

Creep and Shrinkage of Aging Concrete

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Synopsis: A thermodynamically-based creep theory is developed, in which the total incremental strain at any time is composed of mechanical, thermal, hygral and autogenous contributions. Each of these contributions (except the autogenous) consists of an instantaneous part which is reversible (in the working stress regime) and a time-dependent part which may in turn contain both reversible and irreversible components. The theory provides a unifying framework for examining existing methods of modeling creep phenomena and developing new models where experimental data reveal deficiencies in current models.

Keywords: age; concretes; creep properties; deformation; mass concrete; pressure vessels; shrinkage; strains; stresses; thermodynamics

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INTRODUCTION

In spite of the extensive body of literature that has developed since the turn of the century, particularly during the last decade, ability to describe definitively the behaviour of concrete subjected to mechanical and environmental loading histories is at best limited. Complexity in the structure of concrete as an engineering material stands as the root cause of the often highly controversial nature of both theoretical and experimental contributions to the literature. The recent survey of Bazant [1] provides an excellent perspective in this regard. In addition it brings into sharp focus the extraordinary breadth required to deal properly with the subject-ranging from physical chemistry and continuum mechanics through numerical analysis and codified procedures for structural design.

The present paper provides a conceptual framework within which the requisite yet more or less disjoint viewpoints of physical chemistry, mechanics and structural analysis may be accommodated. Since analysis and design procedures will likely continue to be based on structural variables such as stress and strain, mechanics will play the dominant theoretical role. Nevertheless, the complex microstructure of concrete on the one hand, and the need for computationally feasible models for design purposes on the other will require that the disciplines of physical chemistry and numerical analysis play substantial roles in formulation of phenomenological models of behaviour. In the continuum theory of material behaviour a multiphase material such as concrete is properly studied under the rubrics of the theory of mixtures [2]. At present, however, such a treatment requires a degree of sophistication that seems unwarranted for engineering applications. Alternatively, we will adopt the viewpoint of constitutive theory of materials with internal variables. This will enable us to incorporate rationally

into a single-phase material model the evident multiphase microstructure of concrete. In particular the processes of heat and moisture diffusion as well as aging associated with hydration will be accounted for from the viewpoint of internal variables.

Throughout the paper our emphasis will be conceptual and qualitative so as to minimize detraction from the central ideas that otherwise are apt to become obscured by excessive symbolic and numerical detail. Applications may be found in references cited in the final section.

DEFORMATION OF CONCRETE - A THERMODYNAMIC BASIS

Roles of Theory and Experiments

Our choice of a thermodynamic theory for study of deformation of concrete is influenced by the observation that it:

- (1) provides a mathematical structure for simultaneously treating micro- and macro-phenomena,
- (2) provides a proper basis for three-dimensional generalization,
- (3) incorporates irreversible phenomena,
- (4) is compatible with computer-oriented numerical analysis procedures.

A theory is needed both to organize rationally a body of observed phenomena (set of experiments) as well as to permit cautious prediction of what might be expected to occur if certain conditions in the experiments were changed. A theory defines the structure of a mathematical model and defines critical experiments by which the limits of its validity may be established. Properly used, it minimizes the misleading practice of "curve-fitting", in which fitting of data to an empirical model, no matter how carefully done, provides no predictive quality whatsoever with respect to a new set of conditions.

Experiments, on the other hand, constitute the sine qua non of a theory. A set of experiments, used in connection with a theory:

- (1) Determines parameters in the mathematical model.
- (2) Delineates the range of validity of the theory.
- (3) Demonstrates the need for revision of a theory.

Finally, experiments usually require a theory in order to be properly understood. This results from the fact that it is often impossible to conduct experiments in which the states of strain and stress within the test specimen are spatially uniform, i.e., the stress state is statically indeterminate internally. This constitutes a problem of analysis by itself (an initial-boundary value problem for time-dependent creep experiments) in which the as yet unknown material properties are needed as part of the solution algorithm. This leads in turn to an iterative process in seeking to understand the experiment, making apparent again the complementary roles of theory and experiment. Accordingly, it is necessary that at least conceptually a theory be constructed for states of stress and strain, as well as internal variables describing micro-processes, that are spatially uniform; no spatial gradients should appear in such a theory. The problem of treating structural elements (beams,

slabs, etc.) in which non-uniform states occur constitutes a separate problem of analysis that cannot be attempted until the former problem of modeling is completed. Failure to recognize this seemingly straightforward principle has been a source of much ambiguity in interpreting results of experiments. A typical example is that of creep tests on unsealed cylinders during which uncontrolled moisture diffusion occurs. From such experiments it is impossible to isolate the creep strain associated with mechanical stress from that associated with changes in moisture concentration in the test specimen.

Decomposition of Total Strain

For simplicity we will consider a volume element of concrete subjected to uniaxial stress. Our object is to develop a theory in which the corresponding strain can be expressed in terms of the stress and the temperature, moisture content and age of the material. Generalization to a three-dimensional format is formally straight-forward using the rules of tensor algebra. It is convenient to consider the axial strain of the material in the following ways:

- (1) strain is produced by mechanical as well as non-mechanical sources,
- (2) strain contains instantaneous as well as time-dependent components,
- (3) strain components may be reversible (recoverable) or irreversible (irrecoverable),
- (4) the micro-processes responsible for strain may be interdependent (coupled).

To illustrate how these intuitive ideas may be given quantitative expression by a thermodynamic theory, consider first the calculation of instantaneous strain produced by stress, all other variables held constant. Here and in the sequel we will assume that all instantaneous effects are reversible, although this is not a requisite of the theory, merely a tentative simplification for stress levels less than 0.3 to 0.4 f'_c . Referring to Fig. 1a, an axial strain ϵ is produced by a corresponding stress σ as shown. The elastic complementary energy per unit volume is defined by

$$G_e(\sigma) = \int \epsilon d\sigma \quad (1)$$

If the material is linear, i.e. $\epsilon = J\sigma$, where J is the compliance (constant), then it follows from Eq. (1) that

$$G_e(\sigma) = \frac{1}{2} J \sigma^2 \quad (2)$$

We observe that the complementary energy is a potential function for the strain, i.e., from Eq. (2)

$$\frac{\partial G_e}{\partial \sigma} = J\sigma = \epsilon \quad (3)$$

Differentiating Eq. (3) again,

$$\frac{\partial \epsilon}{\partial \sigma} = J = \frac{\partial^2 G_e}{\partial \sigma^2} \quad (4)$$

If the material is elastic but nonlinear, G_e is no longer a quadratic function of σ . Accordingly, from Eq. (4) it follows that J is no longer a constant but a function of stress level. Formally, we can write Eq. (4) in the form

$$\frac{\partial \epsilon}{\partial \sigma} = J(\sigma) = \frac{\partial^2 G_e}{\partial \sigma^2} \quad (5)$$

Figures 1 b and 1 c give a clear geometric interpretation of Eq. (4) and Eq. (5). For the linear material the slope of the strain-stress curve is a constant, J . For the nonlinear material, Eq. (5) shows that $J(\sigma)$ is an incremental (tangent) compliance. For our purposes, and particularly for computation, it is convenient to write Eq. (5) in the incremental form

$$\Delta \epsilon = J(\sigma) \Delta \sigma \quad (6)$$

We turn now to incorporation of non-mechanical sources of instantaneous strain, which will be associated with changes in temperature T and evaporable water concentration w . From thermodynamic theory we replace the complementary energy by the complementary free energy (Gibbs energy) which has the formal expression

$$G_e = G_e(\sigma, T, w) \quad (7)$$

per unit volume of material, and note that

$$\epsilon = \frac{\partial G_e}{\partial \sigma} \quad (8)$$

remains valid.^x The incremental form of Eq. (8) corresponding to Eq. (6), taking into account incremental changes in all variables, follows by using the chain rule in Eq. (8):

$$\Delta \epsilon = d\left(\frac{\partial G_e}{\partial \sigma}\right) = \frac{\partial^2 G_e}{\partial \sigma^2} \Delta \sigma + \frac{\partial^2 G_e}{\partial \sigma \partial T} \Delta T + \frac{\partial^2 G_e}{\partial \sigma \partial w} \Delta w \quad (9)$$

This equation states that the total instantaneous strain increment is the sum of increments produced by changes in stress, temperature and water concentration. It is useful to introduce the following notation arising from Eq. (9):

$$\frac{\partial^2 G_e}{\partial \sigma^2} \equiv J(\sigma, T, w) \quad , \text{ mechanical compliance} \quad (10)$$

$$\frac{\partial^2 G_e}{\partial \sigma \partial T} \equiv \alpha(\sigma, T, w) \quad , \text{ thermal compliance} \quad (11)$$

$$\frac{\partial^2 G_e}{\partial \sigma \partial w} \equiv \beta(\sigma, T, w) \quad , \text{ hygral compliance} \quad (12)$$

The incremental compliances J, α, β are intrinsic material properties which can only be determined by appropriate experiments in which two of the variables σ, T, w are fixed while changing the third. For example, while holding T and w constant, the strain-stress curve can be obtained as in Fig. 1 c, from which J can be found. By repeating for different values of T, w the sensitivity of J to non-mechanical variables can be established. Similar experiments can be used to evaluate α, β . It may be expected that σ will not influence these material functions appreciably, particularly if the magnitude of σ is less than 0.3 to 0.4 f'_c [1].

^x Our treatment of thermodynamics of solids follows the excellent expository paper of Lubliner [3]. More details can be found there.

Effect of Maturity on Material Functions

In the previous discussion it was tacitly assumed that material functions were determined at a given age of the material. However, experience indicates that the values of these functions are dependent on the maturity of the concrete, which is defined here to be a measure of extent of development of the hydration process in the cement paste. It is apparent that maturity is not really a function of the chronological age of the material; rather, it reflects how the histories of temperature and water concentration may have accelerated or decelerated the age of the concrete by affecting the rate of hydration of cement paste. It is evident that maturity is a concept associated with micro-processes whose dynamics are best discussed within the discipline of physical chemistry; obviously, the variables of continuum mechanics are inappropriate here. A thermodynamic theory incorporating internal material variables provides the vehicle for treating such a problem [3]. We introduce a measure of maturity^x q_1 along with the following postulate:

"the rate of change of maturity depends on the present values of temperature, evaporable water concentration and maturity."

Expressed symbolically,

$$\dot{q}_1 \equiv \frac{dq_1}{dt} = f_1 [T(t), w(t), q_1(t)] \quad (13)$$

where $f_1(T, w, q_1)$ is an intrinsic material property function which must be determined experimentally. It may be noted that this idea is conceptually similar to the notion of "co-age" introduced previously by Mc Henry and others [4], although it is introduced here in an entirely different context. If either T or w is fixed in time, q_1 is a surface whose coordinates are $(q_1, w$ or $T, t)$. For a prescribed history $T(t), w(t)$ one would expect the solution of Eq. (13) to have the form shown in Fig. 2. Each history pair T, w would produce a different curve, i.e. T, w are the parameters of a family of maturity curves.

To incorporate the effect of maturity in the thermodynamic theory, we must replace Eq. (7) by

$$G_e = G_e(\sigma, T, w, q_1) \quad (14)$$

in order to introduce the dependence of instantaneous response on age at time of testing. This produces two effects in Eq. (9) for the total incremental instantaneous strain; first an additional term appears:

$$\Delta \epsilon_a = \frac{\partial^2 G_e}{\partial \sigma \partial q_1} \Delta q_1 = \lambda_1 \Delta q_1 \quad (15)$$

where

$$\lambda_1 \equiv \frac{\partial^2 G_e}{\partial \sigma \partial q_1} \quad (16)$$

defines the autogenous incremental compliance, representing a strain increment produced solely by the phenomenon of maturation. Secondly, the material functions

^x This viewpoint was suggested by Professor J. Lubliner in a personal communication.

$J, \alpha, \beta, \lambda$ now must be regarded as dependent on the present values of the macroscopic (observable) variables σ, T, w as well as the present value of the (internal) micro-variable q_1 which reflects maturity. Since q_1 is computed from a differential equation, Eq. (13), whose solution reflects how the past values of temperature and water concentration affect maturity, it is clear that the instantaneous response function $J, \alpha, \beta, \lambda$ are thereby dependent on the past history of T, w as well through their dependence on q_1 . Observe that this is an additional functional dependence beyond that already explicit in their dependence on present values $T(t), w(t)$ as seen in Eqs. (10), (11), (12). Observe, however, that $J, \alpha, \beta, \lambda$ do not depend on the past history of stress in the theory considered here.^x Using Eqs. (9) - (12) along with (15) and (16) the total instantaneous incremental strain can be written.

$$\Delta \epsilon = J \Delta \sigma + \alpha \Delta T + \beta \Delta w + \lambda_1 \Delta q_1 \quad (17)$$

In summary we have illustrated qualitatively how an internal variable measuring micro-processes associated with hydration can be incorporated in a continuum theory of behaviour to account for the effect of maturity on mechanical, thermal and hygral instantaneous response function. We turn now to a similar exposition to account for time-dependent strain phenomena.

TIME-DEPENDENT STRAIN - INELASTIC BEHAVIOUR AND IRREVERSIBILITY

Complementary Free Energy

Up to this point we have accounted for instantaneous strain resulting from stress, temperature and water concentration changes, including the effect of maturity. This has been accomplished by utilizing a complementary free energy function G_e depending on the present values of σ, T, w, q_1 as shown in Eq. (14), resulting in the total instantaneous incremental strain expression given by Eq. (17). We must now incorporate into the complementary free energy necessary structure to account for time-dependent contributions to the total strain. Such contributions may be of mechanical origin, e.g., viscoelastic or viscoplastic, or they may be associated with thermal or hygral phenomena. To accomplish this we introduce a complementary free energy function [3]

$$G = G_e(\sigma, T, w, q_1) + \sigma \epsilon_i(q_\alpha) \quad (18)$$

where G_e has been previously defined and ϵ_i is the total time-dependent, inelastic strain. The inelastic strain ϵ_i depends on a set of internal variables q_α ($\alpha = 2, 3, \dots, n$) which is used to describe the dissipative, time-dependent microprocesses responsible for viscoelastic, delayed thermal and other related types of inelastic response. Then, using Eq. (18) it follows that

$$\epsilon = \frac{\partial G}{\partial \sigma} = \frac{\partial G_e}{\partial \sigma} + \epsilon_i(q_\alpha) \quad (19)$$

^x This is only an assumption which can be removed if necessary. However, it seems plausible for stress levels less than 0.3 to 0.4 f'_c .

