

Similitude Requirements for Dynamic Models

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Synopsis: This paper is a state-of-the-art discussion on dynamic modeling theory and its application to scale modeling of reinforced concrete structures. Dimensional analysis is used to develop similitude requirements according to which the geometry, initial and boundary conditions, material properties and the loading of the model and prototype have to be related so that the behavior of the latter can be expressed as a function of the behavior of the former. The discussion focusses on various types of models that can be utilized to simulate the dynamic response of structures in the elastic and inelastic range. Commonly encountered problems in achieving satisfactory similitude are outlined and assessed as to the effect they may have on the reliability of prototype response prediction.

Keywords: dynamic response; dynamic structural analysis; earthquake resistant structures; elastic analysis; failure; failure mechanisms; loads (forces); models; reinforced concrete; scale (ratio); structural analysis.

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INTRODUCTION

Experimentation with reduced-scale physical models is often a feasible means for predicting the response of structural components, subassemblies and complex structures subjected to gravitational effects, wind or seismic effects, impacts, pressurization, settlements, time and temperature effects, or any combination thereof. The reliability of response prediction depends strongly on the ability to reproduce, at model scales, all prototype features that affect the quantities or parameters which are to be predicted through the model test. These prototype features may vary with the purpose of the model test and consequently may lead to different modeling requirements for static vs. dynamic models, linear elastic vs. ultimate strength models, or models intended for the study of temperature dependent effects.

This paper reviews briefly the principles of modeling theory applicable to the general problem of physical modeling. Emphasis is then placed on developing similitude requirements for various types of ultimate strength models of reinforced concrete structures under the influence of gravitational and dynamic effects. Such models are of much interest in earthquake engineering where post-elastic response characteristics and safety against failure are of primary importance, and where laboratory experimentation of complete structures is in most cases limited to testing at reduced scales, often at rather small scales. While in most cases it is relatively easy to develop theoretical model similitude requirements, great difficulties are usually encountered in fulfilling all of these requirements and certain distortions must often be accepted. Nevertheless, it is often possible to minimize the influence of these distortions or to evaluate their effects on the predicted response, thus, permitting a quantitative evaluation of model test results in the prototype domain. Several of the problems encountered in fulfilling similitude requirements are discussed in this paper.

MODELING THEORY

Modeling theory establishes the rules according to which the geometry, material properties, initial conditions, boundary conditions and external influences (loading) of the model and prototype have to be related so that the behavior of the latter can be expressed as a function of the behavior of the former. The purpose of modeling theory is to develop a complete independent set of correlation functions (similitude laws) defining the model-to-prototype correspondence. Well established principles of dimensional analysis [1,2] can be utilized to derive these correlation functions.

Dimensional analysis is based on the premise that every physical phenomenon can be expressed by a dimensionally homogeneous equation of the type

$$q_1 = F(q_2, q_3, \dots q_n) \quad (1)$$

where n is the total number of physical quantities involved in the phenomenon, q_1 is the quantity which is to be evaluated and q_2 to q_n are the quantities on which q_1 depends. According to Buckingham's Pi theorem, every dimensionally homogeneous equation involving n physical quantities can be reduced to a functional relationship between a complete set of $n-N$ independent dimensionless products (Π -factors) taking the form

$$\Pi_1 = f(\Pi_2, \Pi_3, \dots \Pi_{n-N}) \quad (2)$$

where Π_1 to Π_{n-N} are independent dimensionless products of powers of the physical quantities q_1 to q_n . The number N is the rank of the dimensional matrix and is usually equal to the number of basic units needed to describe the physical quantities.

Since Eq. (2) is identical to Eq. (1), it describes the same physical phenomenon and, because of its dimensionless form, must be equally valid for prototype and model if similitude is to be achieved. A sufficient condition for complete similitude is therefore

$$\begin{aligned} (\Pi_1)_p &= (\Pi_1)_m \\ (\Pi_2)_p &= (\Pi_2)_m \\ &\vdots \\ (\Pi_{n-N})_p &= (\Pi_{n-N})_m \end{aligned} \quad (3)$$

where the subscripts p and m refer to prototype and model, respectively. The first of these equations is often referred to as the prediction equation while the other equations constitute the design conditions for the model.

To assure a reliable prediction of the response quantity q_1 and, at the same time, to minimize the effort in constructing a physical model, extreme care must be exercised in specifying the right number of physical quantities in Eq. (1). Quantities which have insignificant effect on q_1 will impose unnecessary

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on the model design, while neglecting a significant quantity may yield incorrect results. For instance, when only mode shapes and frequencies in the linear elastic range are of interest, a simulation of gravitational acceleration and stress histories is not needed while proper simulation of boundary conditions is essential.

If all similitude requirements (Eq. 3) are fulfilled, the model is often referred to as a true model. In distorted models, where one or more of the similitude requirements are violated, the effects of the distortions must be accounted for in the response prediction. Corrections for a violation of one similitude law can often be made through adjustments of other similitude laws. If only one prediction quantity is of primary interest (e.g., failure load), it is often possible to evaluate the distortion of the predicted value through extrapolation from several model tests at different scales, provided that the failure mode does not change.

The derivation of a complete and independent set of Π -factors is facilitated by the use of the dimensional matrix which is a matrix containing dimensional exponents of the physical quantities expressed in terms of independent basic quantities. The choice of basic quantities is not unique but it is customary to use either force, length, time and temperature (FLT system) or mass, length, time and temperature (MLT system) for this purpose. When the dimensions of the relevant physical quantities are properly arranged in a dimensional matrix, it is reasonably simple to extract dimensionless products by comparing individual quantities as to their dimensional dependence. To assure that these products constitute a complete and independent set and that they permit optimum control of the design and execution of the model experiment, it is helpful to follow a series of guidelines which are discussed in detail in Ref. 3.

An important conclusion can be drawn from the observation that all physical quantities can be expressed in terms of independent basic quantities. Consequently, as many quantities can be scaled arbitrarily (within physical constraints) as there are basic quantities needed to describe the problem. All other quantities are then scaled as functions of these selected scales. The arbitrarily scaled quantities must be independent, but other than that, can be chosen by the model designer.

In order to illustrate the derivation of similitude requirements, let us consider a problem in which it is desired to predict the stress response of a structure under dynamic actions such as an earthquake ground motion. In the simplest case, the stress history σ will be a function of a position vector $\vec{r}$, the time t , the mass density ρ , the material stiffness E , the acceleration time history a , the gravitational acceleration g , a characteristic length parameter ℓ , and initial conditions which may be described by σ_0 and $\vec{r}_0$. Thus, Eq. (1) can be written for this case as

$$\sigma = F(\vec{r}, t, \rho, E, a, g, \ell, \sigma_0, \vec{r}_0) \quad (4)$$

When needed, this relationship can be expanded to include other physical quantities that may enter the problem under consideration.

The dimensional matrix in the MLT system (neglecting dependence on temperature) is of the following form:

	σ	$\vec{r}$	t	ρ	E	a	g	ℓ	σ_o	$\vec{r}_o$
M	1	0	0	1	1	0	0	0	1	0
L	-1	1	0	-3	-1	1	1	1	-1	1
T	-2	0	1	0	-2	-2	-2	0	-2	0

Products of powers of the ten physical quantities can now be assembled to arrive at $n-N = 10-3 = 7$ independent dimensionless terms (Π -factors). In general, the choice of the dimensionless terms is not unique which may affect the similitude requirement and consequently the model design. A suitable set of Π -factors for this example, arranged in the form of Eq. (2), is

$$\frac{\sigma}{E} = f\left(\frac{\vec{r}}{\ell}, \frac{t}{\ell} \sqrt{\frac{E}{\rho}}; \frac{a}{g}, \frac{a\ell\rho}{E}, \frac{\sigma_o}{E}, \frac{\vec{r}_o}{\ell}\right) \quad (5)$$

To achieve true simulation of the stress history by means of a physical model, each term in this equation must be equal in model and prototype.

Equation (5) can be taken as a basis for deriving similitude requirements for a large number of modeling problems involving dynamic effects. However, for some problems (such as linear elastic problems) too many physical quantities are included in this equation which may permit a relaxation of certain similitude requirements. For other problems it may be necessary to include other quantities (such as temperature, viscosity, coefficient of friction, etc.) that affect the stress history, which would lead to additional similitude requirements. A closer examination of Eq. (5) will also show that for ultimate strength models in reinforced concrete it will hardly ever be possible to fulfill all of the similitude requirements for a true model unless gravitational effects are negligible. In addition, by omitting all material properties other than the stiffness E it is implicitly assumed that time and size dependent effects are properly simulated in the component materials as well as at the interfaces between the materials (bond). Thus, a multitude of problems exist in designing feasible models and in simulating material properties. Nevertheless, in many practical model studies it is possible to overcome these difficulties and obtain a reliable prediction of the prototype response. It is the objective of the following discussion to suggest various types of models that may be suitable for predicting dynamic response characteristics, and to outline problems in achieving similitude and suggest solution techniques where possible.

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DYNAMIC MODELS OF REINFORCED CONCRETE STRUCTURES

True Ultimate Strength Models

If all physical quantities listed in Eq. (4) have a significant effect on the stress history, the similitude requirements for a true model can be derived directly from Eq. (5). Since three independent basic quantities (M,L,T) are needed to describe the problem, three physical quantities can be selected for independent scaling. Practically, only two quantities can be selected freely since the gravitational acceleration g is equal in model and prototype, i.e., $g_m/g_p = 1$. If ℓ and E are selected for independent scaling in addition to g , the similitude requirements shown in column (1) of Table 1 need to be fulfilled for a true model. The subscript r used in this table refers to the ratio of units defining the model to prototype relationship for a physical quantity (e.g., $\ell_r = \ell_m/\ell_p$).

The most restrictive requirement comes from the term $a\ell\rho/E$ in Eq. (5) which necessitates that $(E/\rho)_r = \ell_r$, since $a_r = g_r = 1$. This equation requires that model materials have a much smaller stiffness or much larger density than the prototype material. In reinforced concrete, where prototype and model materials are usually of similar stiffness and density, this all but eliminates the use of true models for dynamic studies involving post-elastic behavior and gravitational effects.

In theory it is possible to test true ultimate strength models, even when prototype-like materials are used, by performing model tests in centrifuges capable of generating high uniform acceleration fields. This will permit full control over the scaling of the gravitational acceleration g and eliminate the requirement $(E/\rho)_r = \ell_r$. If prototype material is used, $(E/\rho)_r$ is equal to one and g_r has to be equal to ℓ_r^{-1} which would require very large accelerations for small-scale models.

Ultimate Strength Models With Artificial Mass Simulation

By far the most practical approach to dynamic modeling of reinforced concrete structures is to augment the density of the structurally effective model material with high density material which is structurally not effective but permits the fulfillment of the similitude requirement $(E/\rho)_r = \ell_r$. Practically, this will require the addition of suitably distributed weights which must be attached to the structural elements in a manner that does not change the strength and stiffness characteristics. Since addition of weights will alter the local stress histories close to attachment points, such models are not true models but will provide fully satisfactory results provided that the weights are properly distributed and are attached to points of noncritical stress histories.

If a correct simulation of the mass distribution in space is essential, it will be necessary to decouple the mass density of

the structurally effective material from an additive ρ_1 which is to be built into the model but has no component in the prototype. From the term $\alpha \rho / E$ in Eq. (5), the mass density ρ_1 can be derived as

$$\rho_1 = \left[\frac{E_r}{\ell_r} - (\rho_o)_r \right] (\rho_o)_p \quad (6)$$

For instance, for a 1:4 scale model using prototype material [$E_r = (\rho_o)_r = 1$] the density will have to be increased by a factor of three.

For line elements the amount of mass μ_1 per unit length that must be added to the model is given by

$$\mu_1 = [E_r \ell_r - (\mu_o)_r] (\mu_o)_p \quad (7)$$

where μ_o is the mass per unit length of the structurally effective material.

Most of the dynamic models of bridge structures are designed according to these criteria and excellent response predictions have been achieved (e.g., Ref. 4). In models of building structures these criteria need to be applied when the mass distribution in walls, floor slabs and frame elements has a significant effect on the dynamic response.

When a correct simulation of mass distribution in space is not essential, the design of models can be simplified considerably. It is then often possible to concentrate structural and non-structural masses at specific locations such as floor levels in buildings. The scaling law for these lumped masses is

$$M_r = E_r \ell_r^2 \quad (8)$$

where E_r is the stiffness scale for the structural material. This equation is derived by multiplying $(E/\rho)_r = \ell_r$ by ℓ_r^{-3} . By using a Rayleigh approach it is also possible, in an approximate manner, to include in the lumped masses distributed between the floor levels.

Lumped mass models of building structures are being used extensively for seismic response studies (5 to 7). They are feasible for a simulation of the dynamic response of two-dimensional systems (frames and walls) but cannot account for many three-dimensional effects such as floor slab - frame interaction. If this interaction is important, discrete masses must be distributed over the floor slabs in a manner that permits simulation of gravitational and inertial effects.

