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A RE-EXAMINATION OF THE EI VALUE FOR SLENDER COLUMNS

By

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Synopsis: An incremental rate-of-creep analysis of reinforced concrete columns subjected to sustained loads is developed and used to generate data for a statistical evaluation of the stiffness, EI , of slender concrete columns subjected to short time and sustained loads. The resulting equations are more accurate than ACI Eqn. 10.7 and are as easy to use. The possibility of column failure under very high sustained loads is discussed.

Keywords: building codes; columns (supports); computers; creep properties; deflections; loads (forces); modulus of elasticity; moment-curvature relationships; reinforced concrete; reinforcing steels; statistical analysis; stiffness; stiffness methods; structural analysis; structural design; structural engineering; tied columns.

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INTRODUCTION

The 1971 ACI Building Code (1) presents a moment-magnifier method for analysing the effects of slenderness on the strength of columns. This design procedure is strongly affected by the effective flexural stiffness, EI , used in the calculations and it has been shown that the EI values given in the ACI Code are inaccurate in many cases(2).

The knowledge of the effect of sustained loads on the load capacity of concrete columns has been improved in the past 15 years by experiments (3-8) and analyses of the problem (6-12). Although the analyses now available are reasonably accurate in predicting the behavior of columns in tests, they are generally too complex for design use. However, such analyses are useful in carrying out computer experiments to study the effects of a wide range of variables to help in the preparation of design equations.

The first part of this paper briefly describes a step-by-step Rate-of-Creep analysis of columns subjected to sustained loads. A series of computer experiments, carried out using this analysis are described in the second part. The results of these have been used to derive design equations for flexural stiffness, EI , for use in the ACI Code moment magnifier method for designing slender columns.

ANALYSIS OF CAPACITY OF SLENDER HINGED COLUMNS

Material Properties

The reinforcing steel is assumed to be an idealized elasto-plastic material. The instantaneous stress-strain relationship for concrete is assumed to be:

1. linear in the strain range:

$$\frac{f'_t}{E_c} \leq \epsilon \leq 0,$$

where $f'_t = 6 \sqrt{f'_c}$

E_c = Modulus of elasticity as defined in Section 8.3.1 of ACI 318-71;

2. quadratic parabola in the strain range:

$$0 < \epsilon < \frac{2f'_c}{E_c}$$

3. perfectly plastic in the range:

$$\frac{2f'_c}{E_c} \leq \epsilon \leq \epsilon_u$$

The slope of unloading and reloading branches is taken equal to the modulus of elasticity, E_c . Under short time loads ϵ_u was taken as 0.0035.

Although the tensile strength of concrete is generally ignored in the design of reinforced concrete members, this analysis is intended to represent reasonably closely the true behavior of slender columns so that data from the computer analysis can be used in the same manner as data from real column tests in studying column stability. For this reason the tensile strength of concrete was included in the analysis. The value of f'_t was chosen to agree with Section 11.5.2 of ACI 318-71 (1) rather than Section 9.5.2.2 to give a conservative estimate of the column behavior.

The properties of concrete are affected by time and vary over a wide range depending on the curing and loading conditions. In this investigation, concrete is assumed to be a non-linear, visco-elasto-plastic material. The non-linear, stress dependent creep function used in the analysis was derived from test data in Ref. 11 for plain concrete prisms subjected to different levels of stress which were held constant for each test specimen during the test period. Shrinkage strains were measured on separate unloaded test specimens. The creep strain at time t , $\epsilon_{cr}(t)$, was found by subtracting the initial strain, ϵ_0 , and the shrinkage strain at time t , $\epsilon_s(t)$, from the measured strain at time t , $\epsilon(t)$:

$$\epsilon_{cr}(t) = \epsilon(t) - \epsilon_0 - \epsilon_s(t) \quad (1)$$

ϵ_0 is given by the equation of the short time stress-strain curve:

$$\epsilon_0 = \frac{2f'_c}{E_c} \left(1 - \sqrt{1 - \frac{f_c}{f'_c}} \right) \quad (2)$$

where E_c is the initial tangent modulus of elasticity of the concrete at the time under consideration. The creep function $\phi(t)$ is defined as the ratio between the creep strain and initial strain:

$$\phi(t) = \frac{\epsilon_{cr}(t)}{\epsilon_o} \quad (3)$$

One creep function was obtained from each of twenty test specimens subjected to different constant stress levels in the range of f_c/f'_c from 0.185 to 0.808. By plotting $\phi(t)$ versus f_c/f'_c at different times it was found that the stress dependency of the creep function at any time t can be described by the function:

$$\phi(t) = C_1(t) + C_2(t) \frac{f_c}{f'_c} \quad (4)$$

The coefficients $C_1(t)$ and $C_2(t)$ determined by linear regression analysis carried out for different times after load application are shown in Fig. 1. Since only the effect of f_c/f'_c on the creep function was investigated in Ref. 11 the other variables will be accounted for using the CEB creep functions (13).

For analytical purposes it was assumed that the coefficients $C_1(t)$ and $C_2(t)$ could be represented by a function of the general type:

$$C_n(t) = \bar{C}_n \left[1 - \exp(-\alpha_n t^{\delta_n}) \right] \quad (5)$$

where $\bar{C}_n$, etc. refers to C_1 or C_2 , etc.

The values of the coefficients used in the analysis were based on those derived in Ref. 11 and were taken as:

$$\bar{C}_1 = \bar{C}_n \frac{[\phi(\infty, 0.40) - 0.8]}{3} \quad (6)$$

and

$$\bar{C}_2 = 2.5 [\phi(\infty, 0.40) - \bar{C}_1] \quad (7)$$

In these equations $\phi(\infty, 0.40)$ refers to the value of the creep function ϕ at time $t = \infty$ and $f_c/f'_c = 0.40$ and is taken equal to the value of ϕ given by the CEB Recommendations (13).

The total strain can now be represented by:

$$\epsilon(t) = \epsilon_o \left[1 + C_1(t) + C_2(t) \left(\frac{f_c}{f'_c} \right) \right] + \epsilon_s(t) \quad (8)$$

For columns with an equal amount of reinforcement on both faces it was found that the effect of shrinkage was insignificant and consequently it has not been considered in this study (5, 12).

The increase in the compressive strength of the concrete due to maturing was also included in the analysis. For the design studies carried out later in this paper the strength at $t = \infty$ was conservatively assumed to be 1.10 times the 28 day strength. The decrease in concrete

strength in the presence of very high sustained stresses was assumed to be 20 percent of the short time strength (14).

Method of Analysis for Short Time Loads

In the first stage of the analysis a family of moment curvature relationships is computed. In the second stage the deflections and load capacity of the given column are computed by an iterative numerical integration procedure making use of the stored moment curvature relationships. With the exception of the method used to account for tension stiffening, the analysis resembles the procedures described by Pfrang (15).

The cross section of a column subjected to uniaxial bending and compression is divided into a number of concrete and steel fibres perpendicular to the axis of symmetry and parallel to the neutral axis. The ultimate compression strain ϵ_u is also divided into NE segments of strain. These segments are selected as strain increments at the compressed edge of the cross section as shown in Fig. 2. By assuming an arbitrary strain distribution over the depth of the cross section for any value of ϵ_j the concrete fibre forces C and the steel layer forces S may be computed from the basic stress-strain relationships. Eqns. (9) and (10) give the forces P and M associated with the assumed strain distributions.

$$\sum C + \sum S = P \quad (9)$$

$$\sum (Cy_c) + \sum (Sy_s) = M \quad (10)$$

By varying the strain distribution for a given value ϵ_j at the compressed edge, by a trial and error procedure as shown in Fig. 2 the sum of the internal forces P is made equal to the external force P_1 . By repeating this procedure for all given values of P_1 ($i = 1, 2, \dots, NP$) and all given values of ϵ_j ($j = 1, 2, \dots, NE$) the moment curvature relationship M_1, j, ϕ_1, j , results.

In the procedure just described, the influence of the concrete tensile zone has not been considered. This influence will be included by subtracting a correction $\Delta\phi$ from the computed curvature values, ϕ (Fig. 3). No correction is required if the whole cross section is subjected to compression strains only. The influence of the tensile zone and as a consequence the correction $\Delta\phi$ reaches a maximum at the cracking moment, M_{cc} . To compute the correction $\Delta\phi_{cc}$ the following method has been applied. For any value of the axial force P_1 the cracking moment is computed by a trial and error procedure similar to that shown in Fig. 2 except that the strain distributions are selected with the ultimate tensile strain $\epsilon_2 = f_t/E_c$ at the bottom edge of the cross section. Because the cross section is considered as uncracked in this particular analysis, Eqns. (9) and (10) also include concrete tensile forces. The value $\Delta\phi_{cc}$ can now be found as the difference between the curvature, $\bar{\phi}_{cc}$, computed for the uncracked cross section and the curvature, ϕ_{cc} , computed for the cracked cross section where both $\bar{\phi}_{cc}$ and ϕ_{cc} are evaluated for the moment $M = M_{cc}$. For other values of M the following corrections are then applied:

$$M \leq M_o \quad : \Delta\phi = 0 \quad (11)$$

$$M_o < M < M_{cc} \quad : \Delta\phi = \Delta\phi_{cc} \frac{M - M_o}{M_{cc} - M_o} \quad (12)$$

$$M_{cc} \leq M \quad : \Delta\phi = \Delta\phi_{cc} \frac{M_{cc} - M_o}{M - M_o} \quad (13)$$

where: M_o is the moment for zero strain at the tensile edge.

These equations are compared to analyses and tests in Ref. 11 and good agreement is found.

The resulting moment curvature relationships can be used in an iterative numerical integration procedure (15) to determine the deflections, w , of the column subjected to P_1 . By repeating the computation of deflections for different values of P_1 a relationship P_1, M representing the column behavior, can be found where M is the maximum moment at midheight given by $M = P_1(e + w)$. Failure is assumed to occur at the intersection of this P_1, M curve with the P, M interaction curve as shown in Fig. 4. The interaction curve is given by $P_1, M_{1,j=NE}$ based on $J = NE (\epsilon_1 = \epsilon_u)$ from the moment curvature relationships. The result of the analysis is the ultimate load capacity P_u and the ultimate moment M_u . In the case of stability failures the ultimate load, P_u , was that of instability and the ultimate moment was taken as the point on the P, M interaction diagram corresponding to the computed ultimate load, P_u .

Method of Analysis for Sustained Loads

To include the effect of creep, the moment-curvature relationship is expanded to $M_{1,j}, \phi_{1,j,k}$ where index k stands for time and $k = 1$ indicates values for the short term loading situation so far referred to as $\phi_{1,j}$. The values $\phi_{1,j,k}$ for $k = 2, 3 \dots NT$ are computed by a Rate-of-Creep Method using time as a discrete variable. Fig. 5 shows the strain distribution and curvature $\phi_{1,j,k}$ at the beginning of creep stage k . First the unrestrained creep strain $\Delta\epsilon_{cr}$ is computed for each compressed concrete fibre using Eqn. (3) and (4). Next, residual forces ΔC and ΔS are applied to maintain equilibrium and restore the linear strain distribution. The values of ΔC and ΔS and the strain increments $\Delta\epsilon_1$ and $\Delta\epsilon_2$ follow from the equilibrium conditions:

$$\Sigma\Delta C + \Sigma\Delta S = 0 \quad (14)$$

$$\Sigma(\Delta C y_c) + (\Delta S y_g) = 0 \quad (15)$$

The curvature $\phi_{1,j,k+1}$ at the beginning of creep stage $k+1$ can now be computed from:

$$\phi_{1,j,k+1} = \phi_{1,j,k} + \frac{\Delta\epsilon_1 - \Delta\epsilon_2}{h} \quad (16)$$

This procedure must be repeated for all creep stages.

The influence of the concrete tensile zone is included as described earlier giving rise to $\bar{\Phi}$ as shown in Fig. 3. It is assumed that the cracking moment M_{cc} does not vary with time.

Computation of Column Deflections and Capacities

The time dependent deflections of a column subjected to a sustained load P_s are computed by a Rate-of-Creep Method making use of the $M_{1,j}, \bar{\Phi}_{1,j,k}$ relationship. The length of the column is divided into a number of intervals for the numerical integration of the curvature. The instantaneous column deflection, w_o , is computed from the $M_{1,j}, \bar{\Phi}_{1,j,1}$ relationship. To compute the creep deflections, Δw_1 , occurring in the first creep stage the incremental increase in curvature due to creep is determined from:

$$\Delta \bar{\Phi}_{1,j,1} = \bar{\Phi}_{1,j,2} - \bar{\Phi}_{1,j,1} \quad (17)$$

$\Delta \bar{\Phi}_{1,j,k}$ has to be evaluated for each interval of column length. The creep deflections, Δw_1 , due to $\Delta \bar{\Phi}_{1,j,1}$ will cause an increase of the external moment thereby causing an additional instantaneous deflection, $\Delta w_{o,1}$. It is assumed that this increase in the short term deflections $\Delta w_{o,1}$ takes place at the end of the creep stage. The total deflection at the beginning of creep stage 2 is then given by:

$$w_1 = w_o + \Delta w_1 + \Delta w_{o,1} \quad (18)$$

or generally, at the beginning of creep stage $k + 1$:

$$w_k = w_o + \sum_{m=1}^k (\Delta w_m + \Delta w_{o,m}) \quad (19)$$

A more detailed description of this procedure is presented in Ref. 11. In the case of a homogeneous linear visco-elastic material this procedure converges to the solution of the appropriate differential equation. Because the quality of the convergence decreases with increasing second order deformations, eight creep stages have been used in the present analysis.

