

Shear and Moment Capacities of Steel Fiber Reinforced Concrete Beams

By Roop L. Jindal

Synopsis: Tests were made on 44 beams to study the effect of steel fibers as shear reinforcement and to determine if there was any increase in the shear/moment capacity and change in the mode of failure. Span length of 30 in. (762 mm) was used for shear-span ratios (a/d) of 2.0 and 2.4, and 60 in. (1524 mm) for a/d ratios of 3.6 and 4.8. Steel fibers of 1% by volume were used in all SFRC beams. The variables were type of fibers, aspect ratio (l/d) of the fibers and the shear span ratios. Test results showed that shear and moment capacities of SFRC beams varied from 1.50 to 1.92 and 1.12 to 1.39 times, that of conventional reinforced concrete beam, respectively. Mode of failure changed from shear mode to moment mode when SFRC was used. Steel fibers having aspect ratio of 75 or thin fibers were found to be most effective for increasing the shear capacity of SFRC beams. A design method has been suggested for analysing and designing SFRC beams. Theoretical results based on this method compare favorably with the test results.

Keywords: beams (supports); fiber reinforced concretes; flexural strength; metal fibers; moments; shear properties; structural analysis; structural design.

ACI member Roop L. Jindal is Senior Engineer, Civil & Structures, Bechtel Power Corp. San Francisco. Dr. Jindal worked formerly as Associate Professor of Civil Engineering, Punjab Engineering College, Chandigarh, India, and at University of Sulamaniyah, Iraq. He has published a number of papers on various aspects of concrete and experimental stress analysis.

INTRODUCTION

A number of investigators (1,2) have shown that ultimate tensile strength of SFRC (steel fiber reinforced concrete) varies from 1.5 to 2.5 times that of plain concrete. Various methods have been suggested to predict flexural strength of SFRC (4). Steel fibers can be used effectively in conventional reinforced concrete beams, to increase the flexural and shear strength and crack resistance. Usage of SFRC in sections where shear stresses are very high, can eliminate some congestion of reinforcement. SFRC beams can absorb large amount of energy, which can be very useful in areas of seismic zones. The following investigation was made to study the effect of high strength brass coated and mild strength steel fibers as shear reinforcement, to determine if there was any increase in shear and flexural capacity of SFRC beams as compared to conventional reinforced concrete beam and to propose a suitable, simple method of analysis and design of SFRC beams.

EXPERIMENTAL PROGRAM

Description of the Beams

Tests were conducted on 44 beams. The beam had overall dimensions of 4 x 6 in. (100 x 150 mm) with effective depth of 5 in. (125 mm) and reinforced with 2 bars of 1/2 in. (12 mm) diameter of steel. Yield strength of the main tensile steel was 67.50 ksi (4745 kgf/cm²). The span of the beams was 30 in. (762 mm) for shear span - depth ratios (a/d) of 2.0 and 2.40, and 60 in. (1524 mm) for a/d ratios of 3.6 and 4.80. No vertical stirrups or bent up bars were used in any beam.

Details of Materials

Steel fibers -- Steel fibers, 1% by volume, were used in all the beams. Two different types of fibers were used in this study. Brass coated plain, high strength, circular steel fibers of sizes 1 x 0.01 in. (25.4 x 0.254 mm), 1 x 0.016 in. (25.4 x 0.406 mm) and 0.5 x 0.006 in. (12.7 x 0.152 mm) were used. These fibers were of tensile strength in the range of 270 to 400 ksi. (19,000 to 28,000 kgf/cm²). Mild steel fibers of aspect ratios 10, 25, 75, and 100 were obtained by cutting 30 gage i.e. 0.011 in. (0.282 mm) diameter wire. The wire had following properties:

Yield stress	=	54,015 psi (3818 kgf/cm ²)
Ultimate tensile stress	=	69,500 psi (4913 kgf/cm ²)
Percentage elongation	=	14.6
Modulus of Elasticity	=	32.2 x 10 ⁶ psi (2.27 x 10 ⁶ kgf/cm ²)

Concrete -- The concrete mix used had water/cement ratio of 0.60 by weight and the ratio of 1:3.0 of cement/sand. Natural river sand of fineness modulus of 3.25 was used. Ordinary portland cement was used.

Test details

Series A beams were conventional reinforced concrete. Beams B, C, D, E and F had mild steel wire fibers of aspect ratios 10, 25, 50, 75 and 100 respectively. Beams G, H and J had circular brass coated high strength steel fibers of sizes 1 x 0.01 in. (25 x 0.25 mm), 1/2 x 0.006 in. (12.5 x 0.15 mm) and 1 x 0.016 in. (25 x 0.4 mm) respectively. The beams were cast in steel molds and compacted by internal vibration. The beams were tested at age of 28 days under two point loading. Deflections and strain measurements were made at two sections of the beams as shown in Figure 1, at various stages of loading.

TEST RESULTS AND DISCUSSION

Table 1 gives the test results of different beams. The shear and flexural strength of Series A beams (conventional RCC test beam) were found to be 4.4 kips (2000 kgf) and 96 inch-kips (110.4 m-kgf) respectively. Shear and flexural strength ratios of SFRC beams were determined by dividing ultimate shear and ultimate moment of test beams by 4.4 kips and 96.0 in-kips respectively. Maximum shear stress was calculated as follows:

$$v = VQ/Ib$$

where V = maximum shear
 b = width of beam
 Q = first moment of the area about n.a.
 I = moment of inertia of uncracked transformed section

Shear Strength Ratio

Shear strength of SFRC beams increased up to maximum of 2.05 times that of conventional beams. M.S. (mild steel) wire fibers were quite effective in shear. Maximum shear strength ratio of 1.44 was obtained in case of series D and E beams having M.S. wire fibers of aspect ratios 50 and 75 respectively. In the case of high strength brass coated fibers, maximum shear strength ratio of 2.05 was obtained with SFRC beam having 1/2 x 0.006 in. (12.5 x 0.15 mm) size fibers.