The similitude requirements summarized in column (1) of Table 1 apply to models with artificial mass simulation with the exception of $(E/\rho)_r = \ell_r$ which is replaced by Eqs. (6) to (8). For completeness, the requirements are repeated in column (2) of this table.

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Ultimate Strength Modeling Without Simulation Of Gravity Forces

For certain kinds of structural configurations the stresses induced by gravitational effects may be very small compared to stress histories generated by dynamic effects. In this case the gravitational acceleration g can be omitted from Eqs. (4) and (5) which eliminates the constraint $g_r = 1$ and permits the selection of a third independent model scale. If the material density ρ is chosen for this purpose, the similitude requirements shown in column (3) of Table 1 can be derived. In addition, all masses which are not part of the structural system have to be scaled according to $M_r = E_r \ell_r^3$. If prototype material is used (i.e., $E_r = \rho_r = 1$), all scale factors in this column can be expressed in terms of ℓ_r . Models of this type are easier to build since no artificial mass simulation is needed. However, the large scaling requirements for time ($t_r = \ell_r$) and acceleration ($a_r = \ell_r^{-1}$) may cause problems in measurement accuracy and in reproducing the dynamic excitation, and will amplify material strain rate effects.

In post-elastic tests, gravity effects on stress histories are always present. It is up to the model analyst to decide whether or not the errors introduced by neglecting gravity effects are acceptable. If the errors are too large, true model distortion is evident and either the prediction equation or dimensionless products which depend on a/g have to be modified to compensate for the error. When material and geometric nonlinearities are involved in the problem this will be an extremely difficult task.

Structures that may be suitable for model testing without simulation of gravity forces are, for instance, containment vessels in which the stresses induced by gravity are small compared to stress histories caused by internal pressurization and dynamic effects such as impact or earthquake ground motion. Models of this type may also be useful for dynamic response studies on slender shear walls provided the walls and boundary elements attract small gravity forces.

Linear Elastic Models

When a model test is concerned only with linear elastic behavior, gravitational effects can usually be decoupled from dynamic effects. Thus, dynamic response characteristics can be simulated in the model without regard to the scaling of the gravitational acceleration, and the similitude requirements in column (3) of Table 1 can again be applied. This makes model design for linear elastic tests a simpler task since the only material constraints are that of a linear elastic range characterized by E and, when needed, similitude of Poisson's ratio ν and material damping properties. Secondary effects caused by geometric nonlinearities are, however, not simulated properly in this type of model.

Ultimate Strength Models With Strain Distortion

In true ultimate strength models a requirement for similitude is that all dimensionless quantities must be equal in model and prototype. Thus, strains must be equal ($\epsilon_r = 1$) and the stress-strain curves of model and prototype materials should be identical except for a constant multiplier E_r in the stress direction (see Fig. 1a). It may sometimes be impossible to fulfill this stringent similitude requirement for model materials and strain distortion will have to be accepted. If the strain distortion is constant, that is, if it can be described by a single parameter ϵ_r (see Fig. 1b), it appears possible to account for this distortion by modifying the Π -factors shown in Eq. (5).

Rocha [8] suggested to introduce strain as an additional basic quantity in the dimensional matrix and to assign a dimensional exponent of one to all quantities dependent on ϵ . Doing this will result in the similitude requirements shown in column (4) of Table 1. Since displacement and length are scaled differently, secondary effects caused by geometric nonlinearities are not simulated properly in models with strain distortion. Also, the assumption of linear dependence of physical quantities on strain may need further verification for dynamic problems.

If steel is used as reinforcement for strain distorted models, the reinforcement area should be scaled by $E_r \lambda_r^2$ and the yield ϵ_r . This is necessary to accomplish compatibility in strain similitude between steel and concrete.

The types of models discussed here by no means exhaust the possibilities of dynamic model studies. For specific problems one may take advantage of available knowledge on the physical phenomenon under study which may permit model design with various kinds of distortions. It is then left to the ingenuity of the model designer to minimize the effects of distortions and to evaluate their influence on the response prediction.

ADDITIONAL SIMILITUDE CONSIDERATIONS

The specific purpose of a model study will dictate the degree to which the aforementioned similitude requirements have to be fulfilled and to which additional considerations will enter the model experiment. Such considerations may include simulation of initial and boundary conditions, the dynamic loading mechanism, the time, temperature and size dependence of material properties and their effects on the dynamic response and the failure modes.

The intent of the following brief discussion is to give an overview of simulation problems and solution techniques rather than be specific on any one aspect. For detailed information the reader is referred to the extensive literature in this field of which Refs. 10 to 14 are representative examples.

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Simulation Of Initial Conditions

In theory, simulation of initial conditions will require the tracing of all time history events affecting the prototype, from the time of construction to the time at which the dynamic event takes place. These events will include, amongst others, construction sequence, curing methods, time dependent material effects (creep, shrinkage, increase in compressive strength of concrete), as well as damage accumulation due to previous events.

Modeling of combined creep and shrinkage effects is virtually an impossible task because of incompatibilities in time scaling. Simulation of creep requires equal time in model and prototype ($t_r = 1$) while simulation of shrinkage requires large time scaling. Creep and shrinkage depend strongly also on the size and grading of the aggregates which are not reproduced at model scales. Considering these problems it is usually a futile exercise to attempt modeling of creep and shrinkage phenomena, particularly since they often have a reasonably small influence on the ultimate strength behavior under cyclic loading. It appears to be more appropriate to control the initial conditions in the model experiment by minimizing shrinkage and creep through sealing of the model structure (for instance, by applying coats of shellac) and avoiding long-time preloading prior to the dynamic test.

Simulation of stresses and deformations introduced during construction is an equally difficult task. Unless the self-weight of the model concrete is artificially augmented during construction and creep and shrinkage are properly simulated, the effects of construction sequence are not represented in the model structure. If the selfweight of the model concrete is not augmented, the induced selfweight stresses will usually be small ($\sigma_r^{\text{grav}} \propto \lambda_r$) and will remain up to the time of testing if creep and shrinkage are minimized as was suggested previously. Well defined initial conditions will then exist in the model which facilitates interpretation of test results.

The possibility of simulating damage accumulation due to a series of time history events exists in physical models to a larger degree than in analytical models. Damage due to long-time events can hardly be simulated because of inadequate modeling of creep and shrinkage, but it is possible to subject a model structure to those past transient events that may affect the initial conditions and consequently the response in the dynamic study.

Simulation Of Boundary Conditions

In dynamic model studies a proper simulation of boundary conditions may be even more essential than in static model studies. Taking seismic response studies on shake tables as an example, supporting the model structure rigidly on the table will not allow for the rocking and translational motion at the

foundation-soil interface and for the energy dissipation by hysteretic action and radiation of waves in the supporting medium. It is quite feasible to support model structures on a medium that simulates these phenomena to various degrees [9]. The largest difficulty will be encountered in simulating wave radiation which depends on the wave propagation velocity v and requires a scaling of $v_r = \ell^{1/2} = (E/\rho)^{1/2}$ for a true model. Since the stiffness of the supporting medium must be scaled in the same manner as the stiffness of structural material (usually $E_r = 1$) this will necessitate a very high density medium.

Simulation Of Basic Material Properties

The simulation of those material properties which can be represented to a standard stress-strain diagram has been the subject of many studies and is summarized, for instance, in several papers of Refs. 10 and 11. If microconcrete is used for modeling concrete, the compressive strength can be adequately reproduced through a careful mix design which may have to be based on several trials because of the variable properties of the local materials used in the mix. Problems are often encountered in simulating stiffness properties and small strain distortion may have to be accepted. In most studies, the tensile strength of microconcrete has been found to be relatively high compared to the compressive strength. In the work described in Ref. 12, treatment of the aggregates with mineral oil or silicone resins has been used successfully to reduce the tensile strength to the desired level. This treatment, together with careful grading of aggregates has produced microconcrete which permitted accurate simulation of all basic strength properties.

The effects of specimen size on measured tensile and compressive properties have also been studied extensively. Increases in strength with decreasing size have consistently been reported. The measured increase has varied much from study to study; for instance, by reducing the diameter of a test cylinder from 6 in. (15 cm) to 0.5 in. (1.25 cm), the compressive strength was increased by 15 to 60 percent. Even larger increases were reported for the tensile strength [13]. To overcome this problem in small-scale model studies, it may be best to base the design of the concrete mix on material tests performed on small specimens compatible with the size of the model.

The effects of high strain rates on basic material properties may have to be considered in dynamic model studies. High strain rate compression tests have recently been carried out by the authors of this paper on 2 in. by 4 in. (5 cm by 10 cm) cylinders made of microconcrete. Some of the results are presented in Figs. 2 and 3. Strain values were obtained by averaging the continuous readings of four 2 in. (5 cm) extensometers applied at 90 degree angles to the test cylinders. The applied strain rates were reasonably constant for the increasing branch of the stress-strain diagram but, because of insufficient stiffness of the testing system, increased considerably for the decreasing branch.

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For this reason the stress-strain diagrams shown in Fig. 2 are discontinued soon after f'_c . This figure clearly shows the expected increase in stiffness and strength and decrease in ϵ'_c when the strain rates are increased. However, considering the relatively small difference between prototype and model strain rates ($\dot{\epsilon}_r = \dot{\epsilon}_r^{-1/2}$ for true models), these strain rate effects may not be of major importance.

In dynamic studies which are concerned with large inelastic deformation reversals, the strength, ductility and failure mode of critical parts of the structure are often governed by the properties of the reinforcement, necessitating accurate simulation of the stress-strain behavior in the model reinforcement. This can be achieved by using plain wire, cold working it in the laboratory (through knurling or cold rolling) and annealing it to the desired yield strength. The effects of strain rates on the yield strength of knurled and annealed wires are relatively small as can be seen from Fig. 4 which is based on work reported in Ref. 15.

Simulation Of Failure Modes

It is an unfortunate fact that even the tightest material control does not assure similitude of all possible failure modes. These modes may be affected by crack and bond similitude, cyclic loading effects, confinement and multiaxial states of stresses in concrete, stress and strain gradient effects, fabrication accuracy and many other considerations. Several of these aspects are not fully understood in prototype structures and even to a lesser degree in models.

It is encouraging that nevertheless a successful simulation of complex failure mechanisms has been achieved in many model studies reported in the literature. Although in most cases an exact duplication of crack patterns was not realized in model studies, the essentials of crack formation and the overall load-deformation response often have been reproduced satisfactorily, even at rather small model scales and under complex cyclic loading patterns. However, the number of model studies for which prototype test results were available for direct comparison is small and at this time it is presumptuous to draw definite conclusions regarding the reliability of predictions of detailed failure modes, particularly when shear failure and bond splitting failures are involved.

In view of these arguments it may be helpful to consider two interrelated but still distinguishable objectives of dynamic model studies on complete structural systems. Such studies may serve the purpose of predicting overall response characteristics as well as localized failure mechanisms. With carefully designed models a reliable prediction of overall response characteristics, such as frequencies, mode shapes, distribution of forces, deformations and damage, as well as identification of critical elements can usually be achieved. If doubts exist regarding the reliability

of predictions of localized failures, the model test can always be supplemented by prototype tests on critical components. The results of the dynamic model test can be utilized to decide upon boundary conditions and loading histories for these component tests.

Frequency Effects In Cyclic Component Tests

In earthquake engineering research, a large majority of the cyclic response studies on structural elements is preformed by applying slow quasi-static load or displacement histories to the test specimens. It remains to be answered whether the response characteristics of a dynamically excited element can be predicted confidently from a quasi-static cyclic load test, or to what degree these characteristics are affected by the cyclic frequency. This question was addressed by the authors in a study of twelve identical beam models which were subjected to identical displacement histories but at three different cycling frequencies of 0.0025, 2 and 10 Hz. Four beams were tested at each frequency, using the two-beam specimens shown in Fig. 5.

Representative load-displacement histories at the three frequencies are shown in Fig. 6. Three cycles each at two different displacement amplitudes were applied to each specimen. Only the first cycles at the two amplitudes are shown in the Figure. A relatively small effect of cycling frequency on the strength is noticed at the smaller amplitudes, however, considerably larger strength deterioration is evident for the quasi-static test (0.0025 Hz) during the first large displacement cycle. The effect of cycling frequency on strength deterioration is shown more clearly in Fig. 7 in which each point represents the mean of the peak loads from four tests. The peak loads in the small amplitude cycles (P_{11} , P_{12} and P_{13}) are reasonably insensitive to the cycling frequency, whereas the peak load in the first large amplitude cycle (P_{21}) is affected considerably by the cycling frequency.

The results of this test series indicate that cyclic strength deterioration during large inelastic cycles depends, to some extent, on the frequency of the applied deformation history. For the beams tested in this study, the frequency effects are considerably larger between a representative prototype frequency (e.g., 2 Hz) and a quasi-static cycling frequency (e.g., 0.0025 Hz) than between the prototype frequency and a properly scaled model frequency (e.g., 10 Hz).