For the purpose of computing the final short term load capacity of a column after a period of sustained load, an additional set of moment curvature relationships is required to account for the increase in concrete strength with increasing age. The computational procedure for these short term relationships is the same as for $M_{1,j}, \bar{\Phi}_{1,j,1}$. The analysis after a period of sustained load is carried out in the same way as the computation of the behavior due to an initial short term loading situation but the residual creep deflections (the deflections remaining after unloading the column quickly) must be added to the initial geometrical eccentricity of the load.

The analysis described here is essentially the same as that described in Ref. 11. That analysis is compared in Ref. 11 to the

results of columns tested under short time and sustained loads by Ramu et.al. (5) and gave good predictions of both the load capacity and the load deflection response of these columns.

SUSTAINED LOAD BEHAVIOR OF SLENDER COLUMNS

The typical behavior of a slender hinged column bent in single curvature is shown in Fig. 6. Curve (A) shows the instantaneous short term behavior expressed in terms of load and moment at midheight. Failure is assumed to occur at the point of intersection of Curve (A) with the P, M - interaction curve ($\bar{A}$) computed with the initial 28 day concrete strength. Curve (B) shows the behavior of a column loaded up to failure after being subjected to a sustained load P_s for the period of time indicated by the index k . Failure is now assumed to occur at the intersection with an interaction curve ($\bar{B}$) computed with a slightly increased concrete strength. Curves (C) and ($\bar{C}$) shows a similar behavior for a column loaded to failure after the total amount of creep possible under the sustained load P_1 has occurred.

The interaction curves are given by the values $P_1, M_{1,j=NE,k}$. The fact that the interaction curves have been found from the condition $\epsilon_1 = \epsilon_u = 0.0035$ does not mean that the maximum total strain in the column at failure is equal to ϵ_u . Instead, the total strain minus the residual creep strains at the edge of the failure section is assumed to reach $\epsilon_1 = \epsilon_u$. Thus it is assumed that the creep process does not affect the ability of the concrete to develop its full ultimate short term strain for a final short term load (10). This assumption is in fairly good agreement with test results.

For all eccentrically loaded columns there is a "critical sustained load", P_{scr} . For sustained loads less than P_{scr} the column load can be increased to failure at time $t = \infty$ as shown by curve (C) in Fig. 6. For sustained loads equal to or greater than P_{scr} the column will fail under the sustained load (9, 11).

The strength of concrete tends to decrease under sustained high stresses (14). This decrease has been assumed to be 20 percent of the short-time strength of concrete of the same age. Since the short-time strength at time infinity has been conservatively assumed equal to $1.10 f'_{c28}$ in this investigation, the critical sustained stress was assumed to be $0.8 \times 1.10 f'_{c28}$. The interaction curve ($\bar{D}$) in Fig. 6 then results from the $M_{1,j}, \phi_{1,j,k=NT}$ relationship where j indicates the strain level, ϵ_1 , in the extreme compression fibre which is set equal to the critical sustained load strain ϵ_{us} computed from Eqn. (8) for $f_c/f'_{c28} = 0.88$ and $t = \infty$ in computing the interaction curve. Curve (D) in Fig. 6 represents the maximum possible increase in the moment at failure section due to creep. The intersection of this limiting curve and the interaction curve ($\bar{D}$) yields the critical sustained load which is the lowest load which can cause a creep failure for any given initial eccentricity, e .

Generally, significant differences between the interaction curve

($\bar{A}$) and the curves ($\bar{B}$), ($\bar{C}$) and ($\bar{D}$) are only noticeable for load levels above the balanced load.

DERIVATION OF DESIGN EQUATIONS

Computer Experiment

Earlier studies of the behavior of slender columns (2) suggested that the following five variables would have the most effect on the strength and behavior of such columns: slenderness ratio, ℓ/h ; shape of cross section and reinforcement position; reinforcement ratio, ρ_t ; ratio of the distance between the outer reinforcement layers and the overall thickness, γ ; and eccentricity, e/h .

The principal groups of columns studied were a fully factorial block (16) including:

1. Eighty-one tied columns with bars in 2 faces (Shape A in Fig. 7) and every combination of the following variables:
 $\gamma = 0.6, 0.75$ and 0.9
 $e/h = 0.1, 0.25$ and 0.40
 $\ell/h = 10, 20$ and 40
 $\rho_t = 0.008, 0.040$ and 0.064
 $f'_c = 4$ ksi
 $f_y = 60$ ksi
 (creep coefficient) = 5.0
2. Eighty-one tied columns with bars in 4 faces (Shape B in Fig. 7) and every combination of the variables given above.
3. Sixty-four other columns with 8 cross section shapes, 8 slenderness ratios, 8 values of ρ_t , 8 ratios of e/h , 8 values of f'_c , 8 values of γ and 8 values of the creep coefficient.

The columns in the third group were selected according to a 8×8 Graeco-Latin square (17) to extend the data in a more or less random way to sample the effects of other variables. The combinations of the variables have been chosen according to the pattern of a 8×8 Graeco-Latin square as shown in Table 2. Each of the 64 cells in Table 2 represents one "computer experiment" in the third group of columns and each of the seven numbers in each cell represents the level of one variable listed in Table 1. For example, the combination 4356273 for cross section shape C and slenderness ratio $\ell/h = 20$ in Table 2 means that the properties of this column were:

First Number - slenderness ratio, $\ell/h = 20$ (the 4th level of ℓ/h in Table 1)

- Second Number - cross sectional shape - C (the 3rd level in Table 1 - Fig. 7 shows this to be a spiral column)
- Third Number - $\rho_t = 0.04$ (the 5th value in Table 1)
- Fourth Number - $d_c/h = 0.175$ (the 6th value in Table 1)
- Fifth Number - $f'_c = 3,000$ psi
- Sixth Number - $e/h = 0.35$
- Seventh Number - $\phi = 2.0$ (creep coefficient)

It is a feature of the Graeco-Latin square variable arrangement that each value of each variable appears once with each value of every other variable. Latin squares and Graeco-Latin squares of different sizes are described in Ref. 17. An estimation of the total interaction of the variables is possible from the Analysis of Variance using the residual sum of squares. A residual sum of squares exists in the present case because the 8×8 Graeco-Latin square could contain nine instead of the six or seven variables used.

The ranges of the variables e/h and l/h in this study were chosen on the basis of a study of the range of variables encountered in over 22,000 actual columns in buildings (18).

Thus, a total of 226 columns have been analyzed to determine their capacity under short time loads. In addition, these columns have been studied under varying sustained loads yielding a total of 972 different loading cases.

The study was limited to hinged columns bent in symmetrical single curvature without transverse loads between their ends. The moment-magnifier procedure used in the ACI Code was originally derived for this particular type of column (2). Other loading patterns and other end restraint conditions are accounted for in the ACI Code using correction factors (C_m, k). By considering only hinged columns bent in single curvature, errors in C_m or k will not affect the accuracy of the EI expressions which are derived.

EVALUATION OF EFFECTIVE STIFFNESS EI FOR SHORT TIME LOADS

The maximum moment in an elastic beam-column bent in uniform single curvature can be calculated with sufficient accuracy for design purposes using Eqn. (20) (2):

$$M_u = \frac{P_u e}{1 - \frac{P_u}{P_c}} \quad (20)$$

where

$$P_c = \frac{\pi^2 EI}{\ell^2} \quad (21)$$

In order to obtain rational EI values including the effects of cracking, etc. the following method was used in this investigation: A number of columns are computed using the described method of analysis. From the results P_u and M_u for each column one stiffness value EI can be found by substituting P_u and M_u into Eqn. (20) and solving Eqns. (22) and (21) for EI:

$$EI = \frac{P_u M_u}{M_u - e P_u} \cdot \frac{\ell^2}{\pi^2} \quad (22)$$

The total population of these EI values computed from all the columns analyzed was then used to evaluate a simplified EI equation containing the most significant variables.

VARIABLES AFFECTING EI FOR SHORT TIME LOADS

The effects on EI of the variables considered in the computer column tests were studied using an analysis of variance and co-variance and a multiple linear regression analysis. Depending on the manner in which the effect of the reinforcement is expressed, the following three groups of variables were found to have a significant effect on $EI/E_c I_g$:

1. $E_s I_s / E_c I_g$, e/h , ℓ/h and for tied columns with bars in two faces, γ .
2. ρ_t , γ , e/h
3. $\rho_t \gamma^2$, e/h and ℓ/h .

The terms in each group are listed in order of decreasing significance.

As shown in Figs. 8, 9 and 10 for one family of columns the value of $EI/E_c I_g$ tends to increase as the total steel ratio, ρ_t , and the lever arm of the reinforcement, γ , increase. These figures suggest that ACI Equation 10.7 shows the correct general trend of the data although it tends to overestimate the effect of ρ_t and γ and underestimate the effect of the basic concrete section. A similar trend was observed for the columns with bars in all four faces as shown in Fig. 11. The effect of γ is much smaller for column shapes B to H (Fig. 7) than for shape A.

The values of $EI/E_c I_g$ for all 226 columns are plotted against ρ_t in Fig. 12. There is a definite correlation between EI and ρ_t .

The effect of γ and e/h on EI is shown in Fig. 13 for one set of variables. For small eccentricities the initial softening of the

cross section leading to failure is generally caused by yielding of the compression reinforcement. In such a case, the increase in stiffness due to increasing the value of γ is offset by the earlier yielding of the compression steel as shown by the values of $EI/E_c I_g$ for $\gamma = 0.9$ and 0.75 at $e/h = 0.1$ in Fig. 13.

The value of EI derived from the calculated failure loads of slender columns tended to increase as the slenderness increased as shown in Fig. 14. The ultimate load intensity and hence the degree of inelastic action is lower in a long column than in a short column leading to the observed increase in EI .

Very slender columns may fail due to instability rather than material failure as assumed in the ACI Code. For such columns the material failure moment corresponding to the instability load is considerably higher than the moment at which the column actually becomes unstable. As a result the EI from Eqn. 22 for such columns is lower than that controlling the instability. In every case a material failure has been assumed in evaluating the EI values so that the values obtained will be compatible with the ACI Code.

EVALUATION OF DESIGN EQUATIONS FOR EI FOR SLENDER COLUMNS SUBJECTED TO SHORT TIME LOADS

The value of EI of a slender column is affected by the amount and location of the reinforcement, the degree of cracking, the degree of inelastic action in the concrete and the tensile or compressive reinforcement, and other variables. Because of the complexity of the problem there is considerable scatter in the values obtained when various column sections are considered as is shown in Fig. 12 and a unique expression for EI cannot be derived analytically.

The derivation of the EI equations presented in this paper involved four steps:

1. Statistical selection of significant variables and calculation of a regression equation from the results of the 226 computer column tests.
2. Derivation of EI equations based on the elastic properties of the cross sections.
3. Simplification of the results of steps 1 and 2 to eliminate variables which were difficult to assess or which had limited variation in actual columns.
4. Comparison of the final equations to the results of column tests and to the computer column tests.

A multiple linear regression analysis was used to evaluate equations for EI using the values of EI computed with Eqn. 22 from the computer column tests. The following two equations gave the best

correlation for the various formats and groupings of variables considered.

$$EI = E_c I_g (0.190 + 0.773 E_s I_s / E_c I_g + 0.0072 \ell/h - 0.346 e/h) \quad (23)$$

$$EI = E_c I_g (-0.381 + 1.082 \frac{E_s}{E_c} \rho_t + 0.888 \gamma - 0.0064 \ell/h - 0.458 e/h) \quad (24)$$

The terms in these equations are arranged in order of decreasing significance. The equations had multiple correlation coefficients of 0.903 and 0.800 respectively, representing reasonable correlation.

The use of the term e/h in a design relationship was considered impractical because e/h tends to zero and infinity at its limits, because it is not clearly defined for unequal end eccentricities, and because it had not been checked over the full range of possible variation. If the term ℓ/h is also eliminated to simplify the equations, the following two regression equations result:

$$EI = 0.271 E_c I_g + 0.773 E_s I_s \quad (25)$$

$$EI = E_c I_g (0.317 + 1.09 \frac{E_s}{E_c} \rho_t) \quad (26)$$

These equations had multiple correlation coefficients of 0.845 and 0.648, respectively, representing adequate and fair correlation.