Flexural Strength Ratio

Maximum flexural strength ratio of 1.59 was obtained in case of beams series D and E having M.S. wire fibers of aspect ratios 50 and 75 respectively. In the case of beams having high strength brass coated fibers, maximum flexural strength ratio of 1.38 was obtained in series H beams having 1/2 x 0.006 in. (12.5 x 0.15 mm) fibers.

Mode of Failures

Shear failure -- Plain conventional RCC beams having a/d of 2.0, 2.4 and 3.6 failed in shear. In this case cracks developed along the compressive stress trajectories in the shear-span. The shear crack appeared suddenly and upper portion of the shear-span was violently ejected upwards.

Initial moment and ultimate failure in shear -- SFRC beams of series C of $a/d = 3.6$, G of $a/d = 2.4$, and H of $a/d = 2.0, 3.6$ and 4.8 failed in this mode. In these cases, number of moment cracks formed in the initial stage of loading, but finally the beams failed due to formation of sudden shear crack.

Initial shear and final moment failure -- SFRC beam of series J of $a/d = 2.0$ failed in this mode. In this case there was formation of diagonal shear crack in the beginning somewhere in the shearspan, but subsequently moment cracks appeared and the beam finally failed in flexure. The initial shear crack did not propagate further.

Moment failure -- Most of the SFRC beams failed in flexure. A number of cracks formed in the central region. Near the failure there was an increase in the deflection with little or no increase in the load. The test was stopped at this stage.

Mode of failure changed from shear to moment in most of the beams as steel fibers were used. This shows that steel fibers are very effective as shear reinforcement.

Effect of Aspect Ratio and Type of Fibers

Figure 2 shows the effect of aspect ratio on ultimate shear stress, for M.S. wire fibers for shear span ratios of 3.6 and 4.8. Though most of these beams failed in flexure, maximum shear stress increased from 352 psi (24.8 kgf/cm²) to 622 psi (43.79 kgf/cm²) when aspect ratio was increased from 10 to 75. There was decrease in the maximum shear stress when aspect ratio was increased to 100. The beams having brass coated high strength steel fibers, indicated similar behavior. The maximum shear stress occurred in SFRC beam having $1/2 \times 0.006$ in. fibers of aspect ratio of 83.3. There was decrease in the maximum shear stress value when fibers of size $1 \times .01$ in. having aspect ratio of 100 were used. If aspect ratio is less number of wires increase in a section, for the same amount of fibers. Thus the shear

strength of SFRC beams should increase with the decrease in the aspect ratio of the fibers. However, the efficiency of the fiber decreases with the decrease in length due to poor anchorage. Therefore, there has to be some optimum value of the aspect ratio for the maximum shear strength. Based on the limited test data of this study, it seems that the optimum value of aspect ratio is near about 75 for maximum shear strength.

Mild steel wire fibers seems to be quite effective in increasing shear/ flexural capacity of the SFRC beams. These fibers are cheaper and can be fabricated easily in any workshop, in India or in any other developing country.

Load vs Deflection

Typical load-deflection curves of the conventional RCC beams and SFRC beams having 1/2 x 0.006 fibers, for shear span ratios of 3.6 and 4.8, are shown in Fig. 3. The ultimate deflections of SFRC beams were about 3-4 times that of conventional RCC beams, for shear-span ratios of 3.6 and 4.8. It can be concluded from these curves that SFRC beams have higher toughness. The toughness is defined as the area under the load-deflection curve. Toughness ratio vs shear-span ratio is shown in Fig. 4. The toughness ratio is the ratio of toughness of SFRC beams to that of conventional RCC beam. The maximum toughness ratio of 7.85 was observed in SFRC beam having 1/2 x 0.006 in. fibers, with shear-span ratio of 3.6.

DESIGN METHOD

It can be concluded from the test results that there is considerable increase in moment and shear capacity of beams when SFRC is used. Swamy and Al-Ta'an (3) also reported similar test results. It has been observed that there is difficulty in obtaining uniform SFRC mix if the fiber content is more than 2% by volume (2). It is suggested that the fiber content should be limited to 1% fiber by volume of the concrete. Since the shear failure is primarily a diagonal tension failure, steel fibers increase the shear strength of FRC considerably. Number of methods have been suggested for the design/ analysis of SFRC beams (3). These methods are quite complicated. A new simple method is proposed herein. The ultimate tensile strength of SFRC in flexure can be obtained by using the following equations given by Swamy and Mangat (4):

$$\sigma_{ut} = 0.97 \sigma_m (1 - V_f) + 3.41 V_f (l/d)$$

This equation is in N/mm^2 units.

The following equation is obtained in psi units.

$$\sigma_{ut} = 0.97 \sigma_m (1 - V_f) + 495 V_f (l/d) \dots\dots\dots (1)$$

where,

σ_{ut} = ultimate flexural tensile strength of SFRC

σ_m = ultimate flexural tensile strength of plain concrete

V_f = volume fraction of the fibers

l/d = aspect ratio of the fiber

Method of Analysis

Assumptions

1. The strain diagram is linear.
2. The tensile stress block is rectangular and its depth is 0.85 times the distance between neutral axis and the bottom fiber of the beam. This assumption is similar to the assumption made by Whitney (5) for the compressive stress block for calculating the ultimate strength of rectangular reinforced concrete section. The stress in SFRC is taken as $0.85 \sigma_{ut}$. The assumed stress block is shown in Figure 5.
3. There is no increase in compressive strength of SFRC as compared to plain concrete.
4. Ultimate shear stress of SFRC is equal to $0.41 \sigma_{ut}$, because effective length of any fiber in any direction = 0.41 (length of fiber) as found by Romualdi and Mandel (6).

