

Ferrocement—Past and Present

By Gajanan M. Sabnis

Synopsis: This is an introductory paper to the symposium and should be considered in conjunction with papers by Batson et al., and Romualdi presented in this volume. An attempt is made to review briefly the historical background and to create the stage for the symposium that ferrocement is a useful engineering material rather than a boat-building material.

Keywords: boats; construction materials; ferrocement; history; reinforced concrete; reinforcing materials; welded wire fabric; wire cloth.

Dr. Gajanan M. Sabnis, F ACI is an active member of ACI, serving on ACI committees 549, Ferrocement and also, on Committees 116, 435, 444. He is also active locally in ACI activities and presently is Director of the National Capital Chapter, of which he was a co-founder and past President. Dr. Sabnis obtained his Ph.D. from Cornell University and is registered engineer in the States of California and Maryland. He is currently associate professor in Civil Engineering at Howard University and is editor of Handbook of Composite Construction Engineering to be published by Van Nostrand Reinhold.

INTRODUCTION

ACI Committee 549 on Ferrocement was formed in 1974 and was charged with the mission to study and report on the engineering properties, construction practices and practical applications of ferrocement and similar materials and to develop standards and safeguards for ferrocement construction. The first activity of this committee was to prepare a state-of-the-art report and to review the literature on the subject by holding a symposium that would feature developments around the world. The present symposium reveals the success of the Committee's efforts.

Although ferrocement has been used as a construction material for several decades for various applications in many countries, establishment of the above ACI Committee 549 indicates the recent interest in this country in the use of ferrocement as a structural material. The major use of this material has previously been in the developing countries, where the advantage was taken of a large but unskilled labor force, and where excellent properties of ferrocement and successful field applications were observed with relatively little theoretical basis. As the material technology developed, so did the interest of engineers who began to view this material as a potentially significant material of construction. It is the hope this writer and the committee that such developments will continue and allow engineers to exploit ferrocement through their ingenuity in new structural applications.

This paper provides a brief historical review and discusses the current status of ferrocement. The papers in this symposium will present considerably more information in terms of the many properties and the applications of ferrocement in countries around the world. The last paper will be devoted to the research needs and potential applications of ferrocement.

HISTORICAL BACKGROUND

Ferrocement is a form of reinforcement concrete in the sense of the grouping the terms 'reinforced' and 'concrete'. It differs, however, from reinforced concrete by the manner in which the reinforcement is distributed in the matrix. The general arrangement of closely spaced continuous reinforcement converts the matrix into a composite material unlike conventional reinforced concrete. Because of the closer spacing (of the order of 1 to 2 in. (25 to 50 mm)) and the small size of the reinforcing, the materials are used efficiently in thin sections. The nature of the reinforcement is such that hardly any formwork is required to form complicated shapes. Many of the characteristics of ferrocement are the result of the experience in field and not from theoretical analysis, since no skilled or trained labor was needed. It has been most widely used in the developing countries of Asia.

The use of ferrocement, if considered in the very broad sense of the term, goes back many centuries. One can consider it as reinforcement impregnated with mortar because of the high percentage of reinforcement, strongly influences the mechanical properties. The use of bamboo and mud (concrete) in constructing huts has been common for centuries in villages of India and other Asian countries.

The Europe, although others had attempted using reinforcement in concrete in the early part of the 19th century, credit for using ferrocement should go to J. L. Lambot in France, who constructed a concrete rowing boat in which the reinforcing was the form of interlocking wires and thin bars. His patent was issued in 1847 and it states a fairly accurate description of ferrocement (1):

"My invention shows a new product which helps to replace timber where it is endangered by wetness as in wood flooring, water containers, plant pots etc. The new substance consists of a metal net of wire or sticks which are connected or formed like a flexible woven mat. I give this net a form which looks in the best possible way, similar to article I want to create. Then I put in hydraulic cement or similar bitumin tar or mix, to fill up the joints."

The underlined words in the above extract from his patent clearly indicates that ferrocement is even older material than the conventional reinforced concrete. Probably the use of words like water containers plant pot suggested the use of the material in boat-building, which remained the major application of ferrocement for a long time. In the First World War, large scale use of ferrocement was in the building ships using a combination of lightweight reinforced concrete with

ferrocement, due to the shortage of steel plates for shipbuilding.

In the early 1940's, Nervi in Italy applied ferrocement for roofs of buildings and warehouses. Since then ferrocement has been used in the construction of domes and roofs of stadiums, opera houses, restaurants, etc. in Europe. It is worth mentioning that in spite of Nervi's demonstration of successful use of the material, no systematic investigation was carried out till much later.

It was not until the 1960's, that ferrocement began to be used in the developed countries such as Australia, the United Kingdom and New Zealand. However, it still remained largely a boat building material. But, because of the universal availability of the basic component materials of ferrocements and the low skill needed for construction of structural forms developing countries became more concerned with its use as a general purpose structural material. This resulted in its use (in applications such as) low cost housing, food storage silos, and irrigation troughs.

In 1972 the National Academy of Sciences established the Ad Hoc panel to study the use of ferrocement in developing countries. The final NAS report (2) aroused interest in Ferrocement in the United States. This was followed by the American Concrete Institute (ACI) establishing the Committee 549 on Ferrocement in 1974 with the following mission:

"To study and report on the engineering properties, construction practices, and practical applications of Ferrocement and similar materials; develop standards and safeguards for ferrocement construction."

In the last 4-5 years, considerable effort has been made by the many individuals and organizations around the world to develop ferrocement as a construction material. Another symposium planned by the International Ferrocement Information Center (IFIC) in Bangkok Thailand, later this year is another indication of interest of the professional community in the potential of this material.

DEFINITION OF FERROCEMENT

As the material "Ferrocement" was used for a long time in boat building and similar allied structures rather than in structural applications and a rigorous engineering definition of ferrocement was not essential. Within ACI Committee 549 and after considerable discussion on various definitions from many sources, it came up with a concise, accurate definition that may be acceptable to the engineering profession. Some of the definitions from various sources are presented.

The definition according to the American Bureau of Shipping is:

"A thin, highly reinforced shell of concrete in which the steel reinforcement is distributed widely throughout the concrete, so that the material under stress acts approximately as a homogenous material. The strength properties of the material are to be determined by testing a significant number of samples..."

Although at first glance, the above definition seems a pretty good one, it raises a number of questions about the words underlined that obviously may suggest different meanings to different people. BIGG (3) has discussed the various aspects of this definition and has suggested other ways to define it, including reference to ferrocement as a composite material. He points out how the available engineering concepts in composites of fiber reinforced concrete may be used to come up with a definition of ferrocement.

BEZUKLADOV (4) defined ferrocement in terms of the ratio of the surface area of reinforcement to the volume of the composite. In this manner, ferrocement was distinguished from conventional reinforced concrete by assigning arbitrarily a specific surface greater than $2 \text{ cm}^2/\text{cm}^3$ was considered reinforced concrete.

SHAH (5) defined ferrocement in a manner similar to Bezukladov, and called it a composite made with mortar and a fine diameter continuous mesh as reinforcement. The continuous mesh has higher bond due to its smaller size and a larger surface area per unit volume of mortar. According to him, this ratio may be as much as ten times that which is observed in conventional reinforced concrete. This results in failure of ferrocement in tension by the actual breaking of wire mesh and a much higher cracking strength in the matrix.

Certain characteristics of ferrocement as a composite may be summarized as follows:

- a. Since the wire mesh (reinforcement) is much stronger in tension compared to the matrix (mortar), the role of the matrix is to properly hold the mesh in place, to give a proper protection and to transfer stresses by means of adequate bond.
- b. Compression strength of this composite is generally a function of the matrix (mortar) compressive strength, while the tensile strength is a function of the mesh content and its properties.
- c. It follows from (b) that the stress-strain relationship of ferrocement in tension may show either complete elastic behavior (up to fracture of reinforcement mesh) or some inelasticity depending upon the yielding properties of the mesh.

- d. Since the properties of this composite are very much a function of orientation of the reinforcement, the material is generally anisotropic and may be treated as such in theoretical analysis.

The above discussion indicates the variety of approaches that have been used in a structural definition of ferrocement. ACI Committee on Ferrocement defined it as a construction material as follows:

"Ferrocement is a type of thin wall reinforced concrete construction, where usually a hydraulic cement is reinforced with layers of continuous and relatively small diameter mesh. Mesh may be made of metallic materials or other suitable materials."

ENGINEERING PROPERTIES

Traditionally, ferrocement has been viewed as a material for boat building rather than for building applications. Its use, however, has been extended to many types of structures in the eastern hemisphere. As such, little scientific information on its engineering properties was available. This is not to say that the intent here is to make a simple material used by unskilled labor more complex, but to throw light on its engineering properties in order to assist the profession in the design and use of ferrocement as a viable material for construction.

The main properties which have been studied include:

1. Tensile behavior (or alternatively, flexural strength)
 2. Cracking behavior
 3. Compressive strength
 4. Impact strength
 5. Fatigue strength
- and 6. Water (or liquid) retaining capacity

Since ferrocement is basically composed of reinforcement and mortar one may be tempted to compare it with reinforced concrete. However, from the discussion of its definition, the material differences are clear. Structural properties also differ due to the fine dispersion of the reinforcement in ferrocement.

The above properties are important in many structural applications. Although the first three properties are of general structural use, the rest having use in special applications. These properties will be dealt in detail in the next paper (6) in this symposium.

IMPROVEMENT OF FERROCEMENT PROPERTIES

Attempts have been made recently to improve the properties of ferrocement. Notable improvement is reported by ATCHESON and ALEXANDER (7) in another symposium paper using fibers with ferrocement. Fibrous concrete has also been found to have good structural properties. Combining the two types of concrete, the above authors demonstrated that fibrous ferrocement performed equal, or better than either individually. The fibrous ferrocement produced by using large aperture steel mesh and fibers, thus removing the restriction on ferrocement to have closely spaced wire mesh.

Other possibilities of improving the properties of ferrocement exist such as using polymerization techniques, which have been demonstrated to improve properties of reinforced concrete. As the technology progresses toward more economic use of polymerization, it is hoped that research will encourage its application in ferrocement.

RECENT APPLICATIONS

There have been many applications of ferrocement all over the world, both in developing and developed countries. The particular aspect of using the reinforcing cage as the stiff formwork itself is an important advantage of ferrocement for use in complex structures. Where the cost of forming is sometimes prohibitive. Recent applications have been discussed in some of the papers in this symposium. The universality of the applications of this material is apparent from the contributions from countries around the world. In addition, recent use of ferrocement in North America has been demonstrated for prefabricated thin domes in Quebec, Canada (8). Naaman and Shah (9) also have pointed out some potential of this material in off-shore structures such as buoys, deep submersibles and vessels for transportation of liquid gases.

CONCLUDING REMARKS

This introductory paper presents briefly the past and the present status of ferrocement as a construction material. The next paper will present the detailed version of the properties of ferrocement, while the others include both analytical and experimental evaluations of the various properties. There are a number of application papers, as mentioned earlier, to give one the range of its uses as a potential material in structural applications. Finally, the last paper will take the reader into future of ferrocement to conclude the symposium.

REFERENCES

1. ACI 549, Ferrocement, "State-of-the-Art Report on Ferrocement" To be published in ACI Journal (1978).
2. NAS, "Ferrocement: Applications in Developing Countries", National Academy of Sciences, Washington, D.C., February 1973, 90 pages.
3. Biggs, G.W., "Definition of Ferrocement", Report prepared for SNAME Task Group HS-6-4, April 1972, 28 pages.
4. Bezukladov, V.F., et al, "Korpora Dudov iz Amotsementa" (Ship-hulls made of Ferrocement), Leningrad, 1968, 817 pp. (English Translation: NAVSHIP's Translation No. 1148, NTIS - AD680042).
5. Shah, S.P., "New Reinforcing Materials in Concrete", J. ACI, Vol. 71, May 1974, p. 257.
6. Batson, G., Sabnis, G.M. and Naaman, A.E. "Survey of Mechanical Properties of Ferrocement as a Structural Material", Paper presented at the ACI symposium on Ferrocement-Material and Applications", April 1978.
7. Atcheson, M., and Alexander, D., "The Properties of Fibrous Ferrocement", Presented at the Symposium on "Ferrocement-Material and Applications", held at the Annual Convention, ACI, Toronto, April 1978, 34 pages.
8. Brun, P., Lachance, L. and Picard, A., "Systeme Resille-Coque et Prefabrication pour Voiles Minces en Ferrocement", Proceedings of IASS Conference, Vol. 2, June 1976, p. 989.
9. Naaman, A.E. and Shah, S.P., "Evaluation of Ferrocement in Some Structures Applications", Proceedings of International Symposium on Lower Housing Problems, May 1976, p. 1069.