It is important to note that the effect of the reinforcement as given by the second term of Equation (25) is significantly less than the $1.0 E_s I_s$ currently used in ACI Equation 10.7. This is borne out by the data plotted in Figs. 8 to 11. For very large steel percentages the second term in this equation should tend to $[(n-1)/n] E_s I_s$. If $0.271 E_c$ is used to evaluate the effective modular ratio n in this expression, however, $(n-1)/n$ is about 0.97. For simplicity, and to approach the correct $E_s I_s$ for large steel percentages, the EI of the reinforcement was taken to be $1.0 E_s I_s$. The resulting equation for EI was:

$$EI = C E_c I_g + E_s I_s \quad (27)$$

where C was 0.198 for the tied and spiral columns and 0.205 for all 226 columns. Accordingly, Equation (27) which is the same as ACI Equation (10.7) will give answers similar to Equation (25) with a little more scatter. This equation should also be usable for composite columns and the continued use of ACI Eqn. 10.7 is recommended.

To simplify the calculation of EI using ACI Equation 10.7 (Eqn. 27) this equation can be expressed as:

$$EI = 0.2 E_c I_g + K E_s \rho_t \gamma^2 I_g \quad (28)$$

where K is a constant depending on the reinforcement pattern. Table 3 shows that K varies from 1.4 to 3.0.

Further simplification would result, with a corresponding loss of accuracy if $K\gamma^2$ in Eqn. (28) were taken as a constant as shown in Eqn. (29):

$$EI = 0.2 E_c I_g + 1.2 E_s \rho_t I_g \quad (29)$$

This equation resembles the regression equation (26) except that the first term has been set equal to $0.2 E_c I_g$ to agree with Eqn. (28). This equation is slightly easier to apply than Eqn. (28) and it is proposed that ACI Eqn. 10.8 be replaced with Eqn. (29). For simplified design the designer would guess ρ_t and evaluate EI , correcting his guess later if required. If he felt this was too complicated, he would set ρ_t equal to its minimum value of 0.01, evaluate Eqn. (29) and proceed.

The ultimate loads computed using ACI Eqns. 10.7 and 10.8 and the proposed Eqn. (29) are compared to the theoretical short time ultimate capacities for the 226 computer column tests in Fig. 15 and Table 4. The proposed equation for EI (Eqn. 29) has much less scatter than ACI Eqn. 10.8. For the short time computer tests it is a little more conservative than ACI Eqn. 10.7 although Eqn. (29) has a little more scatter.

The three EI equations, Eqn. (29) and ACI Eqn. 10.7 and 10.8 have been used to compute the ultimate load capacity of 199 columns tested in short term tests in thirteen investigations (3-8 and 19-25). The comparisons have been limited to hinged columns bent in symmetrical single curvature for the reason discussed earlier. All but 18 of the columns were rectangular. The concrete strength has been converted to the cylinder strength f'_{cyl} in cases where cube or prism strength were reported. The cylinder strength has been assumed as $0.85 f'_{cube}$ or $1.00 f'_{pr}$. The cylinder strength for Baumann's (19) tests and Tal and Chistiakov's (25) tests has been assumed as $1.05 f'_{pr}$, where the concrete strength was measured on prisms having dimensions 12 by 12 by 36 cm. and 20 by 20 by 60 cm., respectively. The prism strengths reported by Goyal and Jackson (7) were used in preference to the cylinder strengths as recommended in their paper. The column strength has been taken equal to the cylinder strength, $f'_c = f'_{cyl}$, for horizontally cast columns and as $f'_c = 0.85 f'_{cyl}$ for vertically cast columns.

The results are presented in Fig. 16 and Table 4. The results from the proposed Eqn. (29) compare reasonably with ACI Eqn. 10.7.

The proposed Eqn. (29) has a smaller coefficient of variation than the other two equations. It should be noted that the range of ρ_t and γ in the test specimens was relatively small and this tends to reduce the scatter for Eqn. (29) and particularly for ACI Eqn. 10.8.

EVALUATION OF DESIGN EQUATIONS FOR EI FOR SLENDER COLUMNS
SUBJECTED TO SUSTAINED LOADS

Four different procedures are currently used or have been proposed to account for the effect of creep on the strength of slender columns. These are the Reduced E-Modulus Method used in the ACI Code (1), the Dischinger Method (26), a simplified approximation to the Dischinger Method (8) and the Creep Eccentricity Method (27). Only the Reduced E-Modulus Method will be discussed in this paper since it is a relatively simple procedure to apply and since North American designers are familiar with this procedure.

The Reduced E-Modulus Method as presented in the 1971 ACI Code takes the effect of creep into account by reducing the flexural stiffness to $EI_R = EI/(1 + \beta_d)$. This value of EI_R is then used in a moment magnifier equation to compute magnified moments for the design of the column. In the original derivation of EI_R it was observed that under sustained loads the load transfer to the reinforcement due to creep frequently led to a reduction of the effective value of EI due to premature yielding of the compression reinforcement and for this reason both the concrete and the steel terms in the ACI EI equation were reduced due to creep. With the trend to higher steel percentages this has led to serious questions about this procedure, especially for composite columns. Accordingly Eqns. (30) and (31) will be used to define the reduced modulus in this study:

$$EI_R = \frac{E_c I_g}{5\alpha} + E_s I_s \quad (30)$$

or

$$EI_R = \frac{E_c I_g}{5\alpha} + 1.2 E_s \rho_t I_g \quad (31)$$

where α accounts for the effects of sustained loading. Eqn. (22) can be solved for EI_R and α using the values of M_u , P_u calculated for $t = \infty$. Eqn. (30) was used to define EI_R in this evaluation since this is the basic design equation proposed in this study. The same value of α was found to be usable in both Eqn. (30) and Eqn. (31) for practical design purposes. By computing α for time $t = \infty$ for all the columns analyzed, a population of α values is created which can be used in the statistical derivation of a value of α for use in design.

Because the designer must base his design on the interaction diagram (A) in Fig. 6, the value of M_u used in deriving α was the point on diagram (A) corresponding to P_{su} . This is labelled as M'_{su} in Fig. 6.

Using a stepwise multiple regression analysis the correlation between the creep parameter α and the following variables was investigated:

1. ϕ_∞

2. β_p, β_p^2
3. $\ell/h, (\ell/h)^2, h/\ell, (h/\ell)^2$
4. ρ_t (or $\frac{E_s I_s}{E_c I_g}$)
5. $e, d_c/h, f'_c$, and ℓ/e .

The variable β_p represents the ratio of the sustained load to the required ultimate load for which the designer must design his column. In the regression analysis β_p was taken equal to the ratio P_s/P_{su} . Since the analyzed columns had a constant end eccentricity, $\beta_p = \beta_d$.

The level of sustained load has been expressed in terms of the ratio of loads, β_p , rather than the ratio of moments, β_d , as in the ACI Code for two principal reasons:

1. There are a number of practical cases where the original ACI definition of β_d has broken down including:
 - a) minimum eccentricity.
 - b) sustained moment of different sign from short time moment.
 - c) short time moment reverses in sign.
 - d) short time moments reduce the total moment below the sustained load moment.
2. No comprehensive study has been made of cases involving columns loaded with sustained loads having a different eccentricity than that of the rapidly applied load.

The effects of a change from β_d to β_p can be shown by a number of practical cases:

- i. Column loaded with a constant eccentricity throughout its loading history - both procedures are the same.
- ii. Column in lower floor of a building - such a column would be governed by minimum eccentricity and the code β_d lacks definition. Probably β_d would be taken as P_{sust}/P_{total} .
- iii. Column with wind moments - If wind moments controlled the design of such a column β_d would be negative and therefore 0 for wind blowing in one direction and would equal the sustained load moment over the total moment for wind in the other direction. For this case β_d would be small, say 0.2, for the top storeys and larger, say 0.5 or so for the lower storeys where more of the load is sustained. For such a case β_p defined on the basis of loads instead of moments would follow the same trend and would be small in the upper storeys and larger in the lower storeys.
- iv. A column in a seismic region will generally not be very slender and hence this problem is less important there.

Because, as shown in Fig. 6, the strength behavior of a column failing under a short time load applied after a period of sustained load is different from the strength and behavior of the same column if it fails during the period of sustained loading, the two types of failure have been considered separately in the statistical derivations.

The strength of some of the shorter columns increased during the sustained load period because the effect of the gain in concrete strength exceeded the reduction in strength due to slenderness. In the derivations that follow, only those columns for which the strength was reduced by the sustained load were considered.

DERIVATION OF α FOR COLUMNS FAILING UNDER A SHORT TIME LOAD FOLLOWING A PERIOD OF SUSTAINED LOAD

A statistical investigation of α showed that the order of significance among the variables considered was e/h , β_p , γ , l/h and the creep coefficient ϕ for Eqn. (30) and e/h , ρ_t , β_p , l/h and ϕ for Eqn. (31). The presence of the terms e/h , l/h and γ can be explained in large part by the rejection of these terms when Eqns. (23) and (24) were simplified to Eqns. (27) and (29). The correlation was not very good in this analysis possibly because all the inaccuracies from the use of the moment magnifier, from the EI value and from the effects of strength gain and creep are concentrated in the term for α .

Because North American designers are not accustomed to calculating creep coefficients, ϕ , this variable was not included in the final design equations. The term e/h was eliminated for the reasons given before, and l/h was deleted because the value of α tended to reduce as l/h increased and it was felt desirable to become slightly more conservative as l/h increased. The final equation for α was derived to be:

$$\alpha = 0.75 + 1.8 \beta_p \quad \text{but not less than 1.0.} \quad (32)$$

This equation will be compared to the theoretical strengths of the computer column tests and to actual test results later in this report.

DERIVATION OF α FOR COLUMNS FAILING UNDER A SUSTAINED LOAD

As shown in Fig. 6, columns subjected to high sustained loads may fail under the sustained load. The lowest value of sustained load for which this will happen is called the critical sustained load, P_{scr} . In order to determine a design value of the critical sustained load, the creep parameter α_{scr} has been derived from the values of P_{scr} found in the computer tests. This term was derived in the same manner as α except that $P_{ui,k}$ had to be replaced by P_{scr} and $M_{ui,k}$ by the moment M'_{scr} on the interaction curve ($\bar{A}$) in Fig. 6 at the level of P_{scr} . This

moment has been used because ($\bar{A}$) is the only interaction curve available to the designer and therefore the use of M_{scr} would have led to creep parameters which would overestimate P_{scr} slightly.

Equation (33) was derived from the computer column test results as value of β_p at lower limit of the data for columns failing under sustained loads.

$$\beta_{scr} = 0.85 + 4\rho_t - 0.0075 \ell/h \quad (33)$$

The minimum ratio of critical sustained load to ultimate short time load in the computer column tests was 0.513. For this column it was found that for values of $\beta_p \geq 0.62$ the sustained load exceeded the critical sustained load. For a column designed to carry only dead load the ACI load factor of 1.4 would give $\beta_p = 0.71$. Thus this could be a practical case for slender, lightly reinforced columns.

Considering only the results for columns failing at $t = \infty$ under a sustained load without further loading, it was possible to derive α_{scr} as:

$$\alpha_{scr} = 0.06 \ell/h \text{ but not less than } 1.0. \quad (34)$$

In the design of a column subjected to sustained loads, the column would first be proportioned for the design (ultimate) load using Eqns. (32) and either (30) or (31). If the sustained loads represented a major portion of the total loads the designer would check if the column would fail during the sustained loading period when subjected to the sustained loads only. This check would be carried out using Eqns. (33) and either (30) or (31) with $P_u = P_s$ in Eqn. (20).

When this procedure was investigated, however, it was found that the inherent conservatism in Eqns. (30) to (32) for very slender columns made it unnecessary to carry out the second analysis for high sustained loads in columns with ℓ/h up to 40. Accordingly, a designer need only use Eqns. (30) to (32) to design a slender column under sustained loads. Where possible, β_p should not be allowed to exceed β_{scr} . For very slender columns this may necessitate an increase in ρ_t .

Accuracy of Proposed Equations

The design capacities computed using Eqns. (32 and 30 or 31) and the two ACI equations for EI were compared to the theoretical sustained load capacities of the 972 column loading cases analyzed. The calculated ultimate load and moment result from the point of intersection between the P, M - curves representing the column behavior and the short-time P, M - interaction curve (curve $\bar{A}$ in Fig. 6). The results are denoted by P_{sUD} where the subscript D refers to design values. Results from the theoretical analysis, up to now referred to as $P_{u1,k=NT}$ will be denoted by P_{sUC} in the balance of the paper where the subscript C refer to theoretical values.

Histograms of P_{sUC}/P_{sUD} are presented in Fig. 17 for the two ACI

equations and Eqns. (30) and (31). The understrength factor, ϕ , has been taken equal to 1.0 in these comparisons. Characteristic values of the individual distributions are reported in Table 5. The quality of the distribution measured by the coefficient of variation is best for the design procedure based on Eqn. (30) and poorest for that based on ACI Eqn. 10.8.

It is interesting to note that the mean value of P_{uC}/P_{uD} based on $\phi = 1.0$ was 1.09 and 1.33 for ACI Eqns. 10.7 and 10.8 for short time loads and increased to 1.23 and 1.60 for sustained loads. This suggests that the ACI creep assumption tends to be conservative.