This method was applied to determine shear and moment capacities of SFRC test beam having the following section:

$$\begin{aligned} b &= 4.0 \text{ in.} & d &= 5.0 \text{ in.} \\ D &= 6.0 \text{ in.} & A_g &= 0.352 \text{ in.}^2 \\ f_y &= 70.4 \text{ ksi} & f'_c &= 3 \text{ ksi} \\ \sigma_m &= 0.2 \text{ ksi} \end{aligned}$$

The calculated values of ultimate moment and shear were 106.2 inch kips and 5.3 kips, respectively. The test result values of ultimate moment and ultimate shear were 117.0 inch-kips and 6.6 kips, respectively. The test results of moment and shear values are fairly close to the calculated values.

Design Method

A SFRC beam for given ultimate shear and moment capacities can be designed as follows:

1. Knowing the tensile strength of the plain concrete, fiber size, and content, determine:

$$\sigma_{ut} = 0.97 \quad \sigma_m (1 - V_f) + 495 V_f (1/d) \dots\dots\dots (1)$$

$$\sigma_{us} = 0.41 \quad \sigma_{ut} \dots\dots\dots (2)$$

2. Knowing M_u and V_u for the beam to be designed we have the following relationship.

$$V_u = b_j d (0.41 \quad \sigma_{ut}) \dots\dots\dots (3)$$

$$M_u = 0.261 b d^2 f_c' \dots\dots\dots (4)$$

The width of beam in general lies between $d/2$ to $2/3 d$. From equations (3) and (4) determine b and d . Take higher of values of b and d .

3. Maximum value of 'a' depends upon values of f_c' and f_y . ACI 318.7(7) limits maximum values of a/d for given values of f_c' and f_y . Assume a value slightly less than this value. From the following equation calculate area of tensile steel (A_s):

$$M_u = A_s f_y (d - a/2) + b g (0.85 \sigma_{ut}) (D - a/2 - g/2) \dots\dots\dots (5)$$

4. Knowing A_s , calculate depth of n.a. and determine moment capacity of the section. Revise the section and area of main tensile steel if required.

Application to Other Test Beams

The method of calculating ultimate flexural strength of SFRC beams was applied to SFRC beams reported by Swamy and Al-Ta'an (3), where crimped steel fibers of size 0.02 x 2.0 in. (0.50 x 50 mm) were used. The calculated and test results values were very close as shown in Table 2.

CONCLUSIONS

The following conclusions can be made.

1. Steel fibers are very effective as shear reinforcement. The maximum shear strength ratios were 1.44 and 2.05 in beams having mild steel fibers of aspect ratio of 75 and brass coated fibers of size $1/2 \times 0.006$ in respectively.
2. There was increase in flexural strength of SFRC beams. The maximum flexural strength ratios were 1.59 and 1.38 in SFRC beams having M.S. fibers of aspect ratio 75 and brass coated fibers of size $1/2 \times 0.006$ in respectively.
3. The mode of failure changed from shear in conventional RCC beam to moment or moment - shear failure in SFRC beams.
4. The maximum increase in shear and moment capacity of the beams were obtained using fibers of aspect ratio 75.
5. The ultimate deflection in SFRC beams was 2-3 times that of conventional beams. The maximum toughness ratio of 7.85 was obtained in beam having $1/2 \times 0.006$ in. fibers, tested for shear-span ratio of 3.6.
6. A method for analysis and design of steel fibers reinforced concrete beam has been suggested. This method is simple and values of moment and shear capacities were fairly close to the test results.

ACKNOWLEDGMENTS

Financial aid for this project was provided by Council of Scientific and Industrial Research, New Delhi and Punjab Engineering College, Chandigarh, India. The brass coated fibers were provided by National Standard Company, Niles, Michigan. The author is thankful to the Bechtel Power Corp., San Francisco, for providing facilities for the preparation of the manuscript for this paper.

NOTATION

A_s	=	Area of main tension steel
a	=	Shear-span or depth of the rectangular compressive stress block
b	=	Width of the beam
d	=	Effective depth of the beam
D	=	Overall depth of the beam
f'_c	=	Ultimate compressive strength of the concrete at age of 28 days
f_y	=	Yield strength of steel
g	=	Depth of rectangular tensile stress block for fiber reinforced concrete beam
l	=	Length of the fiber
M_u	=	Ultimate moment
M_{uo}	=	Ultimate moment capacity of RCC beam
M_{uf}	=	Ultimate moment capacity of SFRC beam
V_f	=	Volume fraction of the fiber
V_u	=	Ultimate shear
V_{uo}	=	Ultimate shear capacity of conventional RCC beam
V_{uf}	=	Ultimate shear capacity of SFRC beam
σ_m	=	Flexural tensile stress of the plain concrete
τ_{us}	=	Ultimate shear stress of SFRC
σ_{ut}	=	Ultimate flexural tensile stress of the steel fiber reinforced concrete.

REFERENCES

- 1) Monfore, G.E. "A Review of Fiber Reinforcement of Portland Cement Paste, Mortar and Concrete," Journal PCA Research and Development Laboratories, Vol. 1, No. 3, Sept 1968, pp. 36-42.
- 2) Shah, Surendra P., and Rangan, V. Vijaya, "Fiber Reinforced Concrete Properties." ACI Journal, Proceedings, Vol. 68, No. 2, Feb. 1971, pp. 126-135
- 3) Swamy, R.N. and Al-Ta'an, Sa'ad A. "Deformation and Ultimate Strength in Flexure of Reinforced Concrete Beams Made with Steel Fiber Concrete." ACI Journal, Proceedings, Vol. 78, No. 5, September-October 1981, pp. 395-405.
- 4) Swamy, R.N., and Mangat, P.S., "A Theory for the Flexural Strength of Steel Fiber Reinforced Concrete." Cement and Concrete Research Journal, Vol. 4, pp. 313-325, 1974, Pergamon Press Inc. U.S.A.
- 5) Whitney, Charles S., "Plastic Theory of Reinforced Concrete Design." ASCE TRANS., Vol. 107, 1942, p. 251.
- 6) Romualdi, J.P., and Mandel, J.A., "Tensile Strength of Concrete Affected by Uniformly Distributed and Closely Spaced Short Lengths of Wire Reinforcement." ACI Journal, June 1964, pp. 657-671.
- 7) ACI 318-77, "Building - Code Requirements for Reinforced Concrete" ACI, March 1979.

