed
	3.1.3 lateral building movement	0.15 in. (3.8 mm) offset per story $0.002 \times$ (height)	Five min sustained wind load
	3.1.4 vertical thermal movement	$L/300$ or 0.60 (15.2 mm) max	Full temperature differential
3.2 Ceilings	3.2.1 plaster	$L/360$	Incremental deflection after ceiling is built
	3.3.3 unit ceilings such as acoustic tile	$L/180$	

Table 1

Deflection Limitations from ACI 435 (Continued)

REASONS FOR LIMITING DEFLECTION	EXAMPLES	DEFLECTION LIMITATION	PORTION OF TOTAL DEFLECTION ON WHICH THE DEFLECTION LIMITATION IS BASED
3.3 Adjacent building elements supported by other members	Windows, walls and folding partitions on unyielding supported below the deflecting member	Absolute deflection limited by tolerances built into the element in question	Incremental deflection after building element in question is constructed
4. Effect on a structural elements			
4.1 Deflections causing instability of primary structure	Arches and shells Long columns	Effect of deflections on stresses and stability of the structure should be taken into account in the structural design of the element	
4.2 Deflection causing different force system or change in stresses in some other element	Beam bearing rotation on masonry wall	Effect of deflections on the stresses and stability of the structure should be taken into account in the structural design of the element	
4.3 Deflections causing dynamic effects	Resonant vibrations which increase static deflections and stresses such as those produced by wind, dancing, moving loads and machinery	Dynamic deflections should be added to static deflections and the total should be less than the limitations imposed for other reasons	

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Table 2
Deflection Limitations from the Czechoslovak Code

LINE	STRUCTURAL ELEMENTS			LIMIT DEFORMATION		DEFORMATION CONSIDERED FOR THE CHECK
				SLOPE OF DEFLECTION CURVE AT THE POINT OF LOAD	DEFLECTION AT MIDSPAN	
0	1	2	3	4	5	6
1	Beams of gantries for	hand cranes		0.0030	$l/500$	deformation after placing and leveling of rails
2		electrically powered with max. load	less than 50 t	0.0025	$l/600$	
3			over 50 t	0.0020	$l/750$	
4		suspended cranes		0.0040	$l/400$	
5	Beams of industrial floors	with railway traffic	regular	0.0025	$l/600$	deformation after leveling of floors
6			narrow gage	0.0040	$l/400$	
7		without railway traffic	girders	0.0040	$l/400$	
8			other beams	0.0060	$l/250$	
9	ceilings and walls with compact (for example covered by plaster) surfaces			0.0090	$l/350$	deformation after finishing the surface
10	ceiling and walls mountable surfaces (for example suspended ceilings)			0.0150	$l/200$	

Table 2

Deflection Limitations from the Czechoslovak Code (Continued)

LINE	STRUCTURAL ELEMENTS			LIMIT DEFORMATION		DEFORMATION CONSIDERED FOR THE CHECK
				SLOPE OF DEFLECTION CURVE AT THE POINT OF LOAD	DEFLECTION AT MIDSPAN	
0	1	2	3	4	5	6
11	floors made out of prestressed elements			-	-1.5 cm	short-term part of deflection due to pre- stressing and due to dead load
12	axially loaded elements with lining			-	depends on the kind of surface	after finishing the surface
13	floors in apartment and public buildings and elements of outside walls	$l_v \leq 6 \text{ m}$		0.0150	$l_v/200$	after finishing the ceiling or the floor
14		$l_v > 11 \text{ m}$		0.0100	$l_v/300$	
15	floors under rooms crowded by people	flat reefs and terraces	reinforced concrete	0.0150	$l_v/200$	short-term deformation due to movable live load
16			prestressed concrete	0.0090	$l_v/350$	
17		other	reinforced concrete	0.0090	$l_v/350$	
18			prestressed concrete	0.0055	$l_v/600$	

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Table 2
Deflection Limitations from the Czechoslovak Code (Continued)

LINE	STRUCTURAL ELEMENTS			LIMIT DEFORMATION		DEFORMATION CONSIDERED FOR THE CHECK
				SLOPE OF DEFLECTION CURVE AT THE POINT OF LOAD	DEFLECTION AT MIDSPAN	
0	1	2	3	4	5	6
19	floors under partition walls	compact partition walls	parallel to the plane of bending	0.005 max. 2.5/l	$l/800$ max. 0.8 cm	deformation after placing of partition wall
20			perpendicular to the plane of bending	0.0017	-	
21		flexible partition walls, vertically movable partition walls		0.013 max. 8/l	$l/250$ max. 2.5 cm	
22	horizontal elements deformed due to temperature differences in columns			-	$l/300$ max. 1.5 cm	deformation due to temperature differences
23	columns of the frames (total and relative horizontal displacements of girders			-	$l/500$ max. 0.4 cm in a storey	due to wind effects
24	structural elements adjacent to window and door frames, movable walls, etc.			as specified for the built-in element		after placing the element
25	floors of gymnasia, dancing rooms, etc.	to avoid vibration	reinforced concrete	0.0045	$l/700$	short-term deformation due to live, movable
26			prestressed concrete	0.0025	$l/1200$	
27		to ensure plans surface of floors		0.0055	$l/600$	after finishing the floors
28	floors under all utility spaces			0.0210	$l/150$	after finishing the floors

Table 3

Basic Span-to-Effective Depth (l/d_1) Ratios

Span Meters (Ft.)*	SUPPORT CONDITIONS			
	Cantilever	Simply Supported	Partially Fixed	Fully Fixed
2.5 (8)*	9	20	25	30
3.0 (10)	7	20	25	30
4.5 (15)	5	20	25	30
6.0 (20)	4	20	25	30
7.5 (25)		20	25	30
9.0 (30)		20	25	30
10.5 (35)		17	22	26
12.0 (40)		15	19	22
15.0 (49)		12	15	18
18.0 (59)		10	12	15

* Approximate equivalents in feet

Table 4

Multipliers for Reinforcement

Pt fy (N/mm²)	≤0.5	0.75	1.0	1.25	1.50	2.0	3.0	6.0
250 (36)*	1.80	1.80	1.62	1.49	1.40	1.24	1.11	1.02
300 (44)	1.80	1.57	1.39	1.25	1.13	1.04	0.91	0.85
350 (50)	1.57	1.34	1.19	1.07	0.99	0.89	0.79	0.73
400 (58)	1.38	1.18	1.04	0.93	0.87	0.78	0.69	0.64
420 (60)	1.31	1.12	0.99	0.89	0.83	0.74	0.66	0.61
450 (65)	1.22	1.05	0.92	0.83	0.77	0.69	0.62	0.57
460 (67)	1.20	1.02	0.90	0.81	0.76	0.68	0.60	0.56
500 (73)	1.10	0.94	0.83	0.75	0.70	0.62	0.55	0.51

* Approximate equivalent in ksi

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Table 5

Multipliers for Long-Term Effects

RATIO OF PERMANENT LOAD TO TOTAL LOAD	RATIO OF AREA OF COMPRESSION REINFORCEMENT TO AREA OF TENSION REINFORCEMENT		
	0.3	0.5	0.7
0.85 - 1.00	0.90	1.10	1.30
0.65 - 0.85	1.00	1.20	1.40
- 0.65	1.15	1.35	1.55

Table 6

Multipliers for Concrete Mix and Environmental Conditions

IMPORTANCE DEFLECTION CONTROL	CREEP, SHRINKAGE AND TEMPERATURE EFFECTS		
	Category 1	Category 2	Category 3
CRITICAL	1.00	0.85	0.70
NOT CRITICAL	1.20	1.00	0.85

Category 1: Shrinkage less than 0.0006
Creep coefficient less than 3.5

Category 2: Shrinkage coefficient between 0.0006 and 0.00075
inclusive
Creep coefficient between 3.5 and 4.0 inclusive

Category 3: Shrinkage coefficient greater than 0.00075
Creep coefficient greater than 4.0

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Computation of Deflections For Beams And One-Way Slabs

By

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Synopsis: The paper presents a condensed state-of-the art report on practical procedures for computing short-time and long-time deflections for beams and one-way slabs. For short-time (immediate) deflection, approximate procedures are discussed for continuous beams, using (a) the midspan flexural rigidity only; (b) the simple average (ACI method) considering flexural rigidities at the continuous ends as well as at midspan; and (c) the weighted average where the effect of flexural rigidities at the continuous ends and at midspan are used in proportion to the magnitudes of moments. The creep and shrinkage effects on long-time deflection are discussed, including the simple ACI multiplier method as well as other more exact procedures involving creep and shrinkage considered separately and considered combined. An expression for span-to-depth ratio limitation is presented that includes the effects of cracking and sustained loads. Numerical examples are included illustrating the methods discussed.

Keywords: beams (supports); compressive strength; concrete slabs; continuous beams; cracking (fracturing); creep properties; deflection; flexural strength; loads (forces); modulus of elasticity; moments of inertia; reinforced concrete; reviews; shrinkage; span depth ratio; standards.