Sustained load tests on reinforced concrete columns have been reported by several authors (5-8). The usual test procedure is to subject a column to a constant sustained load; if no creep failure occurs within a certain time period the column is loaded up to failure in a final short term test. To compute the ultimate load of these columns the time dependency of the creep parameters has to be known. In this study it has been assumed that the time dependency of α corresponds to the time dependent development of ϕ can be taken as $\gamma(t)$ times the values at $t = \infty$ where:

$$\gamma(t) = 1 - \exp(-0.1325 \sqrt{t}) \quad (35)$$

In applying the ACI EI Eqns., β_d was taken equal to $\gamma(t)$ times β_p . In Eqns. (30) and (31) the ultimate capacity of a column was computed using $\gamma(t)\alpha$.

The ratios between measured and computed ultimate loads are summarized in the histograms in Fig. 18 and in Table 5 for 74 sustained load column tests (5-8). In every case the columns had hinged ends and were bent in symmetrical single curvature. The concrete strength conversion factors used earlier were also used in this study. The columns tested under sustained load by Gaede (3,4) have not been included because of the relatively large variations in the sustained load during these tests.

Again, Eqns. (30) and (31) gave as good or better agreement with the analyses and tests as the current ACI equations.

Inclusion of Understrength Factor ϕ in Eqn. 22

In the original derivation (2) of the moment magnifier method used in the ACI Code it was assumed that by multiplying the short column interaction diagram co-ordinates by $\phi = 0.7$ and 0.75, the effects of understrength would be adequately accounted for. Subsequently, in response to a discussion (28) of the Proposed Code, the value of P_{cr} in Eqn. 22 was multiplied by ϕ giving the appearance that ϕ is considered twice in the design of a slender column. However, the strength of a short column is governed primarily by the interaction diagram and is relatively insensitive to whether P_{cr} or ϕP_{cr} is used. On the other hand, the strength of a slender column is governed primarily by ϕP_{cr} and is relatively insensitive to the ϕ included in

the interaction diagram. Thus, at the two extremes the manner of including ϕ is essentially correct and no change is proposed at this time.

If the ACI values of ϕ are used in Eqn. 22 and also in the interaction diagram, the minimum, maximum and average values of P_{uC}/P_{uD} for the 972 sustained load computer tests for Eqns. (30) and (32) increase by 1.42 ± 0.01 times. This is approximately $1/\phi$ times the value for $\phi = 1.0$. This suggests it is correct to include ϕ in both places.

DESIGN RECOMMENDATIONS

On the basis of the study reported in this paper it is proposed that lines 10 to 14 of Section 10.11.5 of the 1971 ACI Building Code be changed to read:

"In lieu of a more precise calculation, EI in Eqn. 10.6 may be taken either as:

$$EI = \frac{E I_c}{5\alpha} + E_s I_s \quad (10.A)$$

or:

$$EI = \frac{E I_c}{5\alpha} + 1.2 E_s \rho_t I_g \quad (10.B)$$

where

$$\alpha = 0.75 + 1.8 \beta_p \text{ but not less than } 1.0 \quad (10.C)$$

In Section 10.0, the definition of β_p should be inserted:

" β_p = the ratio of the design sustained load to the total design load."

A corresponding change should be made in Eqn. 10-11 of the ACI Building Code.

CONCLUSIONS

This paper presents a statistical evaluation of the stiffness parameters for use in the design of slender reinforced concrete columns subjected to short time and sustained loads. It is recommended that the current ACI Code Eqns. 10.7 be modified slightly and that the current Eqn. 10.8 should be replaced.

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NOTATION

C	=	force in a concrete "fiber" in the cross-section
C_1, C_2	=	constants used in defining stress dependency of creep function
d_c	=	thickness of concrete cover on reinforcement measured to center of longitudinal bar.
e	=	eccentricity of compressive force measured to centroid of section
e_{cr}	=	residual eccentricity due to creep after a column has been subjected to sustained load is subsequently unloaded
E_c	=	modulus of elasticity of concrete as defined in Section 8.3.1 of ACI 318-71.
E_s	=	modulus of elasticity of steel = 29,000 ksi
EI	=	rotational stiffness of cross-section
EI_R	=	reduced stiffness to account for sustained load effects
f_c	=	stress level in concrete
f'_c	=	standard 28 day compressive strength of concrete
f'_t	=	tensile strength of concrete as defined in Section 11.5.2 of ACI 318-71.
h	=	overall depth of a cross-section
i	=	subscript for axial load increment
I_g	=	gross moment of inertia of uncracked concrete cross section
I_s	=	moment of inertia of the area of the reinforcing bars

j	= subscript for strain increment
k	= subscript for time increment
ℓ	= buckling length of column
M	= moment
M_{cc}	= cracking moment in presence of given axial load
M_o	= moment corresponding to zero tensile stress on one face of a column in presence of given axial load
M'_{su}	= ultimate moment for a given sustained load and short time load to failure computed using design interaction diagram but with understrength factor, ϕ , equal to 1.0.
P	= axial load
P_c	= critical load
P_s	= sustained load
P_u	= short time capacity
P_{scr}	= critical sustained load. Sustained loads equal to or greater than this value will cause failure of the column without any subsequent rapid loading
P_{uC}	= theoretically calculated capacity of a "computer test" column
P_{suC}	= theoretically calculated capacity of a column for time = ∞
P_{uD}	= calculated ultimate load based on design EI values and $\phi = 1.0$
P_{uT}	= ultimate load measured in laboratory column tests
t	= time
S	= force in a steel layer in the cross section
w	= the mid-height deflection of the column
w_o	= instantaneous deflections
w_{cr}	= residual deflection due to creep after a column has been subjected to sustained load and is subsequently unloaded
α	= factor used to reduce modulus of elasticity of concrete to account for sustained loads, Eqn. (32).

α_s	= value of α for columns failing under sustained load, Eqn. (34)
β_{scr}	= critical value of β_ρ above which columns fail under sustained load, Eqn. (33)
β_d	= ratio of maximum design dead load moment to maximum design total load moment
β_ρ	= ratio of design dead load to design total load
γ	= ratio of distance between centroids of outermost bars in section to overall depth of section
γ	= factor used to adjust time effects when comparing analyses with tests, Eqn. (35)
ϵ	= strain
ϵ_{cr}	= creep strain
ϵ_o	= instantaneous strain
ϵ_s	= shrinkage strain
ϵ_1	= strain at most compressed face
ϵ_u	= assumed strain at crushing of concrete under short time loads = 0.0035
$\phi(t)$	= creep function at time (t), Eqn. (3)
$\phi_{\infty, 0.40}$	= creep function for time = ∞ and $f_c/f'_c = 0.40$
ϕ	= understrength factor
ϕ	= curvature
$\bar{\phi}$	= curvature including effect of tension stiffening
ρ_t	= total reinforcement ratio = total area of reinforcement divided by total area of cross section

Table 1
Variables for Columns*

Order in Cell	Variable	Level of Variable							
		1	2	3	4	5	6	7	8
1.	Slenderness Ratio ℓ/h	5	10	15	20	25	30	35	40
2.	Cross-Section Shape (See Fig. 7)	A	B	C	D	E	F	G	H
3.	Reinforcement Ratio ρ_t	0.008	0.016	0.024	0.032	0.040	0.048	0.056	0.064
4.	Reinforcement Cover d_c/h	0.050	0.075	0.100	0.125	0.150	0.175	0.200	0.225
5.	Concrete Strength f'_c (psi)	2000	3000	4000	5000	6000	7000	8000	9000
6.	Load Eccentricity e/h	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
7.	Creep Coefficient ϕ	0.8	1.4	2.0	2.6	3.2	3.8	4.4	5.0

*Values of factors held constant: $f_y = 60,000$ psi, $E_s = 29.0 \times 10^6$ (psi),

$f'_t = 6 \sqrt{f'_c}$ (psi), $\epsilon_u = 0.0035$

Table 2

8 x 8 Graeco-Latin Square Experimental Design for Third Group of Columns

Slenderness Ratio ℓ/h	Cross Section Shapes (See Fig. 7)							
	A	B	C	D	E	F	G	H
5	1111111	1225467	1332685	1443872	1557348	1664753	1778236	1886524
10	2122222	2218643	2371458	2467531	2583765	2646387	2735174	2854816
15	3133333	3274582	3317864	3451627	3542156	3685241	3726718	3868475
20	4144444	4263128	4356273	4415786	4538517	4621835	4787652	4872361
25	5155555	5281376	5348721	543268	5516432	5673614	5762847	5827183
30	6166666	6247215	6384137	6428354	6575823	6612578	6753481	6831742
35	7177777	7236851	7323546	7482413	7561284	7658162	7714325	7845638
40	8188888	8252734	8365312	8476145	8525671	8637426	871563	8813257

Note: The variables listed in each cell were taken from Table 1.

re-examination of ei value

TABLE 3 CALCULATION OF I_s
 note : total area of steel = $A_{st} = \rho_t A_c$

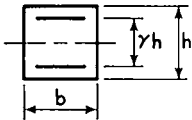
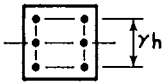
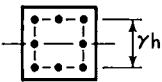
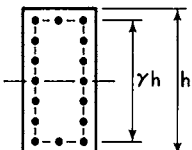
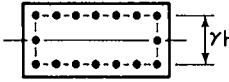
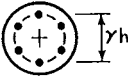
TYPE OF COLUMN	No. OF BARS	I_s	I_s / I_c
	—	$0.25 A_{st} \gamma^2 h^2$	$3 \rho_t \gamma^2$
	3 per face 6 per face	$0.167 A_{st} \gamma^2 h^2$ $0.117 A_{st} \gamma^2 h^2$	$2 \rho_t \gamma^2$ $1.4 \rho_t \gamma^2$
	8 bars (3 per face) 12 bars (4 per face) 16 bars (5 per face)	$0.187 A_{st} \gamma^2 h^2$ $0.187 A_{st} \gamma^2 h^2$ $0.172 A_{st} \gamma^2 h^2$	$2.24 \rho_t \gamma^2$ $2.24 \rho_t \gamma^2$ $2.06 \rho_t \gamma^2$
	$h = 2b$ 16 bars as shown About Strong Axis	$0.128 A_{st} \gamma^2 h^2$	$1.54 \rho_t \gamma^2$
	$b = 2h$ About Weak Axis	$0.219 A_{st} \gamma^2 h^2$	$2.63 \rho_t \gamma^2$
	—	$0.125 A_{st} \gamma^2 h^2$	$2 \rho_t \gamma^2$
	—	$0.125 A_{st} \gamma^2 h^2$	$1.5 \rho_t \gamma^2$

Table 4
Comparison of EI Equations to Analyses
and Tests - Short Time Loads

	Ratio of $\frac{P_{uC}}{P_{uD}}$		
	ACI Eqn. 10.7	ACI Eqn. 10.8	Eqn. 29
<u>Computer Column Tests</u>			
-226 Columns			
Mean	1.09	1.33	1.16
Coefficient of Variation	22.7%	35.0%	24.8%
Minimum	0.715	0.612	0.717
Maximum	2.375	3.608	2.454
Percentage less than 1.0	39.9%	20.2%	25.9%
<u>Laboratory Column Tests</u>			
-199 Columns			
Mean	1.31	1.30	1.39
Coefficient of Variation	24.8%	25.2%	23.5%
Minimum	0.803	0.764	0.804
Maximum	2.669	2.337	2.576
Percentage less than 1.0	17.5%	21.6%	11.1%

Note: The ACI Understrength Factor ϕ was taken equal to 1.0 in all comparisons

Table 5
Comparison of EI Equations to Analyses
and Tests - Sustained Loads

	Ratio of $\frac{P_{uC}}{P_{uD}}$ or $\frac{P_{uT}}{P_{uD}}$			
	ACI Eqn. 10.7	ACI Eqn. 10.8	Eqns. (30 & 32)	Eqns. (31 & 32)
<u>Computer Column Tests</u>				
-972 Cases				
Mean	1.23	1.60	1.11	1.19
Coefficient of Variation	19.7%	40.2%	18.8%	21.8%
Minimum	0.784	0.623	0.708	0.681
Maximum	2.148	6.168	2.015	2.433
Percentage less than 1.0	9.0%	9.4%	28.2%	17.8%
<u>Laboratory Column Tests</u>				
-74 Columns				
Mean	1.58	1.68	1.27	1.40
Coefficient of Variation	27.3%	24.5%	25.0%	27.4%
Minimum	0.895	0.895	0.854	0.874
Maximum	2.982	2.657	2.472	2.603
Percentage less than 1.0	2.7%	2.7%	9.4%	4.1%