Table 1
Test Results

Beam No.	Shear Span Ratio = a/d	Ult. Shear V_u kips	Ult. Moment M_u Inch-kips	Strength Ratio		Maxim Shear Stress = psi	Mode of Failure
				$\frac{V_{uf}}{V_{uo}}$	$\frac{M_{uf}}{M_{uo}}$		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
A	2.0	4.4	44.0	1.0	0.46	433	S
	2.4	4.4	52.0	1.0	0.55	433	S
	3.6	4.4	79.0	1.0	0.83	433	S
	4.8	4.0	96.0	0.91	1.0	394	M
B	4.8	3.6	86.4	0.82	0.9	354	M
C	3.6	4.7	84.6	1.07	0.88	463	M-S
	4.8	5.55	133.0	1.26	1.35	546	M
D	3.6	5.5	99.0	1.25	1.03	541	M
	4.8	6.35	153.0	1.44	1.59	625	M
E	3.6	6.05	109.0	1.38	1.13	595	M
	4.8	6.35	153.0	1.44	1.59	625	M
F	3.6	5.25	94.5	1.19	0.98	517	M
	4.8	5.0	120.0	1.14	1.25	492	M
G	2.0	6.63	66.3	1.51	0.96	652	S
	2.4	6.63	80.0	1.51	0.83	652	M-S
	3.6	6.50	117.0	1.48	1.22	640	M
	4.8	4.4	106.0	1.0	1.1	432	M
H	2.0	9.0	90.0	2.05	0.94	880	M-S
	2.4	8.55	103.0	1.94	1.07	837	M
	3.6	6.45	116.0	1.47	1.21	632	M-S
	4.8	5.5	132.0	1.25	1.38	540	M-S
J	2.0	7.4	74.0	1.68	0.77	725	S-M
	2.4	7.75	93.5	1.76	0.97	760	M
	3.6	5.5	99.0	1.25	1.03	540	M
	4.8	5.0	120.0	1.14	1.25	490	M

Notes: 1. S - Shear failure; M - Moment failure; M-S - Initial failure by moment and ultimate failure in shear; S-M - Initial failure in shear and ultimate failure in moment.

2. All the above results are average of 2 test specimens

Table 2

Calculated and Test Result of Ultimate Flexural Strength of Beams
Tested by Swamy and Al-Ta'an (3).

Beam No.	Fiber Percent by Volume	M_{test} (kN-m)	$M_{calc.}$ (kN-m)	Ratio
(1)	(2)	(3)	(4)	4/3
DR-11	0.5	23.25	24.24	1.04
DR-12	1.0	23.81	24.8	1.04
DR-21	0.5	33.68	30.92	0.92
DR-22	1.0	35.06	32.74	0.93
DR-31	0.5	28.62	27.1	0.95
DR-32	1.0	30.83	28.3	0.92

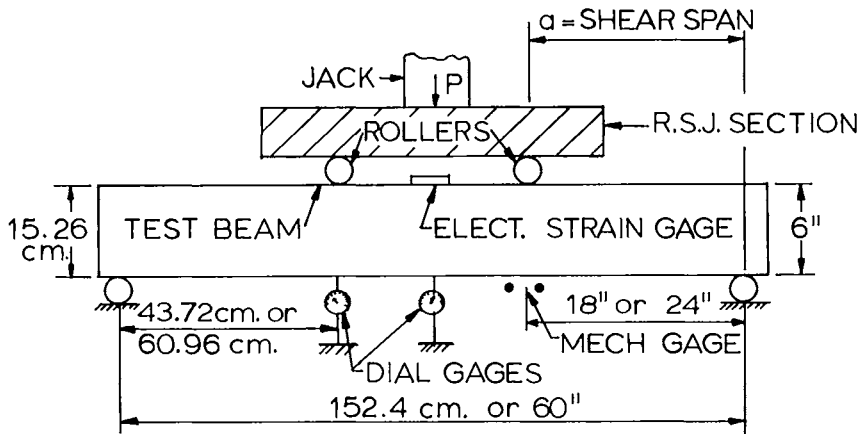


Fig. 1--Details of the specimen, location of gages for deflections and strain measurements