Denoting the instantaneous response by

$$\epsilon_e = \frac{\partial G_e}{\partial \sigma} \quad (20)$$

we can rewrite Eq. (19) in the form

$$\epsilon = \epsilon_e + \epsilon_i \quad (21)$$

In incremental form Eq. (21) is

$$\Delta \epsilon = \Delta \epsilon_e + \Delta \epsilon_i \quad (22)$$

We have already expressed $\Delta \epsilon_e$ in Eq. (17). Noting that ϵ_i depends only on q_α , we can write the inelastic strain increment

$$\Delta \epsilon_i = \sum_{\alpha=2}^n \frac{\partial \epsilon_i}{\partial q_\alpha} \Delta q_\alpha = \sum_{\alpha=2}^n \lambda_\alpha \Delta q_\alpha \quad (23)$$

where

$$\lambda_\alpha = \frac{\partial \epsilon_i}{\partial q_\alpha} = \frac{\partial^2 G}{\partial \sigma \partial q_\alpha}, \quad \alpha = 2, 3, \dots, n \quad (24)$$

denote incremental inelastic compliance functions associated with each internal variable q_α . As noted, the set of internal variables has the task of representing at the macroscopic, continuum level the effects of inelastic micro-processes. We may therefore expect to require mechanical internal variables q_α^m , thermal internal variables q_α^T and hygral internal variables q_α^w .

As in the case of the internal variables q_1 , used to account for maturity, it is necessary to postulate rate equations from which the internal variables can be calculated when the macroscopic state of the material (expressed in terms of σ , T , w) is prescribed.

Rate Equations for Internal Variables

The following postulate is introduced:

"the rate of change of internal variables accounting for inelastic phenomena depends on the present values of stress, temperature, evaporable water concentration, maturity and the internal variables themselves." Expressed symbolically,

$$\dot{q}_\alpha = f_\alpha[\sigma(t), T(t), w(t), q_1(t), q_\alpha(t)], \quad \alpha = 2, 3, \dots, n \quad (25)$$

where the functions f_α reflect intrinsic material properties. A special, though very general form of Eq. (25) results if it is rewritten

$$\dot{q}_\alpha = Q_\alpha(q_1) f_\alpha[\sigma, T, w, q_\alpha], \quad \alpha = 2, 3, \dots, n \quad (26)$$

This permits us to write Eq. (26) as a set of rate equations in terms of equivalent, temperature- and water concentration - compensated time measures $\xi_\alpha(t)$. To see this let

$$\xi_\alpha(t) \equiv \int_0^t Q_\alpha[q_1(s)] ds, \quad \alpha = 2, 3, \dots, n \quad (27)$$

Using this result and changing variables in Eq. (27) yields

$$\frac{dq_\alpha}{d\xi_\alpha} = f_\alpha[\sigma, T, w, q_\alpha] \quad , \quad \alpha = 2, 3, \dots, n \quad (28)$$

where it is understood that σ, T, w, q_α are now expressed as functions of the time variable ξ_α . Observe that the evolution of inelastic behaviour (as measured by q_α) is dependent on the history of σ, T, w through the solution of Eq. (28). Several important points should be noted:

- (1) each internal variable (or each set $q_\alpha^m, q_\alpha^T, q_\alpha^w$) may have its own time scale defined by Eq. (27).
- (2) rate equations (28) for the internal variables may be coupled or uncoupled, i.e., the equations for mechanical internal variables q_α^m may depend on non-mechanical internal variables q_α^T, q_α^w and vice versa.
- (3) the number of internal variables required to describe a particular inelastic phenomenon (permanent flow, delayed elastic recovery, delayed dilatation, etc.) will depend on the intrinsic complexity of the phenomenon and the degree of approximation desired. At least one variable of each type is the obvious minimum.

We can now summarize the contributions of mechanical and non-mechanical effects to the total incremental strain. Using Eqs. (17), (22), (23) we can write

$$\begin{aligned} \Delta \epsilon = & \underbrace{J \Delta \sigma + \sum_{\alpha=2}^k \lambda_\alpha \Delta q_\alpha^m}_{\text{mechanical}} + \underbrace{\alpha \Delta T + \sum_{\alpha=2}^l \lambda_\alpha \Delta q_\alpha^T}_{\text{thermal}} \\ & + \underbrace{\beta \Delta w + \sum_{\alpha=2}^r \lambda_\alpha \Delta q_\alpha^w}_{\text{hygral}} + \underbrace{\lambda_1 \Delta q_1}_{\text{autogenous}} \end{aligned} \quad (29)$$

Eq. (29) shows that the total strain increment at any time is composed of mechanical, thermal, hygral and autogenous contributions. Each of these contributions (except the autogenous) consists of an instantaneous part which is reversible (for working stress regimes) and a time-dependent part which may in turn contain both reversible and irreversible (permanent) components. The time-dependent contributions are determined by internal variables whose values are the solutions of rate equations (28), which may or may not couple together the mechanical, thermal and hygral internal variables depending on the type of model desired.

It may be noted that discontinuous phenomena such as plastic flow, transitional thermal or hygral creep, etc. may be accounted for by requiring that the rate equations (28) display a discontinuity in the function f_α , where the discontinuity would depend on an appropriate threshold ("yield") condition.

Finally, in connection with the thermodynamic theory, it should be noted that the second law of thermodynamics (in the form of the local Clausius-Duhem inequality) must be satisfied for the complementary free energy in every deformation process. In incremental form this leads to [3]:

$$\frac{\partial G}{\partial q_1} \Delta q_1 + \sum_{\alpha=2}^n \frac{\partial G}{\partial q_\alpha} \Delta q_\alpha \geq 0 \quad (30)$$

From Eq. (18) this can be written

$$\frac{\partial G_e}{\partial q_1} \Delta q_1 + \sum_{\alpha=2}^n \sigma \frac{\partial \epsilon_i}{\partial q_\alpha} \Delta q_\alpha \geq 0 \quad (31)$$

Eq. (31) expresses the requirement that micro-processes characterized by internal variables remove free energy from the material. The consequences in the present context is to place restrictions on the manner in which the function G may depend on q_1 and q_α .

This completes our formal treatment of a theory for creep and shrinkage of concrete. To proceed further one must make explicit assumptions, based on intuition or experiments, about the structure of the complementary free energy function G and the rate equations (28). We observe that all of the well-known linear creep models appear as special cases if the function G is taken to be a quadratic polynomial function of variables σ , T , w , q_α and the rate equations are linear with age-dependent coefficients. A summary of these results may be found in [5] and [6]. More important the present theory provides a vehicle for developing models that reflect nonlinearity in any or all variables. However, the planning and execution of a coordinated research program with balanced emphasis on the complementary roles of theory and experiment is essential for further progress in this area.

AGING MODELS FOR CONCRETE CREEP

Different Forms of Aging

We assign now specific structure to the form of aging in order to reproduce familiar creep models of concrete literature. To this end we consider the internal variable description of the time dependent strain increment, Eq. (23)

$$\dot{\epsilon}_i = \sum_{\alpha=2}^n \lambda_\alpha \dot{q}_\alpha \quad (32)$$

where λ_α transforms internal variables into inelastic deformations. For uniaxial conditions a positive value will satisfy the dissipation inequality (31), thus we can assign to

$$\lambda_\alpha = \frac{\partial \epsilon_i}{\partial q_\alpha} = 1 \quad (33)$$

without being too restrictive. For multiaxial conditions, the proper choice for λ_α is very complex and is equivalent to the flow rule postulate in plasticity.

For an assessment of different forms of aging we can now restrict our discussion to specific mathematical forms of the evolution law (25).

Delayed Elastic Model -- To this end we recall the classical formulation of linear viscoelasticity which interprets creep as a delayed elastic phenomenon.

$$\dot{\epsilon}_i = \dot{\epsilon}_d = \sum_{\alpha=2}^n \dot{q}_\alpha, \quad \lambda_\alpha = 1 \quad (35)$$

The growth law is in this case independent of the maturity q_1 and thus the age t . Without being too restrictive we further assume that there is no interaction among the individual internal variables, thus

$$\dot{q}_\alpha = f_\alpha[\sigma(t), T(t), w(t), q_\alpha(t)] \quad , \quad \alpha = 2, 3, \dots, n \quad (36)$$

Let us disregard for a moment the environmental effects, then we recover the classical form of a non-aging Kelvin chain if

$$f_\alpha[\sigma(t), q_\alpha(t)] = \Phi_\alpha \sigma(t) - \mu_\alpha q_\alpha(t) \quad (37)$$

This furnishes the familiar rate equation

$$\dot{q}_\alpha + \mu_\alpha q_\alpha = \Phi_\alpha \sigma \quad (38)$$

which is equivalent to the impulse form of an associated convolution integral. The driving force on the right hand side in Eq. (38) is in this case linear in stress and thus applies to creep in the working stress regime. We readily envision additional driving forces due to temperature and water content, however, they are usually omitted because of negligible aftereffects in the hygrothermal expansion and little interaction between mechanical and environmental strain. In contrast, the environmental effects must be retained in the material parameters of the growth law, i.e. the coefficients Φ_α and μ_α are in general functions of T and w . From the structure of the first order differential equation we recognize that $\Phi_\alpha = \Phi_\alpha(T, w)$ affects the magnitude, i.e. the amount of hygrothermal creep while $\mu_\alpha = \mu_\alpha(T, w)$ only its rate. Therefore, we incorporate temperature and humidity effects in Φ_α rather than in μ_α which would only transform the retardation spectrum of the Kelvin chain. Alternatively, we assign the same environmental dependence to both Φ_α and μ_α and recover in this way the equivalent time shift model of Eq. (28), however, with $q_1 = q_1[T(s), w(s)]$.

Aging Flow Model -- To this end we recall the "rate of creep method" of concrete literature [7,8] which models creep as an aging viscous flow phenomenon without any delayed aftereffects. The rate equation is in this case independent of the internal flow variable, while the dependence on the maturity q_1 , or the time t is equivalent to the time-hardening concept of metal creep

$$\dot{\epsilon}_i = \dot{\epsilon}_v = \dot{q}_2 \quad \text{no sum}$$

where

$$\dot{q}_2 = f_2[\sigma, T, w, t] \quad (39)$$

Disregarding environmental effects for a moment we recover an aging Maxwell unit if

$$f_2[\sigma(t), q_2(t)] = \psi(t) \sigma(t) \quad (40)$$

The rate equation is in this simple case linear in stress and thus

$$\dot{q}_2 = \psi(t) \sigma \quad (41)$$

which applies for creep problems in the working stress regime. Environmental driving forces are again omitted; however, temperature and humidity affects considerably the magnitude of viscous flow, thus the fluidity must be extended to account for T and w aside from aging

$$\psi = \psi(t, T, w) \quad (42)$$

Summation Model -- In this case the creep model is composed of an aging flow mechanism which is superimposed on a non-aging delayed elastic contribution

$$\dot{\epsilon}_i = \dot{\epsilon}_v + \dot{\epsilon}_d \quad (43)$$

The mathematical structure of each component corresponds to that of the previous models where the viscous flow component accounts for aging, i.e.

$$\epsilon_v: \quad \dot{q}_2 = \psi(t, T, w) \sigma(t) \quad , \quad \alpha = 2 \quad (44)$$

and the delayed elastic component for non-aging aftereffects e.g. in form of creep recovery.

$$\epsilon_d: \quad \dot{q}_\alpha = \Phi_\alpha \sigma(t) - \mu_\alpha q_\alpha(t) \quad , \quad \alpha = 3, 4, \dots, n \quad (45)$$

The summation model is in essence the "rate of flow" method of concrete literature. Rheologically speaking it corresponds to a generalized Burgers body with an aging Maxwell unit [9, 10].

Product Model -- In this case aging is restricted to the driving term of the delayed elastic growth law. Analogous to the first possibility of describing environmental effects

$$\Phi_\alpha = \Phi_\alpha(t)$$

and the rate equation for delayed action is

$$\epsilon_d: \quad \dot{q}_\alpha = \Phi_\alpha(t) \sigma(t) - \mu_\alpha q_\alpha(t) \quad , \quad \alpha = 2, 3, \dots, n \quad (46)$$

This weighing of the delayed elastic component corresponds to the classical models for concrete creep as presented by Mc Henry [11] and others [1], [12], [13]. They account for aging by an affine transformation of the creep law at reference age, only that a step-integral formulation is used instead of the differential law (46).

Effective Time Model -- In this case we assume that aging affects both the driving term and the term with the delayed internal variable, in Eq. (37)

$$\epsilon_d: \quad \dot{q}_\alpha = Q_\alpha(t) [\Phi_\alpha \sigma - \mu_\alpha q_\alpha] \quad (47)$$

Analogous to the second possibility of accounting for environmental effects we recover the equivalent time shift model which was examined already in Eq. (28) in context with temperature and humidity compensated time transformation. At this stage it is reasonable to adapt an equivalent time measure which is identical in the individual rate equations, then

$$\epsilon_d: \quad \frac{dq_\alpha}{d\xi} = [\Phi_\alpha \sigma - \mu_\alpha q_\alpha] \quad \text{where} \quad \xi(t) = \int_0^t Q[q_1(s)] ds \quad (48)$$

The time shift hypothesis was originally introduced in the concrete literature in [14] to account for environmental effects, and it was extended to incorporate aging in [4], [15].