CONCLUSIONS AND RECOMMENDATIONS

1. Various types of models may be suited for dynamic model studies of reinforced concrete structures. When gravity and inertia forces have to be simulated, the addition of structurally ineffective masses to the model material provides a practical means for achieving similitude.

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2. Overall dynamic response in the elastic and post-elastic range can usually be simulated in models, provided that initial and boundary conditions, geometry, material strength, stiffness and ductility, and dynamic input are modeled in accordance with similitude requirements.

3. The simulation of localized failure mechanisms is the most difficult task and sometimes may necessitate an auxiliary testing program on prototype components.

4. Time dependent effects can be evaluated as strain rate effects at the material level (Figs. 2, 3 and 4) and as frequency effects at the element level (Figs. 6 and 7). Unless the time is scaled by a large factor, these effects appear to be of minor importance in dynamic model studies.

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Table 1 Similitude requirements for dynamic models

Model Type		True Ultimate Strength Models	Models with Artificial Mass Simulation	Gravity Forces Neglected	Models with Strain Distortion
				Linear Elastic Models	
Scaling Parameters		(1)	(2)	(3)	(4)
length	ℓ_r	ℓ_r	ℓ_r	ℓ_r	ℓ_r
time	t_r	$\ell_r^{1/2}$	$\ell_r^{1/2}$	$\ell_r (E/\rho)_r^{-1/2}$	$(\epsilon_r \ell_r)^{1/2}$
frequency	ω_r	$\ell_r^{-1/2}$	$\ell_r^{-1/2}$	$\ell_r^{-1} (E/\rho)_r^{1/2}$	$(\epsilon_r \ell_r)^{-1/2}$
velocity	v_r	$\ell_r^{1/2}$	$\ell_r^{1/2}$	$(E/\rho)^{1/2}$	$(\epsilon_r \ell_r)^{-1/2}$
gravitational acceleration	g_r	1	1	neglected	1
acceleration	a_r	1	1	$\ell_r^{-1} (E/\rho)_r$	1
mass density	ρ_r	E_r/ℓ_r	**	ρ_r	$\epsilon_r E_r \ell_r^{-1}$
strain	ϵ_r	1	1	1	ϵ_r
stress	σ_r	E_r	E_r	E_r	$E_r \epsilon_r$
modulus of elasticity	E_r	E_r	E_r	E_r	E_r
specific stiffness	$(E/\rho)_r$	ℓ_r	**	$(E/\rho)_r$	$\ell_r \epsilon_r^{-1}$
displacement	δ_r	ℓ_r	ℓ_r	ℓ_r	$\ell_r \epsilon_r$
force	F_r	$E_r \ell_r^2$	$E_r \ell_r^2$	$E_r \ell_r^2$	$E_r \ell_r^2 \epsilon_r$
energy	$(EN)_r$	$E_r \ell_r^3$	$E_r \ell_r^3$	$E_r \ell_r^3$	$E_r \ell_r^3 \epsilon_r^2$

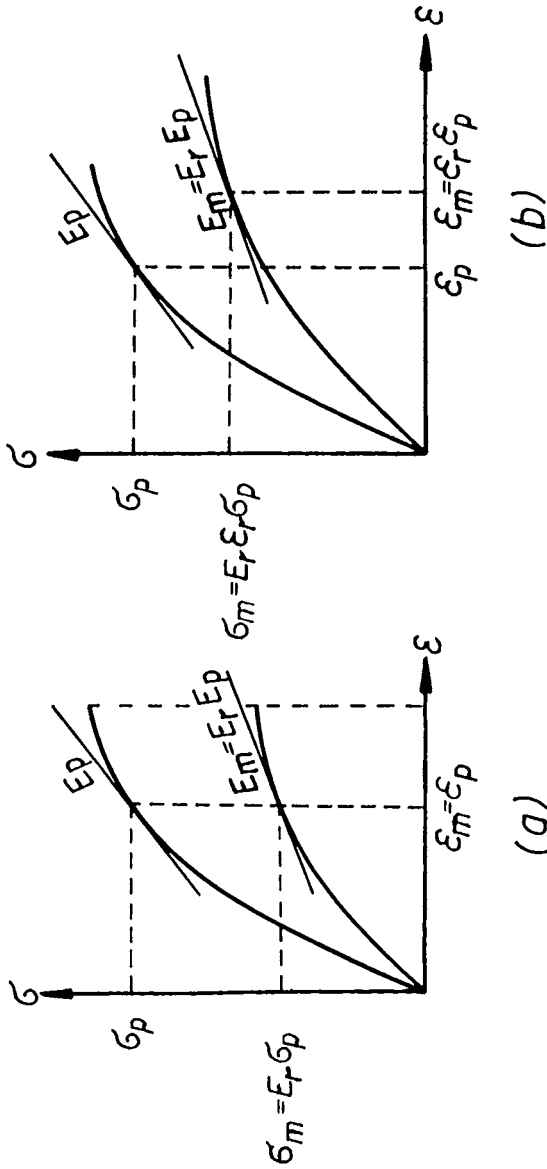


Fig. 1--Stress-strain similitude requirements

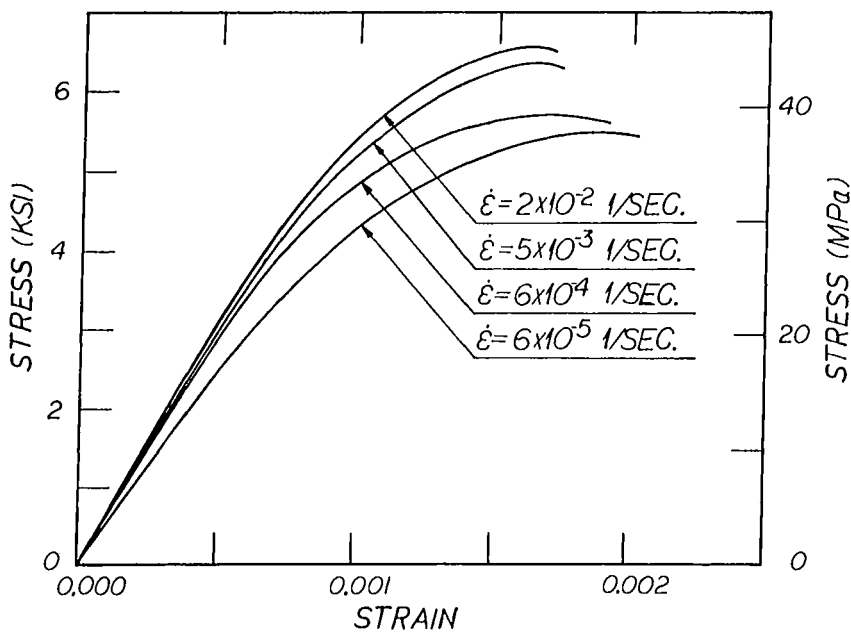


Fig. 2-- Stress-strain curves for microconcrete at different strain rates

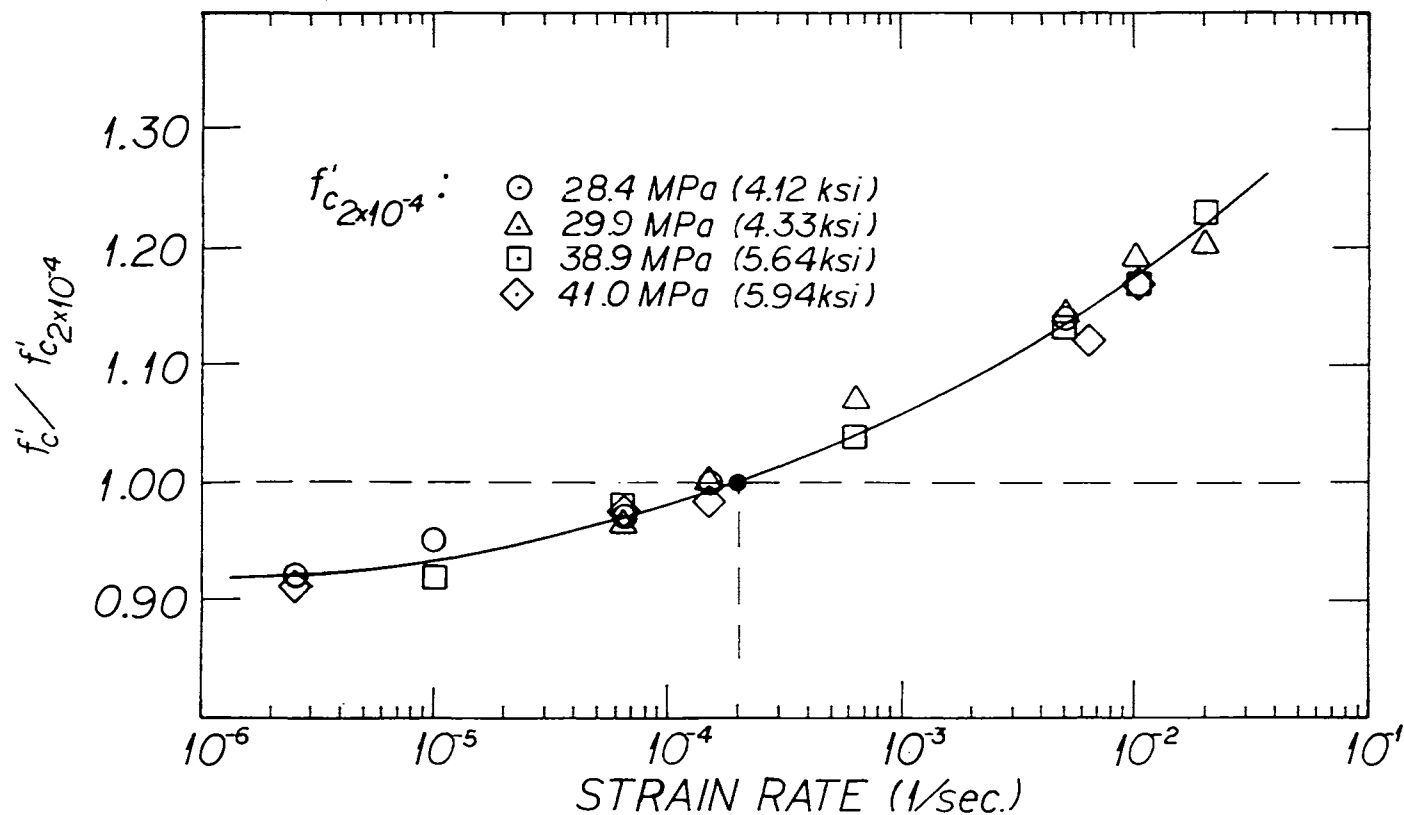


Fig. 3--Effect of strain rates on compressive strength of microconcrete

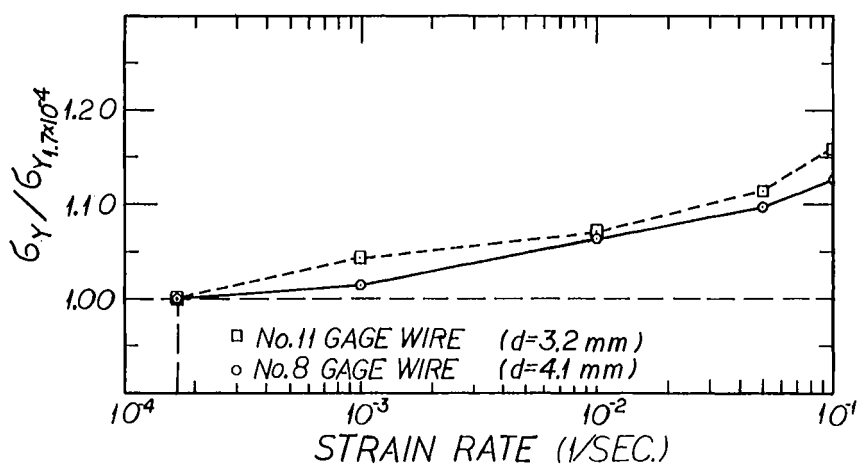


Fig. 4--Effect of strain rates on yield strength of model reinforcement (knurled and annealed wires)