Survey of Mechanical Properties of Ferrocement as a Structural Material

By Gordon B. Batson, Gajanan M. Sabnis,
and Antoine E. Naaman

Synopsis: This is a summary of the mechanical properties of ferrocement as a structural material based on the state-of-the-art report being prepared by ACI Committee 549, Ferrocement. The mechanics of ferrocement is very complex because of the almost infinite variety of size, geometry, fabrication methods, orientation, yield and ultimate stress of the steel wire mesh reinforcement available. The specific surface area of the wire mesh appears to correlate well with the first crack strength and the spacing and number of cracks. The tensile, flexural and compressive strengths depend on the orientation of the mesh, technique of fabricating the mesh and its ultimate strength. It appears that the conventional methods of analysis for reinforced concrete can be used to calculate the flexural strength of ferrocement. Ferrocement has superior impact properties, but its fatigue strength may limit the otherwise high allowable stresses to which it could be subjected. A design approach based on allowable crack width for service loads seems to be practical.

Keywords: compressive strength; construction materials; cracking (fracturing); crack width and spacing; fatigue (materials); ferrocement; flexural strength; hydraulic cements; impact strength; mechanical properties; tensile strength; welded wire fabric; wire cloth.

Gordon B. Batson is Secretary of ACI 549 Ferrocement and a member of ACI Committee 544, Fiber Concrete. He is Professor of Civil and Environmental Engineering, Clarkson College of Technology, Potsdam, New York. He obtained his Ph.D. from Carnegie Mellon University and is a registered professional engineer in New York.

Gajanan M. Sabnis is a member of ACI Committee 549 and also 116, 444, and 435. He is an Associate Professor of Civil Engineering, Howard University, Washington, D.C. He obtained the Ph.D. from Cornell University and is a registered professional engineer in California and Maryland.

Antoine E. Naaman is a member of ACI Committee 549. He is Associate Professor of Structural Design, University of Illinois at Chicago Circle, Chicago, Illinois. Dr. Naaman has had several years of professional experience as a structural engineer in Montreal, Quebec. He received his Ph.D. from Massachusetts Institute of Technology in 1972.

INTRODUCTION

One of the historical difficulties in considering ferrocement as a unique material has been the problem of defining it. There are, in essence, two problems. The very term ferrocement implies the combination of a ferrous product with cement. Yet there are certain characteristics of ferrocement that can be achieved with reinforcement other than steel mesh. The idea of ferrocement boats, for example, can be said to predate Lambot because the Chinese long ago used for a similar purpose burnt lime in combination with a reinforcement of bamboo rods. Also, the ancient and universal method of building huts or reinforced structures in countries like India by using reeds to reinforce dried mud (wattle and daub) is a form of ferrocement.

Thus, the ACI Committee 549, Ferrocement, concluded that a definition of ferrocement should not be limited to steel reinforcing even though it is recognized that most applications emphasize it.

Another difficulty has to do with the spacing or dispersion of the reinforcement. It is generally recognized that ferrocement utilizes "closely spaced" layers of mesh in which the opening or mesh spacing is "small." But how close and how small? In short, if one expands the distances between the individual wires, at what point does ferrocement become reinforced concrete?

In attempting to overcome these difficulties, the Committee has adopted the following definition:

"Ferrocement is a type of thin wall reinforced concrete construction where usually a hydraulic cement is reinforced with layers of continuous

and relatively small diameter mesh. Mesh may be made of metallic materials or other suitable materials."

Ferrocement is a term used to describe a form of reinforced concrete that differs from conventional reinforced or prestressed concrete primarily by the manner in which the reinforcing components are dispersed and arranged. The arrangement of reinforcement is such that a composite material is formed which exhibits behavior so different from conventional reinforced concrete in strength, deformation and potential application that it is classified as a separate and distinct material. It differs from conventional reinforced concrete in that its reinforcement consists of closely spaced, multiple layers of mesh or small bars completely impregnated with cement mortar. It can be formed into thin panels or sections, frequently less than 1 in. (25.4 mm) thick with only a thin mortar cover over the outermost layers of reinforcement.

In essence, ferrocement can be considered as mortar impregnated reinforcement.

COMPOSITION OF MATERIALS FOR FERROCEMENT

For the purpose of this paper the composition of ferrocement will be divided into two components; matrix and the reinforcement.

The matrix of ferrocement is a hydraulic cement binder which may contain fine sand, fibers and admixtures to reduce cost, control shrinkage and set time, increase corrosion resistance and modify the plastic and hardened properties. The binder is itself a composite material consisting of a hydrated cement paste and an inert filler material. The cement commonly used is Portland cement possibly mixed with pozzolanic cement which serves to replace or supplement the Portland cement. Filler material is usually a well graded sand capable of passing a No. 8 sieve and this classifies the binder material as a mortar. However, depending upon the characteristics of the reinforcing material, concrete containing some gravel filler may also be used. The binder is thus not significantly different from that which is used in the production of conventional reinforced concrete.

For a Portland cement binder, the physical properties and micro-structure of mortar depends upon the chemical composition of the cements, the nature of the sand filler, the water to cement ratio used in the production of the mortar, and the conditions under which the finished products are permitted to cure. As the binder represents approximately 95 percent by volume of the resulting ferrocement produced, the properties of the binder component have a great influence on the final properties of the ferrocement product.

Portland cement as the binder material in ferrocement yields a composite in which the binder is considered to have some tensile strength. This results from a composite behavior between the matrix

and the reinforcement. The use of fibers will also affect the tensile properties of the resulting matrix but the definition of ferrocement specifically eliminates fibers as a reinforcement component. Because the addition of fibers favorably affects the control of cracking and the tensile load capacity of the mortar matrix, fibers are included as a part of the binder material. The use of relatively short, slender (aspect ratio of about 100) ($l/d \approx 100$) randomly distributed fibers in hydraulic cement mortars is well known (1)(2). Alexander and Atcheson have reported that fibrous concrete mortars can also be effectively used in the production of ferrocement (3). The overall effect being to improve tensile and shear properties of cement mortar matrix and thus improve the properties of the resulting ferrocement product.

The mixing water should be relatively free from organic compounds. Water to cement ratios commonly used in ferrocement production vary between 0.35 and 0.55, by weight. The average value, 0.45, yields a workable mix which generally will completely penetrate and surround the mesh reinforcement and still result in acceptable levels of shrinkage and porosity. Water reducing admixtures may be used to enhance mix plasticity and retard initial set, as with conventional concretes.

Reinforcement of ferrocement is restricted to layers of continuous mesh formed by welding, twisting or weaving. Specific mesh types include: woven or interlocking mesh (such as aviary wire cloth); woven cloth mesh in which filaments are interwoven and their intersections are not rigidly connected; welded mesh in which a rectangular pattern formed by perpendicular intersecting filament elements are welded or cemented together at their intersections; and specially woven patterns which may include diagonal elements woven through the rectangular mesh pattern. There are at least two other forms of metal reinforcement in use; expanded metal lath, which is formed by slitting thin gage sheets and expanding in a direction perpendicular to the slits; and punched, or otherwise perforated, sheet products. Another form of reinforcement consists of continuous filaments which are randomly, or at least irregularly, assembled into a two dimensional mat. This particular form of reinforcement has been formed using vegetable fiber and glass fiber filaments. It has appeared most often using indigenous materials in developing countries. And lastly, specially designed three dimensional space frame mesh reinforcements have also been assembled using plain or galvanized steel wire.

There is a wide variety of mesh dimensions as well as the amounts, sizes and properties of the materials used. Some mesh filaments are galvanically coated or impregnated with sizing before, or during, assembly of the mesh. Properties of the resulting ferrocement product can be expected to be affected by filament size or gage, strength, ductility, manufacture, and treatment, in varying degrees. It will be important to keep this fact in mind in the discussions of this paper relating to the mechanical properties of ferrocement.

Aside from the water reducing, set retarding, and air entraining admixtures commonly employed in the production of both ferrocement and conventional reinforced concrete, ferrocement may require additives to reduce reaction with galvanized reinforcements. Chromium trioxide added to the mix water has been reported to be useful in this regard (4). The recommended solution concentration depends on the water to cement ratio used and is approximately three hundred parts per million by weight. The reaction between the alkaline cementitious binder and the galvanized reinforcement results in the liberation of gases which can cause disruptive pressures within the ferrocement matrix. No special precautions need be taken when using non-galvanized reinforcing materials.

Many of the same surface and near surface coating treatments available for use with conventional concrete and reinforced concrete installations can be applied to ferrocement. These include polymer impregnation, epoxy impregnation and the use of acrylic, latex, and cement base coatings. These are applied to reduce porosity, repair cracks, or provide a surface of particular functional design.

MECHANICS OF FERROCEMENT

Ferrocement is often thought of as just a variation of conventional reinforced concrete. While this may be true for ferrocement with small quantities of reinforcement, it is not true for the quantity of reinforcement considered in this paper.

The Italian engineer-architect P.L. Nervi is generally credited with the present day usage of the term ferrocement to describe a material system constructed of layers of closely spaced wire mesh separated by skeletal rods and filled with cement mortar. He observed that "the material thus created did not behave like regular concrete, but presented all the mechanical characteristics of a homogeneous material" (5). He had no theoretical explanation, but he did associate the mechanical properties with the close spacing and distribution of the mesh throughout the volume of the matrix. This qualitative perception of the importance of the degree of subdivision of the reinforcement was subsequently confirmed and quantified by the establishment of an inverse relationship between ultimate strength and average spacing of the reinforcement elements (Figure 1), and was supported by a theoretical analysis based on linear-elastic fracture mechanics indicating an inverse relationship between first crack strength and the average wire spacing (1)(2). The concept of a critical value for the maximum average spacing or a minimum volume concentration of reinforcement was established and would differentiate the behavior of a composite either as ferrocement or as conventionally reinforced concrete.

The present state-of-the-art indicates the response of ferrocement to load can be correlated with the specific surface area of reinforcement, S , defined as the total surface area of the wire in contact with the cement mortar divided by the volume of the composite. However, one should note that some investigations use only the surface

area of wire in the load resisting direction, denoted by S_L , as used in Figure 2 showing the experimental relationship between S_L and the first crack stress (9). Also in some of the investigations on the flexure of ferrocement, only the surface area of wire in the tensile region is used.

Bezukladov (6) selected a specific surface area, S_L , equal to $2.5 \text{ in}^2/\text{in}^3$ ($1 \text{ cm}^2/\text{cm}^3$) as the lower limit for a composite to be ferrocement. Figure 2 shows the maximum composite stress at first crack to increase in direct proportion to the specific surface. However, this relationship may be considerably different for other types of reinforcement such as expanded metal lath (7)(8). It is important to note that many of the test data available and for the composites commonly used, S_L values are less than the above suggested minimum value. Perhaps this is one reason why the theoretical results based on an analysis using conventional reinforced concrete design principles agree so well with test data on ferrocement as shown in Figure 3 (9).

The value of S or S_L appears to influence not only the first crack strength in tension, but also the width and spacing of cracks as shown in Figure 4.

The percentage of reinforcement, p , is defined as either the volume of wire per unit volume of composite in the loaded direction or the area of wire per unit cross sectional area in the loaded direction. There is a unique geometrical relationship between S and p , but their relationship to the physical properties are quite different. S is mostly associated with the cracking behavior while p is a direct measure of the ultimate strength of ferrocement because the ultimate load is resisted entirely by the wire mesh.

Other parameters can be found to correlate with some of the strength properties such as the mesh-mortar parameter for the modulus of rupture and ultimate moment in flexure (10).

A typical tension stress-strain curve for ferrocement is shown in Figure 5. The curve was divided into three regions by Walkus (11). In region I, the material is linear elastic because both the reinforcement and matrix deform elastically. Cracking of the cement mortar matrix is the beginning of region II and the slope of the stress-strain curve decreases. There seems to be several definitions of "first crack." It may be the point at which the slope of the stress-strain curve decreases, the first crack visible to the eyes, or the first crack visible with special lighting or optical magnification.

Naaman and Shah (9) have shown that the stress at first crack can be increased by increasing the volume of reinforcement, by decreasing the spacing between wires in the loaded direction, or by increasing the specific surface of the reinforcement in the loaded direction

for the same volume of reinforcement. A lower bound to the modulus of the composite after first crack (the slope of stress-strain curve) can be evaluated by

$$E_C = E_{RL} V_L$$

where

E_{RL} = modulus of the mesh

V_L = volume of percentage of their forcement in the loaded direction.

Walkus and Kowalski (11) divided region II into a "quasi-elastic" portion followed by a "plastic" portion. The separation of the two portions of region II was assumed to occur when the cracks were 50 microns in width to establish limits for resistance to corrosion and water penetration.

In region III, the wire reinforcement supports the total load and the ultimate capacity can be estimated from the maximum load capacity of the wire reinforcement alone.