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INTRODUCTION

The permissible deflection is primarily governed by the amount which can be tolerated by the interacting components of the structure. Both the short-time and the long-time effects must be considered. The acceptable deflection depends on many factors, among which are the type of building (warehouse, school, factory, residence, etc.), the presence of plastered ceilings, the type and arrangement of partitions, the sensitivity of equipment to deflection, and the magnitude and duration of live load. Such factors as these were considered in establishing the maximum allowable deflection limits included in Table 9.5(b) of the 1971 ACI Code [1].

These limitations on deflection are somewhat arbitrary. Historically, $L/360$ has been the accepted limit to prevent the cracking of plastered ceilings. Other limits should be considered as guidelines, with the designer having the responsibility for evaluating the possible adverse effects of excessive deflection in any given situation.

ACI Committee 435 has reported recommendations for allowable deflections for a great variety of situations [2].

DEFLECTIONS FOR ELASTIC SECTIONS

In general, the maximum deflection $a_{\max}$ in an elastic member may be expressed as:

$$a_{\max} = \beta_a \frac{WL^3}{48EI_c} \quad (1)$$

where W = total load on the span; L = span length; E = modulus of elasticity; I_c = moment of inertia of a reference section in the member; and β_a = a coefficient which depends on the degree of fixity at supports, the variation in moment of inertia along the span, and the distribution of loading.

For a uniformly loaded member with unequal end moments, as shown in Fig. 1, the general expression for elastic midspan deflection is

$$a_m = \frac{5L^2}{48EI} [M_s - \frac{1}{10}(M_a + M_b)] \quad (2)$$

The midspan deflection (Eq. 2) may be taken as maximum deflection in the span for practically all prismatic members, even though the absolute maximum deflection will occur at midspan only when the loading is uniform and the end moments are equal.

Continuous Beams Having Variable Flexural Rigidity

When significant changes of gross cross section along the span length are involved, such changes should be included in the analysis for moments. When deflections are desired for such cases, the best approach is to account for the actual variation in flexural rigidity (EI).

As an approximation, it is suggested to use Eq. (2) even when variable flexural rigidity is involved. In such cases, one may use a single adjusted value of flexural rigidity, EI . Three methods have been suggested [3].

Midspan Value Alone--With this assumption the EI for use in Eq. (2) is

$$EI = (EI)_c \quad (3)$$

where $(EI)_c$ = flexural rigidity at midspan. This is the simplest method and when the inflection point is between about $0.2L$ and the support, the results are within ± 20 percent of those obtained using variable EI , as long as $0.33 \leq \alpha \leq 1.0$ (where $\alpha = [EI \text{ at midspan}] / [EI \text{ at end}] = (EI)_c / (EI)_e$. When α lies between 0.50 and 1.0 the results are within ± 5 percent of those obtained using variable EI . A recent study [31] suggests this method may be the most practical for ordinary design situations.

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is Simple Average--With this assumption the average EI

$$EI = \frac{\frac{1}{2}[(EI)_{e1} + (EI)_{e2}] + (EI)_c}{2} \quad (4)$$

where $(EI)_{e1}$ and $(EI)_{e2}$ are the flexural rigidities at the two ends of the span. It is suggested that the use of both $(EI)_{e1}$ and $(EI)_{e2}$ is appropriate only when there are moments at both ends. For end moment at one end, it is appropriate to use $EI = \frac{1}{2}[(EI)_e + (EI)_c]$. ACI-9.5.2.2 suggests using this method.

Weighted Average--In this method the adjusted EI is obtained by weighting the flexural rigidities in accordance with the magnitudes of the end moments [3]; or

$$EI = (EI)_c \left[1 - \left(\frac{M_a + M_b}{2M_o} \right)^\phi \right] + \frac{(EI)_{e1} + (EI)_{e2}}{2} \left(\frac{M_a + M_b}{2M_o} \right)^\phi \quad (5)$$

where $\phi = 4$ has been suggested [3] when the points of inflection lie between $L/8$ and $L/3$ from the supports.

MODULUS OF ELASTICITY

Ordinary beam theory presumes the modulus of elasticity in tension to be the same as in compression, usually using the secant modulus. In a reinforced concrete section, creep and the magnitude of stress affect the apparent modulus on the compression side. On the tension side, at low-moment sections with the concrete uncracked, the secant modulus is essentially the same as in compression; however, it markedly reduces as the stress nears the cracking level.

In design, a statistically determined ACI code-specified [1] elastic modulus is used,

$$E_c = w^{1.5} \quad 33 \sqrt{f'_c} \quad (6)$$

or for normal weight concrete

$$E_c = 57,000 \sqrt{f'_c} \quad (f'_c \text{ in psi}) \quad (7)$$

MOMENT OF INERTIA

The effective moment of inertia, even for sections whose external dimensions are constant, varies considerably along a span. Consider for example a span with a T-shaped section in a continuous beam or frame, as shown in Fig. 2. In such a continuous structure, at or near the support the concrete slab is cracked, and the effective section is as shown in section A-A, while in the positive-moment zone the stem is cracked, with the resulting effective section as shown in section B-B. Furthermore, near the points of inflection nearly the entire section may be uncracked and fully effective, as shown in section C-C.

In an elastic frame analysis, only relative flexural rigidity values are required; but in deflection computations, the actual magnitudes for E_c and I must first be used. Moreover, the amount of deflection under each increment of load is not constant. The flexural rigidity $E_c I$ is greater under low stress, since the fully uncracked section provides the greatest effective moment of inertia.

For loads below the cracking load, deflections may be based on the gross concrete section, with generally a small difference arising from whether or not the transformed area of reinforcement is also included. However, as the load increases above the cracking load, the moment of inertia decreases and approaches that of the cracked transformed section, although it may be greater between cracks.

Prior to 1971, either gross moment of inertia or transformed cracked section moment of inertia was used. Generally, as shown by the typical load deflection curve in Fig. 3, the use of gross section underestimated deflection, and the use of cracked transformed section overestimated the deflection. However, the degree of accuracy was affected by the magnitude of the service load compared to the cracking load.

The 1963 ACI Code prescribed using gross section when ρf_y did not exceed 500, and using transformed cracked section when ρf_y exceeded 500. This procedure provided satisfactory results when ρf_y was either well below or well above 500. However, it gave an abrupt discontinuity instead of a gradual transition for the moment of inertia.

The 1971 ACI Code (Formula 9-4) uses the following as an effective moment of inertia, as proposed by Branson [4] in 1963, which includes the effect of load level and degree of cracking:

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$$I_e = \left(\frac{M_{cr}}{M_{max}} \right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_{max}} \right)^3 \right] I_{cr} \leq I_g \quad (8)$$

where $M_{cr} = f_r I_g / y_t$ = cracking moment

M_{max} = maximum moment for the loading condition

under which deflection is computed

I_g = gross moment of inertia (neglecting reinforcement)

I_{cr} = cracked transformed section moment of inertia

f_r = modulus of rupture (taken by the ACI Code as $7.5 \sqrt{f'_c}$ for normal-weight concrete)

y_t = distance from neutral axis to extreme tension fiber.

The effective moment of inertia I_e from Eq. (8) is to be calculated at the location of maximum moment as a single value for the entire span in the case of simply supported beams, or as a single value between points of inflection in continuous beams. If one wishes to recognize the continuous variation of the moment of inertia along the span, Branson proposed using the fourth power instead of the third power in Eq. (8). In this case, M_{cr} and M_{max} are the cracking moment and the applied moment, respectively, at each section along the span. The span may be broken into segments with each segment having a different moment of inertia and numerical integration used to compute the deflection. A graphical method [30] is available for evaluation of Eq. (8).

Equation (8) was developed from a statistical study of 54 test specimens which had M_{max}/M_{cr} values from 2.2 to about 4 and I_g/I_{cr} values from 1.3 to 3.5. The study included simple-span rectangular beams [5], T-beams [6] and two-span continuous rectangular beams [7]. Branson has provided an excellent summary [8] of background material relating to Eq. (8).

SHORT-TIME DEFLECTIONS IN DESIGN

Throughout the history of reinforced concrete construction, computations of short-time (instantaneous) deflection usually involved using either cracked transformed section (Maney, 1914; Swain, 1924; and ACI Committee 307, 1931) [9], or gross uncracked section (Portland Cement Assn., 1947) [10]. In either case, Eq. (1) is suitable because it may be rewritten as

$$a = \beta \frac{ML^2}{E_c I_e} \quad (9)$$

where β = coefficient based on load and support conditions
 I_e = effective moment of inertia, given by Eq. (8)
 E_c = modulus of elasticity of concrete, as given by
 Eqs. (6) and (7).

Other procedures have been used, primarily varying the expression for the effective moment of inertia I_e [9, 11-14]. The method of Branson [4] was adopted by the 1971 ACI Code as giving the highest accuracy of any of the proposed methods and eliminating the discontinuity in values of the effective moment of inertia resulting from the use of the 1963 ACI Code.

An ACI Committee 435 study [15] indicates that, by using the 1971 ACI Code criteria, for deflection under controlled laboratory conditions "there is approximately a 90 percent chance that the deflections of a particular beam will be within the range of 20 percent less than to 30 percent more than the calculated value."