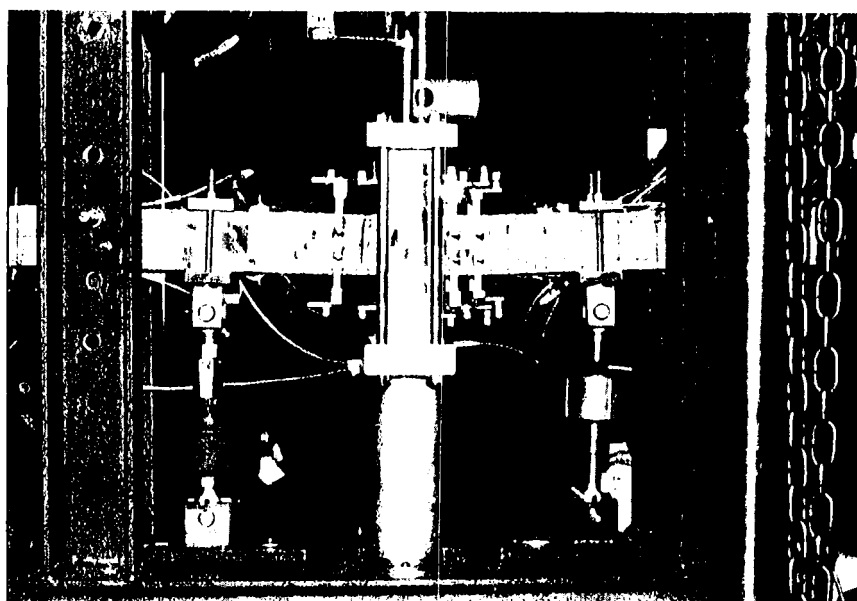


Fig. 5--Beam specimen for quasi-static and dynamic tests

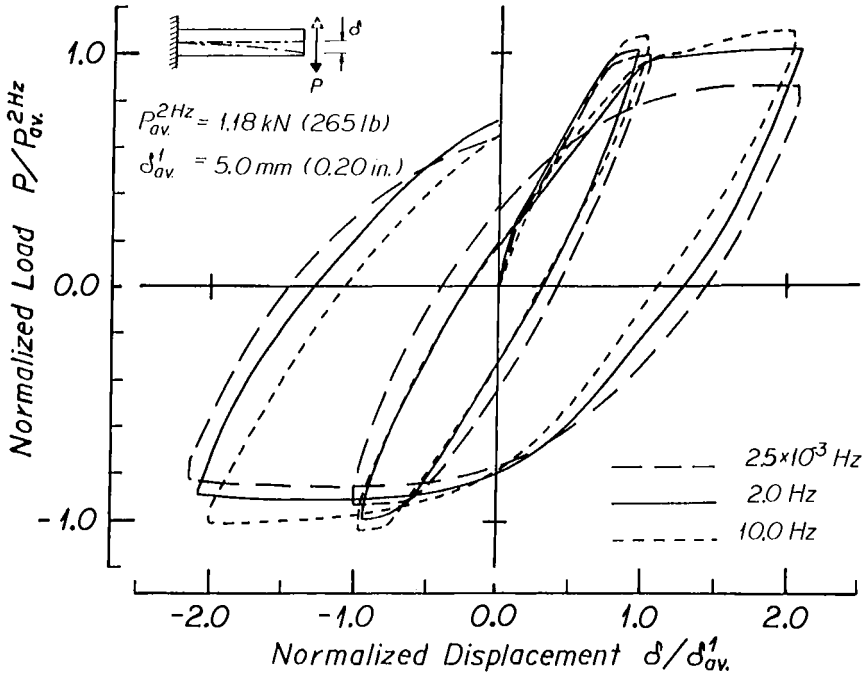


Fig. 6--Load-displacement hysteresis loops at different frequencies

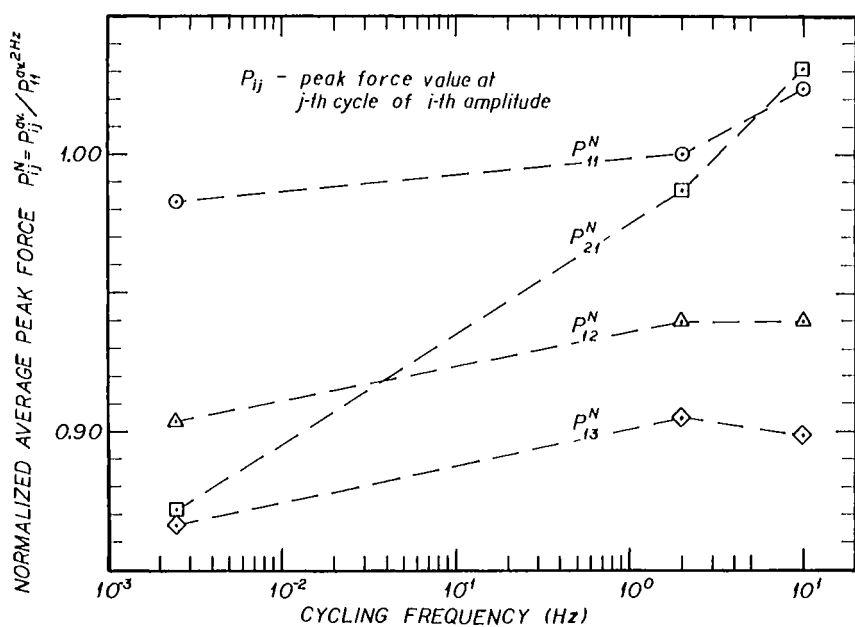


Fig. 7--Effect of cyclic frequency on peak loads in beam tests

Dynamic Tests of Model Concrete

By John Ferritto

Synopsis: The objective of the paper is to evaluate the dynamic properties of model materials. Dynamic tests were conducted on microconcrete with no. 4 maximum aggregate size, and gypsum concrete with no. 4 maximum aggregate size. The effects of strain rate (up to 2.5 in./in./sec) on ultimate compressive strength were obtained. The results are compared with results of dynamic tests conducted on prototype concrete by others. Microconcrete with a no. 4 maximum aggregate gives good correlation with prototype values of dynamic strength increase. The ratio of dynamic to static modulus of elasticity with increasing strain rate and dynamic strength increase factor also gives good correlation. Microconcrete experiences higher strains at ultimate load than the prototype. Gypsum concrete experiences dynamic strength increase factors of approximately half those of the prototype. It may be significant that the increase in modulus of elasticity with increasing strain rate for gypsum concrete is not similar to that of prototype concrete. Strains in gypsum concrete at ultimate load are slightly higher than those for prototype concrete.

Keywords: aggregate size; compressive strength; concretes; cylinders; dynamic modulus of elasticity; dynamic tests; gypsum; microconcretes; models; modulus of elasticity; strains.

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INTRODUCTION

The application of small-scale models to problems in the design and analysis of structures has been increasing in recent years. The structural model can be used to check the validity of analytical solutions, or to form a basis for establishing simplifying assumptions for mathematical solutions. The usefulness of models is especially apparent in the design of highly complex structures where mathematical analysis may be difficult, time consuming, or inaccurate. Another feature of models is the potential ease with which they can be altered for comparative studies of several designs or for studies at different stages of construction. Small-scale models can be tested in the laboratory and their response to dynamic blast loads can be determined. Otherwise the field testing of large-scale structures, involving great expenditures of time, effort and money, would be required.

A research project was initiated at CEL to further the development of modeling techniques. The primary objective of the first phase of the project was to develop a mix design procedure for model concrete used in small-scale models. A technical report was written which establishes a procedure for properly proportioning a model concrete mix that produces strengths in the model essentially the same as those in the prototype (Ref 1). In this procedure, cylinder sizes for compression testing are chosen such that the cross-sectional dimensions are approximately the same as those for a model. For example, 1x2-inch (2.5x5.1-cm) cylinders are used with a model having 1-in.² (2.5-cm²) columns. It has been shown (Ref 2) that with decreasing specimen size an increase in ultimate compressive strength is observed. It is felt that by keeping the test cylinder in proportion to a key dimension of the model as is done in prototype construction using a standard size 6x12-inch (15.2x 30.5-cm) cylinder, the model strength may be predicted more accurately. The maximum aggregate size was established as one-eighth of the minimum dimension of the model; this proportion is used in large-scale prototype concrete. Standard aggregate gradations were established for various maximum sizes.

The rate at which a concrete cylinder gains strength depends on cylinder size, with smaller cylinders gaining strength faster. Therefore, to accurately predict compressive strength, it is necessary to test smaller cylinders at an earlier age. It was experimentally determined that

$$\text{Test Age} = 12 + 4d \quad (1)$$

where time is expressed in days and d is the cylinder diameter in inches.

PRESENT WORK

The primary objective of this phase of the project was to evaluate the dynamic properties of modeling materials when compared to the characteristics of large-scale prototype concrete, and to determine if strength prediction relationships developed for prototype concrete under dynamic loads were applicable to model materials.

Having established a procedure which duplicates the strength of a prototype concrete in a model, it was of interest to determine if the increase in strength in the model obtained by increasing the rate of loading would be the same as in the prototype. Graphs were developed (Refs 3,4,5) that predict increases in ultimate compressive strength of prototype concrete with an increase in loading rate. According to the principles of dimensional analysis the testing of a dynamic model requires that the length scale factor, S_L , equal the time scale constant, S_t . To directly model the prototype concrete, the ultimate compressive stress of the model should be the same as the prototype. Thus, the model material must have the same strength properties as the prototype concrete, but the similitude relationship requires the strain rate in the model to be S_L times the strain rate in the prototype. Since the ultimate strength of concrete increases with strain rate, the model material must have a lower static ultimate strength than the prototype concrete so that at their corresponding strain rates they will have the same ultimate strengths.

The dynamic increase factors to be applied to the static ultimate strengths of the model so as to produce dynamic strengths equal to the prototype at differing strain rates must be evaluated. Size effect (the increase in strength observed by reduction in size) (Ref 2) complicates the evaluation of the dynamic strength increase factor of a model. A series of tests was conducted by varying the rate of loading in order to evaluate the dynamic characteristics of model concrete.

TEST PROCEDURE

Casting

Test cylinders were cast using the same materials, aggregate gradations and procedures described in Reference 1. Type III Portland cement and Ultracal 30, manufactured by the U.S. Gypsum Company, were used. The aggregate was San Gabriel River wash sand and gravel sieved to standard gradations. All specimens were cast in specially constructed plexiglass forms. Batches of model concretes were mixed, cast into cylinders, cured, and capped as described in Reference 1 and outlined in Table 1. The cylinders were tested at ages predicted from Equation 1.

Testing

Four cylinders from each set of sixteen were tested on a Universal Testing Machine using a stress rate of 35 psi/sec (240 kPa/sec) as the static values. Load-strain diagrams were automatically recorded on a plotter using 1-inch strain gages mounted on the cylinders as inputs to the plotter.

The twelve remaining cylinders were tested in three groups on the dynamic testing machine with three different loading rates. A pneumatic-hydraulic machine which utilizes air pressure in hydraulic accumulators supplied the necessary energy. The hydraulic pressure was equalized on the top and bottom of the main piston before each test. The head velocity, which controls the loading rate, was varied by regulating the rate of flow of hydraulic fluid on the bottom of the main piston. Because the main piston operated downward, compression tests were, therefore, conducted in the lower section of the machine. The specimen was centered on a load cell. The top bearing head, which was attached to the lower piston rod, was screwed down upon the specimen. A more detailed description of the machine and its operation may be found in Reference 6. Readings of the load cell, head movement, and strain were automatically recorded on tape and observed on an oscilloscope during testing. Strain rates varied from 10^{-4} in./in./sec to over 1.0 in./in./sec (see Figures 2 and 3 and Table 2).

Test Results

The raw data readings of the load cell, head movement, and strain were reduced using an analog-to-digital converter and were automatically plotted with a sampling rate of 4,000 samples per second. A total of 200 samples were taken for each curve. A typical plot is shown in Figure 1. From these curves the stress and strain rates were computed by taking the ordinate value and dividing it by time. The stress rate and strain rate were computed directly, independently of the material properties. It should be noted that the head movement and load cell readings are linear; the strain rate is based on an approximation of a linear range. A computer was also programmed to give stress-strain plots from which the secant modulus of elasticity was obtained. Typical curves are shown in Figure 2. A summary of the test results is presented in Tables 2 and 3.

DISCUSSION

The test results from this report are compared to prototype concrete results given in References 3, 4, 5, and 7; the plots are shown in Figures 3, 4, 5, and 6.

Microconcrete

It appears that the dynamic strength increase factor (f'_d dynamic/ f'_s static) for microconcrete with a no. 4 maximum aggregate, gives good correlation with prototype values, considering the variation in the prototype results (Figures 3 and 4). For this set of tests, the effect of reducing the size of the specimen on the ultimate compressive strength was found to be adequately compensated for by employing the procedure of testing established in Reference 1. A smaller specimen will give strengths comparable to the prototype concrete even under dynamic loads if it is tested at the appropriate age indicated in Equation 1. The curves used as standards in prototype design procedures

for predicting increases in ultimate strength with increasing strain rate are sufficiently close to be utilized in model design and analysis with the same degree of uncertainty.

The increase in the ratio of dynamic to static modulus of elasticity with increasing strain rate and dynamic strength increase factor (Figures 5 and 6) gives reasonable correlation with prototype concrete results. Considering its dynamic characteristics, this size microconcrete is well suited for use in direct models involving dynamic effects without special adjustment by a dynamic scale factor. It is important to note the increase in strain at failure and the slightly lower modulus of elasticity in the static and the dynamic ranges. These deviations are characteristic of the material's "microstructure." The reduction in size of the aggregate in the microconcrete will increase the surface area to be covered by the cement paste. Thus an increase in cement which has a lower modulus of elasticity than the aggregate is required in order to maintain strength. The consistency of the microconcrete and volume of aggregate are major factors in determining the amount of ultimate strain and the modulus of elasticity.