The great variety of wire mesh size, geometry, method of fabrication, orientation of mesh in the composite, yield and ultimate strength of the wire can shift the range of the three regions of the stress-strain curve shown in Figure 5. For example, the termination of region I can be changed by changing the spacing of the wires in the loaded direction (9). Likewise the region III would depend on the yield and ultimate stress of the wires. It is entirely possible that the break in the stress-strain curve between region I and II could vanish and the composite would have a stress-strain curve similar to conventional reinforced concrete.

Johnston's (7)(8) data for tensile and flexural strength of ferrocement clearly show the influence of the geometry, orientation and fabrication method (woven or welded) for wire mesh and expanded metal lath as illustrated in Figures 6a and 6b. The expanded metal lath and the welded wire mesh provide approximately the same strength in uniaxial tensile loading. However the expanded metal lath suffers a much greater reduction in strength than the welded mesh when its orientation is changed. The orientation is even more important for the expanded metal in flexure, the strength in the weak direction being only about 13 percent of that for the longitudinal direction of slots in the direction of the loading (10). There is also an important reduction in the flexural strength for the welded wire mesh when the wires are not parallel and perpendicular to the flexural stresses. However, woven wire mesh is nearly independent of orientation, probably because the longitudinal and transverse wires are not rigidly connected. In summary, expanded metal lath is superior to welded mesh and welded mesh is superior to woven mesh for uniaxial bending, but welded mesh is superior to expanded metal lath for biaxial bending. However the anisotropy of the expanded metal lath can be countered by alternate orientation of several layers just as is done in the fabrication of plywood.

A ferrocement beam with several layers of mesh is generally over reinforced compared to a typical conventionally reinforced concrete beam. However, it should be noted in Figure 7 (15) that there were no sudden failures typical of over reinforced concrete beams, supporting the concept that the mechanics of ferrocement are different than for conventional reinforced concrete.

The compressive strength of ferrocement is contributed primarily by high compressive strength of the mortar. Johnston and Mattar (7) have shown that significant compressive strength gains can be realized by using a closed box or cylindrical arrangement of mesh which provides lateral restraint and develops a triaxial stress condition. The welded wire mesh is superior to expanded metal lath as shown in Figure 7. A rectangular mesh with greater amount of transverse reinforcement is more cost effective in increasing compressive strength (by creating a triaxial stress condition) than the addition of longitudinal reinforcement.

Very little information is available on the shear strength of ferrocement, perhaps because ferrocement is generally used in thin panels where the span-depth ratio in flexure is large enough so that shear does not govern failure.

The survey of the mechanical properties of ferrocement so far has assumed static or slowly applied loads. Limited data on the impact and the fatigue strength suggest potential applications and possible restriction to the use of ferrocement.

Shah and Key (12) indirectly determined the impact strength by measuring the damage caused by a swinging pendulum impacting on ferrocement slabs. The damage was assessed by the relative discharge of water through the damaged surface for a fixed amount of impact energy. The damage decreased (i.e. the discharge was reduced) for increasing specific surface and ultimate strength of the mesh. The result agrees with the observation of the minimal damage done to ferrocement boat hulls in collisions with rocks and other boats. Ferrocement has good resistance to dynamic punching loads.

Flexural fatigue properties of ferrocement beams reinforced with square woven and square welded mesh have been investigated by Balaguru, Naaman and Shah (13). It was generally observed that the deflection and crack width increased exponentially with the number of loading cycles for a constant load as shown in Figure 8. Therefore, the fatigue properties may limit the otherwise possibly high design stresses that can be used in ferrocement elements.

Present applications of ferrocement are generally in region II of the stress-strain curve shown in Figure 4. The size (width) and spacing of the cracks are controlled by various characteristics of the reinforcing mesh. Thus a design procedure for ferrocement in region II could be based on a criteria of maximum allowable crack width under service loads. Figure 9 shows data for typical average crack width versus steel stress for several layers of woven wire

mesh (16). A practical application of the suggested design method is for example in ferrocement water tanks. The average crack width for ferrocement water tanks has been shown by Guerra, Naaman and Shah (14) to be an order of magnitude smaller for the same steel stress than that of conventional reinforced concrete. Figure 10 shows the average crack width for various steel stresses compared to recommended values for silos containing moisture sensitive materials and water retaining structures.

CONCLUSIONS

Although ferrocement is basically layers of closely spaced mesh impregnated with a Portland cement mortar, the mechanics of ferrocement and the mechanical properties are very complex because of the almost infinite variation of size, geometry, fabrication method, orientation, yield and ultimate stress of steel wire mesh that could be used as reinforcement.

Beyond a minimum reinforcement content, the tensile stress-strain diagram for ferrocement exhibits three regions. Region I, the matrix and reinforcement act together with linear elastic behavior up to the "first crack." Region II begins with a reduced modulus compared to region I and cracking continues until the number of cracks in the matrix has stabilized and the reinforcement has yielded. In region III the load is carried entirely by the yielded reinforcement and is for all practical purposes the ultimate load.

It appears that the specific surface area of the mesh correlates best with the first crack strength, spacing of cracks and width of cracks in tension.

The tensile, flexural and compressive strength of ferrocement can be greatly varied by the orientation of the mesh, by its method of fabrication (welding or weaving of wires or slitting and expanding a metal sheet) and by its ultimate strength.

It appears that the flexural strength of ferrocement can be calculated by the conventional methods of analysis for reinforced concrete and the tensile strength by laws of composite materials. The compressive strength depends primarily on the compressive strength of mortar and to some extent on the confining action of the wires transverse to the load.

Ferrocement has superior impact properties, but fatigue strength may limit otherwise high allowable design stress.

A design approach for ferrocement based on allowable average crack widths for service load conditions is practical.

ACKNOWLEDGMENT

The contents of this paper are based on the state-of-the-art of ferrocement being prepared by ACI Committee 549. Particular recognition is given to Dr. Ronald Zollo, Dr. Colin Johnston, Dr. James P. Romualdi and Dr. Suru Shah for their contributions to the preparation of the state-of-the-art report and its summary in this paper.

References

1. Romualdi, J.P. and Batson, G.B., "Mechanics of Crack Arrest in Concrete," Proceedings, ASCE, V. 89, EM3, June 1963, pg. 147-168.
2. Romualdi, J.P. and Mandel, J.A., "Tensile Strength of Concrete Affected by Uniformly Distributed Closely Spaced Short Lengths of Wire Reinforcement," ACI Journal, Proceedings V. 61, No. 6, June 1964, pg. 657-671.
3. Alexander, D.J. and Atcheson, N.G.A., "Fibrous Ferrocement for Commercial Vessels," Journal of Ferrocement, Vol. 5, No. 2 and 3, March-May 1976, pg. 25-48.
4. Haynes, H.H. and Guthrie, G.S., "Ferro-cement Construction Panels," Technical Note N-1341 Civil Engineering Laboratory, Naval Construction Battalion Center, Port Hueneme, California, April 1974.
5. Nervi, P.L., *Structures*, translated by S. and M. Salvadore, F.W. Dodge, New York, 1956.
6. Bezukladov, K.K., Vanovich, A., et al., "Ship Hulls Made of Reinforced Concrete," (Korpusa Dudov iz Amotsementa) translated from Russian, Navships Trans No. 1148.
7. Johnston, C.D. and Mattar, S.G., "Ferrocement - Behavior in Tension and Compression," Proceedings ASCE Journal of Structural Division, Vol. 102, No. ST5, May 1976, pg. 875-889.
8. Johnston, C.D. and Mowat, D.N., "Ferrocement - Material Behavior in Flexure," Proceeding of ASCE, Journal of Structural Division, Vol. 100, No. ST10, Oct. 1974, pg. 2053-2069.
9. Naaman, A.E. and Shah, S.P., "Tensile Tests of Ferrocement," ACI Journal, Proceedings Vol. 68, No. 9, September 1971, pg. 693-698.
10. Kumar, G.V., Narayanaswamy, V.P. and Sharma, P.C., "Ferrocement a Survey of Experimental Investigations," Journal of Structural Engineering (India), Vol. 1, No. 4, January 1974, pg. 167-182.
11. Walkus, R. and Kowalski, T.G., "Ferrocement: A Survey," Concrete (London), Vol. 5, No. 1, Feb. 1971, pg. 1-5.

12. Shah, S.P. and Key, W.H., Jr., "Impact Resistance of Ferrocement," Proc. of ASCE, Structural Division, Vol. 98, No. ST1, Jan. 1972, pg. 111.
13. Balaguru, P.N., Naaman, A.E. and Shah, S.P., "Ferrocement in Bending Part II: Fatigue Analysis," Dept. of Materials Engineering, University of Illinois at Chicago Circle, Report No. 77-1, Oct. 1977.
14. Guerra, A.J., Naaman, A.E. and Shah, S.P., "Ferrocement Cylindrical Tanks: Cracking and Leakage Behavior," ACI Journal Proceedings, Vol. 75, No. 1, January 1978, pg. 22-29.
15. Balaguru, P.N., Naaman, A.E., and Shah, S.P., "Analysis and Behavior of Ferrocement in Flexure," ASCE Journal of the Structural Division, Vol. 103, No. ST10, Oct. 1977, pg. 1937-1951.
16. Naaman, A.E., "Reinforcing Mechanisms in Ferrocement," M.S. Thesis, Department of Civil Engineering, Massachusetts Institute of Technology, Sept. 1970.

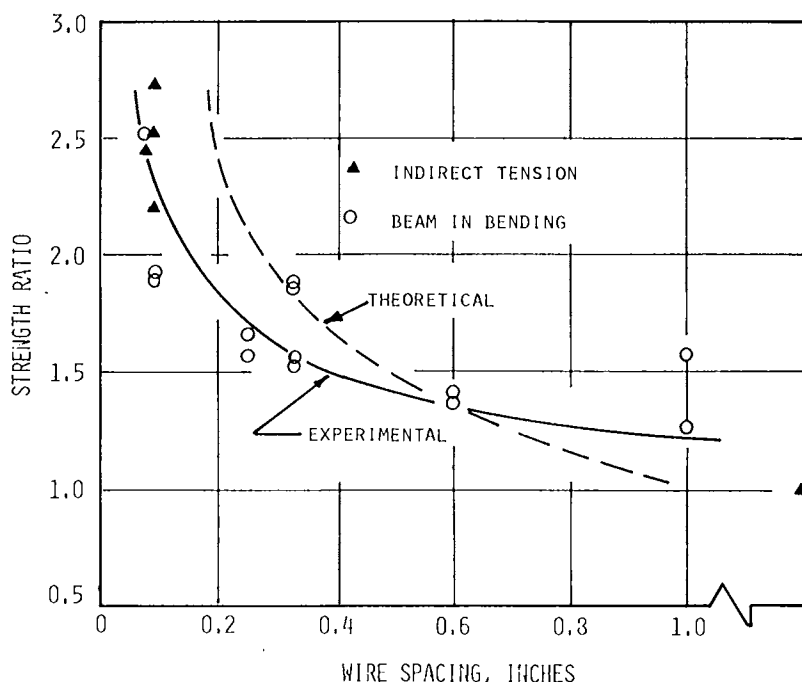


Fig. 1--Theoretical and experimental strength ratio as a function of wire spacing (Ref. 2)

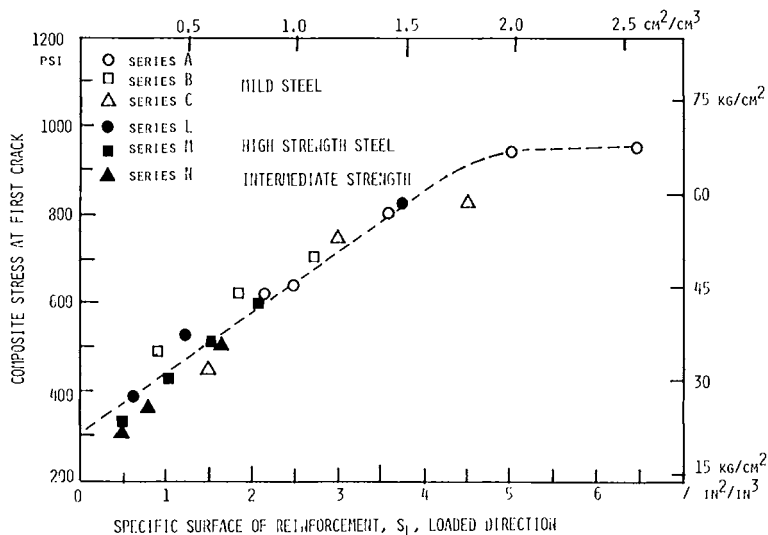


Fig. 2--Stress at first crack versus specific surface of reinforcement (Ref. 9)