Code Recommendation for Mass Concrete

For comparison three different creep laws are evaluated for a stress pulse with identical hold time, but two different loading ages. In particular, the recommendations for the creep of mass concrete are computed as proposed by the CEB [16], the DIN 1045 [17] and the ACI Committee 209 [18]. The individual parameters of the underlining internal variable models are recorded in [5]. In summary

- 1) The CEB recommendation is analogous to our summation model with aging flow (hyperbolic law) and non-aging delayed action (single term exponential series).
- 2) The DIN code requirement is described by the product model without flow, but with an aging delayed contribution (two term exponential series).
- 3) The ACI formulation is interpreted here as an aging flow model without delayed elastic contribution.

We recall that a creep law is primarily the result of curve-fitting creep data under constant loading. Therefore, it applies a priori only to the same class of loading history. For variable stress conditions the creep function must be augmented by an additional superposition postulate which is the essential ingredient in the overall creep model. Our interpretation of e.g. the ACI-creep law in form of a viscous flow mechanism is thus somewhat arbitrary, it is used for the sake of illustration.

The creep response is shown in Fig. 3 for loading at 28 and 365 days. There are substantial deviations between the three code recommendations for mass concrete even during the constant load regime. In the unloading regime these discrepancies are further emphasized by the additional assumptions which are inherent in the creep model.

The CEB-prediction exhibits considerable creep recovery because of non-aging delayed action and the limited amount of viscous flow during the one-year loading period. In contrast, the DIN 1045 norm predicts rather little recovery because of the reduced amount of creep capacity and considerable aging. The viscous flow interpretation of the ACI-code does not allow for any recovery which underscores the serious shortcoming of the rate of creep method during unloading.

At this stage we appreciate the differences among the code predictions even during constant stress which underline the need for additional test data if accurate creep predictions must be ensured.

Assessment of Aging Models

Different forms of the internal variable model are now applied to identify the creep data of the Wylfa reactor concrete [19]. The effect of loading age ranging from 7 days to 12.5 years is reproduced in Fig. 4. In order to simplify the parameter identification we separate the instantaneous response ϵ_e from the time dependent response ϵ_i . The aging elastic behaviour is described e.g. by an exponential series expansion

$$\epsilon_e(t) \Big|_{t=1} = J(q_1) = \sum_{i=1}^I a_i e^{-c_i t} \quad \text{with } q_1 = t \quad (49)$$

The parameters a_i are readily determined via least square identification which furnishes a best fit of the experimental data. The best approximation is attained with $I = 4$ terms of the exponential series expansion which is shown in Fig. 5. The relative root mean square error is in this case $\delta_M = 2.8\%$.

In the next step we face the intricate problem of fitting the inelastic creep $\epsilon_i = \epsilon - \epsilon_e$ as function of two independent variables, the age of loading τ and the load duration $(t - \tau)$. To this end we discretize the creep surface by a 4×14 mesh, i.e. the design values for the identification of the aging creep model are extracted at 14 load intervals and four loading ages.

Let us now examine the accuracy of the aging models discussed previously when applied to approximate the creep surface of Fig. 4. The free parameters are evaluated again via least square identification, whereby the number of exponentials are optimization variables as much as the exponent in the power law for viscous flow. The accuracy of the surface fitting is measured by the relative root mean square error δ_M which is composed of the error in instantaneous and in time dependent deformations. The average value Y of M data values is used to normalize the total error and to express the relative error in percent

$$\delta_M = 100 \left[\text{RMS}(\epsilon_e) + \text{RMS}(\epsilon_i) \right] / Y \quad \begin{array}{l} \text{where } M = 14 \times 4 = 56 \\ \text{and } Y = 64.69 \end{array} \quad (50)$$

The results of least square identification with different aging models are summarized in Table 1. We observe considerable improvement of accuracy as we proceed from the delayed elastic model over the aging flow model to the summation, product and time shift models. The last three require identification of two functions, the dependence on loading age and the dependence on load duration, and thus provide almost the same degree of accuracy. For illustration the summation model is compared in Fig. 6 with the input data. Clearly, there is satisfactory agreement even for such a spectrum of loading ages. If we would resort to non-aging elastic behaviour, we severely restrict the creep model and obtain very large discrepancies as reported in [20].

CREEP AND SHRINKAGE ANALYSIS OF A PCRV

Let us turn to the application of the internal variable model to the time history analysis of a complex concrete structure. To this end we examine the example of a prestressed concrete reactor vessel which is subjected to a realistic loading history. In the first part we have to compute the history of hygrothermal expansion due to drying and heating of the vessel. In the second part these environmental loads are input into subsequent creep and shrinkage analysis of the PCRV, i.e. the computation of stress and deformation histories due to shrinkage and thermal loadings as well as mechanical prestress and pressure. We utilize the finite element software package SMART [22] into which the internal variable creep model has been incorporated. SMART features both static and diffusion analysis capabilities with compatible element libraries.

Finite Element Idealization of PCRV

Fig. 7 shows the geometric configuration of a typical HTR concrete vessel and the axisymmetric finite element idealisation. Note that a thin element layer is placed along the surface where large gradients arise due to local shrinkage effects.

The vessel is exposed to the loading history shown in Fig. 8 involving mechanical prestress, pressure cycles with extended hold times, as well as environmental effects due to shrinkage and heating. In the following we decouple the hygrothermal from the mechanical process, thus we assume that moisture migration and heat conduction analysis can be separated from stress analysis.

Drying of Concrete under Thermal Environments

Lest us examine the moisture transport in the concrete vessel due to drying on the outside surface (relative room humidity $H_0 = 60\%$) and due to the temperature gradient across the containment wall ($\Delta T = 50^\circ\text{C}$ with room temperature on the outside). Let us assume that the hygral process and the thermal process are coupled such that drying is affected by the temperature field, but heat-conduction evolves independently of the moisture field. The hypothesis of weak coupling is justified because moisture migration proceeds at a much slower rate than heat conduction. In this case the humidity transport $H(t)$ is governed by Fick's law of diffusion which is coupled with the thermal field $T(t)$ in form of thermal driving forces due to temperature gradients and non-isothermal conditions. On the finite element level moisture migration, i.e. drying of the idealized structures, is controlled by a set of coupled first order differential equations with unknown nodal humidities H_I

$$C_H \dot{H}_I + K_H H_I = R_H + \underbrace{L_H T_I + M_H \dot{T}_I}_{\text{Thermal Coupling}} \quad (51)$$

In analogy, heat transport is described by the following finite element formulation of Fourier's law of heat conduction with unknown nodal temperatures T_I

$$\mathbf{C} \dot{\mathbf{T}}_I + \mathbf{K} \mathbf{T}_I = \mathbf{R}_I \quad (52)$$

The matrices $\mathbf{C}$ and $\mathbf{K}$ denote the finite element idealization of capacity and conductivity, the load vector $\mathbf{R}$ describes the driving forces in form of fluxes and convective boundary conditions. The hygrothermal coupling matrices $\mathbf{L}_H$ and $\mathbf{M}_H$ define the effects of temperature gradients and temperature changes on the moisture transport. Nonlinear material characteristics [21] are considered iteratively in the incremental solution algorithm for transient analysis. The thermal solution is illustrated in Fig. 9, where the temperature distribution is shown at four sections of the vessel. The resulting moisture distribution is illustrated in Fig. 10 which indicates the localization of drying on the inside and outside surface. We observe that during initial heating there are large temperature gradients at the inside wall while drying is confined first to the outside and starts on the inside only after heating has taken place.

Creep and Shrinkage Behaviour

With the nodal temperature and humidity histories we can now proceed with the time dependent analysis of the PCRV for the loading history shown in Fig. 8. To this end we have to trace a rate process which is expressed in terms of the unknown nodal velocities $\dot{\mathbf{r}}$ of the finite element idealization

$$\mathbf{K}_0 \dot{\mathbf{r}}_I = \dot{\mathbf{R}}_I + \dot{\mathbf{J}}_I \quad (53)$$

$\mathbf{K}_0$ denotes the initial stiffness of the idealized structure which remains constant during the entire process. $\mathbf{R}_I$ are the increments of mechanical loading and the increments of initial loading which arise due to the evolution of inelastic strains

$$\dot{\boldsymbol{\eta}} = \dot{\boldsymbol{\epsilon}}_a + \dot{\boldsymbol{\epsilon}}_T + \dot{\boldsymbol{\epsilon}}_w + \dot{\boldsymbol{\epsilon}}_v + \dot{\boldsymbol{\epsilon}}_d \quad (54)$$

To account for temperature and humidity effects aside from aging, creep parameters have been identified within the frames of the summation model from the creep data Seki- and Kawasumi for $T = 20^\circ, 70^\circ$ and $w = 50\%, 100\%$ [23]. The results of the least square parameter fitting are illustrated in Fig. 11 for different temperature conditions where linear interpolation was used to obtain the values between $20 \leq T \leq 70^\circ \text{C}$.

In conclusion we illustrate the creep response for the combined loading history in Fig. 12 at point P. The shrinkage strain enhances the deformations while the moisture effect in the creep law leads to very pronounced stress relaxation which enlarges the effects due to temperature on the inside wall. Upon heating stresses redistribute due to the temperature increase on the inside wall. However, the effects of thermally increased creep under prestress is counteracted by the thermal expansion. We observe considerable relaxation before pressurization at 528 days which counteracts prestressing and is fully reversible upon the release of pressure at 828 days.

The circumferential stress distribution is shown in Fig. 13 at four sections of the vessel. The stress gradients due to temperature relax until full loading is reached at 1128 days because of the temperature dependent creep properties. This effect is strongly accentuated due to shrinkage because of the strong influence of humidity on the creep behaviour. For this reason the stresses fully relax in the zones of high drying, i.e. in a very narrow region along the inside and outside surface of the vessel, while the stress level within the vessel is hardly reduced.

CONCLUDING REMARKS

The objective of the paper was to develop a constitutive model for incremental solution of creep and shrinkage problems in aging concrete structures. The previous work of the authors [5], [6] is extended to formulate directly an incremental material law for the underlying rate process. The material parameters are identified from creep experiments (constant stress) which are initiated at different loading ages and which are carried out under different, however constant temperature and humidity conditions.

Application of such a constitutive model to variable loading history requires an additional postulate, i.e. the "superposition in time". More out of convenience rather than supporting experimental evidence we resort normally to time superposition without considering the interference between the response for individual step inputs. For the time being we still adopt this concept even though we observe from creep tests and from recovery tests [24] that the material structure changes with the loading history and thus introduces history dependence in form of mechanical aging similar to the physico-chemical maturing.

Clearly, the quantification of this phenomenon is only one topic for a continued research effort in the area of concrete creep which leads immediately to the question of corresponding transitional creep effects for variable temperature and humidity conditions.

Nonlinear creep of concrete is another class of problems which appears to become significant already at intermediate stress levels around $0.45 f_c'$ [24]. Formally, the extension of the present constitutive formulation to include nonlinear effects poses no conceptual difficulties, however, a proper multiaxial formulation is complicated even if we consider only the "simple" instantaneous deformation components. If we want to include different creep in tension and compression, the modelling problems are further aggravated by nontrivial loading conditions.

We can conclude that at the time being several rational models are available for concrete creep which account for the dominant material phenomena such as aging and environmental conditions, however, there is ample need for further refinement and quantification of constitutive parameters from test data.

ACKNOWLEDGEMENT

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REFERENCES

- 1) Bazant, Z., "Creep and Shrinkage of Concrete: A Precis of Recent Developments", Mechanics Today, V.2, S. Nemat-Nasser (ed.), Pergamon Press, 1974, pp. 1-93.
- 2) Truesdall, C. and Noll, W., The Nonlinear Field Theories of Mechanics, S. Flügge (ed.), Encyclopedia of Physics, V.III, Pt. 3, Springer-Verlag, Berlin, 1965, pp. 537-541.
- 3) Lubliner, J., "On the Thermodynamic Foundations of Nonlinear Solid Mechanics", Int. J. Nonlinear Mechanics, V. 7, 1972, pp. 237-254.
- 4) Mukaddam, M.A. and Bresler, B., "Behaviour of Concrete Under Variable Temperature and Loading", ACI Seminar on Concrete in Nuclear Reactors, Berlin, 1970; ACI Special Publication 34, 1972.
- 5) Argyris, J.H., Pister, K.S. and Willam, K.J., "Thermomechanical Creep of Aging Concrete, a Unified Approach", IABSE Publ. 36-1, 1976, pp. 23-57.
- 6) Argyris, J.H., Pister, K.S., Szimmat, J. and Willam K.J., "Unified Concepts of Constitutive Modelling and Numerical Solution Methods for Concrete Creep Problems", ISD-Report No. 185, Univ. of Stuttgart, 1976.
- 7) Dischinger, F., Untersuchungen über die Knicksicherheit, die elastische Verformung und das Kriechen des Betons bei Bogenbrücken, Der Bauingenieur, V.18, 1937, pp. 487-520, 539-552, 595-621, and V.20, 1939, 53-63, 286-294, 426-437, 563-572.
- 8) Ross, A.D., Creep of Concrete Under Variable Stress, J. ACI, V.29, 1958, pp. 739-758.
- 9) Illston, J.M., The Components of Strain in Concrete under Sustained Compressive Stress, Mag. of Conc. Res. V.17, 1965, pp. 21-28.