The following two examples demonstrate the computation of short-time (instantaneous) deflection.

Example 1--Investigate the instantaneous (short-time loading) deflection for the simply supported beam of Fig. 4 over a span of 40 ft. Assume that the member has been designed by the strength method using $f'_c = 4,000$ psi, $f_y = 60,000$ psi, and the ACI Code.

Solution. According to ACI-Table 9.5(a) the minimum depth unless deflection is computed is $L/16 = 40(12)/16 = 30$ in. Thus, a deflection computation is required.

For total dead load plus live load deflection

$$M_{\max} = \frac{28(40)}{4} + \frac{0.35(40)^2}{8} = 280 + 70 = 350 \text{ ft-kips}$$

For gross section,

$$I_g = \frac{1}{12}(14)(24)^3 = 16,128 \text{ in.}^4$$

For the cracked transformed section, as shown in Fig. 4, the neutral-axis position is

$$\frac{14x^2}{2} = 57.3(21.5 - x)$$

$$x = 9.78 \text{ in.}$$

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$$I_{cr} = \frac{1}{3} (14) (9.78)^3 + 57.3 (21.5 - 9.78)^2 = 12,233 \text{ in.}^4$$

$$M_{cr} = f_r I_g / y_t = [0.474 (16,126) / 12] / 12 = 53 \text{ ft-kips}$$

$$f_r = 7.5 \sqrt{f'_c} = 7.5 \sqrt{4000} = 474 \text{ psi (ACI-9.5.2.2)}$$

$$\frac{M_{cr}}{M_{max}} = \frac{53}{350} \left(\frac{\text{dead load}}{\text{+ live load}} \right) = 0.151, \quad \left(\frac{M_{cr}}{M_{max}} \right)^3 = 0.0035$$

Using ACI Formula (9-4), Eq. (8)

$$I_e = \left(\frac{M_{cr}}{M_{max}} \right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_{max}} \right)^3 \right] I_{cr}$$

$$= 0.0035 (16,128) + 0.9965 (12,233) = 12,245 \text{ in.}^4$$

Note that in this case, I_e is only slightly larger than I_{cr} .

$$E_c = 33w^{1.5} \sqrt{f'_c} = 57,000 \sqrt{4,000} = 3.64 \times 10^6 \text{ psi}$$

$$a(\text{beam weight}) = \frac{5WL^3}{384EI} = \frac{5(0.35)(40)(40)^3(1,728)}{384(3.64)(10^3)(12,233)} = 0.45 \text{ in.}$$

$$a(\text{concentrated load}) = \frac{PL^3}{48EI} = \frac{28(40)^3(1,728)}{48(3.64)(10^3)(12,233)} = 1.45 \text{ in.}$$

The total instantaneous deflection under dead load and live load is

$$(a_i)_{D+L} = 0.45 + 1.45 = 1.90 \text{ in.}$$

For calculating live load deflection, it is necessary to first obtain dead load deflection; thus,

$$M_{max} = \frac{10(40)}{4} + 70 = 170 \text{ ft-kips}$$

$$\frac{M_{cr}}{M_{max}} = \frac{53}{170} \left(\frac{\text{dead}}{\text{load}} \right) = 0.312; \quad \left(\frac{M_{cr}}{M_{max}} \right)^3 = 0.030$$

$$I_e = (0.030)(16,182) + (0.970)(12,233) = 12,255 \text{ in.}^4$$

Even under dead load alone, $I_e \approx I_{cr}$; so the answer may be obtained by proportion

$$(a_i)_D = 0.45 + 1.45(10/28) = 0.97 \text{ in.}$$

Though in this example it makes no appreciable difference, consistent logic dictates that live load deflection must be obtained indirectly as

$$\begin{aligned}(a_i)_L &= (a_i)_{D+L} - (a_i)_D \\ &= 1.90 - 0.97 = 0.93 \text{ in.}\end{aligned}$$

This approach gives the live load deflection occurring during the first application of live load; a conservative approach. After cracking occurs, removal of live load will leave a dead load deflection larger than the originally computed dead load deflection.

From a practical viewpoint, one may use $I_e = I_{cr}$ whenever $(M_{cr}/M_{max})^3$ is less than about 0.1, in which case live load deflection may be obtained by proportioning instead of an indirect computation as illustrated above.

Example 2--Investigate the instantaneous deflection of the continuous beam of Fig. 5 using the 1971 ACI Code with $f'_c = 3,000$ psi and $f_y = 40,000$ psi.

Solution. a) Section at the left support. For gross section, (see Fig. 6)

$$x = \frac{36(18)18 + 90(4.5)38.25}{36(18) + 90(4.5)} = \frac{27,160}{1053} = 25.79 \text{ in.}$$

$$\begin{aligned}I_g &= \frac{1}{3}(18)[(25.79)^3 + (14.71)^3] + 72(4.5)(12.46)^2 + \frac{1}{12}(72)(4.5)^3 \\ &= 172,870 \text{ in.}^4\end{aligned}$$

For transformed cracked section (see Fig. 7a),

$$\begin{aligned}18x(x/2) + 40.6(x-2.6) &= 40.9(36.65-x) \\ x &= 9.56 \text{ in.}\end{aligned}$$

$$I_{cr} = \frac{1}{3}(18)(9.56)^3 + 40.6(9.56-2.6)^2 + 40.9(36.65-9.56)^2 = 37,200 \text{ in.}^4$$

It is noted that the modular ratio n is used to transform the steel in the compression zone, instead of $2n$ which would be used in working stress method to check stresses. The use of n is believed to give the more accurate short-time deflection result, though using $2n$ also gives answers believed to be sufficiently accurate.

Compute the cracking moment capacity of the beam with tension on the flange.

$$f_r = 7.5 \sqrt{f'_c} = 7.5 \sqrt{3000} = 411 \text{ psi}$$

$$M_{cr} = f_r I_g / y_t = [0.411(172,870) / 14.71] / 12 = 402 \text{ ft-kips}$$

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$$\frac{M_{cr}}{M_{max}} = \frac{402}{89.6} \left(\frac{\text{dead}}{\text{load}} \right) > 1 ; \quad \text{Use } I_g = 172,870 \text{ in.}^4$$

$$\frac{M_{cr}}{M_{max}} = \frac{402}{200} \left(\frac{\text{dead load}}{\text{+ live load}} \right) > 1 ; \quad \text{Use } I_g = 172,870 \text{ in.}^4$$

b) Midspan section (Fig. 7b).

Obtain transformed section moment of inertia by locating the neutral axis,

$$90(4.5)(x-2.25) + 18(x-4.5)^2(1/2) = 86.6(36.8-x) \\ x = 8.09 \text{ in.}$$

$$I_{cr} = \frac{1}{12}(72)(4.5)^3 + 72(4.5)(5.84)^2 + \frac{1}{3}(18)(8.09)^3 + 86.6(36.8-8.09)^2 \\ = 86,140 \text{ in.}^4$$

The cracking moment capacity of the beam with tension in the stem is

$$M_{cr} = f_r I_g / y_t = [0.411(172,870)/25.79]/12 = 229.5 \text{ ft-kips}$$

$$\frac{M_{cr}}{M_{max}} = \frac{229.5}{229} \left(\frac{\text{dead}}{\text{load}} \right) > 1 ; \quad \text{Use } I_g = 172,870 \text{ in.}^4$$

$$\frac{M_{cr}}{M_{max}} = \frac{229.5}{510} \left(\frac{\text{dead load}}{\text{+ live load}} \right) = 0.45 ; \quad \left(\frac{M_{cr}}{M_{max}} \right)^3 = 0.091$$

$$I_e = \left(\frac{M_{cr}}{M_{max}} \right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_{max}} \right)^3 \right] I_{cr} \\ = 0.091(172,870) + 0.909(86,140) = 94,000 \text{ in.}^4$$

c) Section at the right support (Fig. 7c)

Use transformed cracked section and locate the neutral axis.

$$18x(x/2) + 52.3(x-2.6) = 122.6(35.4-x) \\ x = 14.60 \text{ in.}$$

$$I_{cr} = \frac{1}{3}(18)(14.60)^3 + 122.6(35.4-14.60)^2 + 52.3(14.60-2.60)^2 \\ = 79,240 \text{ in.}^4$$

$$M_{cr} = 402 \text{ ft-kips (same as left support)}$$

$$\frac{M_{cr}}{M_{max}} = \frac{402}{377} \left(\frac{\text{dead}}{\text{load}} \right) > 1 ; \quad \text{Use } I_g = 172,870 \text{ in.}^4$$

$$\frac{M_{cr}}{M_{max}} = \frac{402}{591} (\text{dead load} + \text{live load}) = 0.68 \quad ; \quad \left(\frac{M_{cr}}{M_{max}} \right)^3 = 0.314$$

$$I_e = 0.314(172,870) + 0.686(79,240) = 108,700 \text{ in.}^4$$

The effective values are

	For DL	For DL+LL
Left end	$I = 172,870 \text{ in.}^4$	$I = 172,870 \text{ in.}^4$
Midspan	$I = 172,870 \text{ in.}^4$	$I = 94,000 \text{ in.}^4$
Right end	$I = 172,870 \text{ in.}^4$	$I = 108,700 \text{ in.}^4$