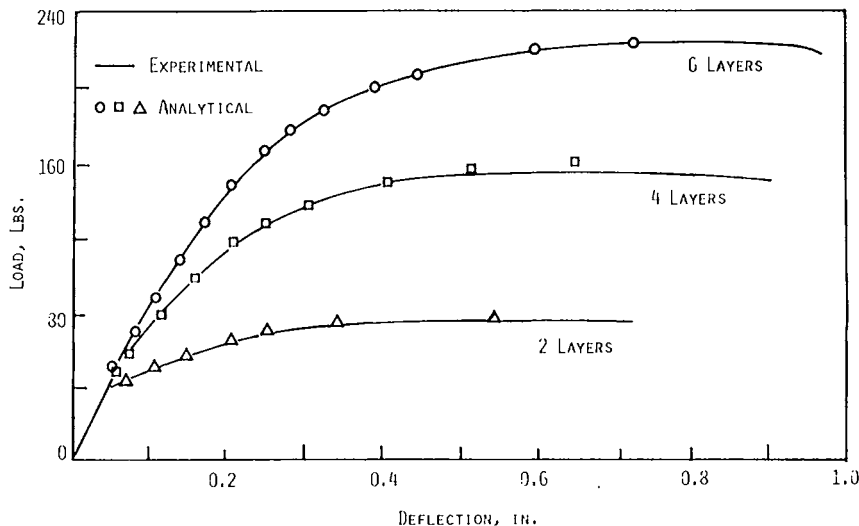


Fig. 3--Experimental and analytical load deflection curves for ferrocement (Ref. 15)

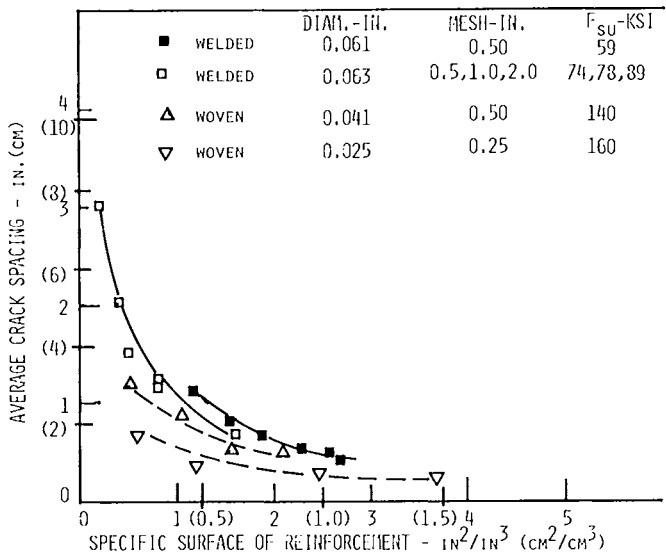


Fig. 4--Average crack spacing in tension as a function of specific surface (Ref. 9)

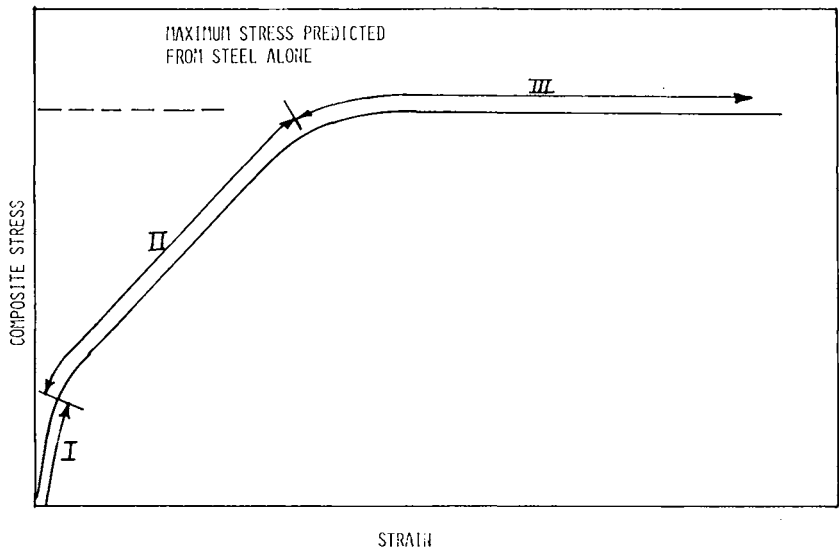


Fig. 5--Typical tension stress strain curve (Ref. 11)

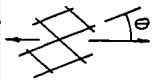
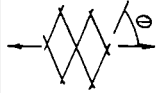
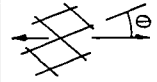
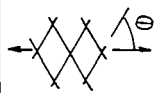
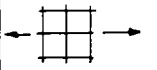
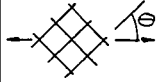
	θ	TYPE	F_{su} KSI (MPa)	AREA IN ² (CM ²)	LOAD KIPS (KN)
	24°	EXM	90 (618)	0.118 (0.761)	12.8 (57.3)
	66°	EXM	90 (618)	0.022 (0.142)	1.2 (5.4)
	31°	EXM	57 (391)	0.152 (0.98)	9.6 (93.0)
	69°	EXM	57 (391)	0.053 (0.342)	1.8 (8.0)
	90°	EXM	59 (410)	0.119 (0.768)	8.0 (35.8)
	45°	EXM	58 (410)	0.119 (0.768)	4.5 (18.8)

Fig. 6a--Orientation of reinforcement versus tensile strength (Ref. 7)

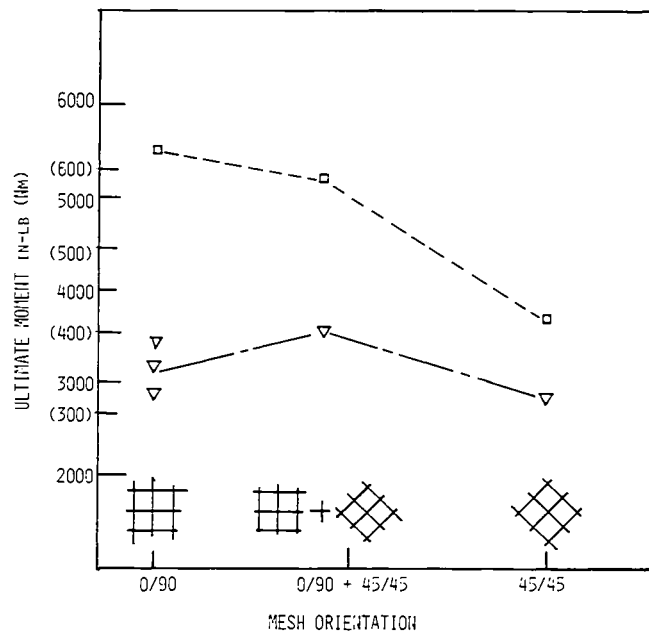


Fig. 6b--Orientation of reinforcement versus tensile strength (Ref. 8)

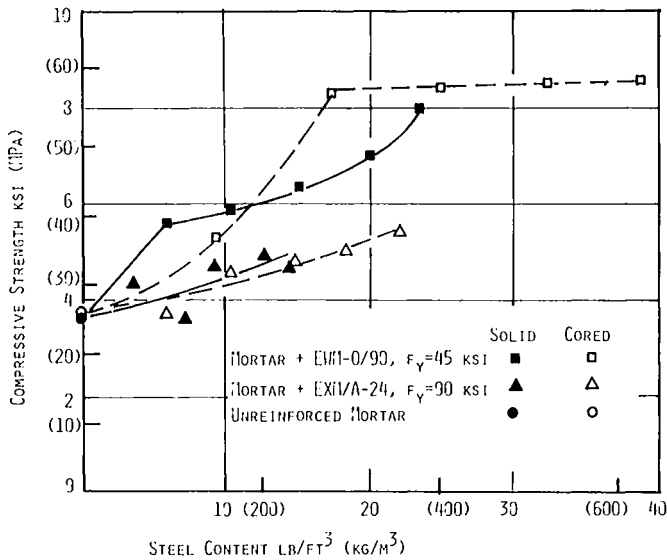


Fig. 7--Compressive strength versus steel content (Ref. 7)

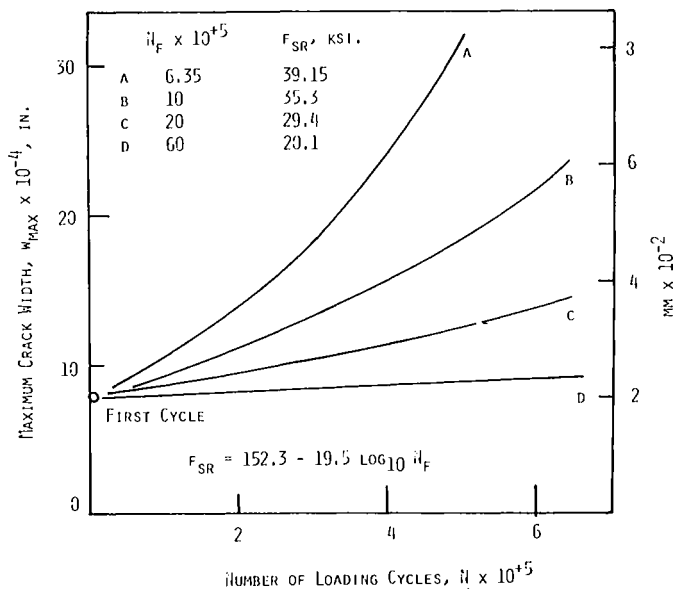


Fig. 8--Crack width in flexure under cyclic loading (Ref. 13)

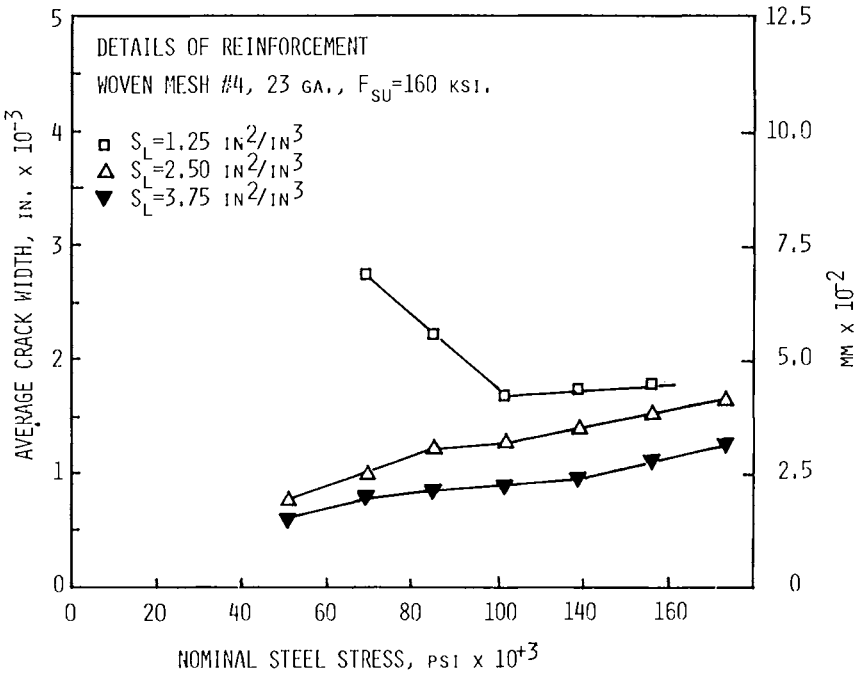


Fig. 9--Typical average crack width versus steel stress

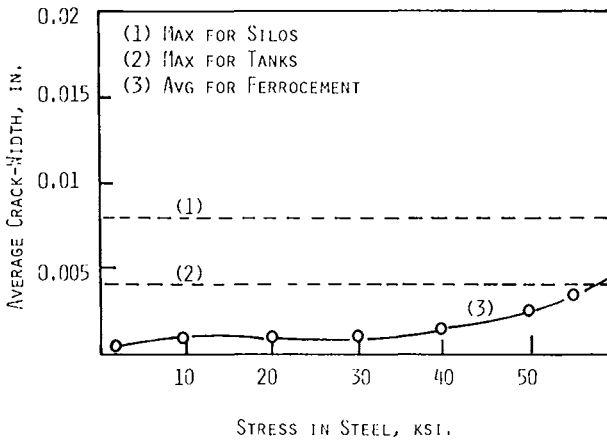


Fig. 10--Average crack width versus steel stress for ferroceement and reinforced concrete (Ref. 14)

Design Predictions of Crack Widths in Ferrocement

By Antoine E. Naaman

Synopsis: One of the distinguishing features of ferrocement as compared to reinforced concrete is the larger number of finer cracks for the same stresses in the steel reinforcement. Designing ferrocement structures to satisfy recommended serviceability criteria in a manner similar to the design of reinforced concrete structures is a logical and rational approach to follow. Serviceability is greatly dependent on cracking and crack width under working load conditions. Thus the prediction of crack widths (average and/or maximum) is an important consideration in design.

This paper is based on a number of recent investigations on the cracking of ferrocement elements: they include tests on flexural beams under static and fatigue loadings, tests on ferrocement prisms with and without transverse reinforcement subjected to direct tensile loads, and tests on internally pressurized model ferrocement cylindrical water tanks. The following aspects of cracking are clarified for both flexure and tension: cracking behavior; influencing parameters; observed variations of crack widths; crack width predictions and suggested design approach.