- 10) Rüsch, H., Jungwirth, D., Hilfsdorf, H., Kritische Sichtung der Verfahren zur Berücksichtigung der Einflüsse von Kriechen und Schwinden des Betons auf das Verhalten der Tragwerke, Beton und Stahlbetonbau, V.68, 1973, pp. 49-60, 76-86, 152-158.
- 11) Mc Henry, D., A New Aspect of Creep in Concrete and its Application in Design, ASTM, V.43, 1943, pp. 1069-1084.
- 12) Arutyunyan, N.Ch., Some Problems in the Theory of Creep, Pergamon Press, 1969.
- 13) Trost, H., Auswirkungen des Superpositionsprinzips auf Kriech- und Relaxationsprobleme bei Beton und Spannbeton, Beton und Stahlbetonbau, V.62, 1967, pp. 232-238, also V. 65, 1970, pp. 155-179.
- 14) Sackman, J.L., Creep in Concrete and Concrete Structures, Proceedings Princeton University Conference on Solid Mechanics, Princeton, N.J., 1963, pp. 15-48.
- 15) Gamble, B.R., Jordaan, I.J., A Direct Method of Viscoelastic Analysis for Aging Concrete, Matl. and Struct., RILEM, V.7, 1974, pp. 37-43.
- 16) CEB, Bulletin d'Information No. 80 and No. 94, Paris, 1972, 1973.
- 17) Haas, W., Über ein EDV geeignetes Verfahren zur Erfassung des Kriechens und Schwindens von Beton, Dissertation, University of Stuttgart, 1974.
- 18) ACI Committee 209, Prediction of Creep, Shrinkage and Temperature Effects in Concrete Structures, Designing for Effects of Creep, Shrinkage and Temperature, ACI Special Publication, 27, 1971.
- 19) Browne, R.D. and Bamforth, P.B., The Long Term Creep of the Wylfa P.V. Concrete for Loading Ages up to 12.5 Years, Paper H 1/8, 3rd SMIRT Conf., London, Sept. 1-5, 1975.
- 20) Bazant, Z.P., Osman, El.M., On the choice of Creep Function for Standard Recommendations on Practical Analysis of Structures, Cement and Concr.Res., V.5, 1975, pp. 129-138.
- 21) Bazant, Z.P. and Najjar, L.J., Drying of Concrete as a Nonlinear Diffusion Problem, Cement and Concr. V.1, 1971, 461-473.
- 22) Argyris, J.H., Faust, G., Szinmat, J., Warnke, E.P. and Willam, K.J., SMART-Strukturmechanische Analyse in der Reaktortechnologie, DAStb Proceed., SBB-Seminar, No. 2/1, Berlin, 13-14 Oct. 1975.

- 23) Seki, S. and Kawasumi, M., Creep of Concrete at Elevated Temperature, ACI Seminar on Concrete in Nuclear Reactors, Berlin, 1970; ACI Special Publication 34, 1972.
- 24) Komendant, G.J., Polivka, M., Pirtz, D., Study of Concrete Properties for Prestressed Concrete Reactor Vessels, Part II, UC-SESM 76-3, Report, Univ. of Calif., Berkeley, 1976.

Table 1 Accuracy of Different Creep Models

Aging Model	Deformation Components	Relative Error
Delayed Elastic	$\dot{\epsilon} = \dot{\epsilon}_e(t) + \dot{\epsilon}_d(t - \tau)$	26.8%
Aging Flow	$\dot{\epsilon} = \dot{\epsilon}_e(t) + \dot{\epsilon}_v(t)$	14.2%
Summation	$\dot{\epsilon} = \dot{\epsilon}_e(t) + \dot{\epsilon}_v(t) + \dot{\epsilon}_d(t - \tau)$	9.3%
Product	$\dot{\epsilon} = \dot{\epsilon}_e(t) + \dot{\epsilon}_d(t, t - \tau)$	10.9%
Time Shift	$\dot{\epsilon} = \dot{\epsilon}_e(t) + \dot{\epsilon}_d(\xi - \xi')$	10.7%

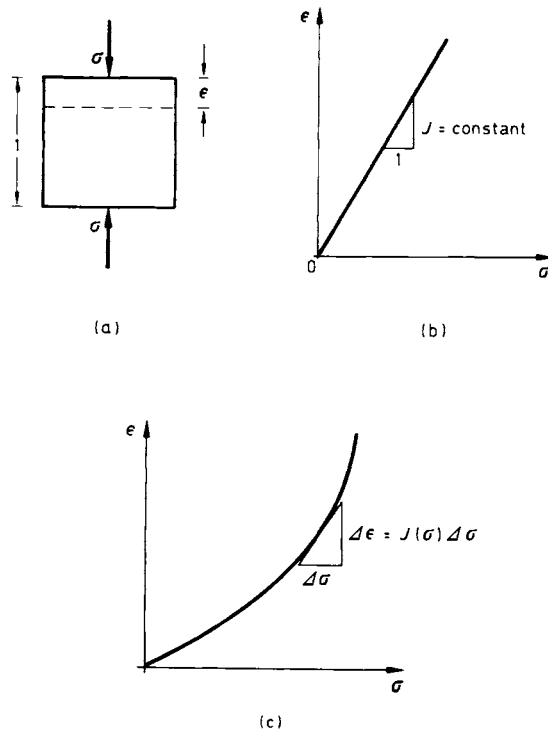


Fig. 1--Uniaxial behavior of linear and nonlinear elastic materials

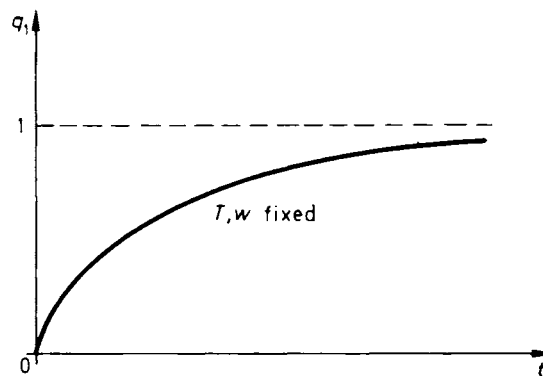


Fig. 2--Concrete maturity as a function of age

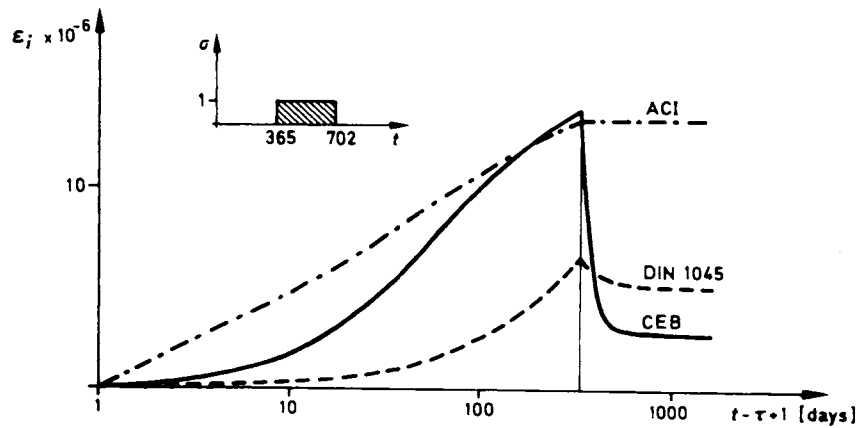
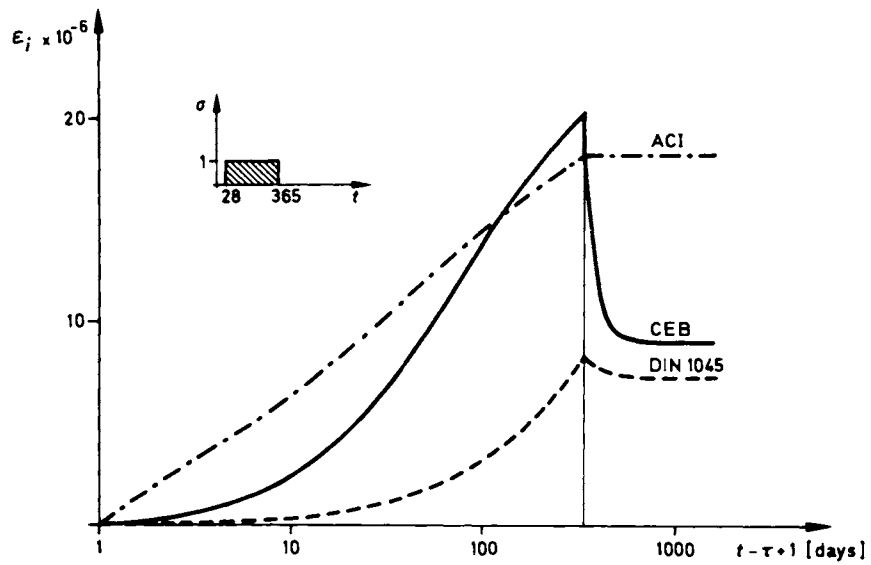


Fig. 3--Code recommendations for mass concrete

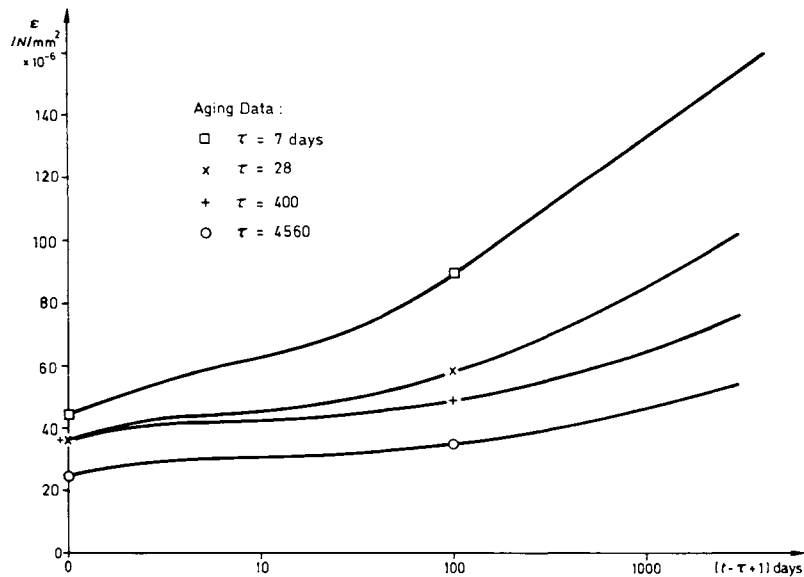


Fig. 4--Aging creep, Wylfa concrete data (Browne and Bamforth)

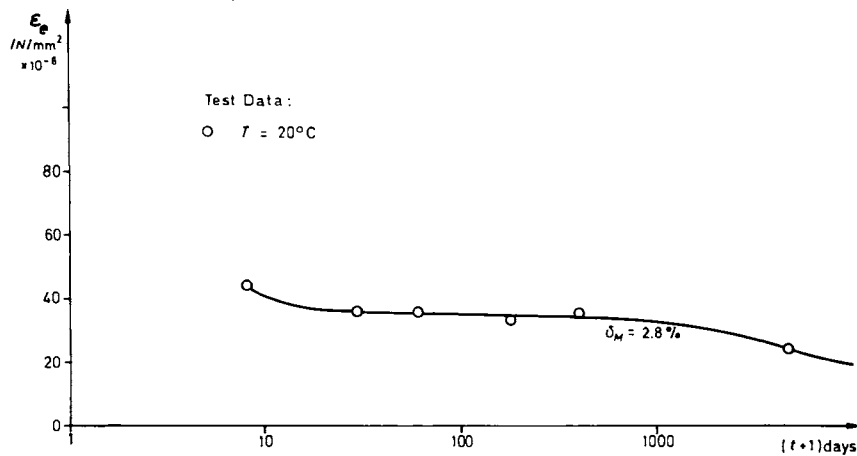


Fig. 5--Exponential fit of elastic compliance, Wylfa concrete data (Browne and Bamforth)

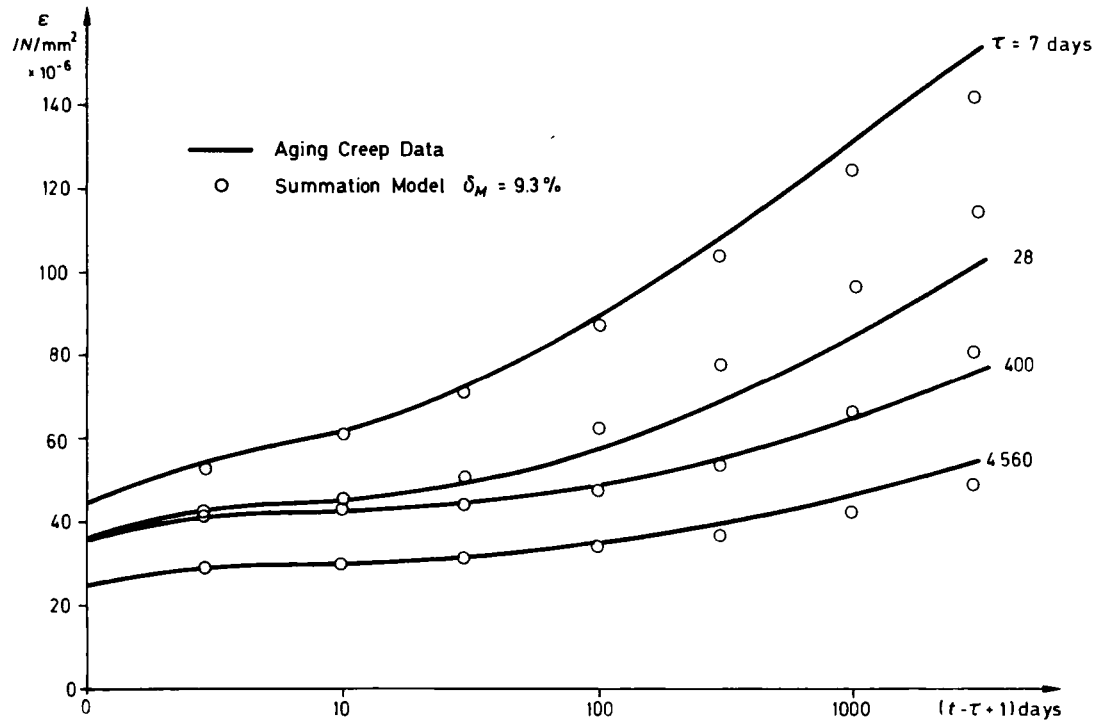
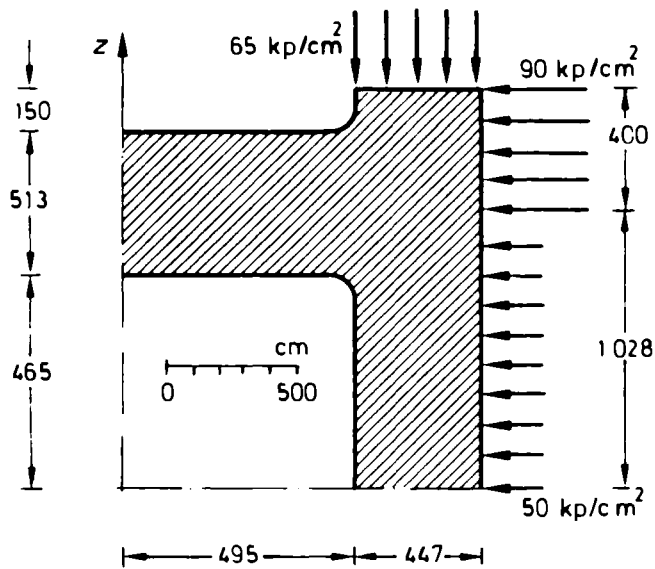