It should be noted that using a strain rate S_0 times faster in the model will affect the final value of the modulus of elasticity. A model having a lower static strength and static modulus of elasticity than the prototype will have a higher dynamic increase factor to produce equal dynamic strength and a higher ratio of dynamic to static modulus of elasticity. This increase in the dynamic-to-static-modulus-of-elasticity ratio for the model over that for the prototype will partly offset the lower initial static value of the model.

Gypsum Concrete

The dynamic strength increase factor for the gypsum concrete cylinders is somewhat less than that expected by comparing it to the prototype curves, Figures 3 and 4. The ratio of modulus of elasticity for increasing load rates does not follow the same pattern as that of the prototype concrete (Figures 5 and 6). This may be a significant factor in the design and analysis of a model. The static strength of a gypsum concrete model must be selected so that the dynamic strength increase factor for the strain rate to be used will produce strengths equal to the strength of the prototype as its strain rate. The increase in modulus of elasticity for this rate will probably vary significantly from that of the prototype. This variation will be most critical when the loading rate varies significantly with time, such as a blast load rather than a controlled constant load rate in a testing machine. The variation of strain rate with time will produce a modulus of elasticity in the model which is initially greater, reaches a maximum, and then decreases. The prototype modulus of elasticity will gradually increase for all increasing strain rates.

Gypsum concrete experiences higher ultimate strains than those for the prototype concrete, but the strains are not as large as those for microconcrete. There is an increase in ultimate strain with an increase in strain rate for gypsum concrete; this is not noticeable for microconcrete. The suitability of this gypsum concrete will depend on the particular model and loading rate. Scale factors which vary with strain rate must be applied to predict strengths and modulus of elasticity.

FINDINGS AND CONCLUSIONS

1. The dynamic properties of microconcrete with no. 4 maximum aggregate give good correlation with prototype concrete values.
2. The dynamic strength increase factors determined from prototype design curves are adequate for use in designing a model constructed from microconcrete with no. 4 maximum aggregate.
3. The increase in strength of the concrete with reduction in size of the specimen is adequately compensated for by testing at an earlier age, even for dynamic loadings.
4. Strains at ultimate in microconcrete are greater than strains at ultimate in the prototype concrete.
5. Gypsum concrete experiences dynamic strength increase factors somewhat less than those of the prototype concrete; the ratio of dynamic to static modulus of elasticity with increase in strain rate does not behave like the ratio for the prototype concrete.

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Table 1. Model Concrete Specimens

Type of Cylinder	Number of Cylinders	Size of Cylinder (in.)	Cement	Maximum Aggregate Size	Age at Test (day)
solid	16	1-1/2 x 3 ^a	Type III portland	no. 4	18
solid	16	1-1/2 x 3 ^a	Ultracal 30 gypsum	no. 4	2

^a Diameter x height

1 in. = 2.54 cm

Table 2. Results From Compression Tests on Solid Microconcrete Cylinders With No. 4 Aggregate

Cylinder No.	Stress Rate (psi/sec)	Strain Rate (in./in./sec)	Compressive Strength (psi)	Secant Modulus of Elasticity ^a (psi x 10 ⁶)	Strain at Maximum Stress (in./in. x 10 ⁻⁶)
1	35	approximately 1.5 x 10 ⁻⁵	4,100	2.51	3,640
2	35		4,240	2.19	4,200
3	35		4,280	2.25	4,210
4	35		3,970	2.13	3,710
5	2.20 x 10 ⁴	9.23 x 10 ⁻³	5,320	2.66	3,430
6	2.23 x 10 ⁴	9.97 x 10 ⁻³	4,960	2.49	4,120
7	2.15 x 10 ⁴	10.28 x 10 ⁻³	5,100	2.56	3,720
8	2.21 x 10 ⁴	8.85 x 10 ⁻³	5,490	2.84	3,900
9	2.77 x 10 ⁵	0.148	5,280	2.79	3,520
10	2.70 x 10 ⁵	0.153	4,950	2.55	3,820
11	2.71 x 10 ⁵	0.157	5,680	2.92	4,110
12	2.62 x 10 ⁵	0.151	5,550	2.70	4,160
13	4.75 x 10 ⁵	2.98	6,120	2.56	4,020
14	5.47 x 10 ⁵	3.73	5,810	2.99	3,150
15	5.35 x 10 ⁵	2.98	6,430	2.76	4,150
16	5.45 x 10 ⁵	3.30	6,020	2.23	4,550

^a To strain of 0.1%

1 psi = 6.89 kPa = .07 kg/cm²

1 in. = 2.54 cm

Table 3. Results From Compression Tests on Solid Gypsum Concrete Cylinders With No. 4 Aggregate

Cylinder No.	Stress Rate (psi/sec)	Strain Rate (in./in./sec)	Compressive Strength (psi)	Secant Modulus of Elasticity ^a (psi × 10 ⁶)	Strain at Maximum Stress (in./in. × 10 ⁻⁶)
1	35	approximately 1.7 × 10 ⁻⁵	3,890	1.93	3,000
2	35		4,040	1.94	—
3	35		4,200	2.06	—
4	35		3,980	2.07	—
5	1.30 × 10 ⁴	6.84 × 10 ⁻³	4,330	2.42	2,880
6	1.10 × 10 ⁴	4.62 × 10 ⁻³	4,260	2.65	2,270
7	1.00 × 10 ⁴	4.86 × 10 ⁻³	4,180	2.44	2,750
8	1.04 × 10 ⁴	5.02 × 10 ⁻³	4,250	2.65	2,450
13	5.53 × 10 ⁵	0.377	4,710	2.62	3,210
14	5.21 × 10 ⁵	0.357	4,550	2.60	2,900
15	4.11 × 10 ⁵	0.255	4,880	2.64	3,080
16	3.90 × 10 ⁵	0.328	4,410	2.60	2,950
9	4.52 × 10 ⁶	2.68	5,170	2.82	3,600
10	4.40 × 10 ⁶	3.01	5,400	2.10	4,400
11	4.86 × 10 ⁶	2.93	5,590	1.74	4,600
12	4.66 × 10 ⁶	2.62	5,410	2.21	3,660

^a To strain of 0.1%

1 in. = 2.54 cm

1 psi = 6.89 kPa = .07 kg/cm²

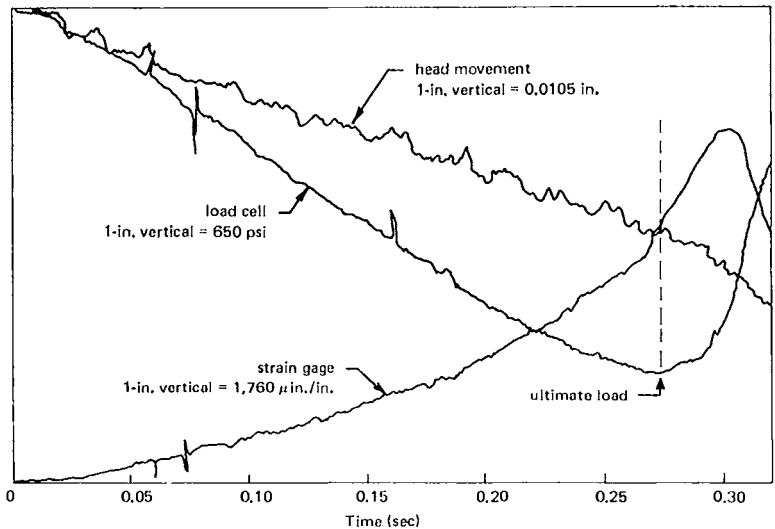


Figure 1. Typical computer plots of raw data for dynamic compression tests.

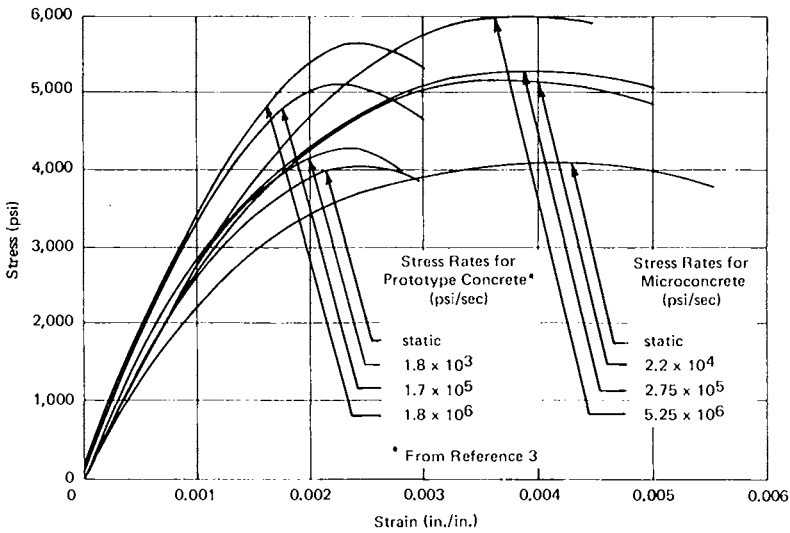


Figure 2. Comparison of stress-strain curves for various rates of loading.

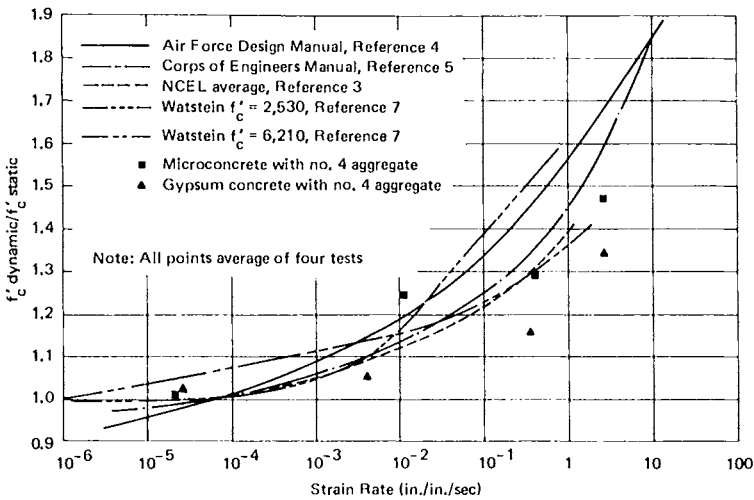


Figure 3. Ratio of compressive strengths versus strain rate.

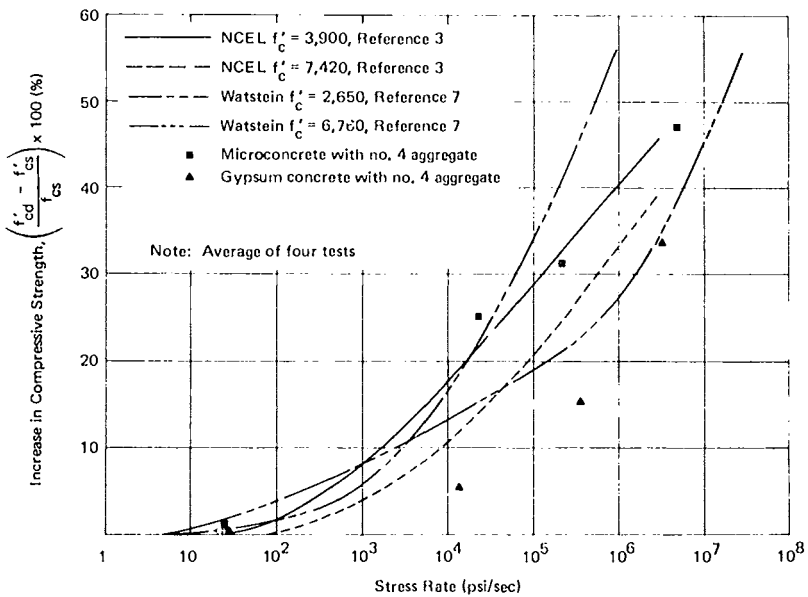


Figure 4. Percentage increase in compressive strength versus stress rate.