Keywords: cracking (fracturing); crack width and spacing; ferrocement; flexural strength; serviceability; structural design; tensile stress; tension tests; wire cloth.

A. E. Naaman is associate professor of structural design, Department of Materials Engineering, University of Illinois at Chicago Circle. Dr. Naaman received his Ph.D. degree from the Massachusetts Institute of Technology in 1972 and is a member of technical committees of ACI, PCI and ASCE.

INTRODUCTION

One of the distinguishing features of ferrocement as compared to reinforced concrete is the larger number of finer cracks for the same stresses in the steel reinforcement. The development of fine cracks under loading makes it possible to build safe ferrocement structures in thin sections and still maintain acceptable appearance, corrosion resistance and virtual watertightness.

Designing ferrocement structures to satisfy recommended serviceability criteria in a manner similar to the design of reinforced concrete structures is a logical and rational approach to follow. Serviceability is greatly dependent on cracking and crack width under working load conditions. Thus the prediction of crack widths (average and/or maximum) is an important consideration in design.

Cracking in reinforced concrete has been the subject of a large number of investigations [1 to 8], which have added to the better understanding of its behavior. Although ferrocement is a type of reinforced concrete--and is expected a priori to behave similarly--its cracking behavior is substantially different [9]. This seems to be due to at least three major differences: 1) ferrocement comes in thin shell-like elements with its reinforcement distributed throughout the section, 2) its specific surface of reinforcement is an order of magnitude higher than that of reinforced concrete (for equal amounts of steel), and 3) the presence of transverse wires in the mesh reinforcement of ferrocement enhances bond transfer properties and cracking characteristics.

As the terms volume fraction and specific surface of reinforcement will be often used in this paper, their meanings are intended as follows: the volume fraction of reinforcement is the ratio of volume of reinforcement per unit volume of composite; it may indicate the total volume V_f or the volume in the loading direction V_{fL} ; note that V_{fL} is quantitatively equal to the reinforcement ratio of the identical reinforced concrete section; the specific surface of reinforcement is the lateral surface (bonded area) of reinforcement per unit volume of composite; it may refer to the total area S_R or to the area in the loading direction S_{RL} . The specific surface and the volume fraction of reinforcement are related by the following relationship:

$$S_R = \frac{4V_f}{\phi} \quad (1)$$

where ϕ is the diameter of the wire used. Note that a similar relationship exists between S_{RL} and V_{fL} .

This paper is based on a number of recent investigations on the cracking of ferrocement elements reinforced mostly with square, galvanized steel meshes; they include tests on flexural beams under static and fatigue loading [10,11,12], tests on prisms subjected to uniaxial tension [13,14] and tests on internally pressurized model ferrocement cylindrical water tanks [15,16]. Based on the observed results, the following aspects of cracking will be clarified for both flexure and tension: cracking behavior; influencing parameters; observed variation of crack widths; crack width prediction and suggested design approach.

UNIAXIAL TENSION

Tensile tests were carried out on prisms which were 12 in. (30 cm) long, 1 in. (2.5 cm) wide and 3/8 in. (≈ 1 cm) thick. The reinforcement consisted of various amounts of either square wire meshes or smooth longitudinal wires (no transverse wires). Only two mesh openings (spacing center-to-center of wires) were tested: 1/4 in. (6.3 mm) and 1/2 in. (12.7 mm). Thus "1/4" or "1/2" will refer to the mesh size or opening. Most of the meshes used were welded meshes and the woven meshes used had a high modulus. The elongation (and equivalently the strain) in the ferrocement composite under load was measured over a 6 in. (15 cm) gage length using a device directly attached to the specimen and equipped with an LVDT. The load-elongation curve was automatically recorded on an electronic XY recorder. Cracks along the observed gage length were detected, counted and their widths measured at different loads using an internally illuminated microscope equipped with a micrometer that allowed an accuracy of about 50 microinch (1.27×10^{-3} mm). Details of the above investigation are given in Ref. 14. Only some of the salient remarks and conclusions are described below. They help reinforce some of the findings reported in an earlier investigation on tensile cracking in ferrocement [13] while bringing new and accurate results on crack widths.

1. The tensile stress-strain curve of ferrocement can be generally divided into three stages: a) elastic behavior until the first matrix cracking; b) multiple cracking stage--in this stage the number of cracks keeps increasing with an increase in tensile stress or strain; their width, however, may only slightly increase depending on the reinforcing parameters used, but not in direct proportion to the stress increase; and c) crack widening stage--in this stage no or few additional cracks are formed, the existing cracks become wider and the behavior of the specimen is controlled by that of the reinforcement alone. Typical stress-strain curves of tensile specimens reinforced with longitudinal wires only and with wire meshes are shown in Figs. 1 and 2, respectively.

Figure 1 shows that the number of cracks increases from 1 to 17 while the stress in the longitudinal steel varies from about 20 to 50 ksi (138 to 345 MN/m²); the average crack width, however, increases only slightly from 4×10^{-4} to 5.6×10^{-4} in. (0.010 to 0.014 mm). An identical specimen with smaller number of wires (smaller V_f and S_{RL}) will show a substantially higher increase in crack width for the same stress range. A mixed trend is observed in Fig. 2 for a specimen reinforced with square wire meshes (i.e., having transverse reinforcement); here the number of cracks keeps increasing even in the nonlinear range of behavior of the steel and thus crack width is greatly influenced by the strain increase in the reinforcement. The stress at which the number of cracks stops increasing is termed here the stabilization stress f_{sta} . Depending on the reinforcing parameters, namely specific surface and transverse wire spacing, the stabilization stress may be somewhere along the elastic or nonelastic range of behavior of the steel reinforcement.

2. Everything else being equal, the higher the specific surface and the smaller the transverse wire spacing, the larger the extent of the multiple cracking stage, the larger the number of cracks developed in the same gage length, the smaller the crack spacing and the smaller the crack widths. Thus for specimens having identical longitudinal reinforcement, better cracking characteristics are obtained when transverse reinforcement is used (i.e., meshes versus wires). Figure 3 shows the variations of the number of cracks developed as a function of steel stress for typical tensile specimens with and without transverse reinforcement. A similar curve is also shown for a flexural specimen for comparison.
3. A direct relationship between the average crack spacing $(\Delta\ell)_{av}$ at crack stabilization and the specific surface of reinforcement seems to exist. An analytical evaluation of this relationship has been proposed in [13] and is given by

$$(\Delta\ell)_{av} = \frac{\theta}{\eta} \frac{1}{S_{RL}} \quad (2)$$

where $(\Delta\ell)_{av}$ = average crack spacing

θ = a factor relating average crack spacing to maximum crack spacing

η = ratio of bond strength to matrix tensile strength

S_{RL} = specific surface of reinforcement in the loading direction

As the factor θ/η can be approximated for design purposes a direct relation between $(\Delta\ell)_{av}$ and S_{RL} can be obtained. The observed values of average crack spacing at crack stabilization

are shown (Fig. 4) versus the specific surface of reinforcement in the loading direction S_{RL} for both the series of specimens reinforced with smooth longitudinal wires and those reinforced with square wire meshes. It can be seen that for the same specific surface, smaller spacings are obtained when the reinforcement contains transverse wires (i.e., meshes). The analytical equation (2) is also plotted with the values of $(\theta/\eta) = 1$ and 2.7. It can be observed that for $(\theta/\eta) = 1$, Eq. (2) predicts very well the experimental data for square meshes while for $(\theta/\eta) = 2.7$ it predicts the data when smooth longitudinal wires are used. Thus Eq. (2) with the above values of the factor θ/η can be used as a design equation to predict crack spacing in ferrocement at crack stabilization.

4. A large number of data on crack widths were generated. They are quite complex to analyze as crack widths depend on many parameters and in ferrocement the extent of the multiple cracking region may have a significant effect on the results. However, as in reinforced concrete the stress in the steel after cracking is of major importance, data on crack widths, average and maximum, were plotted versus the stress in the steel. Typical results are shown in Fig. 5 for one series of tests having six identical specimens. It can be seen that for stresses below 60 to 65 ksi (which are much above recommended design or service stresses), there is little increase in crack width, at least for this series of tests. This is essentially due to the effect of the extent of the multiple cracking stage (which can be visualized in Fig. 2 for one specimen of the above series).
5. The ratio of maximum crack width to average crack width was computed for steel stresses less than 60 ksi, and for all series of tests using square wire meshes. It was generally observed that the higher the specific surface of reinforcement, the smaller the ratio. The mean value of that ratio was equal to 1.85 with a standard deviation of 0.51.
6. For the purpose of design a simple approach to predict maximum crack widths is necessary. A method is proposed here for ferrocement specimens reinforced with square wire meshes. It consists of first predicting a "stabilization stress" f_{sta} and then predicting a maximum crack width value which applies for any stress f_s less than f_{sta} . For stresses above f_{sta} it is expected that the crack width will increase in direct relation to the increase in steel strain; thus it will also directly increase with the steel stress if the latter is less than the yield strength. It will be assumed here that the yield strength will be of the order of 60 ksi. The proposed design approach is summarized as follows:

For $S_{RL} \leq 3 \text{ in.}^{-1}$

- a) Determine stress in the steel at crack stabilization from

$$f_{sta} = 20 S_{RL} \leq 60 \text{ ksi} \quad (3)$$

where f_{sta} is given in ksi for S_{RL} in in.^{-1} .

- b) For any stress less than f_{sta} determine the maximum crack width using

$$w_{max} = \frac{1}{S_{RL}} \frac{f_{sta}}{E_R} = \frac{20}{E_R} \quad (4)$$

where E_R is the modulus of the reinforcing system.

- c) For any stress f_s larger than f_{sta} but less than the yield strength given in ksi use:

$$w_{max} = [6.9 + (f_s - f_{sta})] \frac{29000}{E_R} 10^{-4} \quad (5)$$

where w_{max} is in inches.

For $S_{RL} > 3 \text{ in.}^{-1}$ and for any stress less than the yield strength or about 60 ksi use the following:

$$w_{max} = (15.5 - 3 S_{RL}) 10^{-4} \geq 0.0004 \text{ in.}$$

Figure 6 shows the above relations for various values of S_{RL} and steel stresses.

It is noted that the above proposed approach has provided a reasonable upper bound value to all the data of this investigation. A typical representation of the above equations (3,4,5) is shown in Fig. 5 for the series of tests described in that figure and can be compared to actual values of maximum and average crack widths.

TENSILE CRACKING OF FERROCEMENT WATER TANKS

The detailed information on tests of model ferrocement water tanks has been already published in [15] and [16]. As the crack widths, average and maximum, for the tanks were sometimes higher (up to two or three times) for the same steel stress than those reported above for prisms, a note of clarification is necessary. It is believed that the difference is due to several variables as follows:

- a) The hoop direction of the mesh in the tank was the longitudinal direction of the roll of mesh used, which generally shows a substantially lower modulus than the transverse direction.
- b) The average crack width was calculated as the mean value of only 10 cracks under observation. These cracks (among more than 100 cracks) were detected first and thus had a larger width than the average of all cracks. Thus their average widths is in effect an upper bound measure.
- c) One or two cracks occurred generally in the overlap region of the mesh reinforcement and led to unusually high crack widths, thus substantially influencing, in some instances, the value of maximum and average crack widths.
- d) The ferrocement walls of the water tanks were in effect subject to biaxial tension instead of uniaxial tension. Furthermore, the thickness and reinforcement cover of the walls were not as controlled and constant as in the experiments on prisms.

CRACKING IN FLEXURE

A large number of flexural specimens having overall dimensions of 18 x 5 x 1/2 in. (457 x 127 x 12.7 mm) were tested, in third point bending, under static and fatigue loads. Details are given in [10,11,12]. The reinforcement consisted of three types of square galvanized steel meshes, 1/2 woven, 1/2 welded and 1/4 woven (1/2 and 1/4 refer to the mesh size or spacing center-to-center of the wires in inches in either direction). For each type of mesh three volume fractions of reinforcement corresponding to 2, 4 and 6 layers of mesh were used. The reinforcing layers were cut along the transverse direction of the mesh rolls to assure a better modulus of the reinforcing system. Generally the layers were stacked in such a way that the transverse wires of the different layers were along the same cross sections. Cracks were measured at different levels of loadings using an internally illuminated microscope fitted with a micrometer that allowed a measuring accuracy of 50 microinch (1.27×10^{-3} mm). The analysis of the data generated by the above investigation leads to a number of pertinent results summarized as follows:

- 1. The transverse wires of the outermost layer of mesh are preferential locations for cracks. In most specimens the number of cracks in the constant moment zone reached a steady value soon after the occurrence of first cracking (independently of the number of layers used). This steady value corresponds to an average crack spacing equal to the spacing of the transverse wires (Fig. 3).
- 2. The specific surface of reinforcement did not seem to have as strong an influence on the cracking behavior in flexure as in tension. This may be because in flexure the specific surface in the tensile zone of mortar around the outermost layer of mesh does not increase substantially when the number of mesh layers used increases.