Fig. 6--Aging summation model for $T = 20^\circ C$, Wylfa concrete data (Browne and Bamforth)



GEOMETRY AND MESH LAYOUT

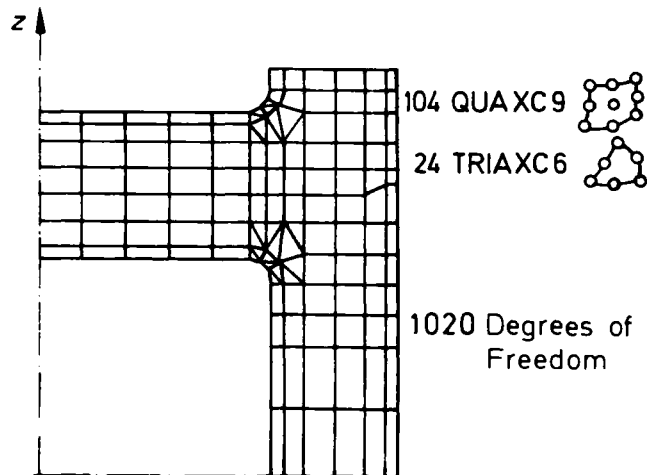


Fig. 7--Prestressed concrete reactor vessel

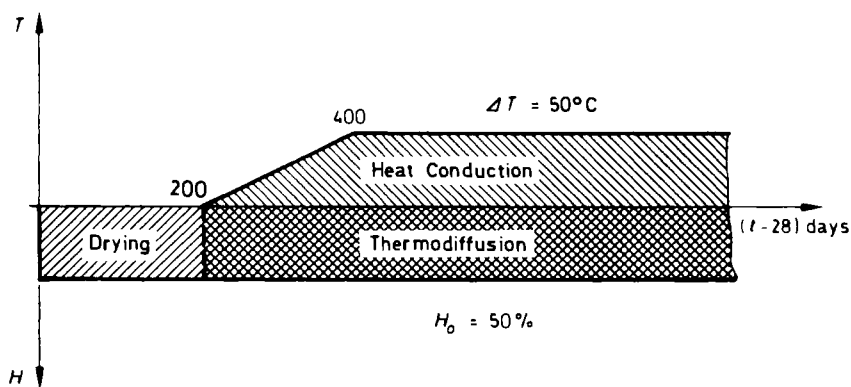
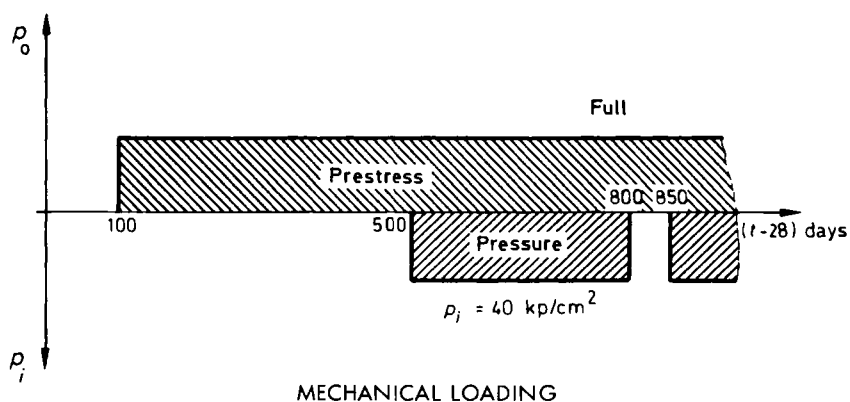


Fig. 8--Loading history

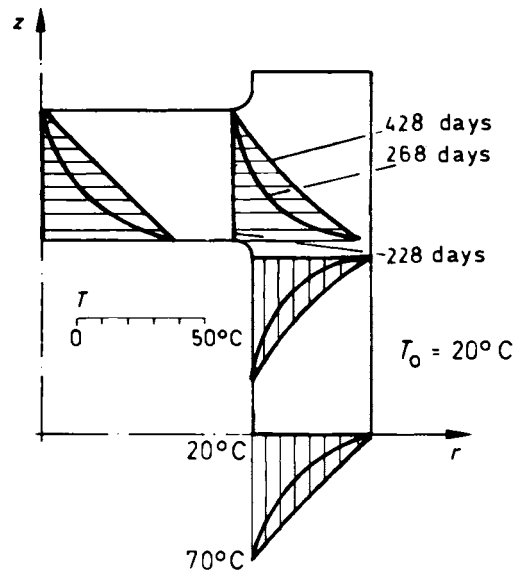


Fig. 9--History of temperature distribution

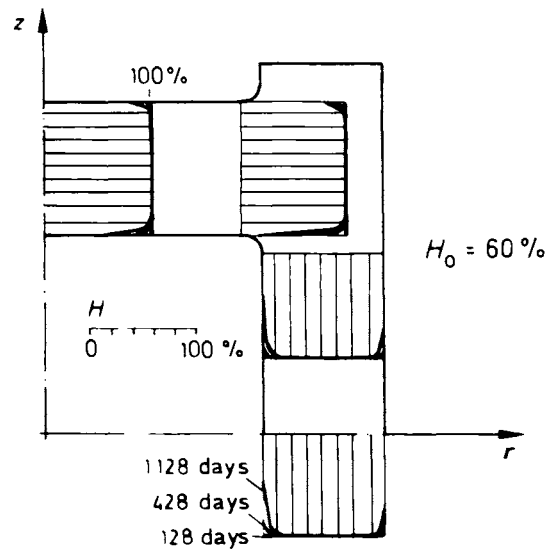


Fig. 10--History of moisture distribution

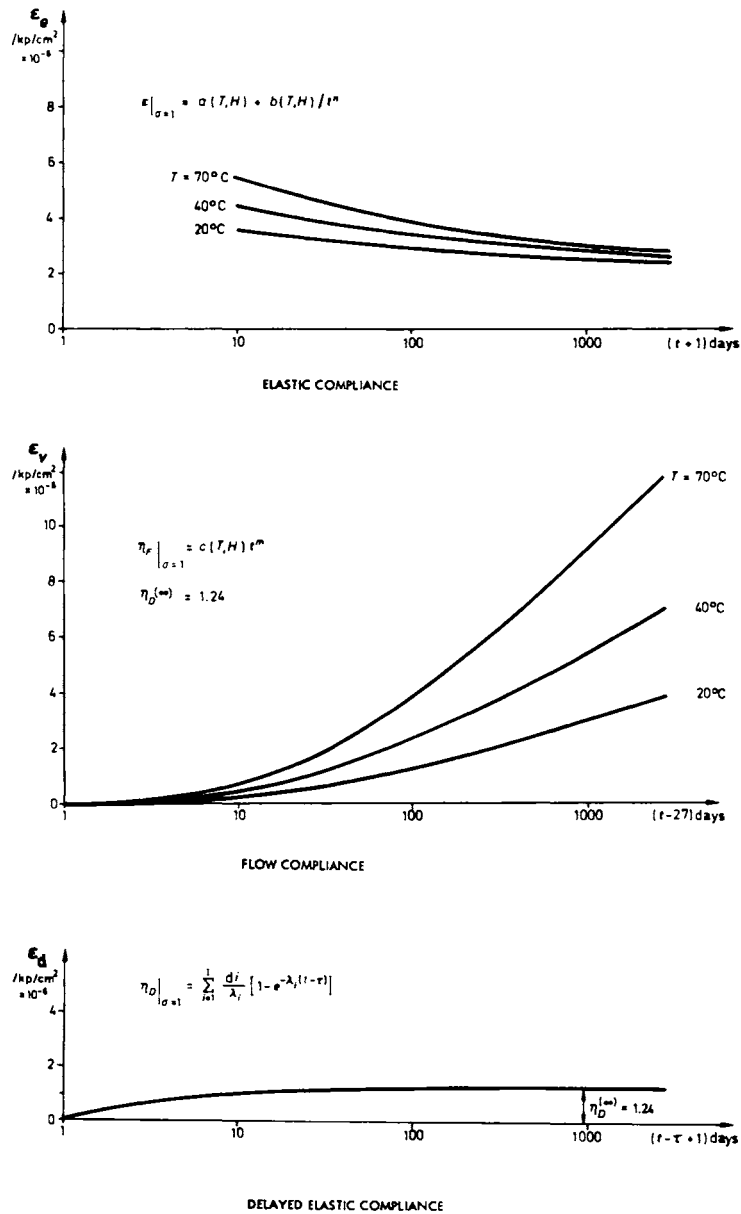


Fig. 11--Deformation components of summation model creep data (Seki and Kawasumi)