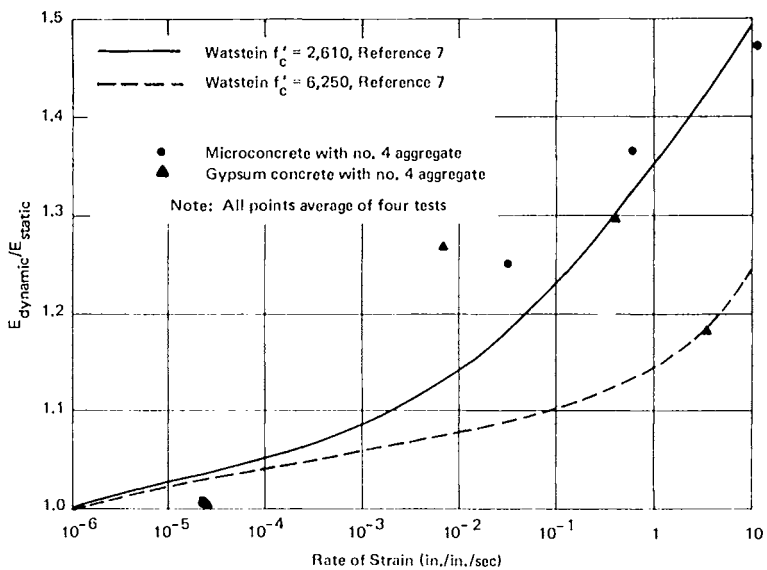


Figure 5. Strain rate versus ratio of modulus of elasticity.

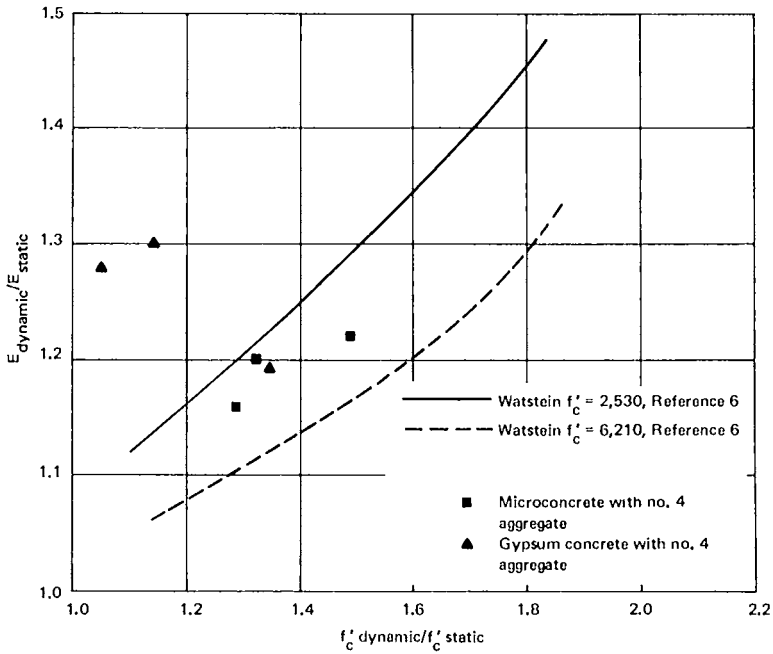


Figure 6. Ratio of dynamic strength increase factor versus ratio of modulus of elasticity.

Static and Dynamic Loading Tests on Two Small Three-Dimensional Multistory Reinforced Concrete Frames

By Grant K. Wilby, Robert Park,
and Athol J. Carr

Two identical small scale six storey three dimensional reinforced concrete framed structures were constructed consisting of columns, beams and floor slabs. Each structure had a single bay in each direction and was approximately one fifth of full size. All members were designed for ductility according to the seismic design requirements of ACI 318-71. One structure was subjected primarily to static lateral loading and the other structure was subjected primarily to dynamic loading on a shaking table.

The lateral load strength of the statically loaded structure was accurately predicted when the actual properties of the structure were taken into account. Severe stiffness degradation occurred during cyclic loading in the inelastic range. The displacement response of the dynamically loaded structure was compared with the displacement response predicted by a dynamic frame analysis computer program and the accuracy of the predicted response was found to be extraordinarily dependent on the stiffness and damping. The tests revealed that under lateral loading significant torsion is induced into the beams at right angles to the direction of loading and this may lead to severe torsional cracking of those beams and a consequent decrease in stiffness and strength of the framed structure.

Keywords: beams (supports); columns (supports); cracking (fracturing); ductility; dynamic loads; dynamic response; dynamic tests; frames; models; reinforced concrete; static loads; strength; structural design.

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INTRODUCTION

In recent years the advent of large digital computers and the development of numerical integration techniques have enabled major advances to be made in the nonlinear dynamic analysis of structures responding to earthquake motions. In turn, the mathematical model used to represent the structure has markedly increased in complexity as the effect of further parameters which affect the response of the structure have been included. Given the complexity of the mathematical model and the unique transient nature of the earthquake to which it is subjected, it is difficult for the designer to develop a feel as to whether the response predicted by a nonlinear frame analysis is reasonable. Ultimately, the only means available to verify this predicted response is to carry out tests on the completed structure. Existing structures can be used for this purpose if it is desired to test the structure using small amplitude sine waves in the elastic range, but unfortunately those results cannot be extrapolated to the nonlinear behaviour of a structure subjected to a major earthquake. The choice is then left of either testing small scale models of the prototype or testing full scale models of selected critical parts of the structure and then deducing the behaviour of the complete structure from the behaviour of the parts of the structure. The latter method has generally been restricted to static tests and suffers in that the testing of only a part of the structure is necessarily an approximation of the real situation, and important interaction



features of the behaviour of the structure may not be recognised because their effects were not apparent in that portion of the structure selected for testing. The testing of small scale models introduces the problem of accurately modelling the prototype structure and the small size of the models often means that measurement of their response is difficult.

The aim of the research reported in this paper was to study experimentally the behaviour of small scale reinforced concrete framed structures and to examine the accuracy of some of the available analytical techniques. Two identical small scale six storey reinforced concrete framed structures were constructed consisting of columns, beams and floor slabs. A complete three dimensional structure was tested, rather than a two dimensional frame, in order that the influence of the floor slabs and lateral beams on the response of the structure could be evaluated. Each complete small structure was six storeys in height, had a single bay in each direction, and was approximately one fifth of full size. This was the largest size feasible which could be tested using the available shaking table. One structure was subjected primarily to static lateral loading and the other structure was subjected primarily to dynamic loading on the shaking table. All lateral loading was applied in the direction of one principal axis of the building. The description of the small structures and the results of the tests may be seen reported in more detail elsewhere (1).

DESIGN OF THE SMALL BUILDING STRUCTURES

Initial Considerations

Ideally, for model reinforced concrete structures it is convenient to use steel reinforcing and microconcrete with the same density and strength values as is used in prototype structures. However, if a perfect model is to be constructed it is necessary to satisfy the similitude relation $\rho_m = (f/L)\rho_p$, where ρ_p = density of prototype material, ρ_m = density of model material, f = prototype to model stress scaling factor, and L = prototype to model geometrical scaling factor. If $L = 5$ and $\rho_m = \rho_p$ are used, then it is obvious that this similitude requirement can only be met by a substantial alteration in the stress scaling ratio f , which would require the use of unreasonably weak model materials. If the same density and strength of materials is to be used for model and prototype one answer to the similitude requirement is to increase the mass of the model by placing additional weights on the model structure. If $L = 5$ this means that the weight of the model structure would have to be increased fivefold.

An alternative is to increase the accelerations applied to the model structure rather than to attach additional weights. This presents little difficulty in so far as the horizontal accelerations are concerned, since the earthquake motions applied to the shaking table can be "speeded up". However the increase

in accelerations in the vertical direction would require the introduction of an auxiliary gravitational field, such as placing the model in a centrifuge which is not feasible for reasonably large models. Nor is it practical to test the model immersed in an inverted position in a dense fluid so that the upthrust experienced by the model more than compensated for the weight of the model. Other possible means of increasing the vertical accelerations are to use magnets to exert a vertical force on the steel contained in the structure, or to increase the vertical compression in the columns using prestressing. The first of these has the difficulty of achieving an even magnetic field. Also prestressing of columns is only a part solution; for example, it provides a constant axial compression up the height of the columns, rather than one which decreases up the height, and problems may be encountered in fitting the prestressing wires.

The solution adopted for this test program was not to model a prototype structure but to design and construct a small reinforced concrete building, the response of which is analysed strictly in terms of its own behaviour. A consequence of building a structure at a smaller scale, using materials of prototype density and strength, is that the dead load of the structure decreases more rapidly than the strength of the sections, because the dead load moment reduced by a factor of $1/L^4$ whereas the moment of resistance of the section reduces by a factor of only $1/L^3$, if the ratio of steel to concrete area remains the same, where L is the geometrical scaling factor. If the steel area is reduced by a factor of $1/L$ this results in undesirably low steel percentages and so the size of the section must be reduced. This results in more slender members in the small structure than in the prototype structure.

Description of the Small Structure

The small structure finally evolved was a building six storeys high, with a 24 in (610 mm) storey height between beam centres. It had a single bay in each direction with a column centre to centre distance of 40 in (1016 mm). The columns were 2 in (50.8 mm) square and were connected by edge beams 2.4 in (61.0 mm) deep overall and 1.2 in (30.5 mm) wide. A 1.0 in (25.4 mm) thick slab was cast monolithically with the columns and edge beams. Two such structures were constructed. The dimensions of each are given in Fig. 1.

Because of the requirement of as much mass on the small structure as possible, in order to develop significant inertia loads under dynamic loading, it was decided to add some additional dead loads which would normally occur in a structure, such as finishes on floors, suspended ceilings and partition walls which may account for about 35 lb/ft^2 (1.68 kN/m^2). For the small structure 7 lb/ft^2 (0.34 kN/m^2) was allowed for these features. In addition, the New Zealand Code then current (2) considered a live load of 20 lb/ft^2 (0.96 kN/m^2) to be present in typical office building during seismic loading, and so a load of 27 lb/ft^2

(1.29 kN/m^2) was applied at each floor level by means of attached weights.

The applied gravity loads were still very much less than that which would have been required for a perfect model. A perfect one fifth scale model would have required the attachment of about 1,744 lb (7.76 kN) of weights per floor, compared with the 282 lb (1.25 kN) per floor actually attached to each small structure. For a perfect model the attachment of the required weights to each floor level, such that the distribution of mass over the structure remained unaltered and no undesirable structural actions were introduced, would be difficult to achieve satisfactorily; also, such significant weights would have interfered with the measurement and/or visual observation of the response of the structure.

Aspects of the Design of the Small Structures

Each small structure was designed using the following load factor equations: $U = 1.4D + 1.7L$, $U = 1.05D + 1.3L_e + E$, and $U = 0.9D + E$, where U = required strength, D = service dead load, L = service live load, L_e = live load present during seismic loading = $L/3$, E = seismic load = CW_t , W_t = total gravity load present during seismic loading = $D + L_e$, and C = seismic coefficient = 0.15. In the design the small structure D was considered to be the actual weight of the structure plus 7 lb/ft^2 (0.34 kN/m^2), and L was taken as 60 lb/ft^2 (2.87 kN/m^2). 10% of the total lateral seismic load acting on the building was applied at the top of the building and the remaining 90% was triangularly distributed up the height of the building. These design loading requirements were typical of the then current New Zealand code (2).

The seismic loading was considered to act in the direction of one principal axis of the building. The design moments and forces for the frame members under gravity and seismic loading were obtained from structural analysis assuming linear elastic behaviour. The design moments for the floor slabs were found by yield line theory assuming fixed edges with a ratio of maximum negative to positive moments of 0.71. For the gravity loading case $U = 1.4D + 1.7L$ it was assumed that the torsional moments induced in the edge beams by the loading on the slabs was equal to that required to resist the moments from a negative moment yield line in the slab along the slab-beam junction. The beams were designed for that torsion as well as flexure and shear force. However, as is customary in design, no consideration was given to the torsion induced in those beams perpendicular to the direction of seismic loading when the beam-column joints rotate during the sidesway deflection of the structure.

All members were designed using the strength method of ACI 318-71 (3), and the special provisions for seismic design of Appendix A of the Code were conformed with. Fig. 1 shows the reinforcing details for the frames of the small structures in the

direction of the seismic loading. The beams in the direction of seismic loading were referred to as "longitudinal beams". For the beams at right angles to the direction of seismic loading, referred to as "lateral beams", the distance between the top and bottom longitudinal reinforcing steel was two diameters less than in the longitudinal beams, due to cross over of the steel in the beam-column joint core. Note that the slab, beam and column reinforcing steel were identical in each storey; that is, there was no variation of reinforcing with height of the structure. The steel was of Grade 40 quality. The columns had a reinforcement ratio of $\rho_t = 2.43\%$ and the beams had an equal tension and compression steel ratio of $\rho = \rho' = 2.01\%$ based on the web width. The stirrups in beams and the ties in the columns, particularly the beam-column joint cores, were as required by Appendix A of ACI 318-71 (3) to prevent shear failure and to ensure adequate ductility in the event of the formation of plastic hinges.