3. A systematic regression analysis of observed crack widths showed that the average crack width is primarily a function of the tensile strain in the extreme layer of mesh and does not depend on the other parameters found important in reinforced concrete. This may be because of the relatively small cover and high specific surface of reinforcement in ferrocement beams.
4. A computerized nonlinear analysis in bending of the ferrocement section showed that the stresses in the mesh reinforcement up to about yielding of the steel can be predicted, with a good approximation, using linear elastic analysis of the cracked section.
5. A linear regression analysis of all the data on crack widths for all the series of tests on static bending leads to the following relationships between crack widths and the stress f_s in the outermost layer of steel mesh:

$$w_{av} = (0.271 f_s - 3.73) 10^{-4} \quad (6)$$

and

$$w_{max} = (0.324 f_s - 4.36) 10^{-4} \quad (7)$$

where w_{av} = average crack width, in.

w_{max} = maximum crack width, in.

f_s = stress in the steel in ksi

The above equations as well as the corresponding data are plotted in Figs. 7 and 8. Note that the average secant modulus at 45 ksi of all the meshes used in the above tests was around 23,000 ksi. Note also that each data point of Fig. 7 has a substantial weight as it represents the average reading for at least three specimens, at a given steel stress and for a large number of cracks each of which characterized by two measurements along its length. For example, the data point representing two layers, 1/2 welded at 50 ksi represents the average of about 60 measurements.

A number of linear relationships between crack widths and steel stress, similar to the above, were developed for each type of mesh in Ref. 10 and can be used if a better accuracy is desired.

6. The overall average ratio between maximum crack width and average crack width, for 32 sets of data and for the overall range of steel stress considered, was determined and found equal to 1.21 with a standard deviation of 0.14. It can be observed that the above ratio and the coefficient of variation are substantially smaller than those observed in tensile tests. From the above results it can be concluded that generally the maximum crack width in flexure for ferrocement is, with 98% probability, less than 1.5 times the average crack width.

7. A simple design equation is proposed in Ref. 10 to predict average crack width as a function of steel strain in the outermost layer of mesh. However, when compared to the experimental data the equation gave a useful upperbound value and could be used in effect to predict maximum crack widths. If, in the elastic range of behavior, the stress in the steel is expressed in function of the strain, and $W_{\max}$ is used instead of W_{av} , the prediction equation given in Ref. 10 takes on the following form:

$$W_{\max} = \beta S \frac{f_s}{E_R} \quad (8)$$

where

$W_{\max}$ = maximum crack width, in.

S = mesh size or spacing center-to-center of mesh wires, in inches

β = ratio of distances to the neutral axis from the extreme tensile fiber and from the outermost layer of steel (elastic cracked section analysis)

f_s = steel stress in ksi

E_R = modulus of reinforcing system in ksi

Note that E_R is the same as the modulus of elasticity of the steel for welded meshes but can be substantially lower for woven meshes depending on the pitch of the weave and other parameters. It has been also observed that E_R may be substantially different if measured along the transverse direction instead of the longitudinal direction. For the types of square woven meshes generally used in ferrocement E_R can vary between 15,000 and 29,000 ksi (103,500 to 200,100 MN/m²).

Assuming a value of $\beta = 1.2$, the line corresponding to Eq. (8) is plotted (Fig. 9) for different hypothetical meshes and compared with the regression equations (Eqs. (6) and (7)) from all the data. It can be seen that in view of the variability expected in crack widths, both approaches are acceptable as a design prediction.

Note that the overall average modulus E_R for the flexural experiments described in [10,11] was around 23,000 ksi (158,700 MN/m²). Thus if one can determine the modulus E_R of a given mesh, Eqs. (6) and (7) can be modified to more accurately predict crack widths (given in inches) as follows:

$$W_{av} = \frac{23000}{E_R} (0.271 f_s - 3.73) 10^{-4} \quad (9)$$

and

$$W_{\max} = \frac{23000}{E_R} (0.324 f_s - 4.36) 10^{-4} \quad (10)$$

for stresses higher than the stress at cracking and smaller than the yield strength and given in ksi.

8. Test results on fatigue loading [12] have indicated that the average and maximum crack width at a given load keep increasing with the number of loading cycles.
9. The following relationship was found [12] acceptable to represent crack width data, average or maximum, under fatigue loading up to about 90% of fatigue life:

$$W = Ae^{Br} \quad (11)$$

where

W = average or maximum crack width

A = value of crack width at maximum load under static test. It could be predicted from Eqs.

e = base of Napierian logarithms

r = cycle ratio; i.e., number of cycles at which crack width is calculated to the number of cycles to failure N_f

B = a parameter determined from the experimental data and described in [12]. A constant value of $B = 1.67$ is proposed for $N_f \geq 450,000$ cycles which is generally the case for most design situations.

In order to use Eq. (11) the number of cycles to failure N_f is needed. A regression analysis of the data [12] between stress range in the steel and the number of cycles to failure has led to the following relation:

$$f_{sr} = 152.3 - 19.5 \log_{10} N_f \quad (12)$$

where

f_{sr} = stress range in the outermost layer of steel in ksi

N_f = number of cycles to failure

Thus if f_{sr} is known, N_f can be predicted, or if a certain value of N_f is required by the design the corresponding limit on stress range can be determined.

A typical example of crack width increase with number of cycles is shown in Fig. 10 curve (a) for a flexural specimen from the tests with the following parameters: 2, 1/2 welded layers of steel mesh (subjected to a fluctuating load of ± 36 lbs; steel

cover $\approx 1/8$ in. (3.1 mm)); maximum tensile stress in the steel = 37.81 ksi (261 MN/m²); minimum stress = 1.34 ksi (9.2 MN/m²) (compression); stress range = $f_{sr} = 39.15$ ksi (270 MN/m²); mean stress = 18.23 ksi (126 MN/m²). From Eq. (12), the number of cycles to failure can be estimated as $N_f \approx 635,000$ cycles. The maximum crack width at maximum stress can be predicted for the first cycle from Eq. (7) and leads to $w_{max} = 7.88 \cdot 10^{-4}$ in. (0.020 mm). This gives the value of A of Eq. (11). The maximum crack width at any cycle ratio r can then be calculated from Eq. (11) as $w_{max} = 7.88e^{1.67r}$ in 10^{-4} in. and is plotted on Fig. 10. Note that Eq. (7) has been used to predict $A = 7.88 \cdot 10^{-4}$. If Eq. (8) was used the result would have been $7.82 \cdot 10^{-4}$. Also on Fig. (10) a number of curves are shown assuming exactly the same specimen subject to the same maximum stress but different stress ranges. It can be seen that for higher values of N_f the increase in crack width at same number of cycles becomes less significant. For example, if $N_f = 6 \times 10^6$ cycles, the increase in crack width at 600,000 cycles is only about 18% while if $N_f = 667,000$ cycles the increase in crack width is about 449%. This observation can be beneficially used in design to reduce the effect of fatigue on crack widths.

CONCLUSION

The subject of cracking is a complex one for both reinforced concrete and ferrocement; exhaustive experimental investigations combined with scrupulous analysis are still necessary in ferrocement to better ascertain the influence of major parameters and separate their effects. The above study is based on experimental tests of ferrocement specimens reinforced mostly with square meshes of relatively small sizes and represents in a way the present state of knowledge. Similar studies exploring different types of reinforcement (such as expanded metal, chicken wire meshes, a combination of mesh and rod-type reinforcement) and a wider range of sizes are needed before generalizing some of the above derived results.

ACKNOWLEDGEMENT

This study was made possible by a grant (ENG 74-20829) from the National Science Foundation (Engineering Mechanics Section, Structural Materials and Geotechnical Engineering Program) to the University of Illinois at Chicago Circle, Department of Materials Engineering.

REFERENCES

1. Rilem, Proceedings of the Symposium on Bond and Crack Formation in Reinforced Concrete, Stockholm, Sweden, 1957.
2. Ferry, Borges, J., "Cracking and Deformability of Reinforced Concrete Beams," International Association for Bridge and Structural Engineering, Vol. 26, Zurich, 1966, pp. 75-95.

3. "Causes, Mechanism and Control of Cracking in Concrete," American Concrete Institute Special Publication SP20, Detroit, Michigan, July 1968.
4. Gergely, P. and Lutz, L. A., "Maximum Crack Width in Reinforced Concrete Flexural Members," American Concrete Institute, Publication SP20, Detroit, Michigan, 1968, pp. 87-117.
5. Atlas, S., Siess, C. B. and Kesler, C. E., "Control of Cracking in Slabs Reinforced with Welded Wire Fabric," American Concrete Institute, Publication SP20, 1968, pp. 205-209.
6. Nawy, E. G. and Huang, P. T., "Crack and Deflection Control of Pretensioned Prestressed Beams," Journal of the Prestressed Concrete Institute, Vol. 22, No. 3, May/June 1977, pp. 30-47.
7. "Control of Cracking in Concrete Structures," ACI Committee 224, Journal of the American Concrete Institute, Vol. 69, No. 12, Dec. 1972, pp. 717-753.
8. Proceedings of the American Concrete Institute Symposium on "Practical Control of Cracking in Concrete Structures," Annual Convention, New Orleans, Louisiana, Oct. 1977.
9. Shah, S. P. and Naaman, A. E., "Crack Control in Ferrocement and Its Comparison with Reinforced Concrete," Presented at the ACI Symposium on "Practical Control of Cracking in Concrete Structures," New Orleans, Louisiana, Oct. 1977.
10. Balaguru, P. N., Naaman, A. E. and Shah, S. P., "Analysis and Behavior of Ferrocement in Flexure," ASCE Journal of the Structural Division, Vol. 103, No. ST10, Oct. 1977, pp. 1937-1951.
11. Balaguru, P. N., Naaman, A. E. and Shah, S. P., "Ferrocement in Bending, Part I: Static Nonlinear Analysis," Dept. of Materials Engrg., Univ. of Ill. at Chicago Circle, Report No. 76-2, Aug. 1976, 144 pp., NTIS #PB-262155/AS.
12. Balaguru, P. N., Naaman, A. E. and Shah, S. P., "Ferrocement in Bending, Part II: Fatigue Analysis," Department of Materials Engineering, Univ. of Ill. at Chicago Circle, Report No. 77-1, Oct. 1977, 246 pp.
13. Naaman, A. E. and Shah, S. P., "Tensile Tests of Ferrocement," Journal of the American Concrete Institute, Vol. 68, No. 9, Sept. 1971, pp. 693-698.
14. Somayaji, S., Shah, S. P. and Naaman, A. E., "Tensile Cracking and Stress Strain Curve of Ferrocement, Part I: Effect of Longitudinal Reinforcement," Dept. of Materials Engrg., Univ. of Ill. at Chicago Circle, Report in Progress.

15. Guerra, A. J., Naaman, A. E. and Shah, S. P., "Ferrocement Cylindrical Tanks: Cracking and Leakage Behavior," Journal of the American Concrete Institute, Vol. 75, No. 1, Jan. 1978, pp. 22-30.
16. Guerra, A. J., Naaman, A. E. and Shah, S. P., "Cracking and Leakage Characteristics of Ferrocement Water Tanks," Dept. of Materials Engrg., Univ. of Ill. at Chicago Circle, Report #76-3, Sept. 1976, 54 pp., NTIS #PB 258003/RC.

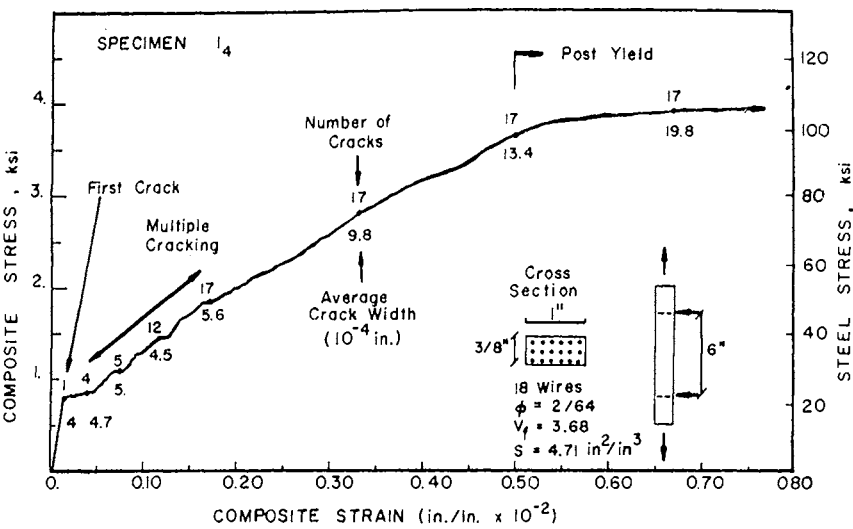


Fig. 1 Typical stress-strain curve in tension of composite without transverse reinforcement.