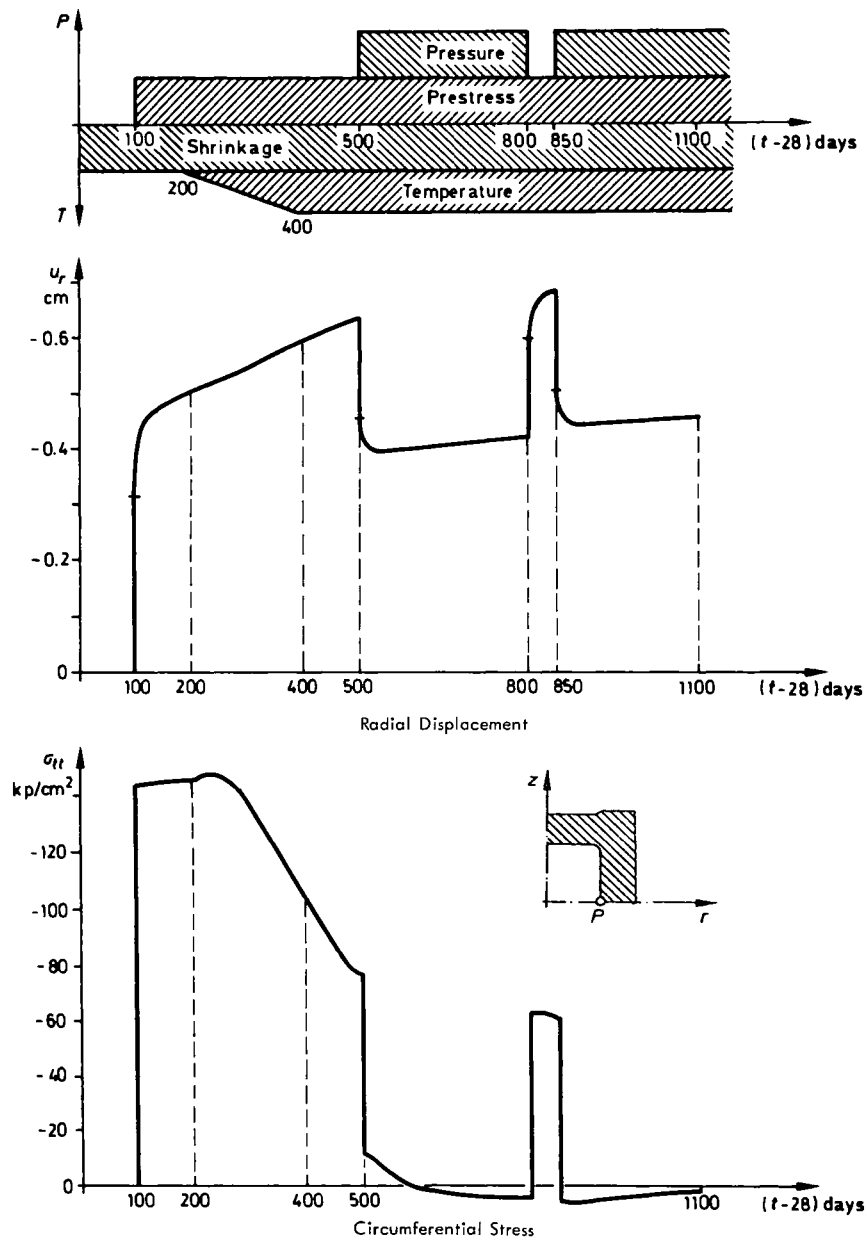


Fig. 12--Creep response and stress relaxation at point P

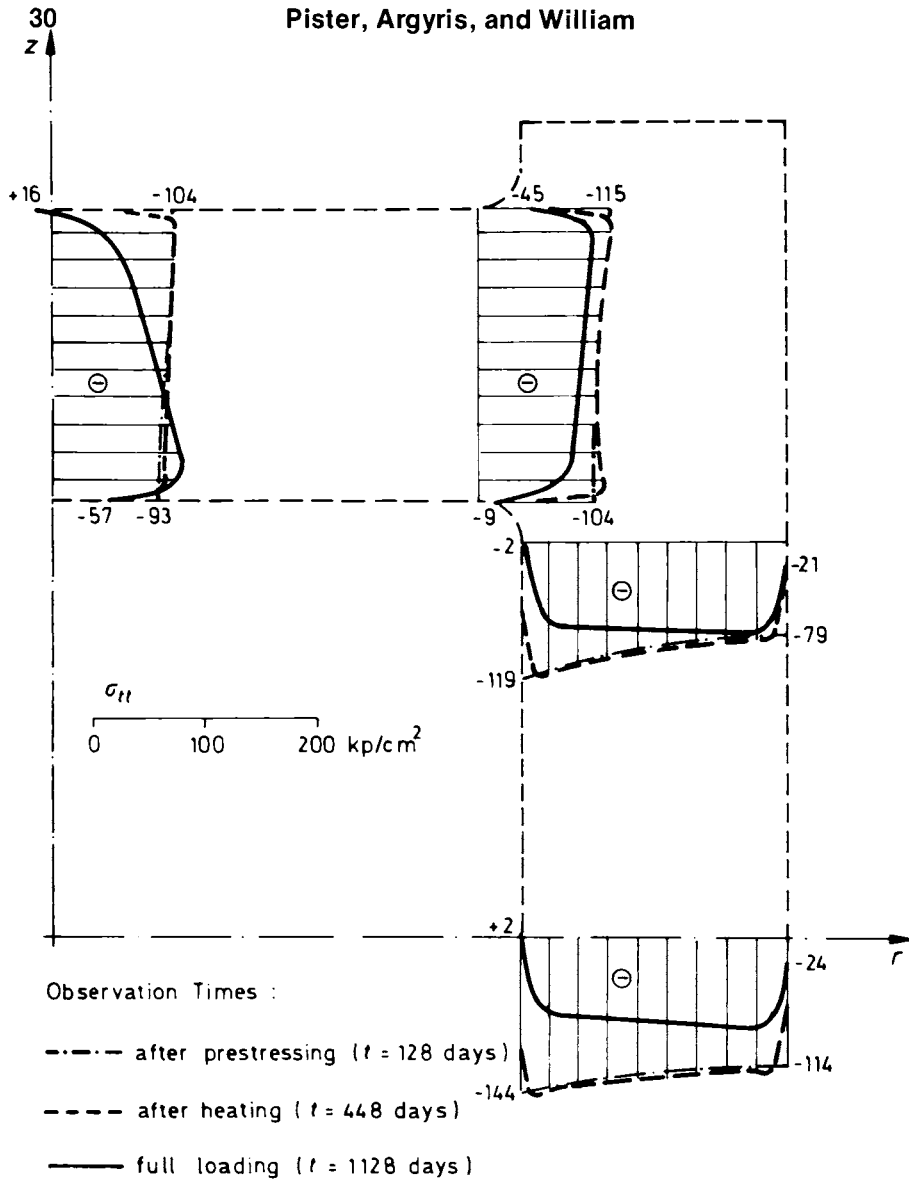


Fig. 13--History of circumferential stress distribution

Creep of Concrete under Various Temperature, Moisture, and Loading Conditions

By J.E. McDonald

Synopsis: An investigation was conducted to develop information on the time-dependent deformation behavior of concrete in the presence of temperature, moisture, and loading conditions similar to those encountered in a prestressed concrete reactor vessel (PCRV). Variables were one concrete strength (6000 psi (41 MPa) at 28 days), three aggregate types (chert, limestone, and graywacke), one cement (Type II), two types of specimens (as-cast and air-dried), two levels of temperature during test 73 F and 150 F (23 C and 66 C), and four types of loading (uniaxial, hydrostatic, biaxial, and triaxial). There were 66 test conditions for creep tests and 12 test conditions for unloaded or control specimens.

Experimental results are presented and discussed. Comparisons are made concerning the effect of the various test conditions on the behavior of concrete and general conclusions are formulated.

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Keywords: creep properties; creep recovery; creep tests; curing; deformation; loads (forces); modulus of elasticity; moisture content; nuclear reactors; Poisson ratio; pressure vessels; prestressed concrete; shrinkage; strains; stresses; temperature

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INTRODUCTION

One of the most important aspects in the design and safety evaluation of a prestressed concrete reactor vessel (PCRV) is the time-dependent deformation behavior of concrete in the presence of varying temperatures, moisture, and loading conditions. Creep and shrinkage in the concrete can have serious consequences because they produce (a) a loss of initial prestress, (b) large deformations of the vessel, which could result in misalignment of the control rod passages, and (c) residual stresses that may be introduced as the loading conditions change. Consequently, a basic research program formulated and directed by the Oak Ridge National Laboratory to develop and improve the technology of PCRV's in the United States included a sizeable investigation of the time-dependent conditions existing in a PCRV. This program was intended to provide basic information that could be used, first, as a basis for developing analytical techniques for examining the time-dependent behavior of a PCRV, and second, as the data necessary when using these techniques for design and safety evaluations. One of the projects included in this effort was a multiaxial creep program performed at the Concrete Laboratory, U. S. Army Engineer Waterways Experiment Station (WES)¹ and summarized generally herein.

TEST CONDITIONS

Although numerous factors affect the time-dependent deformation behavior of concrete, this study concerned only four of the more important variables, i.e., modulus of elasticity of the aggregate, curing history prior to loading, temperature during loading, and state of stress during loading. Sixty-six test conditions involving loaded specimens were investigated. In addition, 12 test conditions involving unloaded or control specimens were investigated. The 78 test conditions are summarized in Table 1.

Concrete made with crushed limestone aggregate (3/4-in. (19 mm) maximum size) was chosen as the main mixture since it was representative of mixtures that might be used in a PCRV in most sections of

the United States. Two other mixtures containing crushed graywacke and chert gravel, aggregates with elastic moduli lower and higher, respectively, than that of limestone, were used to provide information for comparison.

The two curing histories selected for this study were designated "as-cast" (AC) and "air-dried" (AD). The AC specimens were sealed shortly after casting and remained so throughout the test to prevent evaporation losses. The resultant saturated concrete was representative of the interior of a mass of concrete such as a PCRV. After 7 days curing, the AD specimens were allowed to dry in air at 73 F (23 C) and 50 percent relative humidity for the remainder of the 90-day period preceding testing. These specimens exhibited considerable moisture loss and were representative of concrete near the exterior of a PCRV.

The temperature levels during loading were selected as representative of the limits of the range of concrete temperatures experienced in a nuclear reactor containment vessel during normal operation, 73 F and 150 F (23 C and 66 C) corresponding to temperatures expected at the outer and inner surfaces, respectively.

Until recently, the majority of creep work has concerned uniaxial loading only; however, the reinforcement of a PCRV is such that other states of stress exist. As a result, test specimens were loaded in uniaxial, biaxial, hydrostatic, and triaxial states of stress with both axial stress and radial confining stress ranging from 0 to 2400 psi (0 to 17 MPa).

MIXTURE PROPORTIONS

Three concrete mixtures were proportioned with Type II portland cement and 3/4-in. (19 mm) maximum size aggregates, the moduli of elasticity of which ranged from 3.8 to 13.65×10^6 psi (2.6 to 9.4×10^4 MPa), to have compressive strengths of 6000 psi (41 MPa) at 28 days. Proportions of the resultant concrete mixtures, designated high, main, and low modulus according to aggregate moduli were previously reported.^{1,2}

SPECIMENS

Creep and control specimens were 6 in. (152 mm) in diameter by 16 in. (406 mm) in length and were attached to 1-in. (25 mm) thick steel end plates. Two vibrating wire strain gages were embedded in each specimen to measure axial and radial strain. Ninety-eight such specimens were cast from eight batches of concrete. In addition, 249 6-by-12-in. (152- by 305-mm) strength cylinders were cast. Specimen preparation procedures were previously described in detail.^{1,2}

LOADING SYSTEM

In general, creep specimens were placed in the test units located in the proper testing temperature for 7 days prior to loading at 90-days age. All loads were applied incrementally with a hydraulic hand pump at a rate of approximately 35 psi/min (0.24 MPa/min). When the desired maximum load was attained, the test unit was switched to a pressure distribution system which maintained a constant load using an oil reservoir under regulated high-pressure gas.

STRAIN MEASUREMENTS

Strain and temperature measurements were obtained on the creep and control specimens throughout the 90-day curing period, immediately prior to loading, upon attaining full load, and throughout the test period. In addition, strain readings were obtained on the creep specimens at each load increment. After applying the full test loads, gages were generally read every 15 to 30 min for the first 2 to 3 hr or until the readings began to stabilize and then daily for one week after loading; as the readings tended to further stabilize with increased time under load, the time between readings was increased.

EXPERIMENTAL RESULTS

Detailed results of tests conducted to determine the elastic and time-dependent deformation behavior of concrete subjected to a variety of test conditions are reported in Reference 1. These results are summarized in the following and a number of general comparisons are made concerning the effects of the various test conditions on the behavior of concrete.

Strength and Elastic Deformation

Even though the concrete mixtures contained aggregates with a wide range of moduli, they did have similar compressive strengths. The compressive strength of AC specimens from each of the three concrete mixtures was lower than that of AD specimens at both 28 and 90 days. However, after 90 days curing, both curing procedures resulted in concretes which exhibited lower strengths than concrete cured in limewater (standard specimens) until tested. Beyond 90 days both AC and AD specimens were sealed in copper and, after approximately 1 year of additional curing at 73 F (23 C), the compressive strengths of AC and AD concrete were essentially equal.

Elastic strains were determined by taking the difference in strain measurements on the creep specimens immediately before and after loading. These elastic strains were used to compute Poisson's ratio and the modulus of elasticity as follows:

$$\mu = \frac{\sigma_a \epsilon_r - \sigma_r \epsilon_a}{2\sigma_r \epsilon_r - \epsilon_a (\sigma_r + \sigma_a)}$$

$$E = \frac{\sigma_a - 2\mu\sigma_r}{\epsilon_a}$$

where

μ = Poisson's ratio

σ_a = axial stress, psi

ϵ_r = elastic radial strain

σ_r = radial stress, psi

ϵ_a = elastic axial strain

E = modulus of elasticity, psi

Average values for the various test conditions indicate the modulus of elasticity of concrete is proportional to the modulus of elasticity of the aggregate (Fig. 1). A similar relation would appear to exist for Poisson's ratio with the exception of the high-modulus mixture where Poisson's ratio for the concrete was higher than that for the chert aggregate (Fig. 2).

Time-Dependent Deformation

Strains were monitored intermittently during the curing period to determine concrete deformations due to autogenous volume changes. The reference reading for calculating these strains was the 7-day reading because this was the earliest time at which the various environmental test conditions could be considered comparable and stable. Axial and radial strains for the AC control specimens were similar in magnitude and remained essentially constant during the curing period. Each of the three concrete mixtures exhibited very small shrinkage strains at the end of the curing period with an overall average of 13 millionths. The AD specimens were allowed to air-dry at 50 percent relative humidity and 73 F (23 C) from 7 to 90 days age. The resultant moisture loss caused significant shrinkage strains in all AD control specimens (Fig. 3). The shape of the relation between the shrinkage of AD concrete and the modulus of elasticity of aggregate is similar to that previously reported by Rüschi, et al.³

The strain data obtained from the loaded creep specimens during the course of the testing period represented total strains, i.e., those which included the elastic deformation upon application of load, and the time-dependent deformations due to load and chemical or physical volume changes within the specimen. Specific creep strains for a given time were determined by subtracting the elastic strain from the total strain, correcting this value for the appropriate volume-change (control) strain, and dividing it by the applied load. To form a numerical basis of comparison for the various test conditions, logarithmic creep expressions of best fit based on a least squares analysis were computed for the creep strain-time relationships. These equations were then used to compute specific axial and radial creep strains at 1 year after loading.

Temperature

There were 5 and 12 cases where the effect of testing temperature could be compared directly for tensile and compressive creep, respectively. At 150 F (66 C), the tensile creep was larger in four of five cases as compared with 11 of 12 cases for the compressive creep. In these cases, the increase in creep at the higher temperature was slightly higher for the compressive mode, averaging 79 and 51 percent for the compressive and tensile creep, respectively. AD concrete at 150 F (66 C) exhibited a greater increase in compressive creep than did the AC concrete whereas the opposite was true for tensile creep.

Specimens tested by Nasser and Neville⁴ were stored continuously in a water bath at the desired testing temperature from 24 hr after casting until they were loaded at 14 days. Results of specimens loaded (underwater) to a stress-strength ratio of 0.35 indicated the uniaxial compressive creep after 15 months under load at 160 F (71 C) was 1.75 times that of specimens at 70 F (21 C). In comparison, AC specimens loaded uniaxially to an average stress-strength ratio of 0.31 at 90 days indicated the compressive creep after 12 months under load at 150 F (66 C) was 2.0 times that at 73 F (23 C). Overall for the 17 cases where effect of temperature could be compared directly for tensile and compressive creep, the absolute average creep at 150 F (66 C) was 1.76 times that at 73 F (23 C).

Moisture

It is generally concluded that creep depends on the loss of water under load and the amount of water present at the time of application of load, with the latter possibly having the larger effect. Thus, the environment for specimens during the curing period prior to application of load and the environment during the sustained loading period are important. AD specimens would be expected to have a lower moisture content upon loading than AC specimens, and therefore might be expected to exhibit less creep. However, in 12 of 16 cases where a direct comparison between AC and AD specimens was possible, the higher compressive creep was associated with the AD concrete. AD specimens of the high- and main-modulus concretes averaged 27 percent greater compressive creep strain than comparable AC specimens. Three

of the four cases where creep was greater for AC concrete occur in the low-modulus concrete. The AC specimens of low-modulus concrete averaged 45 percent greater compressive creep strain than comparable AD specimens. It should be noted that shrinkage strains in AD control specimens averaged 127, 178, and 552 millionths at the time of loading for high-, main-, and low-modulus concrete mixtures, respectively. If the significantly larger compressive strains in the low-modulus concrete are indicative of a greater moisture loss compared with the remaining mixtures, then this might explain why the compressive creep behavior of the low-modulus specimens was opposite the other two mixtures. During the loading period, AD control specimens generally exhibited tensile (expansion) strains, the magnitudes of which were essentially proportional to the concrete modulus of elasticity ranging up to approximately 130 millionths. This would indicate moisture movement within the AD specimens, and since the direction of the moisture movement is immaterial as far as its effect on creep is concerned,⁵ drying creep in the loaded AD specimens would tend to increase the ultimate creep.