Note: The ACI Understrength Factor ϕ was taken equal to 1.0 in all comparisons

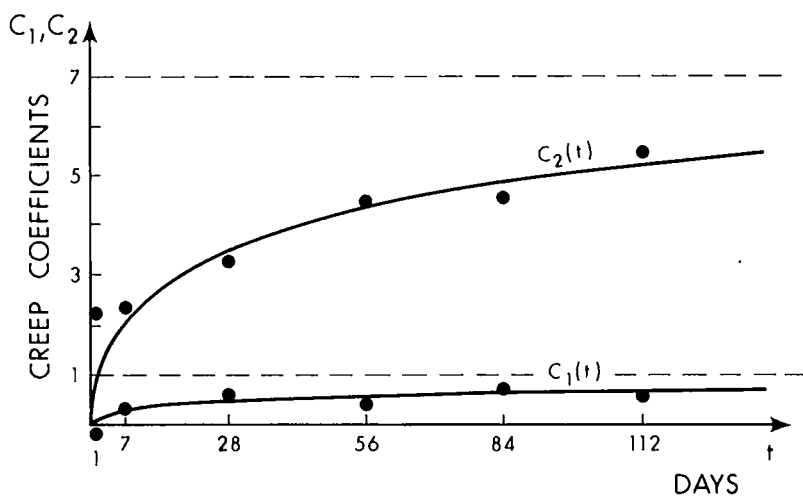


Fig. 1--Creep coefficients

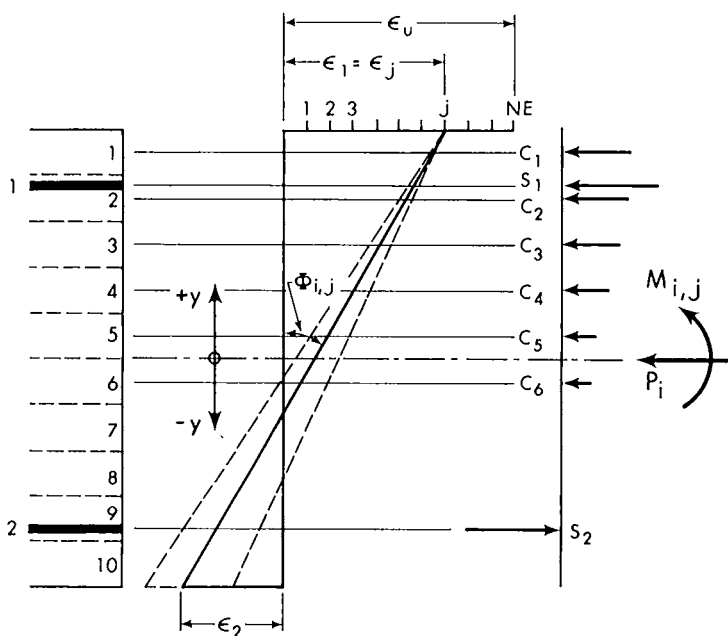


Fig. 2--Iterative computation of the curvature

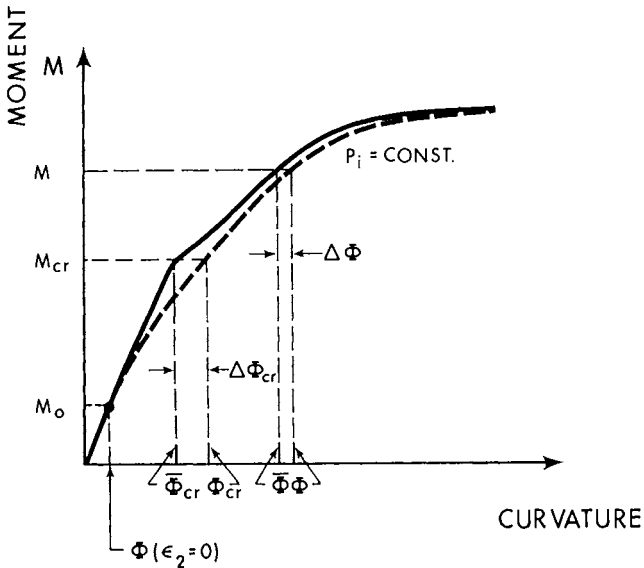


Fig. 3--Influence of the concrete tensile zone on the moment-curvature relationship

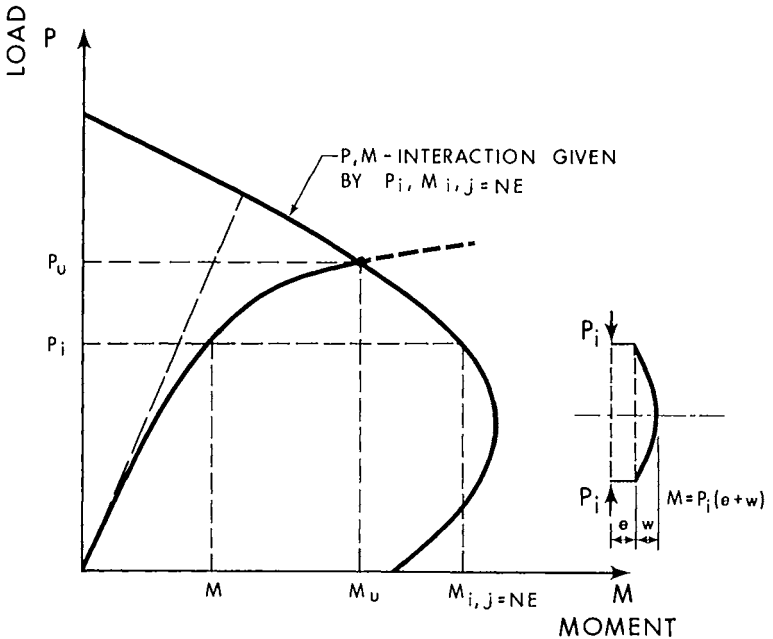


Fig. 4--Load and moment in a slender column under short term load conditions

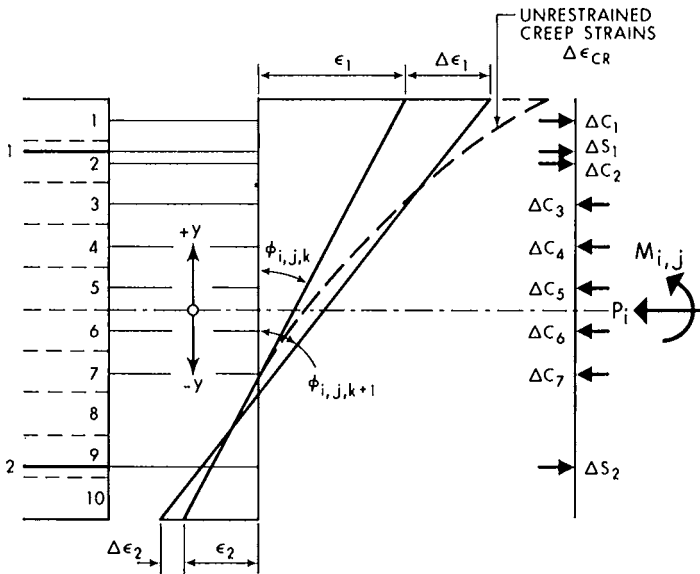


Fig. 5--Change in curvature and residual forces in fibers during creep stage k

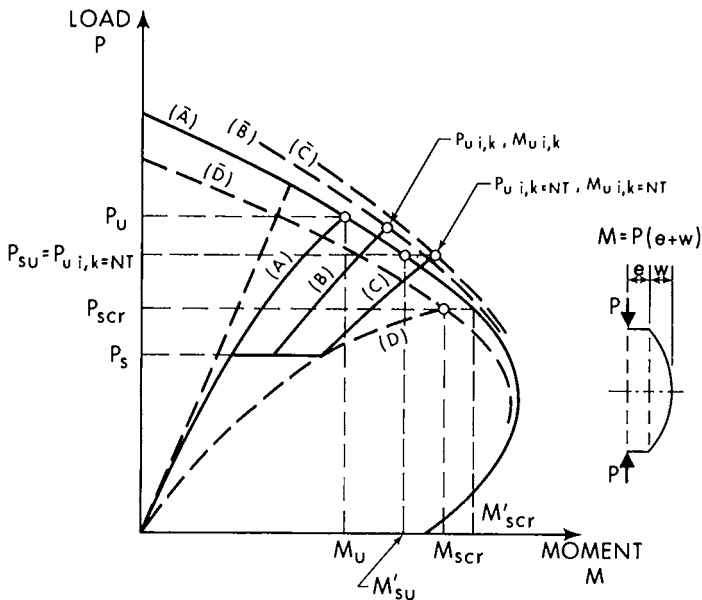


Fig. 6--Load and moment in a slender column under short term and sustained loads

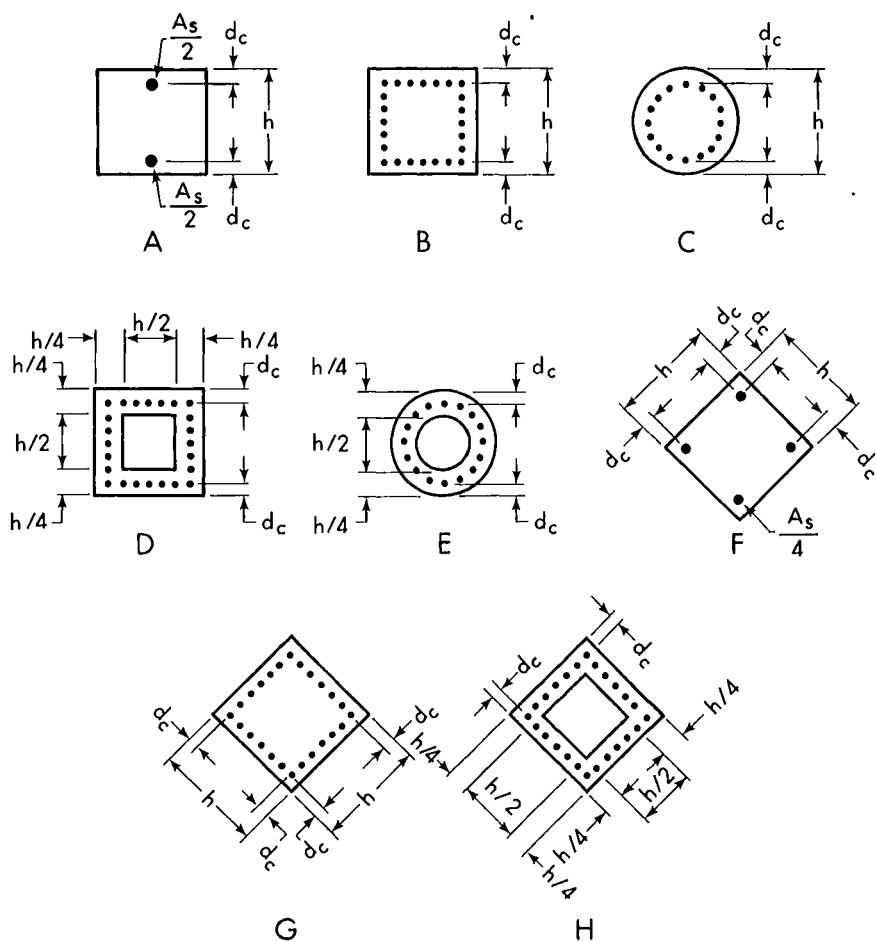


Fig. 7--Cross section shapes investigated in the analysis

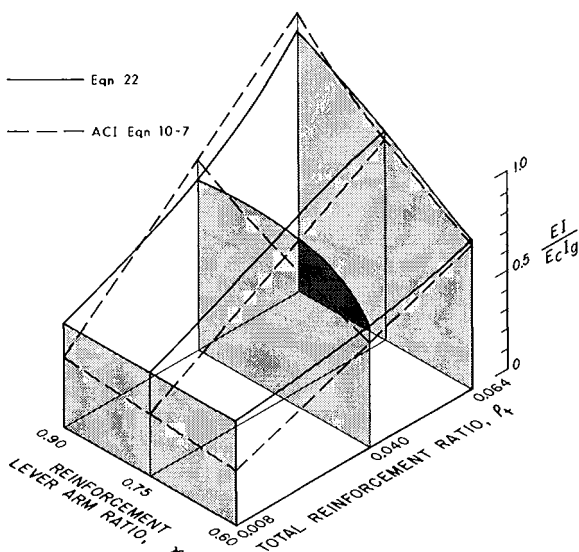


Fig. 8--Effect of steel ratio ρ_t and reinforcement lever arm, γ , on EI for tied columns with bars in two faces, $e/h = 0.1$ and $l/h = 20$

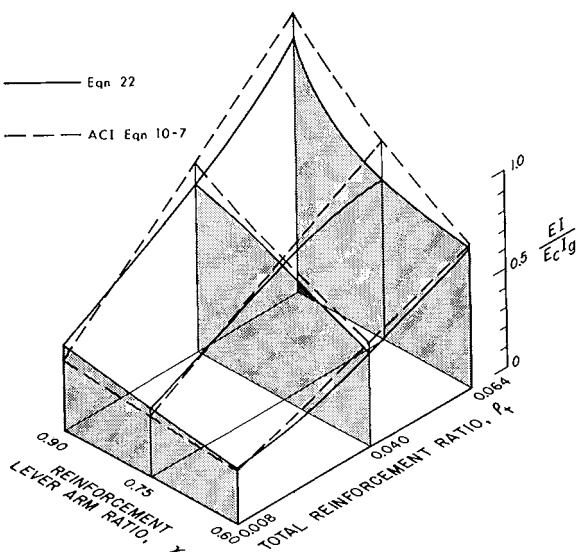


Fig. 9--Effect of steel ratio ρ_t and reinforcement lever arm, γ , on EI for tied columns with bars in two faces, $e/h = 0.25$ and $l/h = 20$