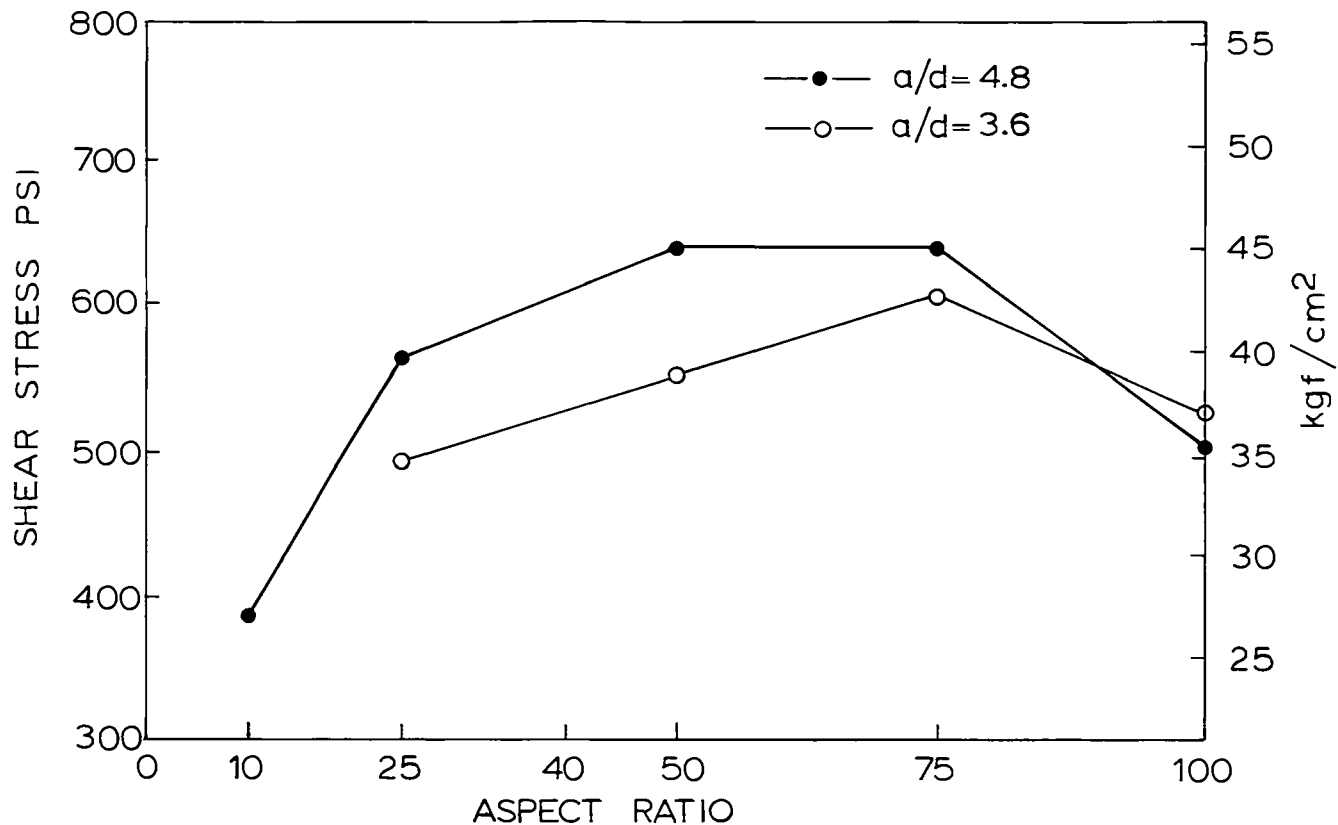


Fig. 2--Effect of aspect ratio on ultimate shear stress

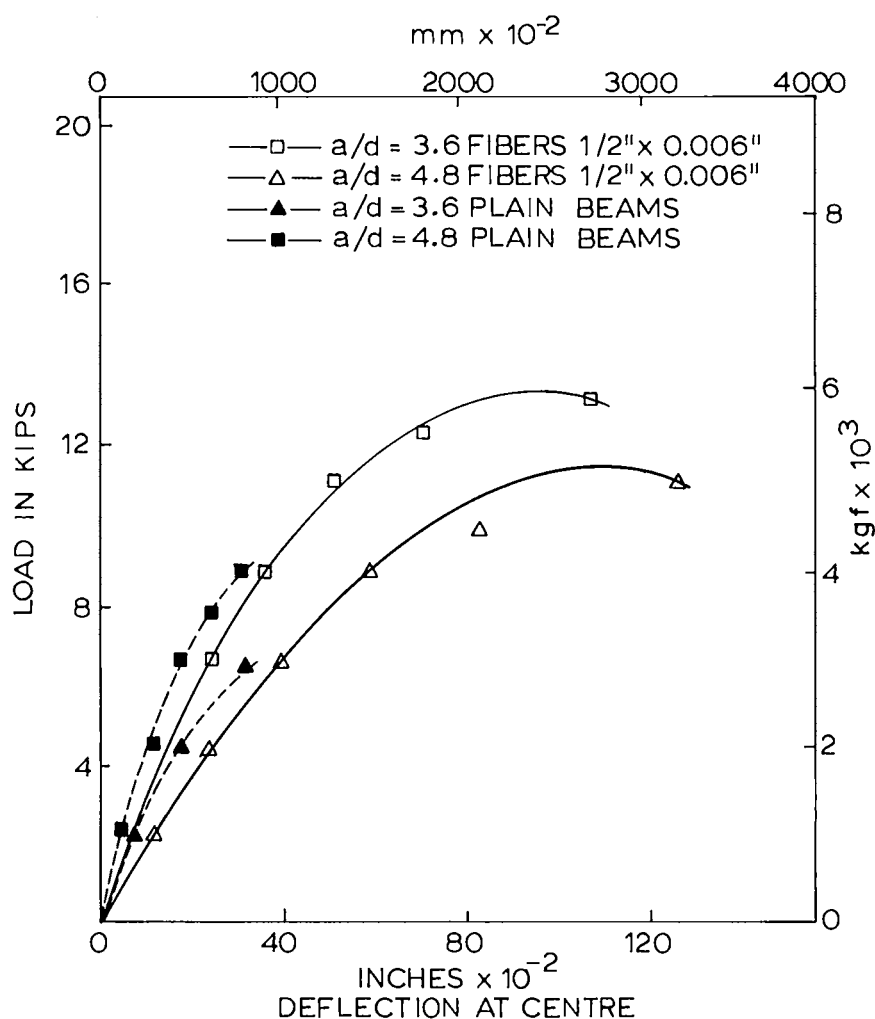


Fig. 3--Load-deflection curves of beams having $1/2 \times 0.006$ in. fibers a/d 3.6 and 4.8