In general, tensile creep strains behaved in a manner opposite that of compressive creep. AC specimens of the high- and main-modulus concretes averaged 11 percent greater tensile creep strain than comparable AD specimens, whereas AD specimens of the low-modulus concrete averaged 17 percent greater tensile strain than comparable AC specimens. However, it should be noted that in three of the four low-modulus cases, AC strains averaged 16 percent greater than AD, and it was the remaining case in which the AD strain was 201 percent greater than AC that caused the overall average to be reversed. In comparison, York, et al.,⁶ in similar tests on the main-modulus mixture, found that compressive and tensile creep strains were generally larger for AD concrete than for AC concrete except in the case of low tensile creep, where the reverse was true.

Modulus of Elasticity

Except for some unusual aggregates, the magnitude of creep in the aggregate at stresses to which aggregate particles are subjected in concrete is very small, particularly in comparison with the creep of concrete. The most probable explanation of the influence of aggregate type on creep involves the modulus of elasticity of aggregate; the higher the modulus, the greater the restraint offered by the aggregate to the potential creep of the cement paste.⁷ This restraining effect of aggregate on the deformation of concrete is largely independent of the cause of the deformation (i.e., shrinkage or creep).

There were five cases in which loading, moisture, and temperature conditions allowed a direct comparison of compressive creep strains concerning the effect of concrete modulus. The average strains for these cases were 0.092, 0.118, and 0.270 millionths/psi (13.3-, 17.1-, and 39.2-millionths/MPa) for high-, main-, and low-modulus concrete, respectively. These strain values correlate generally with the inverse of the modulus of elasticity of the concrete and hence the

inverse of the modulus of elasticity of the aggregate (Fig. 4). The shapes of the curves relating compressive creep and the elastic modulus of concrete and aggregate are similar to those previously reported^{3,8} for longitudinal and lateral creep. Also, the shape of the curves relating creep and modulus of elasticity are similar to those relating shrinkage and modulus of elasticity (Fig. 3). There was only one case in which a direct comparison could be made concerning the effect of concrete modulus on tensile creep. In this case, strains were 0.0202, 0.0321, and 0.0561 millionths/psi (2.9-, 4.7-, and 8.1-millionths/MPa) for high-, main-, and low-modulus concrete, respectively (Fig. 5). In comparison, an overall average gave values for tensile creep of 0.0183, 0.0302, and 0.0705 millionths/psi (2.7-, 4.4-, and 10.2-millionths/MPa) for the three concretes. Obviously, with only one direct comparison, the data are limited and the averages are slightly biased; however, the data do offer a general indication that tensile creep also increases as the elastic modulus of the aggregate and concrete decreases.

State of Stress

There were only two cases in which moisture and temperature conditions allowed a direct comparison concerning the effect of the four loading modes on compressive creep; both of these cases involved main-modulus concrete. Average values of compressive creep for these two cases differed by a factor of 1.00:0.45:0.69:0.84 for uniaxial, hydrostatic, biaxial, and triaxial loading, respectively (Fig. 6). In comparison, the overall averages for all values of compressive creep in the main-modulus concrete differed by a factor of 1.00:0.50:0.84:0.90 for the same states of stress, which is in fairly good agreement with the direct comparison, particularly considering the slight bias in the overall average resulting from differences in moisture and temperature conditions for the various states of stress. On the same basis, overall average values for the high- and low-modulus concrete differed by factors of 1.00:0.48:1.10:0.80 and 1.00:---:0.92:0.89 for the various states of stress. No measurements were available for hydrostatically loaded low-modulus concrete.

With the exception of biaxially loaded high-modulus specimens, creep under multiaxial stress was less than that associated with uniaxial stress. It should be noted that this result was particularly biased by temperature difference, i.e., all available biaxial results were for the 150 F (66 C) environment, whereas at least half of the measurements for each of the remaining states of stress were for 73 F (23 C) where strains were generally lower. Thus the overall results are in general agreement with results previously reported by Hannant⁹ and Gopalakrishnan.¹⁰

There were 13 cases in which a direct comparison could be made concerning the effect of increased stress level on compressive creep for a given state of stress. In 9 of the 13 cases, the higher compressive creep strain was associated with the higher stress level. For the uniaxial, hydrostatic, and biaxial loadings, the specific creep strain at the 2400-psi (17 MPa) stress level averaged 1.17

times that at the 600-psi (4 MPa) stress level. This increase in strain was essentially the same for all three states of stress, ranging from 1.15 for biaxial loading to 1.23 for the hydrostatic case. The higher stress level resulted in an average stress-strength ratio upon loading of 0.31, which is, as the data would indicate, approaching the generally accepted limit for creep-stress proportionality. The application of a 600-psi (4 MPa) radial stress reduced an average of 14 percent the compressive creep strains in the direction of the axial load relative to compressive creep strains in comparable uniaxially loaded specimens.

The effect of uniaxial and biaxial loadings on tensile creep could be compared directly in seven cases; in six of these cases, the higher tensile creep strain was associated with the biaxial stress condition. For these seven cases, the tensile creep strains for biaxially loaded specimens averaged 1.26 times that obtained for comparable specimens uniaxially loaded. There were eight cases in which a direct comparison could be made concerning the effect of increased stress level on tensile creep for the uniaxial and biaxial stress states. In these eight cases, the higher tensile creep strain was equally divided between the two stress levels. Averaging the two states of stress, there was less than 1 percent difference in tensile creep strain for the two stress levels. However, the tensile creep strain at 600 psi (4 MPa) was 1:35 times that at 2400 psi (17 MPa) for the uniaxial stress state, whereas the tensile creep at 2400 psi (17 MPa) was 1.20 times that at 600 psi (4 MPa) for the biaxial state of stress.

Creep Poisson's Ratio

Different methods have been used to compute the magnitude of Creep Poisson's Ratio (CPR) under multiaxial stresses, depending on the investigator's definition of CPR. In this paper values of CPR were calculated by the same equation used to calculate elastic Poisson's ratio (EPR) by substituting the appropriate creep strains for elastic strains, a method previously used by York, et al.⁶ On this basis CPR values were calculated for the various environmental conditions at selected intervals during the test period. Although average CPR values over the entire test period were somewhat biased due to differences in loading conditions, they should offer general indications as to the effects of the various test conditions.

Overall, CPR's averaged approximately 0.10, 0.15, and 0.11 for high-, main-, and low-modulus concrete, respectively. CPR was generally smaller than EPR, averaging 68, 66, and 92 percent for the high-, main-, and low-modulus concrete, respectively. York, et al.⁶ reported an average CPR of 0.16 which was approximately 65 percent of the average EPR for main-modulus concrete. This finding is almost identical to the results of this investigation on the same concrete mixture.

It was shown previously that tensile creep strains for AD specimens were smaller than those of comparable AC specimens of the high- and main-modulus concretes. Therefore, it would be expected that CPR would be less for AD concrete; for the limited data available, the CPR for AC specimens averaged 23 percent more than for comparable AD specimens. Similarly, it was previously shown that average tensile creep strains for AD specimens were slightly higher than for comparable AC specimens of low-modulus concrete, and as might be expected, the CPR for AD specimens averaged approximately 5 percent higher than for comparable AC specimens.

The limited data available indicated the CPR at 73 F (23 C) averaged 34 percent higher than at 150 F (66 C). This apparent trend agrees with results reported by others.^{6, 9, 11}

The state of stress also appeared to influence the magnitude of CPR. For each concrete mixture, the higher overall average CPR was associated with the biaxial loading and the lower CPR with the triaxial loading. Values for uniaxial loading ranged between these two bounds. The overall average CPR's for all three concrete mixtures were in the proportion 1.00:0.80:0.54 for biaxial, uniaxial, and triaxial loading, respectively.

The effect of time on CPR is shown in Fig. 7. While the CPR of low-modulus concrete appears to remain essentially constant throughout the test period, CPR for the two remaining mixtures would appear to be increasing slightly with time. In all three cases, CPR after 1 year under sustained load is still slightly lower than CPR. The data, particularly for the high- and low-modulus concretes in the later part of the sustained load period, are limited; however, it would appear, particularly for the main-modulus concrete, that CPR increases somewhat with time under load. In similar tests, York, et al.,⁶ did not draw definite conclusions due to lack of sufficient data; however, they believe that time does not significantly affect CPR.

Elastic Recovery Strains

Strain measurements were made immediately prior to and after unloading at the end of the sustained load test period to determine elastic recovery. Modulus of elasticity and Poisson's ratio values for elastic recovery were calculated in a manner similar to that used for calculating elastic values.

Overall for the three mixtures, elastic recovery strains in the axial direction ranged from 77 to 121 percent of elastic strains upon loading with an average of 99 percent. While the range of recovery strains in the radial direction was much greater (61 to 257 percent), the overall average (101 percent) was again essentially the same as for the elastic strains.

Elastic recovery strains for the main-modulus concrete averaged 8 percent larger than elastic strains at loading. This finding agrees with the results of similar tests on the same concrete mixture at the University of Texas.⁶ In comparison, average elastic recovery strains for the high- and low-moduli concretes were 2 and 13 percent less than elastic strains, respectively. Gopalakrishnan¹⁰ found that, for concrete cubes subjected to multiaxial stresses, the elastic recovery strains were less than the elastic strains upon loading in all cases.

Elastic recovery strains for three of the four temperature and curing conditions investigated were essentially the same, averaging approximately 3 percent higher than elastic strains upon loading. The exception was AD specimens loaded at 150 F (66 C) in which elastic recovery strains averaged 12 percent less than elastic strains upon loading. The modulus of elasticity and Poisson's ratio for elastic recovery were essentially the same as those at the time of loading, except for those for low-modulus concrete which were somewhat higher than at the time of loading.

Creep Recovery

After unloading, strains in the creep and control specimens were monitored intermittently for a period of approximately 4 months to determine creep recovery. Results indicate that from 4 to 60 percent of the creep strain at time of unloading, an average of 31 percent, was recovered 117 days after the loads were released. This recovery, still continuing at the time comparisons were made, was in the direction opposite that of creep strains with the exception of three cases in which there were small strain recoveries in the same direction as the creep strain.

Overall, creep recovery for AC and AD specimens was essentially the same, averaging 29 and 32 percent of the creep strains at time of unloading, respectively. Results reported by Gopalakrishnan, et al,¹² indicated that, for uniaxially loaded specimens, the percentage of axial creep strain recovered was slightly higher for wet-stored than dry-stored cylindrical specimens. Under comparable test conditions in this investigation, results indicate that the percentage of axial creep recovered for the AC specimens was 4 percent greater than that for AD specimens.

Although only limited test results were available for the 150 F (66 C) test condition, the data indicate no significant difference in the percentage of creep recovered at the two test temperatures. Average creep recovery at 73 F (23 C) and 150 F (66 C) was 31 and 30 percent, respectively.

Creep recovery of main- and low-modulus concrete specimens was essentially the same, averaging 27 and 28 percent, respectively, of the creep strain at the time of unloading. In comparison, creep recovery of high-modulus concrete specimens averaged 35 percent of the creep at the time of unloading.

The percentage of creep recovered was essentially the same for both stress levels, averaging 33 and 35 percent of the creep at the time of unloading for the 2400- (17-) and 600-psi (4-MPa) stress levels, respectively. The type of stress applied also appeared to have only a small effect on the percentage of creep recovered, with multiaxial states of stress averaging 5 percent more creep recovery than uniaxial states. In comparison, York, et al,⁶ in similar tests on the main-modulus concrete mixture, reported no significant effect of type stress applied, whereas Gopalakrishnan¹⁰ found that the percentage of creep strain recovered was approximately 20 percent higher for multiaxial than uniaxial states of stress. It should be noted that in the latter tests the age at loading was 8 days and the duration of load ranged from 22 to 98 days, with the majority of the tests being 28 days duration.

CONCLUSIONS

For the concretes used and the actual test conditions investigated, it appears that the following general conclusions can be drawn.

a. Strength and elastic deformation

- (1) It is possible to proportion concrete mixtures containing widely varying aggregate moduli with subsequent variations in concrete moduli to obtain similar compressive strengths.
- (2) The secant modulus of elasticity of concrete was proportional to the modulus of elasticity of the aggregate.

b. Time-dependent deformation

- (1) Average axial and radial shrinkage strains for AD specimens at the end of the curing period were inversely proportional to the concrete and aggregate moduli, ranging from 127 to 552 millionths for the high- and low-modulus concretes, respectively.
- (2) During the testing period, axial and radial expansion strains in sealed AD specimens at 73 F (23 C) were generally inversely proportional to concrete and aggregate moduli, ranging up to approximately 130 millionths for the low-modulus specimens at the end of the test period.
- (3) During the curing and test periods, axial and radial strains for AC control specimens at 73 F (23 C) were similar in magnitude with a maximum recorded strain of 26 millionths shrinkage.

- (4) Specific creep strains were larger for the 150 F (66 C) condition during loading, averaging 1.79 and 1.51 times that at 73 F (23 C) for the compressive and tensile creep, respectively. AD concrete exhibited a greater increase in compressive creep than AC concrete at 150 F (66 C), whereas the reverse was true for tensile creep. Overall, the absolute average creep at 150 F (66 C) was 1.76 times that at 73 F (23 C).
- (5) AD specimens of the high- and main-modulus concrete exhibited greater compressive creep strain than comparable AC specimens whereas AC specimens of the low-modulus concrete exhibited greater compressive creep strain than comparable AD specimens. The reverse was true for tensile creep.
- (6) Compressive and tensile creep strains were generally proportional to the inverse of the elastic modulus of both concrete and aggregate.
- (7) Compressive creep under multiaxial stress was less than that under uniaxial stress. Values of compressive creep for main-modulus concrete differed by a factor of 1.00 (uniaxial:0.45(hydrostatic):0.69(biaxial):0.84(triaxial)).
- (8) The increase in compressive specific-creep strain at the 2400-psi (17 MPa) level was essentially the same for the uniaxial, hydrostatic, and biaxial states of stress, averaging 1.17 times that at the 600-psi (4 MPa) level.
- (9) Creep Poisson's ratio was generally smaller than elastic Poisson's ratio, averaging 68, 66, and 92 percent of the elastic values for high-, main-, and low-modulus concretes, respectively.

c. Elastic and creep recovery strains

- (1) Overall, average elastic recovery strains for the three concrete mixtures were slightly less than elastic strains except for the main-modulus concrete, where the reverse was true.
- (2) The modulus of elasticity and Poisson's ratio for elastic recovery were essentially the same as those at the time of loading, except for those for low-modulus concrete which were somewhat higher than at the time of loading.
- (3) From 4 to 60 percent of the creep strain at time of unloading, an average of 31 percent, was recovered 117 days after the loads were released. This recovery, still continuing at the time comparisons were made, was in the direction opposite of the direction of creep strains, except for three cases in which there were small strain recoveries in the same direction as the creep strain.