Having obtained the above values, usual practice is to use a single adjusted value of flexural rigidity. Here, the simple average will be used as suggested by ACI-9.5.2.2. For dead load,

$$I_{avg} = 172,870 \text{ in.}^4$$

For dead load plus live load,

$$I_{avg} = \frac{1}{2} \left(\frac{172,870 + 108,700}{2} + 94,000 \right) = 117,400 \text{ in.}^4$$

d) Immediate dead load deflection

$$M_a = 89.6 \text{ ft-kips} \quad M_b = 377 \text{ ft-kips}$$

$$w = 0.76 \text{ kip/ft} \quad P = 24.48 \text{ kips}$$

It is to be noted that the span is taken as that measured between the centerlines of supports, and the end moments are those computed for the same locations. Equally acceptable results are obtained by using the clear span and the face-of-support moments. Referring to Fig. 5 the total midspan deflection is

$$a = \frac{10^6}{E_c I} (3.64P + 1.333wL - 0.164M_a - 0.164M_b)$$

$$E_c = 57,000 \sqrt{3,000} = 3.15 \times 10^6 \text{ psi}$$

$$(a_1)_D = \frac{1}{3.15(172,870)} [3.64(24.48) + 1.333(0.76)(39) - 0.164(89.6 + 377.0)] = \frac{89.2 + 39.5 - 76.4}{545} = \frac{52.3}{545} = 0.10 \text{ in.}$$

e) Immediate dead plus live load deflection

$$M_a = 200 \text{ ft-kips} \quad M_b = 591 \text{ ft-kips}$$

$$w = 0.91 \text{ kip/ft} \quad P = 56.33 \text{ kips}$$

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$$(a_i)_{D+L} = \frac{1}{3.15(117.0)} [3.64(56.33) + 1.333(0.91)(39) - 0.164(200 + 591)] = \frac{205.0 + 47.0 - 129.7}{369} = \frac{122.3}{369} = 0.33 \text{ in.}$$

f) Immediate live load deflection

$$(a_i)_L = (a_i)_{D+L} - (a_i)_D = 0.33 - 0.10 = 0.23 \text{ in.}$$

If this is a floor which does not support frangible partitions, the limiting permissible deflection is $L/360$, or

$$\frac{L}{360} = \frac{39(12)}{360} = 1.3 \text{ in.} > 0.23 \text{ in.} \quad \text{OK}$$

By making use of the moment envelope, using the maximum positive moment of 555 ft-kips (at right hand concentrated load) and the associated end moments, $M_a = 200$ ft-kips and $M_b = 591$ ft-kips, Eq. (2) may be directly applied to give 0.35 in. as the approximate immediate deflection due to dead load plus maximum live load.

CREEP EFFECT ON DEFLECTIONS UNDER SUSTAINED LOADING

The total long-term deflection includes the instantaneous elastic deflection plus the contributions from creep and shrinkage.

For deflection purposes, it is frequently desirable to use a creep coefficient C_t defined as the ratio of creep strain ϵ_{cp} to elastic strain, ϵ_i ; thus,

$$C_t = \frac{\epsilon_{cp}}{\epsilon_i} \quad (10)$$

ACI Committee 209 has recommended [16] the equation of Branson et al [17-21] for the creep coefficient, as follows:

$$C_t = \frac{t^{0.60}}{10 + t^{0.60}} C_u \quad (11)$$

where C_t = ratio of creep strain to elastic strain at any time t after a basic curing period

t = time in days after loading

C_u = ultimate creep coefficient; recommended average value is 2.35 for 40 percent humidity.

Eq. (11) applies to the standard condition of 40 percent ambient relative humidity, 4 in. or less slump, minimum thickness of member 6 in. or less, and loading age of 7

days for moist cured or 1-3 days for steam cured concrete. For other conditions, the standard condition value is to be multiplied by the following correction factors (CF):

(a) Age at loading.

For moist cured concrete:

$$(CF)_a = 1.25 t_a^{-0.118} \quad (12a)$$

For steam cured concrete:

$$(CF)_a = 1.13 t_a^{-0.095} \quad (12b)$$

In the above two equations, t_a is the age at loading, in days after the initial period of curing. Several useful values of Eqs. (12a,b) appear in Table 1.

(b) Humidity. For $H \geq 40$ percent,

$$(CF)_h = 1.72 - 0.0067H \quad (13)$$

where H is the ambient relative humidity in percent. Values for the correction factor appear in Table 2.

(c) Other correction factors. Additional correction factors are available to account for variations in slump, cement content, percent of fine aggregate, and air content [18]; however, these tend to be either small or offset one another and may generally be neglected. Also, when member thickness greatly exceeds 12 in., a correction factor [16] may be applied to correct for the effect of size and shape by considering the volume to surface ratio. Ref. 21 provides a simple chart to determine various correction factors.

Compression Steel Effect on Creep

The presence of compression steel A'_s decreases the deformation due to creep (and shrinkage as discussed in the next article). Evaluation of the effect of compression steel has been reported by Washa and Fluck [5] and a multiplier factor has been given by Committee 435 [9].

$$k_r = 0.85 - 0.45 (A'_s/A_g) \geq 0.40 \quad (14)$$

The creep deflection may then be expressed as

$$a_{cp} = k_r C_t a_i \quad (15)$$

SHRINKAGE EFFECT ON DEFLECTIONS UNDER SUSTAINED LOAD

Shrinkage of concrete in beams may have a similar effect on the deflection as creep. Shrinkage of an isolated plain concrete member would merely shorten it without

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causing curvature. When steel reinforcement is added, however, bond between concrete and steel restrains the shrinkage. Thus, a singly reinforced beam, having its shrinkage restrained at the reinforced face and unrestrained at the unreinforced face, will have considerable curvature. Generally, it is difficult to separate the effects of creep and shrinkage. Shrinkage is more pronounced during the first few months than creep. Typically, 90 percent of the shrinkage will have occurred at the end of one year, while not until the end of five years will 90 percent of the creep have occurred. A number of investigators have studied shrinkage effects separately from those of creep [16-25]. Recent statistical studies [17-20] have resulted in expressions considered acceptable [16] to predict shrinkage strain, ϵ_{sh} .

If the free shrinkage strain is known, shrinkage curvature ϕ_{sh} must be determined as a function of shrinkage strain. Such curvature will depend on the relative amounts of compression and tension steel and also on the geometry of the support system. Shrinkage deflection a_{sh} may be expressed [9,16] as

$$a_{sh} = \alpha_1 \phi_{sh} L^2 \quad (16)$$

where α_1 is a factor relating to the geometry of the support system and may be taken as the following:

$$\begin{aligned} \alpha_1 &= 0.50 \text{ for cantilever beams} \\ &= 0.125 \text{ for simply supported beams} \\ &= 0.086 \text{ for beams continuous at one end only} \\ &= 0.063 \text{ for beams continuous at both ends} \end{aligned}$$

and L is the span length of the beam.

Shrinkage Strain, ϵ_{sh}

ACI Committee 209 has recommended [16] that the following expressions may be used for shrinkage strain:
For any time after age 7 days for moist-cured concrete,

$$\epsilon_{sh} = \frac{t}{35+t} (\epsilon_{sh})_u \quad (17a)$$

For any time after age 1-3 days for steam-cured concrete,

$$\epsilon_{sh} = \frac{t}{55+t} (\epsilon_{sh})_u \quad (17b)$$

where ϵ_{sh} = shrinkage strain at any time t after initial curing

t = time in days after initial curing

$(\epsilon_{sh})_u$ = ultimate shrinkage strain; average value

suggested is 800×10^{-6} in./in. for 40 percent humidity.

For conditions other than those of 40 percent ambient relative humidity, 4 in. or less slump, and minimum thickness of members 6 in. or less, the standard condition value of Eqs. (17a,b) is to be multiplied by the following correction factors (CF):

$$(a) \text{ Humidity. For } H \geq 40 \text{ percent,} \\ (CF)_h = 1.40 - 0.010H, \quad 40 \leq H \leq 80 \text{ percent} \quad (18a)$$

$$(CF)_h = 3.00 - 0.030H, \quad H \geq 80 \text{ percent} \quad (18b)$$

where H is the relative humidity in percent. Values for $(CF)_h$ appear in Table 3.

Other corrections such as for minimum thickness greater than 6 in. and slump greater than 4 in. normally can be neglected.

Shrinkage Curvature, ϕ_{sh}

Several investigators [4,9,16,24] have developed expressions for curvature due to warping that arises from non-uniform shrinkage. Reinforcement of different amounts in the two faces of a beam is the principal cause of shrinkage warping.

Branson [4] has modified the equation given by Miller [24] and empirically extended the results to give equations including the effects of compression steel.

$$\phi_{sh} = 0.7 \frac{\epsilon_{sh}}{h} (\rho - \rho') \frac{1}{3} \left(\frac{\rho - \rho'}{\rho} \right)^{\frac{1}{2}} \text{ for } (\rho - \rho') \leq 3\% \quad (19a)$$

$$\phi_{sh} = \frac{\epsilon_{sh}}{h} \text{ for } (\rho - \rho') > 3\% \quad (19b)$$

Note that ρ and ρ' are in percent, 100 (A_s or A'_s)/(bd). Equations (19a,b) are recommended [9,16] as the most accurate relationships available.