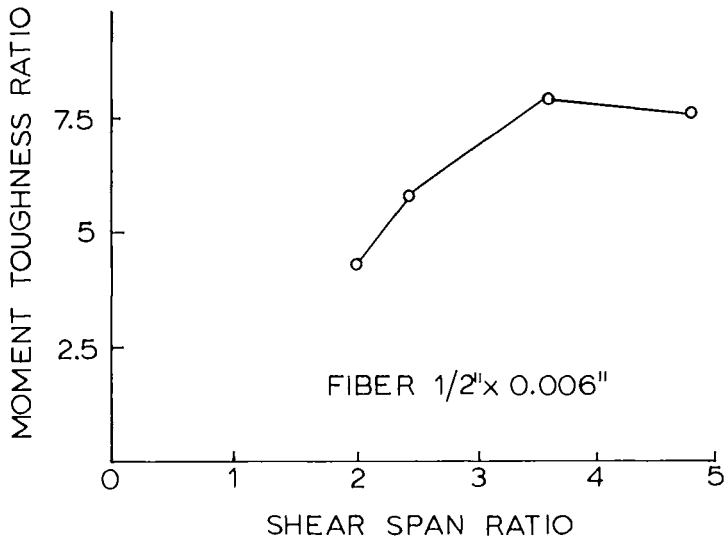


Fig. 4--Toughness ratio versus shear span depth ratio

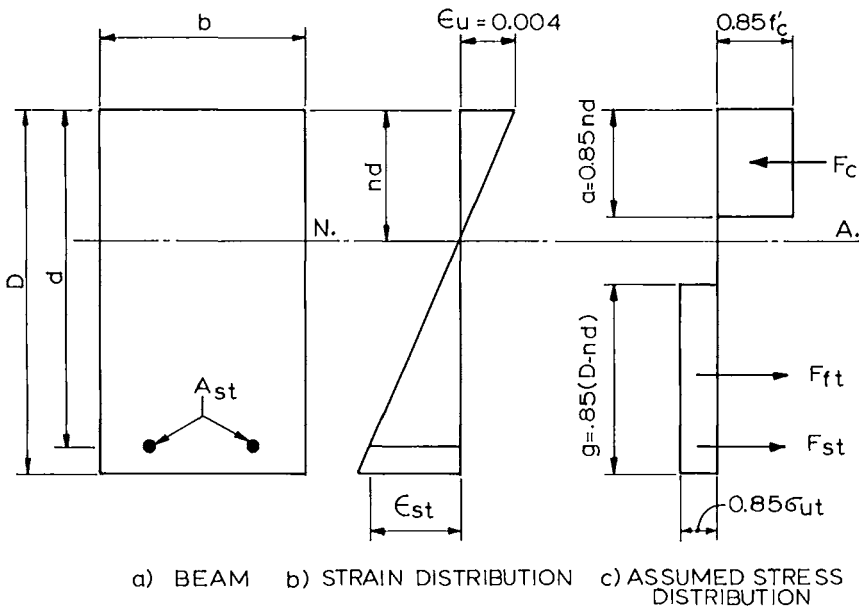


Fig. 5--Assumed stress and strain diagrams for SFRC beam

Torsional Behavior of Reinforced Fibrous Concrete Beams

By R. John Craig, S. Dunya, J. Riaz, and H. Shirazi

Synopsis: The objective of this study was to examine the torsional behavior of reinforced fibrous concrete beams under pure torsion, and to develop a theoretical approach to analyze these beams.

The addition of steel fibers to the concrete mix has been receiving more attention as a way of improving the properties of the concrete. This study has two objectives: 1) to observe the behavior of fibrous reinforced concrete in torsion, and 2) to present a realistic method for analysis of fibrous reinforced concrete in torsion.

Nine beams were analyzed and experimentally tested. There were two beams with no fibers, seven of them with different percentages of fibers by volume (two different types). All beams had the same longitudinal bars but the hoop spacing was varied.

Keywords: beams (supports); compressive strength; fiber reinforced concretes; hoops; metal fibers; reinforcing steels; shear stress; tensile stress; torsion.

ACI member Robert John Craig, is an Associate Professor and Researcher in the Structural and Concrete Laboratory at the New Jersey Institute of Technology, Newark, New Jersey. He is engaged in research of fibrous concrete and in teaching related to reinforced concrete. He is a graduate of Purdue University, West Lafayette, Indiana; BSCE, 1966; MSCE, 1969; and Ph.D., 1973. He is a member of Committees 444 and 544, and chairman of Committee E-801.

S. Dunya, J. Riaz and H. Shirazi are graduate students at the New Jersey Institute of Technology.

INTRODUCTION

The use of fibers in fibrous reinforced concrete is gaining increased importance in the research laboratories as a way of improving the properties of concrete. It is well known that the tensile strength of the plain concrete is very low. This weakness can be improved by the addition of steel fibers in the concrete.

This paper will discuss results from tests run at the New Jersey Institute of Technology on the structural uses of steel fibers in fiber reinforced concrete beams under pure torsion in order to illustrate their behavior. Nine beams were tested. The beams were 6 in. x 12 in. x 72 in., with #4 reinforcing bars as longitudinal steel and #3 bars as hoop steel. Different fiber content and hoop spacing were used in this study (see Table 1). The beams were loaded, using hydraulic rams as shown in Figure 3. The rotation was measured between the beams' third points at each load increment. The cracking was also observed during the testing.

The theoretical analysis was developed in two types of beams: 1) fibrous reinforced concrete beams (no deformed reinforcing steel bars); and 2) reinforced fibrous concrete beams (fibers and reinforcing steel bars). With the addition of fibers there is an increase in the ultimate torque of the beam for the two different types. The magnitude of this increase is dependent on the type and amount of fiber. The ultimate torque for fibrous reinforced concrete beams was found to be given as:

$$T_o = 1.56 (x^2 + 10)y \sqrt[3]{f_r^2} \quad (1-a)$$

$$\text{or} \quad T_o = 1.67 (x^2 + 10)y \sqrt[3]{f_{ct}^2} \quad (1-b)$$

While the second type of beams, the reinforced fibrous concrete

beams, was found to have an ultimate torque given by the equation (2-a).

$$T_u = \alpha_t \frac{(x_1 y_1 A_t f_y)}{s} + \alpha_c (x_2 y_2 \sqrt{f'_c}) \quad (2-a)$$

Where α_t can be found experimentally, which depends upon the type and amount of fiber used in the beam, and α_c was found to have the value of

$$2.0 + \frac{.12 P_{fn}}{\sqrt{f'_c}} \quad (2-b)$$

In developing the above equations, the tensile and shear forces across the crack due to the presence of fibers were considered. The theoretical development will be explained.