The diameter of longitudinal reinforcing (0.176 in = 4.47 mm) used, and thickness of concrete cover to that steel (0.3 in = 7.6 mm) may appear to be large relative to the size of the beam and column sections (see Fig. 1), but this is a consequence of using slender members. A prototype structure five times larger would, using this steel, have bars of size $5 \times 0.176 = 0.88$ in (22 mm) which would be No. 8 bars, and a concrete cover of 1.5 in (38 mm). Also, the long anchorage length of beam steel indicated in Fig. 1 was mainly a consequence of the use of plain round bars and the small member sizes. According to the ACI Code (3) the required development length for deformed Grade 40 bars of 0.176 in (4.47 mm) diameter is $l_d = 2.8$ in (71 mm); for plain round bars in a confined region then, approximately $l_d = 2.8 \times 2 \times \frac{3}{4} = 3.7$ in (94 mm). Conservatively, if an anchorage length of $l_e = 1.1$ in (28 mm) is allowed for the right angle bend used, the further 2.6 in (66 mm) required beyond the bend results in the length of anchorage shown in Fig. 1.

Fig. 2 shows a photograph of the reinforcing details of a typical beam-column joint and Fig. 3 shows another view of the reinforcing during construction.

CONSTRUCTION OF THE SMALL BUILDING STRUCTURES

Concrete

The maximum size aggregate used was $\frac{3}{16}$ in (4.76 mm) and the aggregate grading is shown in Table 1. The aggregate/cement ratio was 4.0 and the water/cement ratio was 0.7, both by weight. The high cement and water content was necessary to obtain reasonable workability with the relatively small aggregate with its greater surface area to volume ratio.

The average compressive strength given by 6 in diameter x 12 in (154 mm x 305 mm) cylinders at 28 days was $f'_c = 4,600$ psi (31.7 MPa). Some 2 in diameter x 4 in (51 mm x 102 mm) cylinders were also cast to check any differences in strength given by the

two size cylinders but no discernible trend could be established, in part because of the extremely variable results given by the smaller cylinders. The average modulus of rupture as found from 3 in x 3 in x 12 in (76 mm x 76 mm x 305 mm) prisms at 28 days was $f_r = 590$ psi (4.07 MPa), and thus $f_r = 8.7\sqrt{f'_c}$ psi ($0.73\sqrt{f'_c}$ MPa).

The effect of scaling a concrete mix to produce micro-concrete is often to increase the ratio of tensile to compressive strengths of the concrete. This results in fewer cracks in the model structure and hence cracking similtude between prototype and model is often poor. However, the modulus of rupture of the microconcrete used in these tests was not unduly high, $f_r/f'_c = 0.128$, and hence this difficulty was not expected.

Steel

The reinforcing steel was from plain round wire. The wire had been produced cold-drawn but was then annealed by the manufacturer to achieve a relatively flat yield plateau and coiled into bundles for delivery. The wire was straightened on arrival in the laboratory using a series of rollers. The yield strength was defined as the stress corresponding to a permanent strain of 0.2% and is given in Table 2 for each size of wire used. Fig. 4 shows a typical stress-strain relation measured for the longitudinal steel used in the beams and columns. The wire was not deformed and was supplied with a relatively smooth surface. To improve the bond characteristics the longitudinal reinforcing wire was placed in a fog room for a period in order to rust and pit the surface. Deformed bars were not available at the reduced size required for the small structures.

Construction Procedure

It is desirable to place construction joints in such positions that they have minimal effect on the behaviour of the structure. Normal multistorey construction methods were ruled out because of the difficulty of accurately placing reinforcing steel in the members, the number of construction joints involved, and the difficulty of ensuring good concrete compaction at the bottom of columns. The method of construction finally evolved was to cast the two longitudinal frames of the structure in a horizontal plane and then to stand these vertically while the floor slabs and lateral beams were cast between those frames. Thus there were two construction joints per floor, one on each side of the slab.

The longitudinal column steel comprised of single lengths of wire. The longitudinal column steel was anchored in a 2.5 in (64 mm) high column stub at the top of each column. At the base of each column the longitudinal column steel was welded to a steel footing plate which was to be bolted on to the shaking table. A small shear key $\frac{1}{2}$ in (12.7 mm) high was also present at each column footing to help transfer the shear force from column concrete to steel footing. Reinforcing cages for the slabs

longitudinal beams and the twelve one half lateral beams, required for each one half structure, were constructed initially and were then tied to the longitudinal column reinforcing. The column ties and most of the slab reinforcing were then tied into position. Over 3000 ties and stirrups were required for each small structure. The construction of the reinforcing cage for a one half structure is shown in Fig. 3. Once the reinforcing cage for the one half structure was completed it was lifted and placed into a mould for the longitudinal frame. Only the longitudinal frame was cast in this mould; the lateral beam steel and the slab steel protruded out of the top of the mould during this operation. Perspex was chosen as the main material for the moulds. The perspex was bolted together and on to the steel backing frames.

The concrete for the longitudinal frames for the two one half structures was cast on consecutive days. After 7 days the perspex moulding was stripped and the concrete frames were clamped to the steel backing frames for support and lifted to their final vertical position on the shaking table where the steel footing plates were bolted down. The longitudinal reinforcing in the lateral beams, which protruded from each side of the concrete frames, were butt welded together at mid span. Slab reinforcing in the lateral direction were lapped and the remaining slab reinforcing in the longitudinal direction was placed. Spacer wires were used to ensure correct thickness of concrete cover. The perspex and plywood moulds for the slabs and the lateral beams were then placed and the concrete cast at the rate of two floors per week. All concrete was kept wet for at least 7 days after casting. Finally the steel backing frames were removed to leave the completed small structure standing. The structures were painted white to facilitate crack detection.

The weights attached to each floor consisted of twenty five steel blocks, each 4.4 in (112 mm) square and weighing 11.3 lb (50.3 N), which were attached at 7.8 in (198 mm) centres to each floor slab. Each steel block was attached by a single bolt with a steel washer between block and concrete to reduce the contact area.

DETERMINATION OF BEAM AND COLUMN PROPERTIES

Test Specimens

It was considered advisable to check on the accuracy with which the failure loads and deformation characteristics of the elements of the small structure could be predicted by theory, since this would govern the accuracy of any analysis carried out on the complete structure. Beam and column specimens, typical of those present in the complete structure, and to the same scale, were tested. Moment-curvature, load-deflection, crack patterns, and crack widths were measured (1).

Beam and Column Tests

Four beams were cast with the cross section shown in Fig. 5 and with a column stub present at midspan. The beams were tested simply supported over a span of 40 in (1.016 m), which is the centre to centre distance between columns in the complete structure, and subjected to a central point (vertical) load. The load was applied monotonically to two specimens and cyclically to the two other specimens using a screw jack. Thus displacements were controlled accurately. The load was measured by a load cell. Electrical resistance strain gauges with a 5 mm gauge length were placed on the top and bottom longitudinal wires on both sides of the column stub in the plastic hinge region and were used to calculate the curvatures at the critical regions.

Five columns were cast with the cross section shown in Fig. 6 and with a beam stub at midspan. The columns were tested horizontally, simply supported over a span of 24 in (0.61 m), which is the interstorey height. The columns were subjected to a central point (vertical) load and to an axial (longitudinal) load using screw jacks. The axial load was maintained constant at a particular level (0, 1000, 2000, 3000 or 4000 lb, where 1 lb = 4.45 N) and the vertical load was applied either monotonically or cyclically. Electrical resistance strain gauges on the longitudinal reinforcing wires each side of the column stub were again used to measure curvatures.

Test and Theoretical Results

Typical measured moment-curvature relations for a beam and column are shown in Figs. 5 and 6. Also shown are the theoretical moment-curvature relations for monotonic loading derived assuming plane sections remain plane and using the equilibrium requirements and mathematical functions which represent the stress-strain curves for the steel and concrete. The stress-strain curve for concrete used was that due to Kent and Park (4); for the steel a series of piece-wise functions was fitted to the actual stress-strain curve. An iterative procedure was used to calculate the moment-curvature relation which calculates the moment and curvature corresponding to a range of extreme fibre concrete compressive strains.

The theoretical and measured monotonic moment-curvature curves generally agreed well. The experimental moments varied by no more than 6% from the theoretical moments and they erred on both the high and low side indicating that there was no consistent error involved. Some previous investigators, for example (5,6), have found that size effects may cause the experimental moment strength of small members to be consistently greater than theoretical member strength, but that was not found here.

A greater variation between the measured cyclic loading moment-curvature relations and the theoretical monotonic loading

relation was observed, as is seen in Figs. 5 and 6. Generally the experimental moments were higher and this could be accounted for by strain hardening of steel. Nevertheless, it is evident that the monotonic relation gives a reasonable envelope curve. These measured cyclic load curves were strongly influenced by the stress-strain curve for the steel, due to full depth cracks, which in turn is strongly influenced by the Bauschinger effect (7).

DYNAMIC TESTING FACILITY AND FRAME ANALYSIS COMPUTER PROGRAM

The Shaking Table

The shaking table facility used is similar to one which had been installed at the University of Calgary (8). The table was activated by an MTS closed loop electro-hydraulic dynamic testing system. The controlled parameter of the shaking table was the displacement. The table is 4 ft 6 $\frac{3}{4}$ in (1.39 m) square and is supported on two opposite sides by bearings which run on 1 $\frac{1}{2}$ in (38 mm) diameter steel shafts. The maximum capabilities of the system are: table displacement (double amplitude) = 4 in (102 mm), table velocity = 12 in/sec (305 mm/sec), and table driving force = 5 tons (50 kN). The displacements of the table and the structure during dynamic loading were measured using Hewlett Packard linear variable differential transformers with a stroke range of 6 in (152 mm) double amplitude; displacements were recorded using a Rapt chart recorder.

Before the six storey small structures were constructed and tested a single storey structure was constructed to gain experience in the techniques to be used and tested to check the performance of the testing equipment. The results of this preliminary test are not reported here but the details may be seen elsewhere (1).

The Dynamic Frame Analysis Computer Program

For the analysis of the small structures use was made of a versatile two dimensional dynamic inelastic frame analysis computer program which has been developed by Sharpe (9). The program achieves a time-wise solution for the equations of motion of a structure for a given earthquake record in a step by step manner. The program permitted modelling of elasto-plastic, bilinear and Ramberg Osgood moment-curvature loops for the beam plastic hinges; for the columns an elasto-plastic loop was used with allowance made for variation of moment capacity with axial load. The program enables allowance to be made for shear deformations, axial deformations and rigid end blocks. The program produces tabulated values of deflections and moments and axial forces in members at specified time steps, and a display of plastic hinge development. The program could be used for static analysis by applying a step by step incremental static loading to the structure.

TEST RESULTS FOR SMALL STRUCTURE 1

Tests Conducted

Small Structure 1 was tested initially under static cyclic lateral loading applied along one principal axes of the building in the direction of the longitudinal frames. This static load test was to check the strength and stiffness of the multistorey structure against theoretical predictions, and to determine the degradation of these properties with cyclic loading. Upon completion of the static load tests, the structure was subjected to dynamic sinusoidal shaking tests.

Static Loading - Experimental Results

The static lateral loading was applied by placing yokes around the two nearside columns at each floor level to which were attached short lengths of chain. Each chain was pulled horizontally in the direction of loading by a wire rope which ran over a pulley mounted on a reaction frame. From the wire rope was suspended a bucket into which steel pellets were placed to apply the lateral load. When the direction of lateral loading was reversed the twelve sets of yokes, chains, wires, pulleys and buckets were removed and placed on the other side of the structure. The distribution of lateral loading up the height of the structure followed that assumed in the design code (2) and is illustrated in Fig. 7.

The main testing program consisted of one lateral loading cycle in the elastic range followed by four cycles into the yield range. Some minute shrinkage cracks were present before testing began, grouped mainly in the columns. A 10 minute wait was necessary after applying each lateral load increment for the deflections to stabilize. Fig. 8(a) shows the sections of the frame where yielding of reinforcing was judged to have occurred when a mechanism developed in the cycles in the inelastic range. Little crushing of concrete occurred, even at the column bases, probably because the large permanent tensile strains in the reinforcing steel often means that the concrete was not in compression when the load was reversed. Fig. 9 shows the experimental static lateral load versus lateral deflection measured at the top of the structure.

All beam-column joint cores suffered severe diagonal tension cracking and the cover concrete spalled off there at lower floors particularly on the side opposite to that from which the longitudinal beams protrude. Perhaps the most startling feature of the tests was the damage caused to the lateral beams by torsion. On each cycle large torsional cracks appeared in the lateral beams and diagonally across the corner of each slab; these cracks did not close on the reverse cycle. Concrete cover began spalling off the lateral beams after only a few loading cycles and some of these beams were severely distressed by the completion of the static cyclic load tests. Photographs of some

of the members near the end of the static cyclic load tests are shown in Fig. 10.

The measured static lateral load-deflection curves illustrated in Fig. 9 show a considerable degradation of stiffness during the cycles of lateral loading. The slopes of the lines connecting the extremities of the load-deflection loops indicates that the stiffness of the structure with respect to lateral loading in the last cycle of loading was only approximately 20% of the initial stiffness. The slope of the loops is also very different from elasto-plastic behaviour and show rounding and pinching due to the Bauschinger effect, the opening and closing of cracks and slip of bars in the anchorage zones (1,7). The measured ultimate lateral load of the structure reduced to about 84% of the original measured ultimate load after the four load cycles in the inelastic range. The maximum imposed displacement ductility factor was about 4, based on a displacement at first yield calculated from the initial elastic stiffness and the ultimate load. It is possible that the structure could have attained its original ultimate load on subsequent inelastic cycles, since the load-deflection curves were still rising at the end of the loading runs.