Compression Steel Effect on Combined Shrinkage and Creep

Whenever shrinkage is to be considered in combination with creep in the computation for the deflection due to sustained load, a multiplier similar to that given by Eq. (14) may be applied to the short-time deflection [16]; in this case,

$$k_r = 1.0 - 0.60 (A'_s/A_s) \geq 0.40 \quad (20)$$

The creep deflection would become $k_r Ta_1$, where both k_r and T (see Table 4) include shrinkage as well as creep effects.

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Alternate expressions for k_r have been suggested by Shaikh [26], and the general relationship has been discussed by Branson [27]. Based on additional data involving flanged sections as well as rectangular sections, Branson [27] recommends as more logical the following equation which is a function of the compression steel ratio ρ' rather than the steel area ratio A'_s/A_s .

$$k_r = 1/(1+50\rho') \quad (21)$$

where $\rho' = A'_s/bd$.

CREEP AND SHRINKAGE DEFLECTION - 1971 ACI CODE METHOD

In the 1971 ACI Code method, the creep and shrinkage deflection due to sustained loads is obtained by multiplying the short-time (instantaneous) deflection by a single factor (ACI-9.5.2.3). Thus,

$$a_{cp+sh} = k_r T (a_i)_D \quad (22)$$

where

$$k_r T = 2 - 1.2(A'_s/A_s) \geq 0.6 \quad (23)$$

and $(a_i)_D$ is the instantaneous deflection due to all sustained loads (usually dead load).

Example 3--For the beam of Example 1 determine the creep and shrinkage deflection according to the ACI Code. Assume only the dead load is sustained.

Solution. First it is necessary to compute the short-time (instantaneous) deflection due to all sustained loads; in this case the dead load. From Example 1,

$$(a_i)_D = 0.97 \text{ in.}$$

Since no compression steel is used, Eq. (23) gives

$$k_r T = 2.0$$

Then, from Eq. (22)

$$a_{cp+sh} = k_r T (a_i)_D = 2.0(0.97) = 1.94 \text{ in.}$$

If part of the live load were considered as sustained, such as certain types of equipment whose placement or installation is not expected to change for a period of five years or more, then it would be necessary to compute a_i for the dead load plus the sustained live load. Under the ACI Code, an additional effective moment of inertia I_e would be required using M_{cr}/M_{max} where M_{max} is due to dead load plus sustained live load.

CREEP AND SHRINKAGE DEFLECTION - ALTERNATE PROCEDURES

Separate Creep and Shrinkage Multiplier Procedure

A procedure for computing the deflections due to creep and shrinkage separately was recommended by ACI Committee 435 [9,28], based on the work of Branson [4], as modified by recent improvement in the prediction of creep and shrinkage [16]. Thus,

$$a_{cp+sh} = a_{cp} + a_{sh} \quad (24)$$

where, using Eqs. (15) and (16),

$$a_{cp} = k_r C_t (a_i)_D \quad (15)$$

and

$$a_{sh} = \alpha_1 \phi_{sh} L^2 \quad (16)$$

For evaluation of Eqs. (15) and (16),

α_1 = constant (see Eq. (15))

ϕ_{sh} = shrinkage curvature; Eqs. (18)

L = span length

k_r = compression steel factor, Eq. (14)

C_t = creep coefficient, originally given in tabular form [9], but more recently [16] and more accurately as Eq. (11) with correction factors of Eqs. (12) and (13)

$(a_i)_D$ = instantaneous deflection due to all sustained loads.

Combined Creep and Shrinkage Multiplier Procedure

This method, which is similar to the 1971 ACI method, was proposed by Branson [28], and recommended by ACI Committee 209 [16]. It may be stated:

$$a_{cp+sh} = k_r T (a_i)_D \quad (25)$$

where

$$k_r = 1.0 - 0.60 (A'_s / A_s) \geq 40 \quad (26)$$

T = time-dependent coefficient (creep plus shrinkage) which may be taken from Table 4.

The recently recommended Eq. (21) may also be used for k_r instead of Eq. (26).

Example 4--For the beam of Example 1 (see Fig. 4) determine the ultimate (i.e., 5 years duration of load) creep and shrinkage deflection using several non-ACI procedures. Assume only the dead load is sustained, the ambient relative humidity is 70 percent, the age at loading

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is 20 days after the initial moist curing period.

Solution. It is noted that ACI-9.5.2.3 permits computation of long-time deflection by a "more comprehensive analysis", which could include any of the methods discussed herein.

(a) Separate creep and shrinkage multiplier procedure.

$$a_{cp+sh} = a_{cp} + a_{sh}$$

$$a_{cp} = k_r C_t (a_i)_D$$

where Eq. (14) gives for $\Lambda'_S/\Lambda_S = 0$,

$$k_r = 0.85 - 0.45(\Lambda'_S/\Lambda_S) = 0.85$$

and from Eq. (11), for $t = 5(365)$ days,

$$C_t = \frac{t^{0.60}}{10+t^{0.60}} C_u = 0.90 C_u$$

which for $C_u = 2.35$ as recommended for average conditions gives the basic value of C_t as

$$C_t = 2.12$$

Adjusting for 70 percent humidity, $(CF)_h = 0.80$ from Eq. (12) or Table 2, and for 20 day age of loading after initial moist curing period, $(CF)_a = 0.87$. Thus, the adjusted C_t is

$$C_t = 2.12(0.80)(0.87) = 1.47$$

From Example 1 using the 1971 ACI Code method,

$$(a_i)_D = 0.97 \text{ in.}$$

Then

$$a_{cp} = k_r C_t (a_i)_D = 0.85(1.47)(0.97) = 1.21 \text{ in.}$$

For shrinkage, from Eq. (16)

$$a_{sh} = \alpha_1 \phi_{sh} L^2$$

where

$$\alpha_1 = 0.125 \text{ for simply supported beams}$$

For this beam, since $\rho = 2.38$ percent and $\rho' = 0$, Eq. (19a) gives

$$\phi_{sh} = 0.7 \frac{\epsilon_{sh}}{h} \sqrt[3]{\rho}$$

Using ϵ_{sh} from Eq. (17a)

$$\epsilon_{sh} = \frac{t}{35+t} (\epsilon_{sh})_u$$

which for $t = 5(365)$ days is, for average conditions,

$$\epsilon_{sh} \approx (\epsilon_{sh})_u = 800 \times 10^{-6} \text{ in./in.}$$

Adjusting for 70 percent humidity, $(CF)_h = 0.70$, from Eq. (18a) or Table 3, the adjusted ϵ_{sh} is

$$\epsilon_{sh} = (800 \times 10^{-6}) 0.70 = 560 \times 10^{-6} \text{ in./in.}$$

Then,

$$\phi_{sh} = 0.7 \left(\frac{560 \times 10^{-6}}{24} \right) \sqrt[3]{2.38} = 25.2 \times 10^{-6} \text{ in.}$$

$$a_{sh} = \alpha_1 \phi_{sh} L^2 = 0.125 (25.2 \times 10^{-6}) (480)^2 = 0.73 \text{ in.}$$

$$a_{cp+sh} = 1.21 + 0.73 = 1.94 \text{ in.}$$

(b) Combined Creep and Shrinkage Multiplier Procedure.
Using Eq. (25)

$$a_{cp+sh} = k_t T (a_i)_D$$

$$k_r = 1.0 - 0.60 (A'_s/A_g) = 1.0$$

$$T = \text{value from Table 4} \approx 1.8$$

Note that age at loading is after initial curing period.

$$(a_i)_D = 0.97 \text{ in. (from Example 1)}$$

$$a_{cp+sh} = 1.0(1.8)0.97 = 1.75 \text{ in.}$$

A comparison of computation methods for creep and shrinkage deflection is shown in the following summary:

Method	a_{cp+sh}
1. ACI 1971, using $k_r T = 2.0$	1.94 in. (Ex. 3)
2. Separate creep and shrinkage, as per Eq. (24)	1.94 in. (Ex. 4a)
3. Combined creep and shrinkage, using $k_r T = 1.8$	1.75 in. (Ex. 4b)

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ACI MINIMUM DEPTH OF FLEXURAL MEMBERS

According to the 1971 ACI Code the minimum depths specified in ACI-9.5.2.1 shall apply to all cases of "one-way construction unless the computation of deflection indicates that lesser thickness may be used without adverse effects." The minimum depth (thickness) values apply to members where large deflection is not likely to damage partitions, ceilings, or other frangible attachments; otherwise deflections must be computed whether or not the minimum thickness requirement is satisfied.

The ACI table for minimum depth (Table 5) may be rationalized [29] as representing limits for uniformly loaded beams, stressed to about 60 percent of the 24 ksi allowable working stress for Grade 60 steel, and assuming the creep and shrinkage deflection is of equal magnitude to the short-time elastic deflection. A limit of $L/360$ is used for this total deflection.