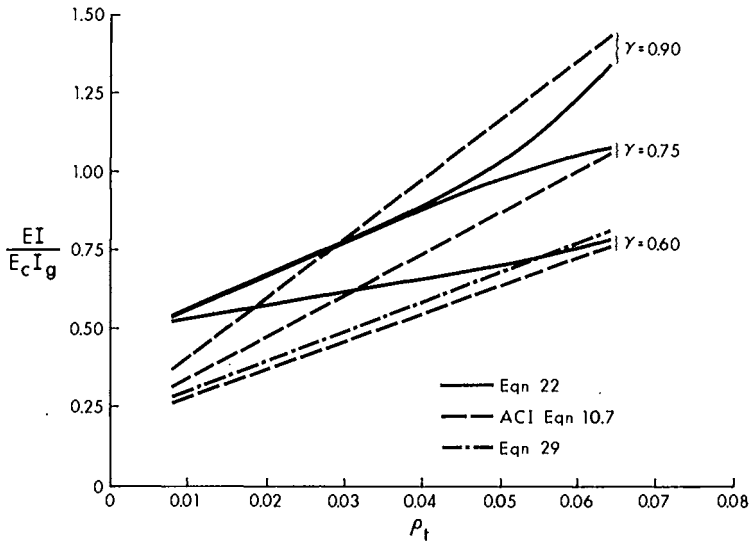


Fig. 10--Effect of steel ratio on EI for tied columns with bars in two faces, $e/h = 0.1$ and $l/h = 20$

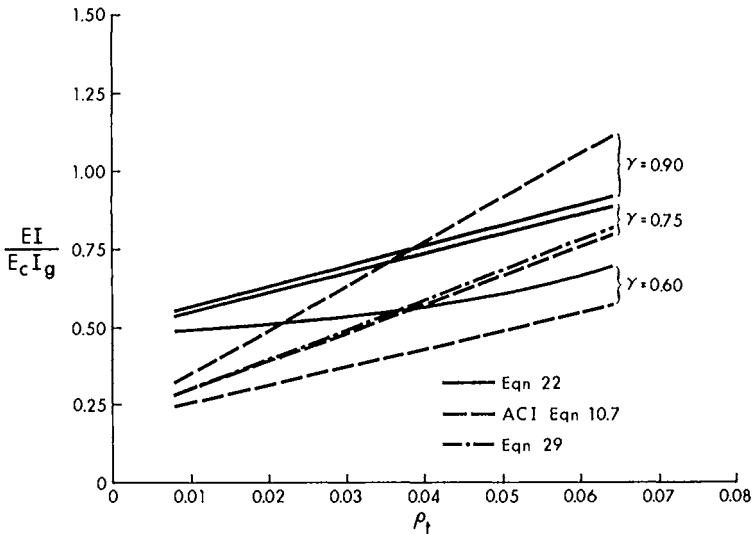


Fig. 11--Effect of steel ratio on EI for tied columns with bars in four faces, $e/h = 0.1$ and $l/h = 20$

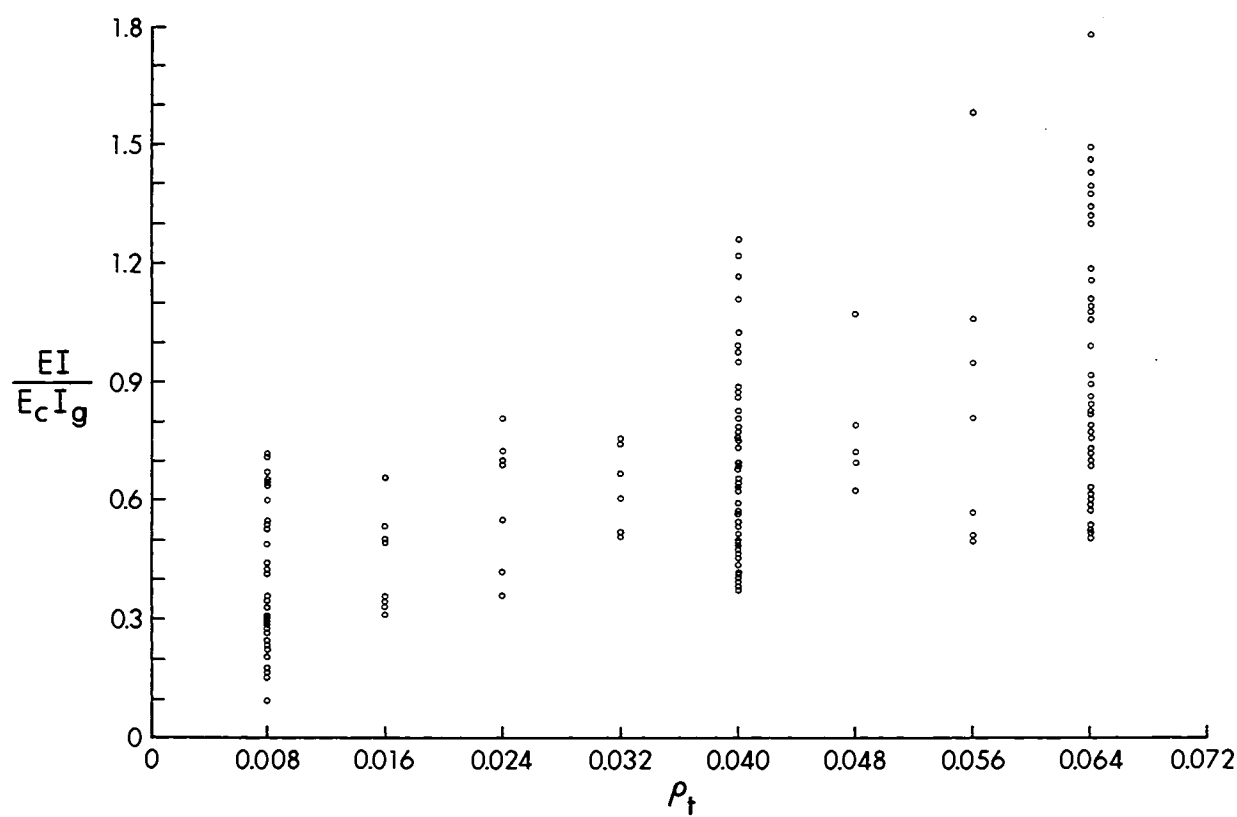


Fig. 12--Effect of steel ratio on EI for all computer column tests

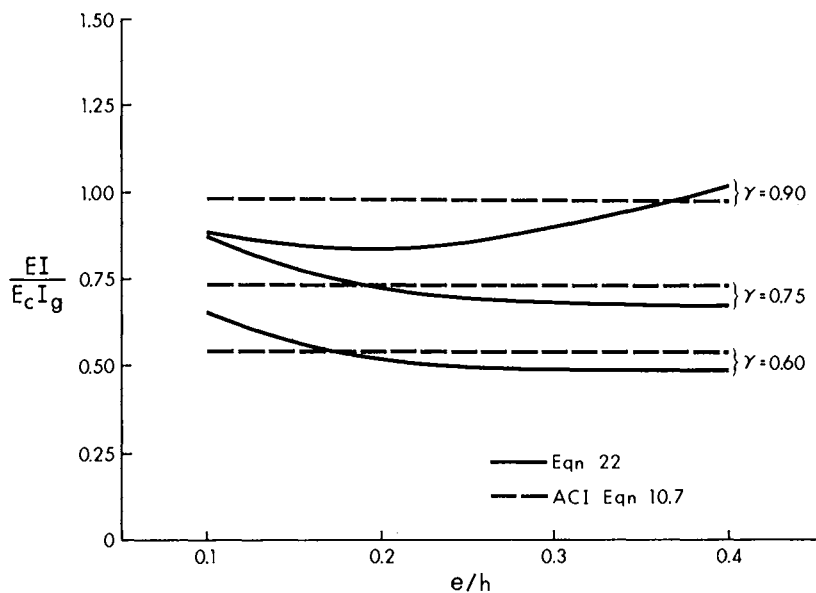


Fig. 13--Effect of γ and e/h on EI for tied columns with bars in two faces, $\rho_t = 0.04$ and $l/h = 20$

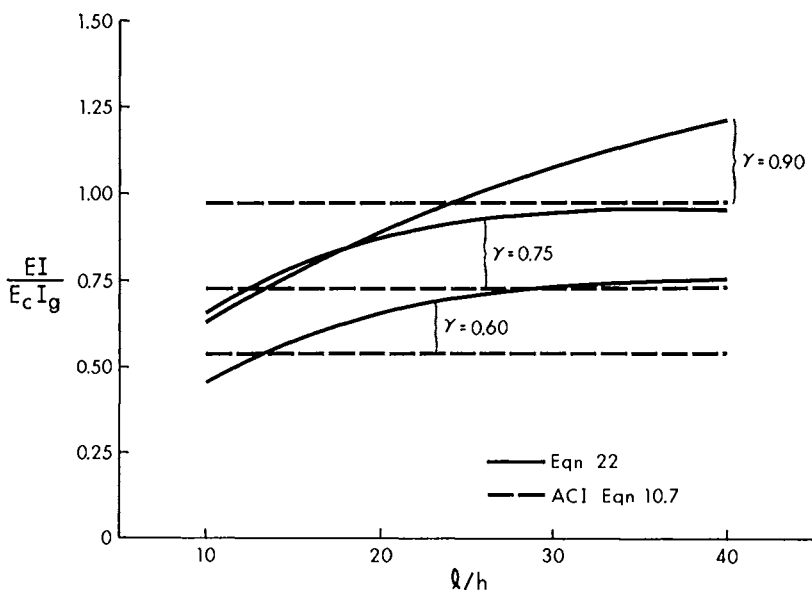


Fig. 14--Effect of γ and l/h on EI for tied columns with bars in two faces, $\rho_t = 0.04$, $e/h = 0.10$

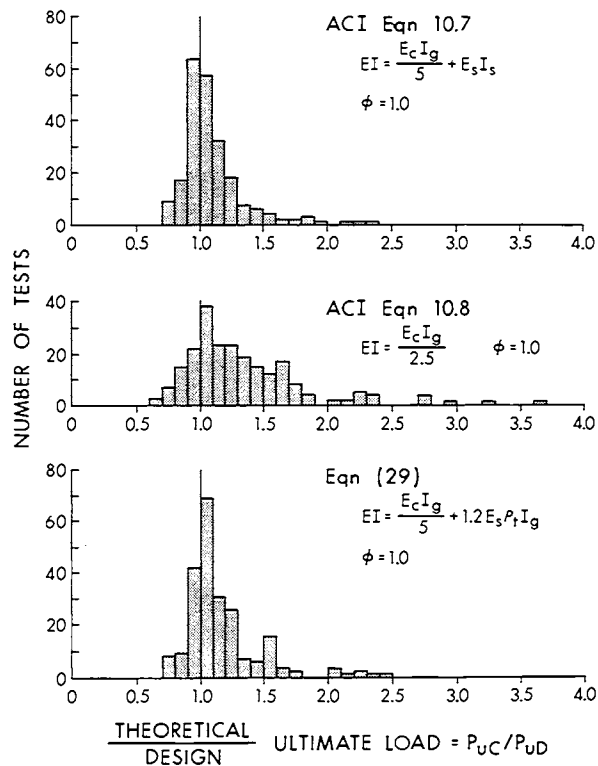


Fig. 15--Comparison of design equations to computer column tests--Short time load

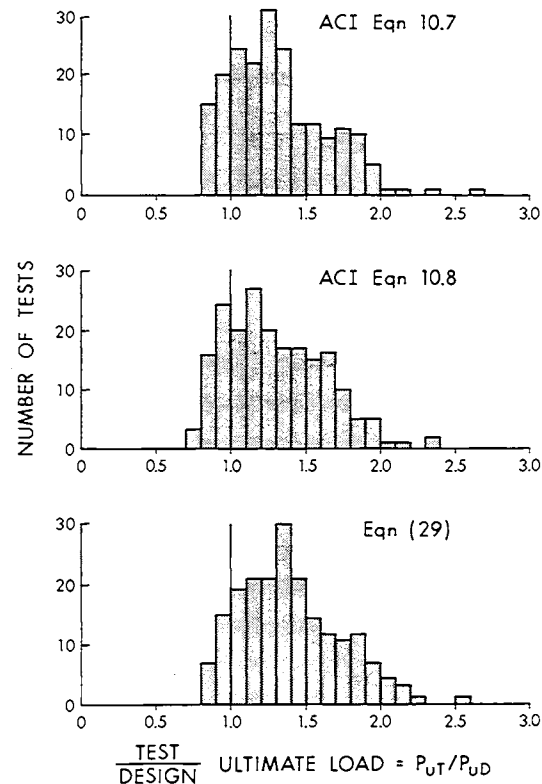


Fig. 16--Comparison of design equations to laboratory column tests--Short time load