- (4) The percentage of creep recovered was essentially the same for variations in curing, temperature, concrete modulus, and stress level investigated, ranging from 27 to 35 percent of the creep at the time of unloading.
- (5) The state of stress applied appeared to have a small effect on the percentage of creep recovered, with multi-axial states of stress averaging 5 percent more creep recovery than uniaxial states.

Table 1

Summary of Experimental Test Conditions

Creep of Concrete

Modulus of Elasticity		High				Main				Low			
Testing Temperature, °F (°C)		73 (23)		150 (66)		73 (23)		150 (66)		73 (23)		150 (66)	
Curing History		AC	AD	AC	AD	AC	AD	AC	AD	AC	AD	AC	AD
Stress, psi (MPa)													
Axial	Radial												
2400 (17)	2400 (17)	*	*	*	*	*	*	*	*	*	*	*	*
	600 (4)	*	*	*	*	*	*	*	*	*	*	*	*
	0	*	*	*	*	*	*	*	*	*	*	*	*
	600 (4)	*	*	--	--	*	*	--	--	--	--	--	--
	0	*	*	*	*	*	*	*	*	--	--	*	*
	0	*	*	*	*	*	*	*	*	--	--	*	*
	2400 (17)	--	--	*	*	*	*	*	*	--	--	*	*
	600 (4)	--	--	*	*	*	*	*	*	--	--	*	*
	0	*	*	*	*	*	*	*	*	*	*	*	*

* Test conditions investigated.

REFERENCES

1. McDonald, J. E., "Time-Dependent Deformation of Concrete Under Multiaxial Stress Conditions," Technical Report C754, US Army Engineer Waterways Experiment Station, Vicksburg, MS, Oct 1975.
2. McDonald, J. E., "An Experimental Study of Multiaxial Creep in Concrete," Concrete for Nuclear Reactors, Vol II, SP-34, American Concrete Institute, 1972.
3. Rüschi, H., Kordina, K., and Hilsdorf, H., "Der Einfluss des mineralogischen Charakters der Zuschläge auf das Kriechen von Beton," Deutscher Ausschuss für Stahlbeton No. 146, 1962.
4. Nasser, K. W., and Neville, A. M., "Creep of Concrete at Elevated Temperatures," Journal of the American Concrete Institute, Dec 1962.
5. Ali, I., and Kesler, C. E., "Mechanisms of Creep in Concrete," Symposium on Creep of Concrete, SP-9, American Concrete Institute, 1964.
6. York, G. P., Kennedy, T. W., and Perry, E. S., "Experimental Investigation of Creep in Concrete Subjected to Multiaxial Compressive Stresses and Elevated Temperatures," Research Report 2864-2, The University of Texas at Austin, Jun 1970.
7. Neville, A. M., Creep of Concrete: Plain, Reinforced, and Prestressed, North-Holland Publishing Company, Amsterdam, 1970.
8. Kordina, K., "Experiments on the Influence of the Mineralogical Character of Aggregates on the Creep of Concrete," RILEM Bulletin No. 6, Mar 1960.
9. Hannant, D. J., "Creep and Creep Recovery of Concrete Subjected to Multiaxial Compressive Stress," Journal of the American Concrete Institute, May 1969.
10. Gopalakrishnan, K. S., Neville, A. M., and Ghali, A., "Creep Poisson's Ratio of Concrete Under Multiaxial Compression," Journal of the American Concrete Institute, Dec 1969.
11. Hannant, D. J., "The Strain Behavior of Concrete Under Compressive Stress at Elevated Temperatures," Research and Development Report, Central Electricity Research Laboratories, London, June 1966.
12. Gopalakrishnan, K. S., Neville, A. M., and Ghali, A., "A Hypothesis on Mechanism of Creep of Concrete with Reference to Multiaxial Compression," Journal of the American Concrete Institute, January 1970.

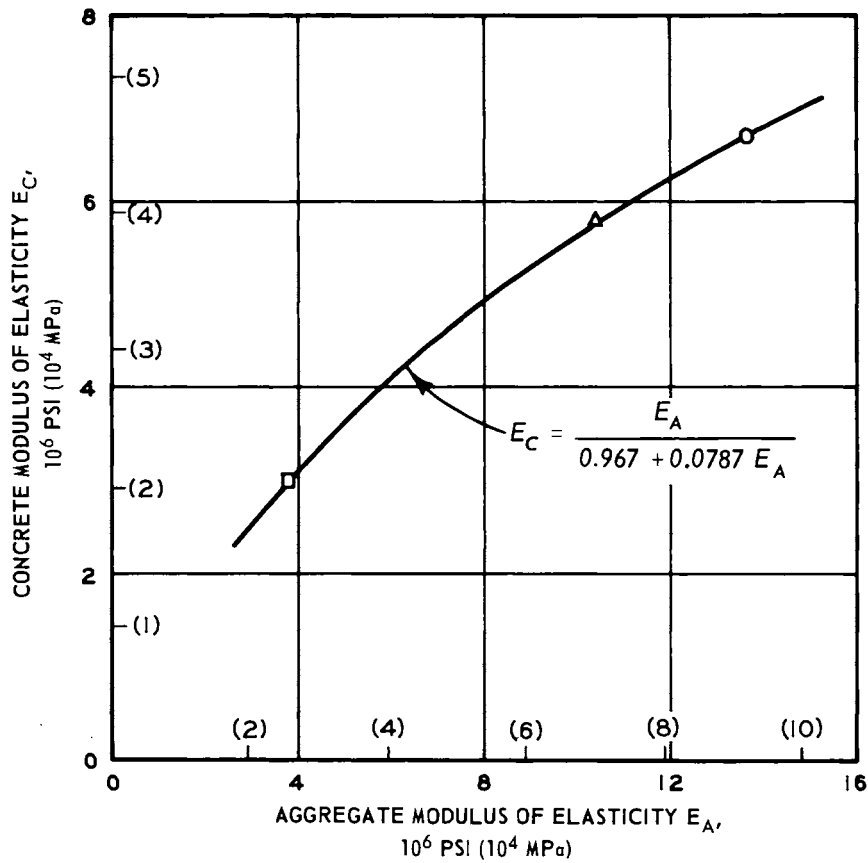


Fig. 1--Relationship of moduli of elasticity of aggregate and concrete

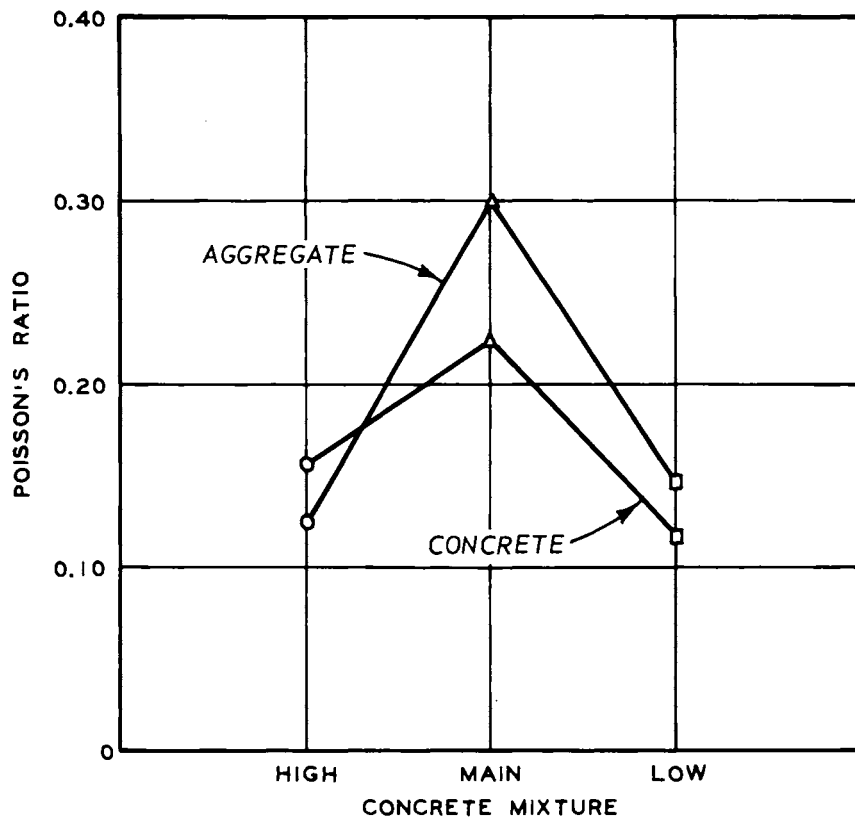


Fig. 2--Relationship of Poisson's ratio of aggregate and concrete

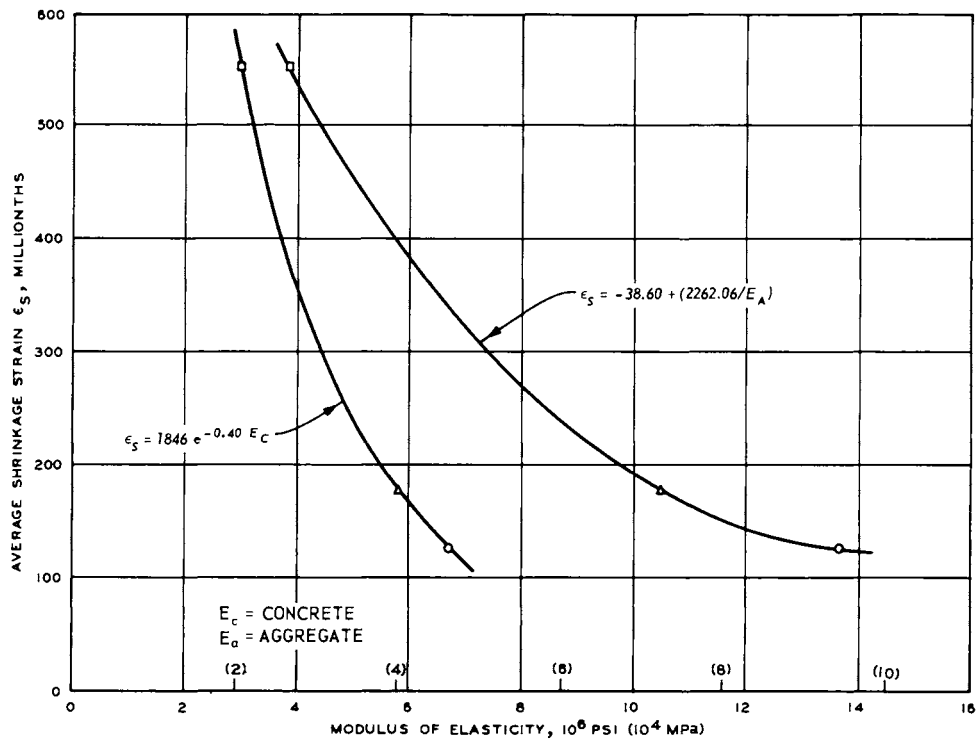


Fig. 3--Relationship of shrinkage strain of air-dried concrete and modulus of elasticity of air-dried concrete

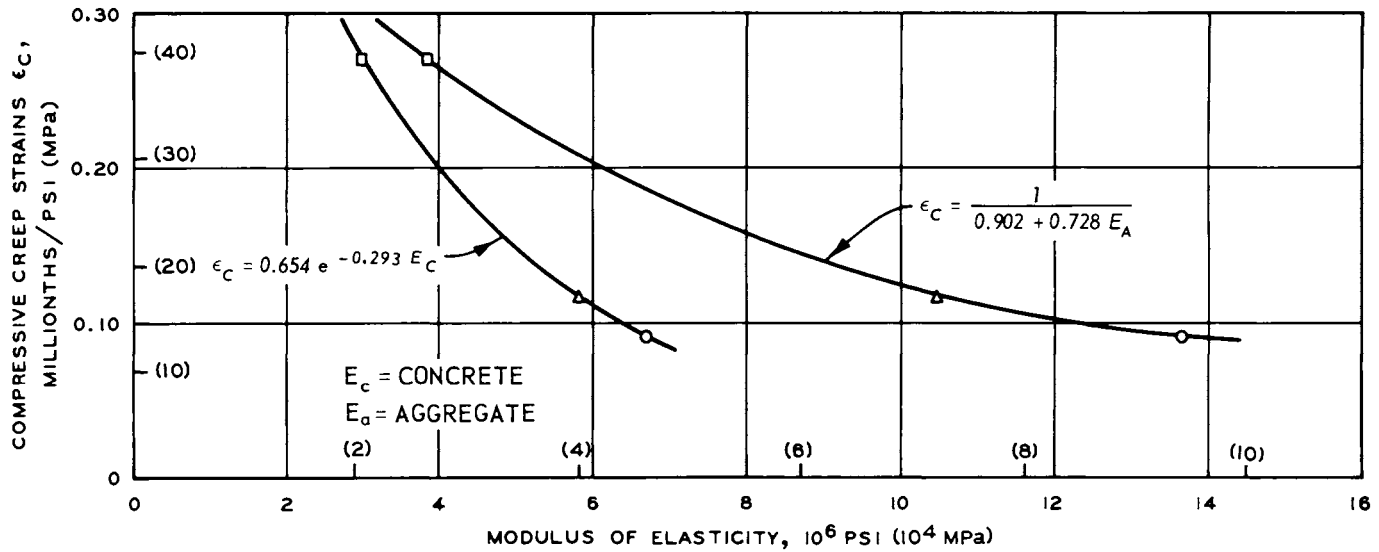


Fig. 4--Relation of compressive creep and elastic modulus of concrete and aggregate