When large deflections may cause cracking of partitions and other frangible attachments, the total deflection (ACI-9.5.2.4) which occurs after installation of such elements is limited to $L/480$. The minimum depths of ACI-Table 9.5a (Table 5) are likely to be too low; hence that table does not apply for such cases, and deflection computations must be made.

SPAN-TO-DEPTH RATIO TO ACCOUNT FOR FOR CRACKING AND SUSTAINED LOAD EFFECTS

In order to include the effects of the many variables, the span-to-depth ratio requirement may be expressed by the following proposed by Branson [25] and developed in detail by Wang and Salmon [29].

$$\left(\frac{L}{y_t}\right)_{\text{limit for } D+L} = \left(\frac{a}{L}\right) \left(\frac{\beta_w}{\beta}\right) \left(\frac{M_{cr}}{M_{D+L}}\right) \gamma_{D+L} \quad (27)$$

where y_t = distance from centroid of gross section to the extreme fiber in tension

$\beta_w = E_c/f_r = 4.4 w^{1.5} = 7680$ for normal weight concrete or $5.86 w^{1.5} = 5860$ for lightweight concrete ($w = 100$ pcf)

β = constant relating to type of loading and support condition as defined in Eq. (9)

$\gamma_{D+L} = I_e/I_g$, where I_e = Eq. (8) with $M_{\max} = M_{D+L}$

In order to include creep and shrinkage effects, one may write

$$\left(\frac{L}{y_t}\right)_{L+cp+sh} = \left(\frac{L}{y_t}\right)_{D+L} \left[\frac{C_L + 1}{C_L + 1 + (k_r T - 1) (\gamma_{D+L}/\gamma_D)} \right] \quad (28)$$

where $C_L = M_L/M_D$

For the situation where the I_e under dead load only is approximately the same as I_e under dead plus live load, $\gamma_{D+L} \approx \gamma_D$, in which case Eq. (28) becomes

$$\left(\frac{L}{y_t}\right)_{L+cp+sh} = \left(\frac{L}{y_t}\right)_{D+L} \left[\frac{C_L + 1}{C_L + k_r T} \right] \quad (29)$$

Charts are available [28] for the L/h ratio for short-time effects of dead load plus live load, Eq. (27). The chart for singly reinforced rectangular beams is given in Fig. 8.

Example 5--Determine the depth of beam required for the loading conditions of Example 1 (Fig. 4) if the sum of the immediate live load plus creep and shrinkage deflection must not exceed $L/480$. Assume only the dead load is sustained and the sustained load factor $k_r T = 2$ as given by ACI-9.5.2.3.

(a) Use ACI-Table 9.5a.

$$\frac{L}{h} = 16$$

This considers average conditions and includes some effect of sustained load deflection.

$$\text{Min. } h = \frac{480}{16} = 30 \text{ in.}$$

(b) Use more accurate procedure with Fig. 8.

$$\rho = 0.0238$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{7.5 \sqrt{4000} (16,128)}{12(12)} = 53 \text{ ft-kips}$$

$$\frac{M_{max}}{M_{cr}} = \frac{180+100+70}{53} = 6.1 > 3 \quad ; \quad \text{Use } 3$$

$$C_L = \frac{M_L}{M_D} = \frac{180}{170} = 1.06$$

From Fig. 8 for $M_{max}/M_{cr} = 3$ and $\rho = 0.0238$, find

$$\left(\frac{L}{h}\right)_{D+L} = 29 \quad , \quad \text{for } \frac{a}{L} = \frac{1}{360}$$

$$\left(\frac{L}{h}\right)_{D+L} = 29 \left(\frac{360}{480}\right) = 21.8, \quad \text{for } \frac{a}{L} = \frac{1}{480}$$

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$$\left(\frac{L}{h}\right)_{L+cp+sh} = 21.8 \left[\frac{C_L+1}{C_L+k_r T} \right] = 21.8 \left(\frac{1.06+1}{1.06+2} \right) = 14.7$$

$$\text{Min. } h = \frac{480}{14.7} = 32.6 \text{ in.}$$

which is somewhat more severe than ACI-Table 9.5a.

(c) Required depth and adequacy of beam in Example 1.
For the given beam with $h = 24$ in.

$$(a_i)_L = 0.93 \text{ in.} \quad (\text{Example 1})$$

$$a_{cp+sh} = 1.94 \text{ in.} \quad (\text{Example 3})$$

$$\text{Allowable deflection, } a = \frac{L}{480} = \frac{480}{480} = 1 \text{ in.}$$

$$\text{Actual deflection, } a = 0.93 + 1.94 = 2.87 \text{ in.}$$

If the beam were 28 in. deep, with the same loading, it would be approximately carrying only $(24/28 = 0.855)$ 85.5 percent of its capacity; i.e., stressed to only 85.5 per cent; in which case,

$$\text{Min. } h = 32.6(0.855) = 27.9 \text{ in.}$$

It would appear that 28 or perhaps 29 in. is required to satisfy deflection limit. Unless the width is reduced, which seems undesirable here, increasing the depth will increase dead load moment, so the 30 in. minimum given by ACI-Table 9.5a seems in this case to be about right.

ACI CODE DEFLECTION PROVISIONS - EXAMPLES

The 1971 ACI Code provisions (ACI-9.5) regarding deflection computations may be summarized as follows. In both the ultimate strength method (ACI "strength" method), and the working stress method (ACI "alternate" method), deflections under service load conditions must be computed:

(a) Whenever excessive deflection may adversely affect the strength or serviceability of the structure at service loads (ACI-9.5.1); and

(b) When minimum thickness used is less than that given by ACI-Table 9.5a for beams and one-way slabs.

The use of minimum thicknesses from ACI-Table 9.5a eliminates the need for computation of deflection only for those cases where partitions, ceilings, and other non-structural elements are not being supported. Beams or one-way slabs supporting such frangible elements are in category (a) above, and deflections must be computed whether or not the minimum thickness limits of Table 9.5a have been met.

Example 6--Investigate the deflection for the beam of Fig. 9 used on a simple span of 25 ft. The maximum bending moments under service load are 158 ft-kips dead load and 105 ft-kips live load, which correspond to $M_u = 400$ ft-kips as used for design by the ultimate strength method. Assume that all loading is uniformly distributed and that none of the live load is sustained. The beam supports partitions and other construction likely to be damaged by large deflections. Use $f'_c = 4,000$ psi, $f_y = 40,000$ psi, and the 1971 ACI Code.

Solution. a) Check minimum thickness Table 5 (ACI-Table 9.5a), for $f_y = 40$ ksi,

$$\text{Min. } h = \frac{L}{20} = \frac{25(12)}{20} = 15 \text{ in.} < 25 \text{ in. used}$$

This would make it seem that deflection is not likely to be excessive; however, deflection computations must be made because of the frangible items the beam supports.

(b) Determine the moment of inertia I_g for gross uncracked section without steel and I_{cr} for the cracked transformed section.

For gross uncracked section,

$$I_g = \frac{1}{12}(14)(25)^3 = 18,230 \text{ in.}^4$$

For cracked section, locate neutral axis under service loads,

$$\frac{14x^2}{2} = 7.22(8)(21.5 - x)$$

$$x = 9.81 \text{ in.}$$

$$I_{cr} = 14(9.81)^3/3 + 7.22(11.69)^2 = 12,300 \text{ in.}^4$$

(c) Determine the effective moment of inertia I_e (ACI Formula 9-4). This may be done graphically using Ref. 30 instead of as illustrated in what follows. The cracking moment is

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{7.5 \sqrt{4000} (18,230)}{12.5(12)} = 15.6 \text{ ft-kips}$$

For dead load deflection,

$$\frac{M_{cr}}{M_{max}} = \frac{57.6}{158} = 0.365 \quad ; \quad \left(\frac{M_{cr}}{M_{max}} \right)^3 = 0.05$$

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$$I_e = \left(\frac{M_{cr}}{M_{max}} \right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_{max}} \right)^3 \right] I_{cr}$$

$$= 0.05(18,230) + 0.95(12,300) = 12,610 \text{ in.}^4$$

For dead load plus live load deflection,

$$\frac{M_{cr}}{M_{max}} = \frac{57.6}{263} = 0.22 \quad ; \quad \left(\frac{M_{cr}}{M_{max}} \right)^3 = 0.01$$

$$I_e \approx I_{cr} = 12,300 \text{ in.}^4$$

(d) Compute immediate deflections.

$$E_c = 57,000 \sqrt{f'_c} = 57,000 \sqrt{4000} = 3.6 \times 10^6 \text{ psi}$$

For dead load,

$$(a_i)_D = \frac{5wL^4}{384EI} = \frac{5ML^2}{48EI} = \frac{5(158)(12)(300)^2}{48(3.6)(10^3)(12,610)} = 0.39 \text{ in.}$$

For dead load plus live load,

$$(a_i)_{D+L} = \frac{5(263)(12)(300)^2}{48(3.6)(10^3)(12,300)} = 0.67 \text{ in.}$$

Then, immediate live load deflection is

$$(a_i)_L = (a_i)_{D+L} - (a_i)_D = 0.67 - 0.39 = 0.28 \text{ in.}$$

(e) Compute creep and shrinkage deflection. From ACI-9.5.2.3 the multiplier is

$$k_r T = 2 - 1.2(A'_s/A_s) = 2.0$$

$$a_{cp+sh} = k_r T (a_i)_D = 2.0(0.39) = 0.78 \text{ in.}$$

(f) Check limitation of ACI-Table 9.5b. For roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflection,

$$(a_i)_L + a_{cp+sh} \leq \frac{L}{480}$$

This limit is for the deflection which is estimated to occur after the nonstructural elements are put in place. Whatever portion, if any, of the live load or creep and shrinkage deflection has occurred prior to the placement of nonstructural elements may be excluded from the $L/480$ limitation. Further, if adequate measures are taken to prevent damage to supported or attached elements, the $L/480$ limit may be exceeded (footnote, ACI-Table 9.5b).