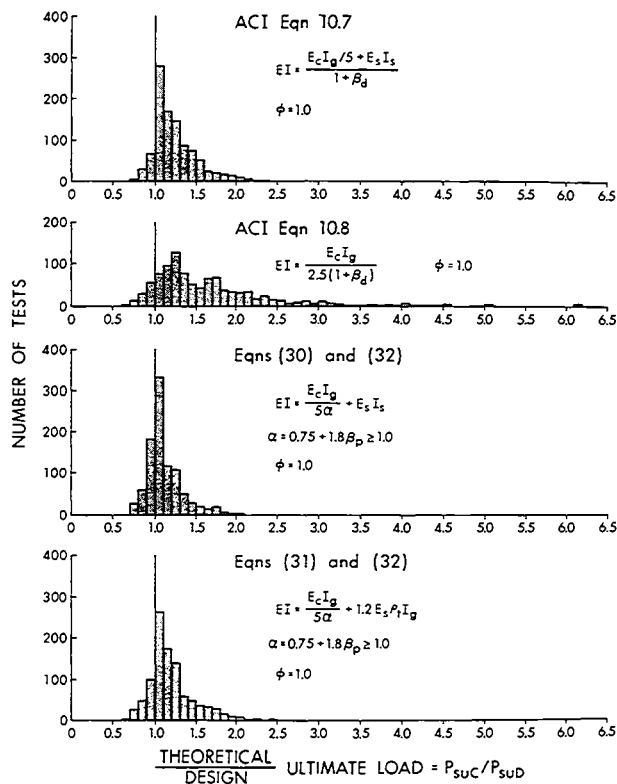


Fig. 17--Comparison of design equations to computer column tests--Sustained load

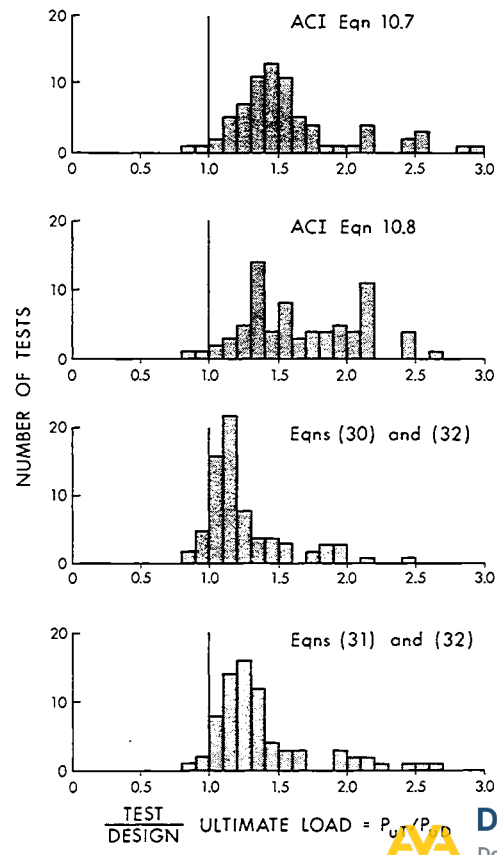


Fig. 18--Comparison of design equations to laboratory column tests--Sustained load

SP 50-2

EFFECT OF YIELDING OF RESTRAINTS ON SLENDER CONCRETE COLUMNS WITH SIDESWAY PREVENTED

By

G. A. Blomeier and J. E. Breen

Synopsis: The effect of yielding of beam end restraints on slender reinforced concrete columns as part of a frame in which sidesway was prevented was studied experimentally and analytically. The columns were part of a three-bay experimental frame that reproduced as realistically as possible conditions in a typical multistory frame. The columns were subjected to single curvature and the degree of end restraint was varied by using different percentages of steel in the beams.

Physical tests involved three model frames and investigated two different degrees of end restraint. The test results were compared with the numerical solution of a mathematical model which considered the nonlinear response of the concrete section and the changes in moments due to deflections and variations in stiffness. The numerical solution was extended to consider the effects of beam hinging and the effects of reinforcement strain hardening.

Keywords: beam-column frame; beams (supports); bending (reinforcing steels); bending moments; columns (supports); compressive strength; deflection; elastic limit; frames; framing systems; hinges (structural); loads (forces); Tong columns; mathematical models; reinforced concrete; reinforcing steels; research; restraints; rigid frames; slenderness ratio; stiffness; strain hardening; structural analysis; structural design.

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INTRODUCTION

In the design criteria for slender concrete columns three different modes of behavior are usually considered:

- (1) Single curvature, produced by checkerboard loading, no significant sidesway permitted.
- (2) Double curvature, produced by fully loading all upper stories (thus restraining the column top) while developing a significant end moment at the column base by alternating live loads on the lower floor, no significant sidesway permitted.
- (3) Sidesway, produced by lateral loading such as wind or earthquake loading, or by unsymmetrical loading or geometry in an unbraced frame.

This investigation studied only single curvature loading with sidesway prevented, as shown in Fig. 1a. The isolated single curvature column A-B is shown in Fig. 1b and corresponding moment diagram in Fig. 1c. The slender column effect is introduced by the column deflection y , which results in an increase of moment $P \cdot y$. This extra moment reduces the capacity of the column to carry axial load. The slender column effect is always present for a single curvature column, but can be neglected for a very short column.

MATHEMATICAL MODEL FRAME

A mathematical model for a portion of a multistory frame with single curvature loading was solved using a highly complex numerical solution developed by Breen (1,2) for columns in reversed curvature based on the procedure presented by Pfrang and Siess (3). The computer program was adapted by Pagay (4) to include columns in single curvature. His version of the program was extended (5) to consider the effect of beam hinging, beam mechanism formation, and reinforcement strain-hardening. This computer program considers the inelastic characteristics of the concrete (by using Hognestad's stress-strain diagram for concrete), the axial load effects on column stiffness, the secondary moments due to column deflections and the reduction of stiffness due

to cracking. For a given load and moment diagram the program determines the member end rotations based on a finite summation of curvatures obtained from the load-moment-curvature diagram for that particular load.

The basic structure is shown in Fig. 1a and the mathematical model frame in Fig. 2a. The moment capability at joint A is shown in Fig. 2b, and the moment diagram for the loaded beam A-E in Fig. 2c. The model is symmetric about the beams' midlength and columns' midheight. The computer solution of the mathematical model initially assumes that no plastic hinge will occur in the loaded beam, and if actual hinging occurs the beam is then artificially stiffened to go on unyielded. The program notes by the printing of a statement the load at which hinging of the beam occurred. The changes in the computer program to take into consideration beam yielding and strain hardening of the steel are explained later.

The key parts of the computer program are as follows:

(a) Moment compatibility at column joints. From Fig. 2b:

$$\sum M_A = 0 \quad M_{AE} = M_{AB} + M_{AF} + M_{AG} \quad (1)$$

Neglecting tensile concrete stresses and assuming no yielding of reinforcement, the stiffnesses of both beams are equal, as are the stiffnesses of both columns. Since equal members are assumed to behave equally, it follows that half the fixed end moment will be resisted by the combination of column AG and beam AF while the other half will be resisted by column AB and beam AE.

$$\text{Thus:} \quad M_{AF} = DM_{AE} \quad (2)$$

where DM represents the moment distributed to the member during the joint rotation.

$$M_{AB} = M_{AG} \quad (3)$$

$$\text{and} \quad M^F/2 = DM_{AE} + M_{AG} = M_{AF} + M_{AG} \quad (4)$$

$$\text{Substituting in Eq. (1)} \quad M_{AE} = M_{AB} + M^F/2 \quad (5)$$

This equation gives the relation between column and beam end moments for a given joint rotation.

(b) Prediction of column deflections and end rotations. The prediction of column deflections and end rotations is accomplished by the deflection and end slope predictors (1,3), which are numerically computed influence coefficients for column end rotations and deflections due to a unit load and unit moments at the column ends. The initial trial values of column deflection and end rotations obtained are calculated from the predictors once for each column load, and the iterative procedure starts from these values.

(c) Iterative procedure. The iterative procedure is started immediately after determining the trial column deflections and end rotations using the predictors. Using a modified Newmark procedure

new trial moments are computed and tested for convergence. If the absolute difference and ratio of these trial moments is within certain allowable limits, convergence is assumed and the program continues to the next column load. If convergence is not obtained, new values for deflections and slopes are estimated and the process repeated.

Once a plastic hinge forms in the beam near the column, a different mathematical model must be solved. Equation (2), which gives the relation between column and beam moments is no longer valid. The new basic structure and mathematical model with plastic hinges at the loaded beam ends is illustrated in Fig. 3a. The mathematical model is shown in Fig. 3b. The behavior of columns B-H and A-G is assumed equal to that of column A-B. Thus, the summation of moments equal to zero at joint A gives:

$$M_P = M_{AB} + M_{AG} + M_{AE} \quad (6)$$

or
$$M_P = 2 \cdot M_{AB} + M_{AE} \quad (7)$$

$$M_{AB} = \frac{M_P - M_{AE}}{2} \quad (8a)$$

similarly
$$M_{BA} = \frac{M_P - M_{BD}}{2} \quad (8b)$$

The value of M_P is taken as the plastic moment capacity of the beam. Equations (8) give the relation between column and beam moments after the formation of a plastic hinge in the loaded beam near the column.

After convergence is attained for each column load, the loaded beam end moment must be compared with M_P and if it is smaller the program follows the usual procedure. The first time the loaded beam end moment is equal to or greater than M_P , a new trial for the same column load is started, using the mathematical model shown in Fig. 3b and Eqs. (8) for the relation between beam and column moment. This model is used for all subsequent column loads until a beam mechanism is formed. When this occurs beam deflections increase with no additional load until the steel reinforcement reaches strain hardening. After this any major increase in internal moment must come from an increase in steel stress due to strain hardening. Since the actual and assumed beams had equal reinforcement at support and midspan, the new "plastic moment" for the beam loading at which a mechanism is formed and any subsequent beam loading is calculated from the equation:

$$2 \cdot M_P = W \cdot L^2 / 8 \quad (9a)$$

or
$$M_P = W \cdot L^2 / 16 < M_U \quad (9b)$$

where M_U is the ultimate moment of the beam cross section considering strain hardening. The value of M_P given by Eq. (9b) is then used in Eq. (8).

In this fashion, steel strain hardening is implicitly considered in the computer program. The importance of this steel property on

frame behavior may be seen from Fig. 4, which shows the computed moments for the column end and center stations of specimen B1 with and without strain hardening.

EXPERIMENTAL FRAME TESTS

A series of physical tests was carried out to document the behavior of columns in frames as affected by variations in the restraining beams. Three frames were designed to illustrate the various behavioral combinations. The experimental frame properties are given in Table 1. Frame B1 had lightly reinforced beams designed to clearly yield prior to column failure. Frame B2 had heavily reinforced beams and very high concrete strength and was designed to have a balanced failure with beams reaching yield at about the point of column failure. Frame B3 had heavily reinforced beams with moderate concrete strength and was designed to undergo a column failure prior to beam yielding. All frames had the same overall dimensions and loading shown in Fig. 5a. Based on a conventional moment distribution analysis (which ignores long column action and uses moment of inertia based on gross concrete area without modification for reinforcement or cracking), Figs. 5b and 5c, respectively, show distribution factors and final distributed moments for an assumed beam fixed end moment of -100 in.-kips (m-kgf). The central beam moments and column moments are exactly the same as those obtained from a moment distribution of the basic multistory structure shown in Fig. 1a ($\psi = 1$, $K_{col} = K_{bm}$). Thus, although the experimental frame used differs slightly from an actual multistory frame because of the omission of a column at each joint, the moment pattern and the beam continuity over the column closely models the restraint conditions of an actual frame.

The loading system shown in Fig. 5a produces single curvature in the columns with symmetry about midheight. Column and beam loads were incremental and were applied simultaneously according to the relation $P_b/P_c = 0.0554$, which was kept constant during the tests and was the same for all the frames.* This load ratio produces a nominal end eccentricity of 0.1h for the column loads.

The experimental frames were tested in a horizontal position. Figure 6 shows a schematic diagram of the test setup and Fig. 7 gives a general view of a test in progress. A steel beam (A) supported on wheels reacted against a fixed concrete beam (B) through a roller nest of steel cylinders. Two steel frames (C) welded to beam (A) supported the bearing heads (D), which were free to move perpendicular to beam (A). Similar column bearing heads (D') were used on the opposite side of the frame, but they were hung with cables from overhead roller supports. The bearing heads (D and D') for each column were connected with steel cables. A column ram E (Fig. 6) loaded the column through knife edges provided at both ends of the column with the applied load monitored by a load cell (F). An additional load cell (G) was used to monitor the lower column reaction.

*This ratio was varied during the test of frame B1 after the full deflection travel of the beam rams was exceeded.