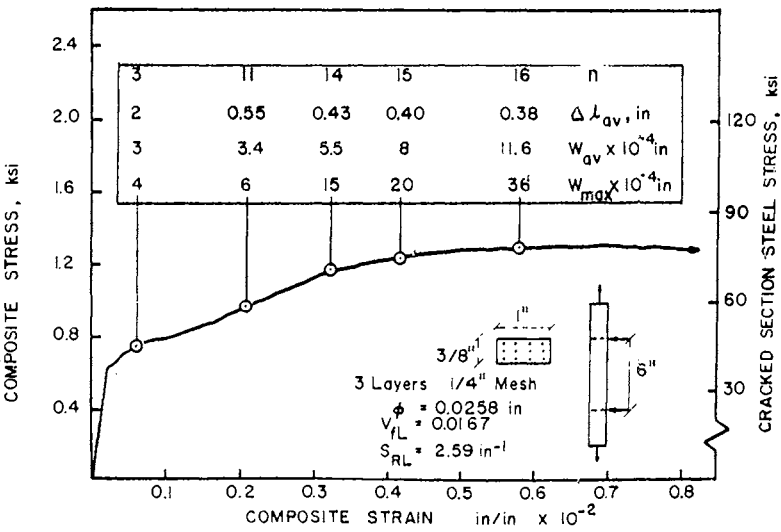


Fig. 2 Typical stress-strain curve in tension of composite reinforced with square meshes.

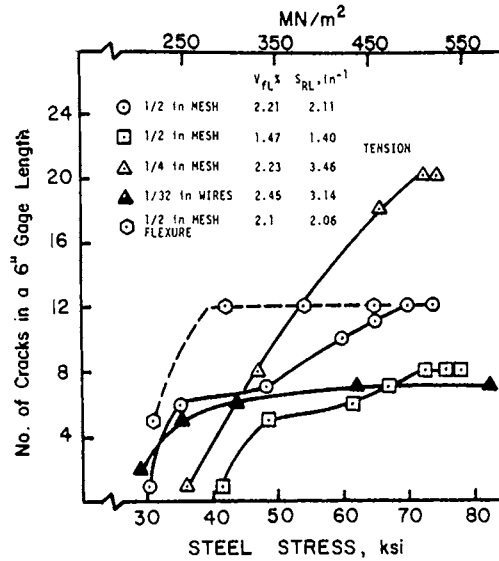


Fig. 3 Typical crack development with steel stress for tensile and flexural specimens.

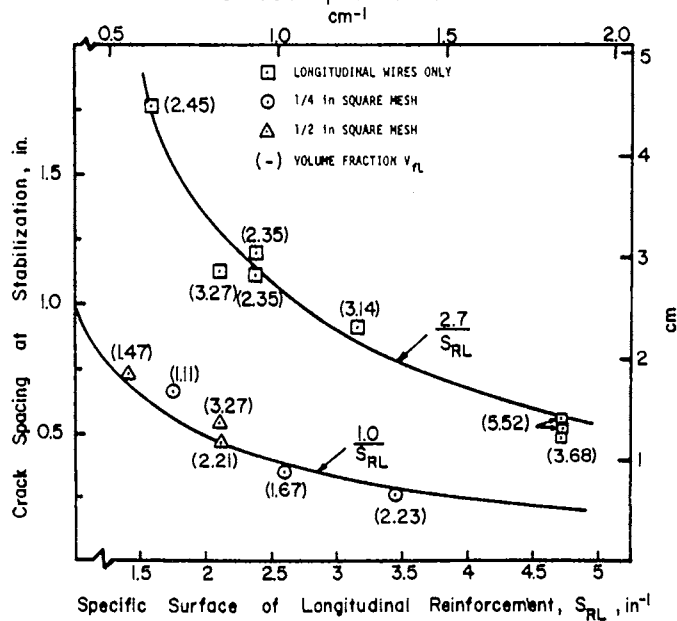


Fig. 4 Variation in crack spacing at crack stabilization with the specific surface of reinforcement.

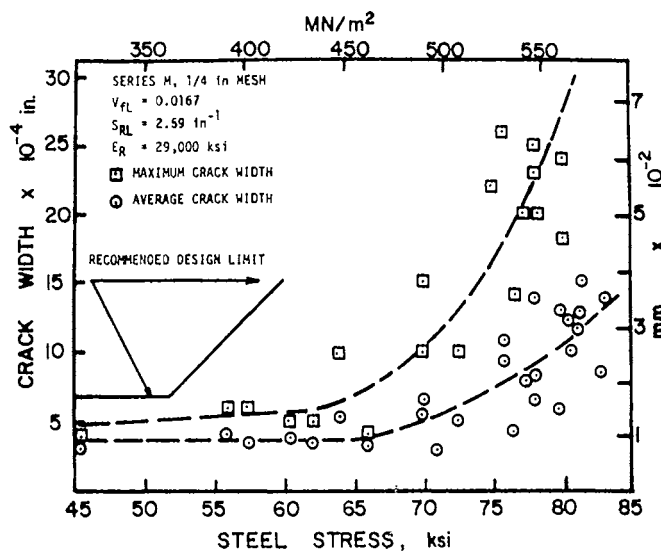


Fig. 5 Typical variations of crack widths in tension versus steel stress.

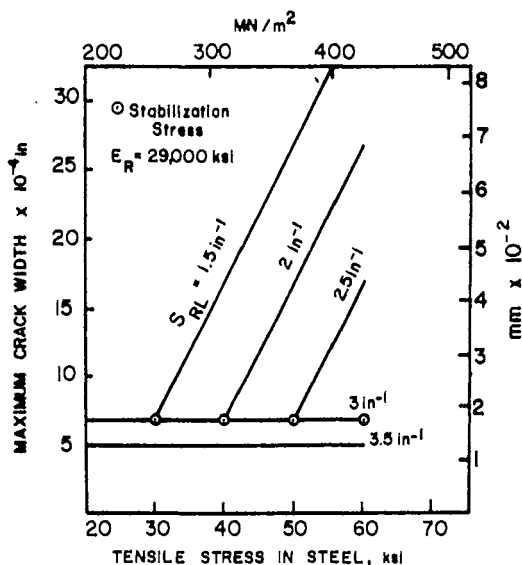


Fig. 6 Proposed maximum crack width prediction in tension for ferrocement reinforced with square meshes.

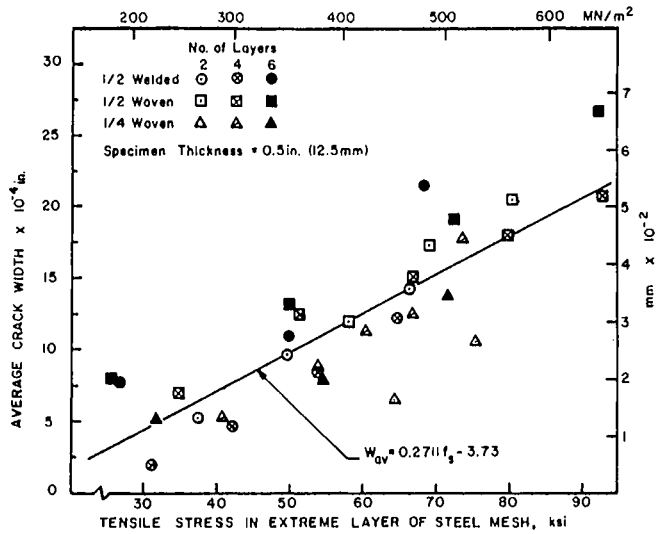


Fig. 7 Observed variation of average crack width in flexure versus steel stress.

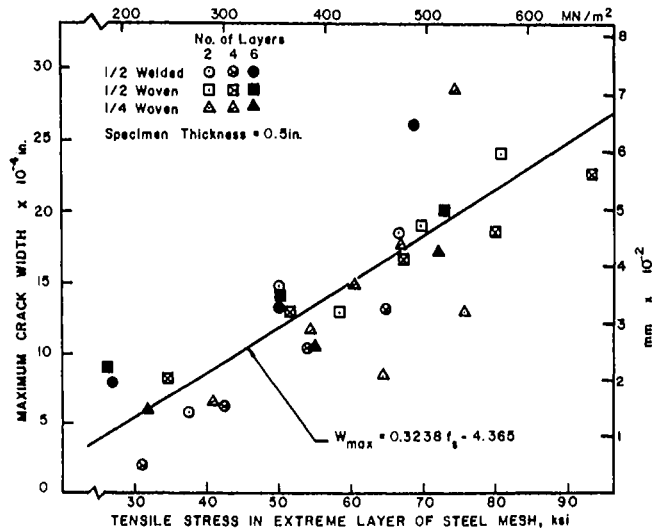


Fig. 8 Observed variation of maximum crack width in flexure versus steel stress.

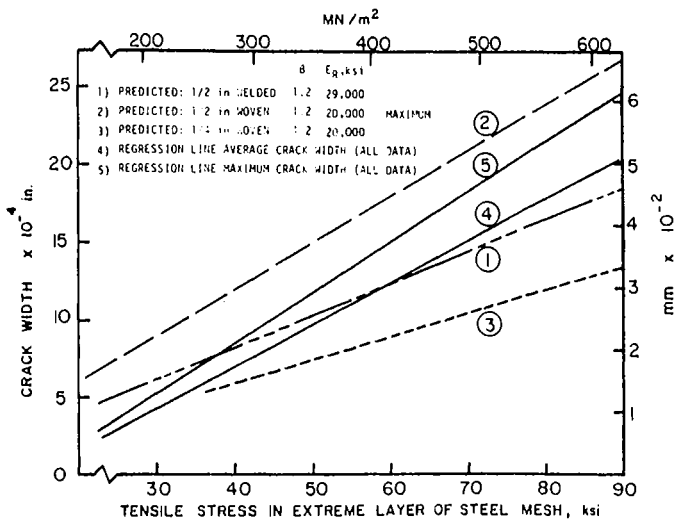


Fig.9 Comparison of predicted and observed crack widths in flexure.

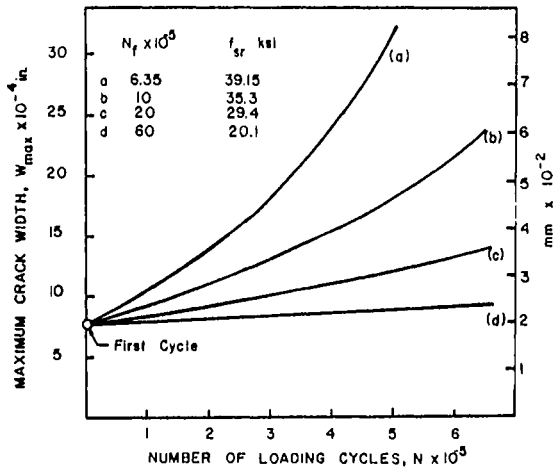


Fig. 10 Typical crack width increase with loading cycles.

Study of Tensile Cracks and Bond-Slip in Ferrocement

By Ricardo P. Pama, Vorachai Intaragumhaeng,
and Bishwendu K. Paul

Synopsis: A theoretical model is presented to predict the width and spacing of cracks in ferrocement elements in tension. A parametric study was also conducted to study the influence of parameters such as bond stress, volume of reinforcement, size of reinforcement, properties of mortar and reinforcement on the width and spacing of cracks.

The theoretical results indicate that the spacing and width of cracks are significantly affected by bond stress, diameter and volume of reinforcement. An experimental investigation was conducted on ferrocement specimens in tension with varying steel content and the results obtained are shown to be in agreement with theoretically predicted values.

Practical guidelines are suggested for the control of tensile cracks in ferrocement elements.

Keywords: bonding; bond (concrete to reinforcement); cracking (fracturing); crack width and spacing; ferrocement; mathematical models; modulus of elasticity; tensile strength; tension tests; wire cloth.

44 Pama, Intaragumhaeng, and Paul

ACI member R. P. Pama is a Professor of Structural Engineering at the Asian Institute of Technology in Bangkok, Thailand. He obtained his Ph.D. from the University of St. Andrews in Dundee in 1968. Dr. Pama is the co-author of three textbooks (Bridge Deck Analysis, Low-Cost Housing and Ferrocement) and has numerous research papers in the field of materials of construction. In addition, he is serving as Regional Director of the International Association for Housing Science (IAHS), Associate Director of the International Ferrocement Information Center, co-editor of the Journal of Ferrocement and a member of the ACI Committee 549 on ferrocement.

V. Intaragumhaeng is a Structural Engineer and presently working in a consulting company in Bangkok, Thailand. He received his M. Eng. degree from the Asian Institute of Technology in Bangkok, Thailand in 1977.

B. K. Paul is a Research Associate of the International Ferrocement Information Center at the Asian Institute of Technology (AIT), Bangkok. He received his M. Eng. degree from the AIT in 1976 and has published papers on low-cost building materials and is co-author of a textbook on Ferrocement.