For this example,

$$0.28 + 0.78 = 1.06 \text{ in.} > \frac{L}{480} = \frac{300}{480} = 0.63 \text{ in.}$$

which is not acceptable.

Example 7--Investigate the deflection for the one-way continuous slab shown in Fig. 10. The slab is 4 1/2 in. thick. Assume that 60% of the 100 psf live load is sustained. Use $f'_c = 3,000$ psi and $f_y = 40,000$ psi.

Solution. Use may be made of Eq. (2) to compute mid-span deflection:

$$a_m = \frac{5L^2}{48EI} [M_s - \frac{1}{10}(M_a + M_b)]$$

Note that clear span is used with a conservative assumption of zero end moment at the exterior support. Thus,

M_s = Net midspan moment

$$= \frac{0.156(11.92)^2}{8} - \frac{1}{2}(2.22) = 2.77 - 1.11 = 1.66 \text{ ft-kips/ft}$$

$$E_c = 57,000 \sqrt{f'_c} = 57,000 \sqrt{3000} = 3.12 \times 10^6 \text{ psi}$$

(a) Determine I_g and I_{cr} , using a one foot width of section.

$$I_g = \frac{1}{12}(12)(4.5)^3 = 91.12 \text{ in.}^4$$

For negative moment region, $d = 3.41$ in., $n = 9$

$$\frac{12x^2}{2} = 9(0.50)(3.41-x)$$

$$x = 1.27 \text{ in.}$$

$$I_{cr} = 12(1.27)^3/3 + 9(0.50)(3.41-1.27)^2 = 28.7 \text{ in.}^4$$

For positive moment region, $d = 3.44$ in.,

$$\frac{12x^2}{2} = 9(0.42)(3.44-x)$$

$$x = 1.19 \text{ in.}$$

$$I_{cr} = 12(1.19)^3/3 + 9(0.42)(3.44-1.19)^2 = 25.87 \text{ in.}^4$$

(b) Determine effective moment of inertia I_e .

$$f_r = 7.5 \sqrt{f'_c} = 7.5 \sqrt{3000} = 411 \text{ psi}$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{0.411(91.12)}{2.25(12)} = 1.39 \text{ ft-kips}$$

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$$I_e = \left(\frac{M_{cr}}{M_{max}} \right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_{max}} \right)^3 \right] I_{cr}$$

In negative moment region; for dead load

$$\frac{M_{cr}}{M_D} = \frac{M_{cr}}{M_{max}} = \frac{1.39}{0.80} > 1 \quad ; \quad I_e = I_g = 91.12 \text{ in.}^4$$

and for dead plus live load,

$$\frac{M_{cr}}{M_{D+L}} = \frac{M_{cr}}{M_{max}} = \frac{1.39}{2.22} = 0.625 \quad ; \quad \left(\frac{M_{cr}}{M_{max}} \right)^3 = 0.244$$

$$I_e = 0.244(91.12) + 0.756(28.7) = 43.9 \text{ in.}^4$$

In positive moment region; for dead load

$$\frac{M_{cr}}{M_D} = \frac{M_{cr}}{M_{max}} = \frac{1.39}{0.60} > 1 \quad ; \quad I_e = I_g = 91.12 \text{ in.}^4$$

and for dead plus live load,

$$\frac{M_{cr}}{M_{D+L}} = \frac{M_{cr}}{M_{max}} = \frac{1.39}{1.66} = 0.838 \quad ; \quad \left(\frac{M_{cr}}{M_{max}} \right)^3 = 0.588$$

$$I_e = 0.588(91.12) + 0.412(25.87) = 64.2 \text{ in.}^4$$

(c) Consider effect of sustained live load which contributes to the creep and shrinkage deflection. The immediate deflection due to all sustained loads is required as the base value on which to apply the time dependent multiplier.

For negative moment region, using sustained load moment of $0.80 + 0.6(1.42) = 1.65 \text{ ft-kips/ft}$

$$\frac{M_{cr}}{M_{max}} = \frac{1.39}{1.65} = 0.843 \quad ; \quad \left(\frac{M_{cr}}{M_{max}} \right)^3 = 0.60$$

$$I_e = 0.60(91.12) + 0.40(28.7) = 66.1 \text{ in.}^4$$

For positive moment region, using sustained load moment of $0.60 + 0.6(1.06) = 1.24 \text{ ft-kips/ft}$,

$$\frac{M_{cr}}{M_{max}} = \frac{1.39}{1.24} > 1 \quad ; \quad I_e = I_g = 91.12 \text{ in.}^4$$

(d) Immediate live load deflection.

$$(a_i)_L = (a_i)_{D+L} - (a_i)_D$$

$$\text{Avg } I_e \text{ (dead load)} = 91.12 \text{ in.}^4$$

$$\text{Avg } I_e \text{ (dead + live load)} = \frac{43.9 + 64.2}{2} = 54.0 \text{ in.}^4$$

$$(a_i)_{D+L} = \frac{5(11.92)^2 144}{48(3.12)(10^3) 54.0} \left(1.66 - \frac{2.22}{10} \right) (12) = 0.22 \text{ in.}$$

$$(a_i)_D = \frac{5(11.92)^2 144}{48(3.12)(10^3) 91.12} \left(0.60 - \frac{0.80}{10} \right) (12) = 0.05 \text{ in.}$$

$$(a_i)_L = 0.22 - 0.05 = 0.17 \text{ in.}$$

(e) Creep and shrinkage deflection. The immediate deflection due to sustained loads,

$$\text{Avg } I_e = \frac{66.1 + 91.12}{2} = 78.6 \text{ in.}^4$$

$$(a_i)_{D+Sust L} = \frac{5(11.92)^2 144}{48(3.12)(10^3) 78.6} \left(1.24 - \frac{1.65}{10} \right) (12) = 0.11 \text{ in.}$$

$$(a_i)_{cp+sh} = k_r C_t (a_i)_{D+Sust L} = 2.0(0.11) = 0.22 \text{ in.}$$

(f) Check deflection if the limit of $L/480$ in ACI-Table 9.5b applies,

$$(a_i)_L + a_{cp+sh} = 0.17 + 0.22 = 0.39 \text{ in.}$$

$$\text{Allowable } a = \frac{L}{480} = \frac{11.92(12)}{480} = 0.30 \text{ in.}$$

which appears unacceptable. If none of the live load was considered sustained,

$$(a_i)_L + a_{cp+sh} = 0.17 + 2(0.05) = 0.27 \text{ in.} \quad \text{OK}$$

The decision regarding whether or not part of the live load is sustained should be made by considering the actual loading and its duration. For instance, a floor system supporting library stacks as the live load might well be considered to have a significant part of the live load treated as being sustained. In most situations it is acceptable to consider that only the dead load is sustained and thereby affects the magnitude of creep and shrinkage deflection.