Static Loading - Theoretical Results

The step by step structural analysis computer program developed by Sharpe (9) was used to determine the theoretical lateral load-deflection relation and theoretical ultimate lateral load. The assumed moment-curvature relations were elasto-plastic and were calculated from the section properties of the small structure. The flexural strength assumed was that calculated at a curvature of 0.01 radian/in (0.004 radian/mm). In the loading runs up to first yield it was found from the load-deflection curves that the structure was slightly less stiff (about 12% at one half of the measured ultimate load) than predicted using the cracked member stiffnesses. The theoretical (predicted) lateral ultimate load was 1010 lb (4.49 kN) which was 77% of the measured ultimate lateral load. The theoretical failure mechanism was of a column sidesway type with plastic hinging occurring at the column bases, at the top of the columns in the third storey, and in the beams of the lower storeys, as illustrated in Fig. 8(b). The effect of torsion occurring in the lateral beams and strain hardening of the longitudinal steel were considered to be the two major reasons for the discrepancy between predicted and measured ultimate loads.

Torsion in the lateral beams effectively increases the flexural strength and stiffness of the longitudinal beams, but does not effect the columns (1). Torsion is induced into the lateral beams due to compatibility of deformations when the longitudinal beam rotates at the negative moment plastic hinge at the column face. This action is illustrated in Fig. 11. Two possible yield mechanisms can occur. In case 1 when plastic rotation occurs in the beam at the column face the tension crack

in the slab will extend across the corner of the slab and cause the slab to pull the top of the lateral beam over resulting in the possible development of the torsional strength of the lateral beam. If the critical section in the lateral beam for torsion is d from the column face, the total moment strength of the longitudinal beam and lateral beam-slab system is $M_u + T_u + m_u d$ where M_u = longitudinal beam flexural strength, T_u = lateral beam torsional strength, m_u = slab flexural strength per unit width and d = effective depth of beam. Alternatively in case 2, if the lateral beam has a high torsional strength a yield line may occur along the slab-lateral beam junction. The total moment strength of the longitudinal beam and lateral beam-slab system is then $M_u + m_u L_1$, where L_1 = one half of the clear span of the lateral beam. The lesser of the total moment strengths given by cases 1 and 2 will govern. For the small structure tested it was found that case 1 was the critical case and that the total flexural strength of the longitudinal beam and lateral beam-slab system was about 16% greater than the longitudinal beam flexural strength. The inclusion of this torsion effect increased the predicted ultimate lateral load from 77% to 84% of the actual failure load.

The effect of strain hardening of the reinforcing steel was not included in the idealized elasto-plastic moment-curvature relations used in the theoretical analysis referred to above. A more refined theoretical analysis was conducted including the increase in flexural strength due to strain hardening of steel after yield and it was found that the measured ultimate lateral load would be reached by the strain hardening structure.

Dynamic Loading - Experimental Results

Shaking table tests, involving a sinusoidal forcing function, were conducted on the small structure when the static loading cycles had been completed. At this stage the nonlinearity of the structure meant that the stiffness varied with the deflection which in turn meant that the structure did not have unique natural frequencies. If the natural frequency of the structure at a particular deflection coincided with the sinusoidal forcing frequency, then the deflection of the structure would increase, altering its stiffness and thus its natural frequency; the natural frequency of the structure would no longer coincide with the forcing frequency so the deflection would subsequently decrease. When the deflection reduced to close to its original value, the natural frequency of the structure would be close to that of the forcing function and so the deflection would increase again. This cyclic process appeared to repeat itself many times.

Thus it was not possible to identify the natural frequencies of the structure absolutely, but only to define a frequency close to the centre of a band of frequencies within which a natural frequency might commonly lie. It was decided to destroy the structure by shaking it as closely as possible to its natural frequencies. The base amplitude and the forcing frequency were

measured accurately; the base acceleration was not measured directly but was calculated from the frequency and displacement. The applied base displacements and accelerations are indicated in Table 3. From free vibration tests before the static cyclic loading had been applied the natural frequency for the first mode of vibration had been found to be 2.67 Hz, compared with the 1.00 Hz shown in Table 3, indicating how the structure had decreased in stiffness as a result of the static cyclic load tests.

The dynamic testing program was simply to shake the structure as severely as possible at its first natural frequency; if collapse did not occur then the structure would be shaken at its second natural frequency, and so on until collapse of the structure was achieved. Only when the structure was shaken at its fourth natural frequency did the structure slowly collapse in a ground floor column sidesway mechanism with plastic hinges at the top and bottom of the columns of the first storey. The length of time for which the structure was severely shaken appeared to be an important factor in determining whether or not collapse resulted. Collapse evidently occurred only after severe strength degradation had taken place.

TEST RESULTS FOR SMALL STRUCTURE 2

Simulated Earthquake Shaking Tests

Small Structure 2 was tested under dynamic loading applied by a shaking table. The direction of motion was along one principal axis of the building in the direction of the longitudinal frames. This test was to check the actual response of the structure against theoretical predictions, and to examine the damage inflicted on the structure by very severe earthquake motions. Only one earthquake record could be used in the yield range, since if the structure yields while responding to that earthquake its response to the second and subsequent earthquakes will be altered.

The N-S component of El Centro May 18 1940 earthquake is the record which has been most often used by researchers over the years and this record was chosen for the simulated earthquake shaking tests. For perfect modelling, the laws of similitude indicate the following relationships between the model and prototype parameters: acceleration $a_p = a_m$, velocity $v_p = L^{1/2} v_m$, displacement $L_p = L L_m$, and time $t_p = L^{1/2} t_m$, where $L = \frac{p}{m}$ the ratio of prototype to model length scale ($L > 1$). These relationships indicate that the earthquake record should be speeded up by a factor of $L^{1/2}$ when used in conjunction with the small building. This means that the frequency components of the earthquakes are increased by $L^{1/2}$, which for $L = 5$ as in this series of tests is 2.24. The fundamental frequency of a perfect model would be increased by $L^{1/2}$ also, but since the small structure actually tested is not a perfect model the actual increase will be less than that. Thus the fundamental frequency

of the structure has reduced in relation to the frequency components of the earthquake.

The El Centro record speeded up by a factor of 2.24 became the standard earthquake used for these tests (run number 1 of Table 4) and any further variations in speed or magnitude introduced were referenced to this waveform. The small structure was subjected to eight simulated earthquake test runs of increasing maximum acceleration, as shown in Table 4. In the initial test runs the maximum accelerations were increased by increasing the displacement of the shaking table until the limit for the table was reached; for the subsequent runs the accelerations were increased by speeding up the earthquake record. The displacement record of the earthquake was used to drive the shaking table in each run.

Experimental Results

Shrinkage cracks appeared in the structure after casting and some cracks were present during all tests. The test program commenced with a free vibration test, followed by sinusoidal forced vibrations applied within the elastic range. Next the structure was subjected to the eight simulated earthquake shaking tests summarised in Table 4. Finally an attempt was made to collapse the structure by sinusoidal excitation at its resonant frequencies. The structure at the completion of the shaking table tests is shown in Fig. 12.

Displacements in the direction of motion were measured at the base and at the top of the second, fourth and sixth storeys. The measured displacements at the top of the structure for runs 1, 4 and 8, representing El Centro N-S 1940 shaking scaled up to 0, 60% and 300%, are illustrated in Fig. 13. The complete set of measured displacements may be seen elsewhere (1).

Calculation of Predicted Displacement

The predicted displacements were computed using the two dimensional dynamic inelastic computer program developed by *type* (9). Generally it was found that the displacement response of the structure could be predicted reasonably accurately providing the actual stiffness and amount of damping of the structure were closely estimated. Initially the predicted displacements of the structure were obtained assuming that the stiffness of the beams and columns were accurately represented by their cracked stiffnesses and that the damping was about 5% of critical damping. Under these conditions the analyses predicted yielding of the structure at a number of locations but examination of the predicted displacement waveforms revealed that they did not resemble the measured ones at all. Next, the stiffness of the structure was measured from the predominant frequency component of the measured displacement waveforms in the initial strong motion part of each test run and a trial and error process used to determine the corresponding amount of damping.

resulting stiffness values found were very much less than the cracked stiffnesses, and the values of damping determined were substantially less than the values obtained from free vibration tests immediately following each simulated earthquake test run. Table 5 gives the damping and stiffness values found to fit the initial part of each test run the best. The displacement response predicted using the damping and stiffness values shown in Table 5 are illustrated in Fig. 13. It is seen that generally while agreement between predicted and measured displacement in the initial part of the earthquake record was assured, the accurate representation over the remainder of the record depended on the extent to which those values represented the state of the structure beyond the initial region. While the structure was barely damaged, such as during test run 1, the predicted and measured displacement waveforms were in excellent agreement. However as the beam reinforcing anchorages began to fail and non-linearity set in, the predicted waveform became accurate for an ever-decreasing length of the earthquake record, finally culminating in test run 8 where only the first few cycles are predicted accurately. The value assigned to the second resonant mode damping in the theoretical analyses was found to have a negligible effect on the predicted displacement waveform, even for extremely large variations in damping.

The extremely large reduction in stiffness of the structure, due primarily to anchorage slip of the main beam reinforcing wires and to the presence of steel couples at the large full depth cracks just above the base of the columns, in combination with the speeding up of the earthquake record, had the unfortunate effect of reducing the fundamental resonant frequency of the structure as compared with those frequencies which contained most of the energy of the earthquake. Consequently, the earthquake energy input to the structure was low and the structure was relatively unaffected by the simulated earthquake test runs, to the extent that yielding of the structure as indicated by the theoretical analyses occurred only during simulated earthquake test run 6, and even then the plastic hinges were confined to the base of the columns. That this situation was occurring was apparent during the course of the dynamic tests, since no displacement drift of the structure was observed on the recorded displacement waveform.

Table 5 indicates that the stiffness of the sections reduced to 16% of the cracked section value by test run 8. One bound to the reduction in stiffness due to cracking is the stiffness given by the steel couple alone at the stage of full depth cracking. For the beam and column sections of the structure tested this stiffness was 45% and 51% of the cracked section values (with neutral axis in the section), respectively. The final stiffness of the model of 16% of the cracked section value is much less than these steel couple values and hence this low stiffness must have been primarily due to anchorage slip of the beams and to softening of the beam-column joint cores.

Sinusoidal Shaking Tests

The simulated earthquake test runs were followed by sinusoidal shaking tests at the resonant frequencies of the structure in an attempt to collapse the frames. The structure finally collapsed after sinusoidal resonance testing at its fourth natural frequency.

COMPARISON OF ACTUAL STRENGTHS OF STRUCTURES

Actual and Design Static Load Strengths

The six storey small buildings studied were designed to withstand an ultimate static lateral load of 0.15 g, distributed as shown in Fig. 7. The ultimate load actually measured (see Fig. 9) for the statically loaded structure was 0.459 g, which was an increase of over 200%. Some of the reasons for this increase are: strength reduction factors were used when calculating steel areas from design moments as in the ACI Code, steel in excess of that actually required for flexure was placed in the beams, column design moments were magnified due to low slenderness ratios, flexural strength of columns were increased to ensure that sum of column strengths at joint exceeded sum of beam strengths in that plane, effective flexural strength of the longitudinal beams increased due to interaction with slab-lateral beam system, actual strengths of concrete and steel were greater than the values assumed in design, and strain hardening of steel occurred. These factors are likely to be present in any building and will add significantly to the actual strength of the structure as built.

Actual Static and Dynamic Load Strengths

The dynamic accelerations at which the two small structures actually collapsed were very much greater than the "static" value of 0.459 g. Both small structures collapsed under sinusoidal resonance testing at their fourth natural frequencies. Small Structure 1, which had been severely damaged during the static tests when it withstood a static load of 0.459 g, collapsed at a base acceleration of 1.83 g, and Small Structure 2 collapsed at a base acceleration of 2.59 g. If a relative intensity index is defined as the maximum base acceleration during dynamic loading divided by the limiting base shear (equivalent static lateral strength) of the structure, then the relative intensity index is 4.0 for Small Structure 1 and 5.6 for Small Structure 2. Similar results have been reported by Otani and Sozen (10), whose small scale 1 bay, 3 storey frames withstood maximum accelerations ranging from 2.1 g to 3.4 g with a maximum relative intensity index of approximately 6.0.

It is apparent that the static lateral load strength of the structure bears no relation to the maximum acceleration that the structure can withstand during dynamic loading from ground shaking. The maximum base acceleration that a structure can withstand varies with the frequency of excitation and the