The addition of fibers in the beams showed: 1) an increase in stiffness; 2) an increase in the ultimate torsional capacity; 3) an increase in ductility and rotational capacity; 4) an increase in cracks with smaller crack widths; and 5) an increase in confinement and damage tolerance of the concrete section over the conventional reinforced concrete beam. The theoretical development showed good correlation with the results obtained in the laboratory.

The use of reinforced fibrous concrete beams has definite advantages as will be described in this paper, but more research is needed to fully develop a method for design. The results of this testing program and theoretical development could be used for this purpose.

EXPERIMENTAL PROGRAM

The results of an experimental testing program were used to come up with the behavior of the reinforced fibrous concrete beams in resisting torsional moments. These results were also used to come up with information which was needed to verify the validity and to evaluate the constants of the analytical method. The procedures, testing, and results will be shown.

Test Beams

Nine full size beams which were 6 in. x 12 in. x 6 ft. were designed for testing in torsion. The test beam series is shown in Figure 1. All the beams were designed to have the same longitudinal reinforcing bars with a varying hoop spacing ($s = 7$ in., 3.5 in., and none). There were two different crimped-end fibers which were used in the testing series at different volumes (0, 1 %, 1-1/2 %, and 2 %) in the concrete. The reinforced fibrous concrete beams were constructed identically to the conventional reinforced concrete beams to assure reliable comparisons.

the same reinforcing steel and concrete. The test beam with cross section and test set-up are shown in Figures 2 and 3, respectively. A photograph of a beam in the test set-up is shown in Figure 4.

Materials and Fabrication

The concrete which was used in constructing the beams used Portland Cement Type I, river sand, and 3/8 inch crushed stone. The mix design used was as follows:

1098 lb/cy	-	coarse aggregate (3/8" max.)
1600 lb/cy	-	fine aggregate
404 lb/cy	-	water
674 lb/cy	-	cement

A superplasticizer was used to help in placing all concrete mixes to avoid problems in constructing the beams.

For the steel fibrous concrete, the mix used the same concrete design as the plain concrete and crimped ended steel fibers which were 30 mm (1.18 in.) long and .50 mm (.0197 in. in diameter) were used in specimens 4 and 5, respectively. The plain concrete was specified to have a minimum 28-day compressive strength of 4000 psi. The fibers which were put into the concrete at 0 %, 1 %, 1-1/2 %, and 2 %, as described in Table 1 had an ultimate tensile strength in the range of 135,000 to 200,000 psi. The typical stress/strain curves for the fibrous concrete and splitting tension values are shown in Figure 5. The values for f'_c , compressive strength, and f_{sp} , splitting tensile strength, are shown in Table 1.

Steel reinforcing bars conformed to ASTM A 615 Grades 40 and 60 for the beams. The Grade 60, having a yield strength of 65,000 psi, was used for the No. 4 longitudinal reinforcing; and the Grade 40, having a yield strength of 46,000 psi, was used for the No. 3 hoop reinforcing. The typical stress/strain curves for the reinforcing are shown in Figure 6.

Fabrication:-- The specimens were cast in plywood forms. The beams were cast in two different castings. The fibers being added to the plain concrete after the conventional beams were cast. Thus the difference in the concrete mixes was due to the presence of fibers only. The beams were cast horizontally so that the fiber orientation due to placing was correct. All concrete was consolidated by an immersion-type vibrator. The specimens were moist-cured for a period of seven days. The forms were stripped after seven days and the beams were tested at about 28 days.

TESTING PROGRAM

The beams were tested, using the testing apparatus shown in Figure 3. The one end of the beam was held fixed while the torque was applied to the other end, using hydraulic rams on both sides of the beam. The loads were applied to the beam on arms which were 18 inches from the center of the beam. The beams were reinforced at the ends to prevent failures in the support areas. Loads were monitored by the use of load cells at the fixed end and hydraulic pressure of the rams at the loading end. The capacity of the hydraulic rams was 16 tons.

The rotation of the beam was measured between the third points of the beam with linear potentiometers. Strain measurements were taken using strain gages on the longitudinal and hoop reinforcing steel. The cracking was observed and marked at each load stage for the beams. The beams were tested at about 28 days after casting.

Test Results

The test results for the nine beams are described in Table 1 and the test beam series is shown in Figure 1. The results of all the torsional resisting moment and rotation are shown in Figure 7. The torsional resisting moment vs. rotation behavior of some typical beams are shown in Figures 8, 9, 10, 11, and 12, and shall be discussed.

Figure 8 shows the results of three beams: 1) a conventional reinforced concrete beam with hoops at 7 inches (T1); 2) a conventional beam with hoops at 7 inches having fibers added (1-1/2% by volume)(T3); and 3) a reinforced fibrous concrete beam (1-1/2% by volume) with longitudinal steel only. The conventional concrete beam ultimate torsional resisting moment was 83,200 in lb as compared with 76,400 in lb, by the ACI 318-77 Code. The ultimate torsional resisting moments were 83,200 in lb, 149,000 in lb, and 98,000 in lb for beams T1, T3, and T2, respectively. The addition of fibers in the beams showed an increase in stiffnesses. The theoretical stiffness was calculated at 15,290 in K/degree as compared to 14,280 in K/degree, 25,470 in K/degree, and 18,430 in K/degree, for beams T1, T3, and T2, respectively. The beams with fibers and longitudinal reinforcing only, showed higher strength than did the conventional reinforced concrete with hoops at 7 in. The presence of fibers increased the ductility and increased the damage tolerance of the beams.

Figures 9, 10, and 11 show the influence of the hoop spacing, s , and the fiber percentage present in the beams. The presence of both fibers and reinforcing steel increases the ductility and strength of the members. The stiffness is also increased.