The beam loads were applied at third points of the central beams by similar yoke and ram units. The load was applied on the upper beam by a hydraulic ram (H, Fig. 6) through a monitoring load cell (I, Fig. 6) and a knife edge to a beam bearing plate. The lower beam load was applied by the yoke directly to a similar knife edge arrangement.

In order to obtain the average concrete strains on the exterior faces of the members as well as the average angle change ϕ over the gage station, concrete strain-rotation meters were used (Figs. 6 and 7) (1,6,7). The strain difference (ϕ) and the measured value of member axial load were used with an interpretive program to determine the effective moment at the section (2). Deflections of various stations along the columns and center beams were measured from a nylon string line suspended between beam-column junction bolts using scales attached to the rotation meters. Corner vertical, lateral, and angular movements were measured utilizing corner meters especially designed for this purpose.

TEST RESULTS

(a) Specimen B1. This frame had columns with a clear height-thickness ratio $l/h = 15$, nominal $e/h = 0.1$, $\psi = 1$, and $f' = 5450$ psi (383 kgf/cm²). The total column reinforcement percentage was $\rho_t = 1.21$ percent, while the beams had a $\rho = \rho' = 0.84$ percent. The specimen had two loading cycles because of a loading cable failure during the first test. Deformation of the specimen near ultimate load is shown in Fig. 8. The observed vs. predicted results of the first test and retest for station 5-6 (failure section) and station 4-5, respectively, are shown in Fig. 9. Station 5-6 was not instrumented during retest. Maximum extension of beam rams occurred in both tests. After this limit was reached the tests were continued with increases only in column loads. This effect is illustrated in Fig. 9 for the retest case by a vertical line separating from the nominal P-M line ($e/h = 0.1$) at the point where constant beam loads were applied. Each point on this vertical line denotes a different nominal value of e , since the nominal moment is constant while the column load is increasing. An explosive type of compression failure occurred at station 5-6 and it included some portions of station 4-5.

The predicted P-M relationships from the mathematical model for the column midheight station with and without consideration of strain hardening are also shown in this figure. The point of departure of these two curves is at a column load of 52 kips (23.6×10^3 kgf), indicating that the beam mechanism was formed between column loads of 52 (23.6×10^3 kgf) and 56 kips (25.4×10^3 kgf). A plot of column load vs. measured midspan deflection for beam A-D (see Fig. 13) indicates that significant beam yielding near the column started at a column load of approximately 52 kips (23.6×10^3 kgf), and the beam mechanism was completely formed by a column load of 58 kips (26.3×10^3 kgf). These values are in good correlation with the numerical results.

For low column loads, Fig. 9 shows good correlation between actual and predicted results. Although the curve for the initial test does

not show the definite point of hinge formation and subsequent effect of steel strain hardening, there is, in general, good agreement between the predicted and actual curves. The retest curve is very smooth and almost of linear variation up to a column load of about 75 kips (34.0×10^3 kgf). This is probably because the yield stress for the beam steel had been increased due to the previous loading, large elongations, and consequent steel strain hardening. Above a column load of 75 kips (34.0×10^3 kgf), the retest curve denotes a sharp decrease in moment. The indicated moment decrease at station 4-5 before failure may be due to two factors. First, since the beam rams were fully extended, the beam deflection was maintained constant from a column load of 76.9 kips (34.9×10^3 kgf) up to column failure. For this loading range, the rate of increase in corner rotation will be much smaller, since it will be due mainly to the P-y effect in the column. Furthermore, since the loaded beam deflection is constant, any increase in corner rotation decreases the column end (M_{DC} , M_{CD}) and outer beam moments (M_{DF} , M_{CF}). Any such decrease in column end moment would be reflected in a decrease in moment at station 4-5. Secondly, the indicated moment decrease may be due to the nature of the computed P-M- ϕ curves. At these axial load levels, these curves indicate that with increasing load increments a moment reduction can occur even with increases in ϕ .

(b) Specimen B2. This frame had columns with $\ell_c/h = 15$, $e/h = 0.1$, $\psi = 1$, and $f'_c = 9100$ psi (640 kgf/cm²). This concrete strength is unusually high, due to an error in weighing materials when mixing. The total column reinforcement percentage was $\rho_t = 1.21$ percent, while the beams had $\rho = \rho' = 1.87$ percent. The specimen had to be loaded a second time because of a failure of the loading rig, due to local buckling of the bearing head of joint A at a column load of 103.8 kips (47.1×10^3 kgf) (Fig. 10). The test apparatus was redesigned and strengthened and the frame retested eight days later.

Results for the first test indicate that the beam was near yielding at a section near the column (see Fig. 13). Retest results indicate formation of a beam mechanism just prior to column failure. An explosive type of compression failure occurred at station 20-21 at a column load of 120.3 kips (54.6×10^3 kgf). Figure 11 shows the column at the instant of failure.

The actual and mathematical model P-M relationships at column mid-height are shown in Fig. 10. The load vs. moment relationship of the first test for both midheight column stations indicates a smooth and almost linear variation throughout the test range. The center beam A-D midspan deflections (see Fig. 13) show a practically constant gain in deflection, indicating that hinging of the beam had not occurred when the test stopped (although beam yielding was incipient).

The retest P-M relationships for both column midheight stations are also shown in Fig. 10. The retest curve of the failure station (20-21) is in excellent quantitative agreement with the mathematical model results for a column load up to about 70 kips (31.8×10^3 kgf) and in good qualitative agreement elsewhere. Results of the computer program for the mathematical model indicate the formation of three plastic

hinges in each center beam near each column at a column load of 105 kips (47.6×10^3 kgf) and at midspan at a column load of 107.5 kips (48.8×10^3 kgf). A plot of the average of the interpreted interior beam (BC, AD) moments at the column centerline for corners C and D vs. column loads indicate beam yielding between column loads of 96 (43.5 kgf) and 107 kips (148.5×10^3 kgf) with a beam yield moment of about 117 in.-kips (53.1×10^3 kgf). Furthermore, results for the center beam midspan station clearly indicate a beam yielding moment of 124 in.-kips (156.2×10^3 kgf) at a column load of 112 kips (50.8×10^3 kgf). Thus, the actual and predicted values for column loads that produced beam hinging at the ends and midspan of the center beams as well as the beam hinging moments are in good agreement. Once the center midspan section yields a mechanism will form, and this occurred in this specimen at a column load of 112 kips (50.8×10^3 kgf). However, because of the extremely high concrete strength and large percentage of reinforcement, the behavior of this mechanism is quite different from that of B1. Due to the high amount of beam compression steel and very high concrete strength relative to B1, the beam is stiffer than B1 after yield of the tension steel and tends to offer greater resistance to increasing beam deflections after the "plastic mechanism" is formed. Thus, the beam did not indicate as well-defined a type of mechanism as specimen B1 indicated (see Fig. 13). The actual column failure load was only 7 percent higher than the load at which the mechanism formed. If even larger column loads could have been applied, quite possibly the center beam would have failed by crushing of concrete at a section near midspan or near the columns.

(c) Specimen B3. This frame had columns with $\ell_u/h = 15$, $e/h = 0.1$, $\psi = 1$, and $f'_c = 5610$ psi (394 kgf/cm²). The total column reinforcement percentage was $\rho_c = 1.19$ percent, while the beams had a $\rho = \rho' = 1.88$ percent. The specimen was similar to B2 except for having more normal concrete strength. The test proceeded smoothly to an explosive compression failure at section 4-5 at a column load of 99.8 kips (45.3×10^3 kgf). Test results indicate that beam hinging did not occur.

Figure 12 shows the actual and predicted P-M relationship for the critical column stations 4-5 (failure station) and 20-21. This figure also shows the predicted P-M relationship for the column end station. This relationship shows a slightly decreasing column end moment for the highest column loads, indicating that the beam relative stiffness actually increases for these loads, thus increasing the amount of moment distributed back to the center beam and decreasing that taken by the column and outer beam. Although the predicted column end moment decreases, the predicted midheight column moment keeps on increasing because of the P-y effect. Although qualitatively the shapes for the actual and predicted curves of the midheight column station are similar up to the nominal failure load, Fig. 12 also indicates that the actual moment values are consistently smaller than the predicted ones. This may have been due to initial accidental eccentricities. This seems to be the case, because all stations along the column gave smaller moments than predicted. The observed P-M relationship for the midheight column stations also indicates a decrease in moment for the highest column loads. This is partially influenced by the same factors which affected the moments at the ends of the columns.

Figure 13 shows plots of column load vs. measured midspan deflection of beam A-D for all specimens. Specimen B1 definitely indicates the formation of a beam mechanism. Results for the first test of specimen B2 indicate no hinge formation while the retest results show the formation of a beam mechanism. However, the behavior of this mechanism was definitely different than that of specimen B1. Specimen B3 indicates no hinge formation.

Table 2 shows comparisons between actual and predicted column loads. Column (2) shows the nominal e/h values for the three frames. Two values are shown for frame B1. The first one is the intended nominal e/h value of 0.1, which could not be maintained above a column load of 77 kips (34.9×10^3 kgf) because of excessive beam deflections. A set of comparisons is made based on this column load (77 kips-- 34.9×10^3 kgf). This can be considered as a load corresponding to a practical frame limit state because of the large deflections of the beam. The second nominal e/h ratio for frame B1 is based on the relative beam to column load ratio at the time of actual column failure. A set of comparisons is also presented for this value of e/h . The ultimate load predicted by the numerical solution [column (10)] is the same for both cases, since the nominal e/h ratio of 0.1 was unchanged up to a load of 77 kips (34.9×10^3 kgf) and the predicted failure load is lower than this value.

The ratio of predicted to actual column loads for beam hinging and beam mechanism formation is shown columns (5) and (8). There is very good correlation for both cases. Column (9) shows the nominal ultimate load (the interaction curve value corresponding to the nominal end eccentricity) and column (10) the numerical solution predicted failure load. The column load at which the measured P-M relationship first crosses the interaction diagram is given in column (11). This is of interest, since it would be the general design failure criteria for section capacity and is the criterion for failure of the mathematical model. Column (13) indicates good agreement between the computer solution and the load at which the interaction diagram is first intersected. Ultimate loads are shown in column (12). Comparisons between the various tabulated loads are given in columns (13) to (15). Column (14) shows that a significant slender column effect existed for specimens B1 and B2. Column (15) shows reasonable correlation between the actual and predicted ultimate loads.

The 1971 ACI Code ultimate load predictions are shown in column (16). These loads were computed by the moment magnification procedures using Code Eq. (10-7) for EI values, $\beta_d = 0$, and $\phi = 1.00$, since all dimensions and material strengths were verified. Comparisons with test results and computer predictions are shown in columns (17) to (19). The Code values are in very good agreement with the measured test values. All column Code ultimate loads are within 10 percent of both the computer solutions and the observed loads at which the interaction diagrams were reached. The actual ultimate loads averaged 3 to 9 percent higher than the Code values and, hence, the Code is slightly conservative.

CONCLUSIONS AND RECOMMENDATIONS

This investigation was very limited in nature since it involved the study of only one major variable (beam steel percentage) and one minor variable (concrete strength) using only three test specimens. Obviously, to arrive at meaningful design conclusions examination of more variables is needed. Since the numerical method was shown to be generally reliable, further tests planned should carefully complement and supplement the less costly computer analysis. Nevertheless, for frames of this general nature with similar material strength the following conclusions and recommendations are warranted:

(1) The influence of beam steel percentage on frame behavior was clearly demonstrated. With identical beam sizes a low percentage of beam reinforcement limited the column load capacity when frame failure criteria were taken as the loads at initial beam hinging or the formation of a complete beam mechanism. With a high percentage of beam reinforcement, beam hinging did not occur and column load capacity was governed by ordinary column strength criteria.

(2) Post-mechanism behavior due to steel strain hardening allowed an appreciable increase in column load, but required large beam deflections which may not be possible or allowable in an actual structure.

(3) The numerical solution is a powerful method that closely predicts the behavior of a column as an integral part of a braced frame. In these tests the column load at which the measured P-M relationship first crossed the interaction diagram averaged only 6 percent higher than the ultimate load predicted by the numerical solution (Table 2, Column [13]), while the observed ultimate load averaged from 9 to 16* percent higher than the numerical solution (Table 2, Column [15]). These values indicate good correlation. The difference in these two ratios occurs because the numerical solution assumes failure when the P-M relationship first crosses the interaction diagram, while in these and other tests (1) the observed P-M relationship has crossed the interaction diagram and the columns continue to take load. This difference is due to a lack of complete understanding of the basic failure criteria for a column as an integral part of a frame.

(4) Ultimate column capacity as predicted by the moment magnification procedures of the 1971 ACI Code gave very fine agreement with both the computer predictions [Table 2, Column (18)] and the measured test results [Table 2, Columns (17) and (19)].

The following observations should be useful in planning further studies and in drawing conclusions for design:

(1) Time effects were not considered in the study. Nevertheless, they are very important and may considerably reduce the column ultimate load capacity. Studies in this area are needed.

*Average varies with failure criteria accepted for frame B1 (see Table 2).