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NOTATION

- a = deflection
 a_{cp} = deflection due to creep
 a_{cp+sh} = deflection due to creep and shrinkage
 a_i = instantaneous deflection (i.e., neglecting sustained load effects)
 a_m = midspan deflection
 a_{max} = maximum deflection
 a_{sh} = deflection due to shrinkage
 $(a_i)_D$ = instantaneous deflection due to dead load only
 $(a_i)_{D+L}$ = instantaneous deflection due to dead plus live loads

- $(a_i)_L$ = instantaneous deflection due to live load only
 b = beam width
 d = beam effective depth
 f'_c = compressive strength of concrete at 28-days after casting
 f_r = modulus of rupture; taken as $7.5 \sqrt{f'_c}$ for normal weight concrete
 f_y = yield strength of steel reinforcement
 h = beam overall depth
 k_r = factor to account for the effect of compression steel on creep and/or shrinkage deflection
 n = ratio of the modulus of elasticity of steel to that of concrete
 t = time in days after a basic curing period
 t_a = age at loading, in days, after a basic curing period
 w = uniform loading
 x = distance from neutral axis to extreme fiber in compression
 y_t = distance from neutral axis of the gross cross section to the extreme fiber in tension
 A_s = area of tension steel
 A'_s = area of compression steel
 C_L = ratio of live load moment to dead load (sustained load) moment
 C_t = creep coefficient; ratio of creep strain to elastic strain
 C_u = ultimate creep coefficient; recommended average value is 2.35 for 40 percent humidity
 $(CF)_a$ = correction factor for the creep coefficient to account for age at loading
 $(CF)_h$ = correction factor for the creep coefficient, and for the shrinkage strain, to account for variation in humidity
 E = modulus of elasticity
 E_c = modulus of elasticity for concrete
 $(EI)_c$ = flexural rigidity at midspan section
 $(EI)_{e1}$ = flexural rigidities at ends 1 and 2 of a member
 $(EI)_{e2}$
 H = relative humidity in percent
 I = moment of inertia
 I_{avg} = average moment of inertia for end regions and midspan region
 I_c = moment of inertia of a reference section in the member
 I_{cr} = moment of inertia of the cracked transformed section
 I_e = effective moment of inertia, given by Eq. (8)

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- I_g = moment of inertia of gross cross section, neglecting reinforcement
 L = span length
 M = bending moment
 M_a, M_b = bending moments at ends a and b of a member; taken as positive if causing tension in the top of the member
 M_{cr} = cracking moment; $f_r I_g / y_t$
 M_{max} = maximum moment for the loading condition under which deflection is computed
 M_O = statical moment, $wL^2/8$
 M_s = bending moment at midspan; taken positive if causing tension at the bottom of the member
 P = concentrated load
 T = time dependent factor to account for both creep and shrinkage
 W = total load on the span
 α_l = factor for shrinkage deflection relating to the geometry of the support system
 β = coefficient based on load and support conditions
 β_a = coefficient depending on the degree of fixity at support, the variation in moment of inertia along the span, and the distribution of loading
 $\beta_w = E_c / f_r$ = 7680 for normal weight concrete, or 5860 for lightweight concrete ($w = 100$ pcf)
 ϵ_{cp} = unit strain due to creep
 ϵ_i = elastic unit strain
 ϵ_{sh} = unit strain due to shrinkage
 $(\epsilon_{sh})_u$ = ultimate shrinkage strain; average value suggested is 800×10^{-6} in./in. for 40 percent humidity
 γ_D = I_e / I_g , where I_e = Eq. (8) with $M_{max} = M_D$
 γ_{D+L} = I_e / I_g , where I_e = Eq. (8) with $M_{max} = M_{D+L}$
 ϕ_{sh} = curvature due to shrinkage
 ρ = reinforcement ratio, $A_s / (bd)$, taken in percent for Eqs. 19 only
 ρ' = reinforcement ratio, $A'_s / (bd)$, taken in percent for Eqs. 19 only

Table 1

Creep Correction Factor, $(CF)_a$ for Age at Loading, Eq. (12)

t_a , Age in Days after Initial Curing Period	Correction Factor, $(CF)_a$	
	Moist Cured for 7 Days Initial Curing Period	Steam Cured for 1-3 Days Initial Curing Period
10	0.95	0.90
20	0.87	0.85
30	0.83	0.82
60	0.77	0.76
90	0.74	0.74

Table 2

Creep Correction Factor, $(CF)_h$ for Humidity, Eq. (13)

Ambient Relative Humidity, H, percent	Correction Factor $(CF)_h$
40 or less	1.00
50	0.94
60	0.87
70	0.80
80	0.73
90	0.67
100	0.60

Table 3

Shrinkage Correction Factor, $(CF)_h$ for Humidity,
Eqs. (18a) and (18b)

Ambient Relative Humidity, H, Percent	Correction Factor $(CF)_h$
40 or less	1.00
50	0.90
60	0.80
70	0.70
80	0.60
90	0.30
100	0

Table 4

Time-Dependent Coefficient T Including Both Creep and Shrinkage Effects, for Both Normal Weight and Lightweight Concrete Members of Common Types, Sizes, and Composition (from Branson [28])^a

Concrete Strength f'_c at 28 days	Average Relative Humidity, Age when Loaded								
	100%			70%			50%		
	$\leq 7d$	14d	$\geq 28d$	$\leq 7d$	14d	$\geq 28d$	$\leq 7d$	14d	$\geq 28d$
2500 to 4000 psi (176 to 281 kgf/cm ²)	2.0	1.5	1.0	3.0	2.0	1.5	4.0	3.0	2.0
>4,000 psi (281 kgf/cm ²)	1.5	1.0	0.7	2.5	1.8	1.2	3.5	2.5	1.5

^aIt is suggested that the following percentages of the values in the table be used for sustained loads that are maintained for the periods indicated: 25% for 1 month or less

50% for 3 months

75% for 1 year

100% for 5 years or more

The 50% values may normally be used for average relative humidities lower than 50%, which might be the case in heated buildings, for example.

Table 5

Minimum Depth h for Beams and One-Way Slabs, for Members Not Supporting or Attached to Partitions or Other Construction Likely to Be Damaged by Large Deflections (from ACI-Table 9.5a)

Type of Members		Simple Support	One End Continuous	Both Ends Continuous	Cantilever
Beams	$f_y = 60$ ksi	$L/16$	$L/18.5$	$L/21$	$L/8$
	$f_y = 40$ ksi	$L/20$	$L/23$	$L/26$	$L/10$
One-way slabs (solid)	$f_y = 60$ ksi	$L/20$	$L/24$	$L/28$	$L/10$
	$f_y = 40$ ksi	$L/25$	$L/30$	$L/35$	$L/12.5$

Note: For structural lightweight concrete having unit weights w from 90-120 pcf, multiply table values by 1.65-0.005 w but not less than 1.09.

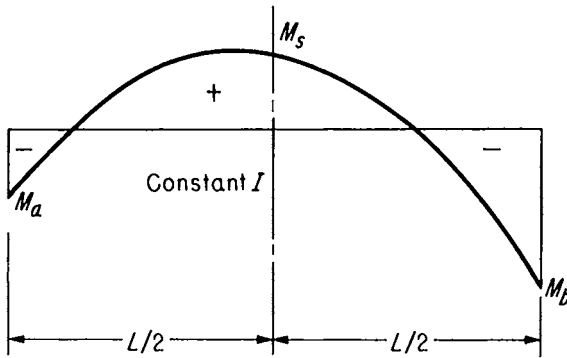


Fig. 1—Typical bending moment diagram for a uniformly loaded span

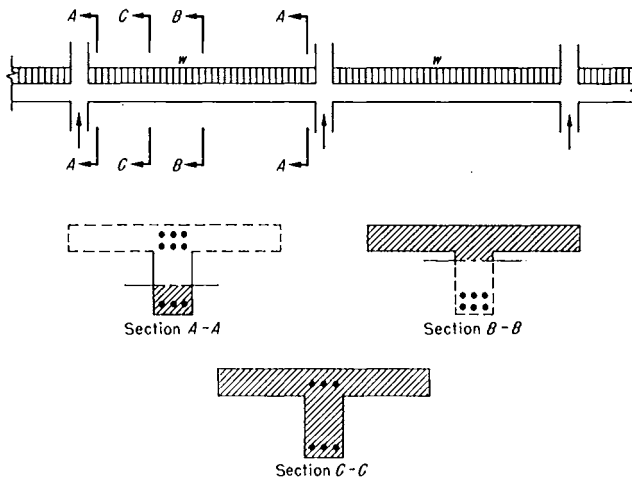


Fig. 2—Effective moment of inertia for continuous T-shaped sections.

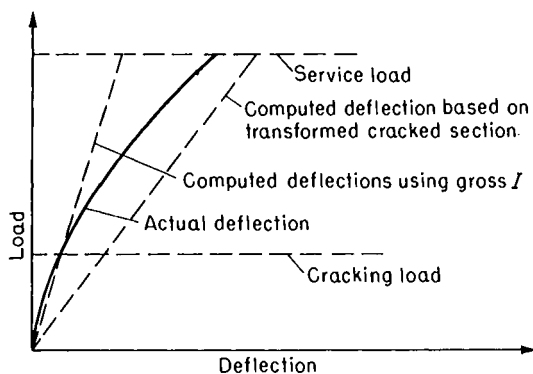


Fig. 3—Typical load-deflection curve for reinforced concrete beams.

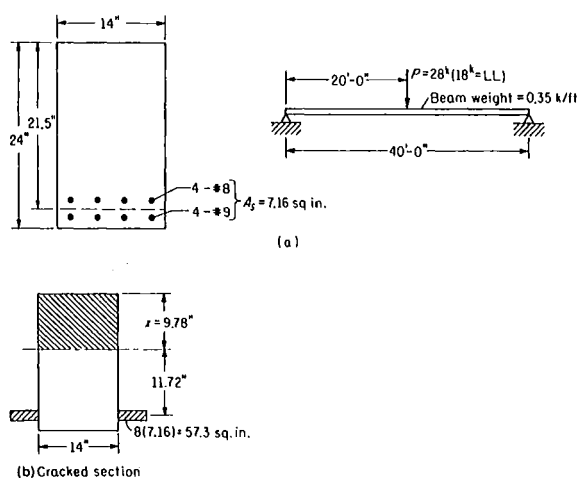


Fig. 4—Beam for Example 1.