INTRODUCTION

In general a composite material consists of a matrix and reinforcement which act together to form a new material which exhibit superior characteristics than either of its constituents alone. Different types of composite materials have been developed and ferrocement is one of them. The history of ferrocement dates back to the last century when Lambot made a boat out of ferrocement in 1848.

Ferrocement contains a high percentage of ductile steel wire mesh with a high surface area to volume ratio embedded in a brittle cement mortar matrix causing the latter to assume the ductile characteristics of the reinforcement.

The fundamental concept behind the fracture of a brittle matrix under load is the initiation and propagation of small cracks and flaws which extend and inter connect until the whole internal structure is completely disrupted. Cracking behavior of composites consists of a combination of sequence of fracture of individual constituents or interface separation between reinforcement and matrix. If these fractures are locally restrained from extending into adjacent materials, then first cracking strength as well as the resistance of the material in the post cracking stage can be significantly improved provided sufficient length and volume fraction of reinforcing fibers (wire mesh) are embedded in the matrix.

An early suggestion of close spacing of reinforcement to arrest the cracks came as a result of the experiments conducted by Nervi on the behavior of concrete slabs reinforced with closely spaced steel wire mesh. The spacing of the wires, 0.5 mm to 1.5 mm in diameter, was in the order of 10 mm apart (1). Early in the last decade an attempt was made by Romualdi and Batson (2) to improve the tensile strength of concrete by using closely spaced short steel wires to prevent the flaws from enlarging and propagating throughout the tension zone. Following this, several investigations have been conducted to develop a mathematical model to study the cracking behavior of fiber reinforced concrete. In ferrocement, several research works (3,4,5,6,7,8,9,10,11) have also been done to study the influence of the different parameters such as volume fraction, type of reinforcement, geometry of reinforcement, properties of mortar and reinforcement on the basic mechanical properties of the material in tension, compression, impact and fatigue loadings. Several experimental investigations have been undertaken to study the cracking behavior of ferrocement (4,9,11,12). To the best of the authors' knowledge no mathematical model to predict the width and spacing of cracks in ferrocement has been published so far. A recent study by Intaragumhaeng (13) on crack and bond slip behavior of fiber reinforced material showed that crack spacing as well as crack width are affected by fiber volume fraction, slip modulus, ultimate bond stress and tensile strength of the mortar. The authors feel that crack width and spacing are two of the main factors which determine the failure ferrocement in tension and bending, and these should be taken into account along with other design parameters in the design process.

In this paper an attempt is made to predict theoretically the tensile property and post-cracking behavior of ferrocement taking into account the stress-strain relation of the wire mesh beyond the initial yield stress. The influence of the various parameters such as bond stress, volume of reinforcement, size of reinforcement, properties of mortar and reinforcement on the width and spacing of cracks were investigated. An experimental investigation was also conducted on ferrocement specimens in tension to verify the theoretical results.

THEORETICAL CONSIDERATION

Assumptions:

The following assumptions are made in the analysis:

1. The stress-strain relation of the wire mesh is idealized as a bilinear curve (Fig. 1).
2. All the steel fibers (wire mesh) are straight.
3. Under tensile stress, cracking in the mortar will occur when the strain of the composite exceeds the cracking strain of the mortar.
4. The tensile stress in the mortar reaches its maximum value at a point halfway between the cracks (14). The greatest possible distance between cracks, for a given set of parameters, occurs when the maximum tensile stress is equal to, but not greater than the ultimate tensile strength of the mortar in tension. The minimum crack spacing is 0.5 times the maximum crack spacing,
5. The bond stress at the instant of cracking is zero.
6. The bond stress at a crack is zero. Immediately adjacent to the crack, the bond stress increases abruptly to its maximum value. The length over which this increase takes place is negligible.
7. The bond slip relation of the steel fiber (wire mesh) and the mortar can be idealized by an elastic-perfectly plastic curve (Fig. 2).

Effective Normal Stress and Young's Modulus:

In the analysis of a ferrocement section in tension, it is assumed that when stressed, the steel fibers are firmly bonded so that no slippage occur at the interface of the steel fibers and the mortar. The load acting on a composite section per unit area carried by the mortar and the steel fibers in direct tension are expressed as:

$$\sigma_c = \sigma_m A_m + \sigma_f A_f \quad (1)$$

in which σ_c , σ_m and σ_f are the average stresses in the composite section, mortar and steel fibers respectively; A_m and A_f are the area fractions of the mortar and steel fibers respectively. Eq. 1 can be expressed in terms of the volume fractions of the constituent materials in the form

$$\sigma_c = \sigma_m V_m + \sigma_f V_f \quad (2)$$

where V_m and V_f denote the total volume fraction of the mortar and steel fiber respectively.

In view of Hooke's Law and the fact that the mortar strain is equal to the strain of the ferrocement, Eq. 2 becomes

$$E_t = E_{mt}V_m + E_fV_f \quad (3)$$

in which E_t , E_{mt} and E_f denote the moduli of elasticity in direct tension of ferrocement, mortar and fiber respectively. The first crack strength is reached when the composite strain exceeds the cracking strain of the mortar, thus

$$\sigma_{cr} = \sigma_{mu}V_m + \epsilon_{mu}E_fV_f \quad (4)$$

where σ_{mu} and ϵ_{mu} denote the cracking stress and strain of the mortar respectively.

The following analysis is based on a one-dimensional model in which the steel wire is assumed to be aligned in the direction of the applied stress. The formulation is based on the bond-slip characteristic of the fiber (14, 15) instead of assuming a bond stress distribution on the fiber. This leads to the bond stress distributions shown in Fig. 3 as the composite stress increases.

The sign convention adopted is such that slip, displacements of mortar and fiber as well as the action of bond stress on the fiber are positive in the x-direction.

Five possible cases will be studied, namely:

1. The maximum bond stress is below the ultimate bond strength (τ_u) and the fiber stress at the cracked section is below its yield point.
2. The fiber stress at the cracked section is above its yield stress, the yield stress is attained at distance ℓ_1 from the cracked section, but the maximum bond stress is still less than the ultimate bond stress.
3. The ultimate bond strength is reached over a length ℓ_0 ; the yield stress is attained at distance ℓ_1 from the cracked section, $\ell_1 > \ell_0$.
4. The ultimate bond stress is reached over a length ℓ_0 but the fiber stress at the cracked section is below its yield stress.
5. The fiber stress at the cracked section is above its yield stress, the yield stress is attained at the distance ℓ ; the ultimate bond strength is reached over a length ℓ_0 , $\ell_0 > \ell_1$.

Case 1: $\tau(0) < \tau_u$ and $\sigma_f^* < \sigma_{fy}$ (Fig. 3a)

In this case the fiber stress $\sigma_f(x)$ and the mortar stress $\sigma_m(x)$ distributions are shown in Fig. 3a. The shear stress $\tau(x)$ at any point can be expressed as

$$\tau(x) = k\{-S(x)\} = -k\{d_f(x) - d_m(x)\} \quad \text{for } 0 \leq S(x) \leq S_y \quad (5)$$

and

$$\tau(x) = \tau_u \quad \text{for } S(x) > S_y \quad (6)$$

where the minus sign in Eq. 5 indicates that negative slip is required to result in a positive bond stress since slip is defined as the difference between the displacement of the fiber and the matrix. The parameter k shown in Fig. 2 denotes the slip modulus, and $d_f(x)$ and $d_m(x)$ the displacements of the fiber and mortar respectively at section x .

Consider the free body of a portion of the fiber under the action of the normal stresses $\sigma_f(x)$ and σ_f^* acting at section x and at the cracked section respectively, and bond stress $\tau(\xi)$ as shown in Fig. 4. The equilibrium of the forces acting on the free body of the fiber requires that

$$\sigma_f(x) = \sigma_f^* - R \int_0^x \tau(\xi) d\xi \quad (7)$$

where R is the ratio of the periphery to the cross-sectional area of the steel fiber.

Differentiating Eq. 7 twice with respect to x yields

$$\frac{d^2\sigma_f(x)}{dx^2} = -R \frac{d\tau(x)}{dx} \quad (8)$$

or

$$\tau(x) = -\frac{1}{R} \frac{d\sigma_f(x)}{dx} \quad (9)$$

and in view of Eq. 5

$$\frac{d\tau(x)}{dx} = -k \left[\frac{d(d_f(x))}{dx} - \frac{d(d_m(x))}{dx} \right] \quad (10)$$

$$\frac{d\tau(x)}{dx} = -k \left[\frac{\sigma_f(x)}{E_f} - \frac{\sigma_m(x)}{E_{mt}} \right] \quad (11)$$

Eq. 8, in view of Eq. 11 becomes

$$\frac{d^2\sigma_f(x)}{dx^2} = Rk \left[\frac{\sigma_f(x)}{E_f} - \frac{\sigma_m(x)}{E_{mt}} \right] \quad (12)$$

The equilibrium of forces acting on the fiber yields

$$\sigma_f^* V_f = \sigma_m(x) V_m + \sigma_f(x) V_f \quad (13)$$

Substituting Eq. 13 into Eq. 12, yields

$$\frac{d^2\sigma_f(x)}{dx^2} - \frac{Rk}{E_f}(1 + mp)\sigma_f(x) = -Rk \left(\frac{mp\sigma_f^*}{E_f} \right) \quad (14)$$

where m is the modular ratio, E_f/E_{mt} and p the ratio of V_f to V_m .

Eq. 14 is the governing equation and the solution can be written in the form

$$\sigma_f(x) = A_1 \cosh(K_1 x) + A_2 \sinh(K_1 x) - \left(\frac{K_2}{K_1} \right)^2 \quad (15)$$

where

$$K_1^2 = \frac{Rk}{E_f}(1 + mp) \quad (16)$$

$$K_2^2 = -Rk \left(\frac{mp\sigma_f^*}{E_f} \right) \quad (17)$$

The constants of integration A_1 and A_2 can be determined from the boundary conditions

$$\sigma_f(0) = \sigma_f^* \quad (18)$$

$$\tau(\ell/2) = 0 \quad (19)$$

In view of Eqs. 18 and 19, Eq. 15 becomes

$$\sigma_f(x) = \left\{ \frac{K_1^2 \sigma_f^* + K_2^2}{K_1^2} \right\} \left\{ \frac{\cosh K_1 (\ell/2 - x)}{\cosh(K_1 \ell/2)} \right\} - \left(\frac{K_2}{K_1} \right)^2 \quad (20)$$

From Eqs. 10 and 20, the bond stress can be expressed as

$$\tau(x) = \frac{k\sigma_f^*}{E_f K_1} \left\{ \frac{\sinh K_1 (\ell/2 - x)}{\cosh(K_1 \ell/2)} \right\} \quad (21)$$

The mortar stress $\sigma_m(x)$ is readily determined by substituting Eqs. 20 and 21 into Eq. 11, thus

$$\sigma_m(x) = \frac{1}{m} \left(\frac{K_2}{K_1} \right)^2 \left[\left\{ \frac{\cosh K_1 (\ell/2 - x)}{\cosh(K_1 \ell/2)} \right\} - 1 \right] \quad (22)$$

By using the fourth assumption, a relation between the fiber stress at first crack and the maximum crack spacing can be derived from

$$pR \int_0^{\ell/2} \tau(x) dx = \sigma_{mu} \quad (23)$$

With Eq. 21, this reduces to

$$\sigma_f^* = \sigma_{mu} \left\{ \frac{1 + mp}{p} \right\} \left\{ \frac{\cosh(K_1 \ell/2)}{\cosh(K_1 \ell/2) - 1} \right\} \quad (24)$$

The crack width w is the difference between the elongation of the fiber and the elongation of the mortar and is expressed as

$$w = 2 \int_0^{\ell/2} \left\{ \frac{\sigma_f(x)}{E_f} - \frac{\sigma_m(x)}{E_{mt}} \right\} dx \quad (25)$$

With Eqs. 11 and 21, the crack width is given by

$$w = \frac{2\sigma_f^*}{E_f K_1} \tanh(K_1 \ell/2) \quad (26)$$

The elongation of the composite, $\Delta\ell$, can be expressed in terms of the elongation of the fiber, thus

$$\begin{aligned} \Delta\ell &= 2 \int_0^{\ell/2} \frac{\sigma_f(x)}{E_f} dx \\ \Delta\ell &= \frac{2}{E_f} \left[\frac{K_2^2 \sigma_f^* + K_2^2}{K_1^3} \tanh(K_1 \ell/2) - \left(\frac{K_2}{K_1} \right)^2 \frac{\ell}{2} \right] \end{aligned} \quad (27)$$

Case 2: $\sigma_f^* > \sigma_{fy}$ and $\tau(0) < \tau_u$ (Fig. 3b)

From Fig. 1, the stress of the fiber above its yield stress, can be expressed as

$$\sigma_f = \sigma_{fy} + E_f'(\epsilon_f - \epsilon_{fy}) \quad (28)$$