Figure 12 shows the influence of having a different fiber type with the same volume of fibers (1 % by volume) with hoop

reinforcing at $s = 7$ inches.

The results of the cracking and ultimate torsional resisting moment vs. hoop spacing of the nine beams are plotted in Figure 13 (T vs. $(x_1 y_1 A_t f_y)/s$).

The presence of both the hoop steel and the fibers will bring about a much smaller spacing and size of cracks. The typical cracking behavior of reinforced fibrous concrete beams subjected to only a torsional moment is shown in Figure 14.

THEORETICAL DEVELOPMENT

Derivation of Ultimate Torque Equation - Fibrous Concrete Beams

When looking at the torsion of fibrous concrete without reinforcing bars, the increase in the ultimate torque is very little. The increase in direct tension and torsional strengths of concrete beams with fibers is only marginal compared to a substantial increase in flexural strength. This marginal increase in tensile strength can be predicted, using the conventional theory of mixtures, when efficiency factors for the fiber orientation and bond strength. These increases in strength are shown in Figure 15 (1).

The ultimate cracking torque T_{cr} for plain concrete beams given by Hsu (3) is:

$$T_u = 6 (x^2 + 10) y \sqrt[3]{f'_c} \quad (1)$$

This equation can be expressed in terms of the modulus of rupture, f_r , and splitting tensile strength, f_{sp} , by substituting $.133 f_r$ and $.147 f_{ct}$ for $\sqrt[3]{f'_c}$ as has been suggested by others (4,5). The following will be found.

$$T_o = 1.56 (x^2 + 10) y \sqrt[3]{f_r^2} \quad (2)$$

$$T_o = 1.67 (x^2 + 10) y \sqrt[3]{f_{ct}^2} \quad (3)$$

Turner and Davis (5) developed the following ultimate torsional capacity T_u expression:

$$T_u = \alpha x^2 y \left[f_t (\max) \right] \left(\frac{3.24}{1 + x/y} \right) \quad (4)$$

where the values for y/x and α are given as follows:

y/x	α
1.0	.208
1.2	.219
1.5	.231
2.0	.246
2.5	.256
3	.267
5	.290
∞	.333

and f_{tmax} equals the maximum principal tensile stress. Considering skew bending of rectangular plain concrete section (according to Hsu (2)), the ultimate torsional capacity, considering the biaxial state of stress condition, is:

$$T_u = \frac{x^2 y}{3} (.85 f_r) \quad (5)$$

Using the test results previously explained, these equations (1, 2, 3, 4, and 5) can be checked with test results. The results of this comparison are shown in Table 2. As can be seen, Equations 2 and 3 show good comparisons.

DERIVATION OF ULTIMATE TORQUE EQUATION - REINFORCED FIBROUS CONCRETE BEAMS

The mathematical analysis which will be presented is based on a similar analysis for reinforced concrete beams by Hsu (7) and represented by Wang and Salmon (8). The analysis presented will reflect the presence of fibers in the reinforced concrete beams.

The failure surface is assumed to be a plane perpendicular to the wider face and inclined at 45° to the axis of the beam. The failure plane may be as shown in Figure 16. The force on the compression side, P , consists of contributions from longitudinal bars and from shear and normal resistances of the concrete. The components of the resultant force, P , are shown in Figure 17. On the tension side, it is assumed that P_v , tensile force in the reinforcing hoops, Q_x and Q_y , horizontal and vertical components of the dowel action, and P_{ct} , compressive force in the longitudinal bars, are assumed to be acting along with a normal and shear resistance contributed by the fiber concrete (P_{TF} and P_{SF} , respectively). On the tension side, there will exist a longitudinal force unless P_{TF} and P_{SF} are equal.

First, let's consider the torque which is developed by the reinforcing. Equilibrium of the free body must be considered to determine the ultimate torque. On the tension side of the beam, all hoops are assumed to yield. The force in the hoops or closed vertical stirrups across a given skew bending failure section is

$$P_v = \left(\frac{y_1}{s} \right) \Lambda_t f_y \quad (6)$$

where y_1/s is the number of hoops intercepted by the 45° failure plane.

The tensile force P_{ℓ_c} in the longitudinal bars intercepting the compression concrete zone is:

$$P_{\ell_c} = \phi (\Lambda_{\ell} / 2) f_y \quad (7)$$

The tensile force P_{ℓ_t} in the longitudinal bars intercepting the tensile fiber concrete zone is:

$$P_{\ell_t} = \bar{\phi} (\Lambda_{\ell} / 2) f_y \quad (8)$$

where ϕ and $\bar{\phi}$ are efficiency factors to account for longitudinal bars being at two or more points within the compression and tension zones, respectively.

The dowel forces Q_x and Q_y may be written:

$$Q_x = q \left(\frac{\Lambda_{\ell}}{4} \right) \Delta_x \quad (9)$$

$$Q_y = q \left(\frac{\Lambda_{\ell}}{4} \right) \Delta_y \quad (10)$$

$$Q_x = q \left(\frac{\Lambda_{\ell}}{4} \right) K_3 \left(\frac{y_2}{2} \right) \quad (11)$$

$$Q_y = q \left(\frac{\Lambda_{\ell}}{4} \right) K_3 \left(\frac{x_2}{2} \right) \quad (12)$$

where q is a constant representing the dowel stresses in pounds per cubic inch; Λ_{ℓ} is the sum of tension and compression longitudinal steel; Δ_x and Δ_y are the bar displacements resulting from dowel action in the x and y directions, respectively. The displacement Δ 's are proportional to the distance from the center of twist to the bars and K_3 is a proportionality constant.

Now, let m equal the ratio of the volume of longitudinal bars to the volume of closed hoops such that:

$$m = \frac{\Lambda_{\ell} s}{2 \Lambda_t (x_1 + y_1)} \quad (13)$$

$$\begin{aligned} \Lambda_{\ell} &= \Lambda_{\ell_c} + \Lambda_{\ell_t} \\ \Lambda_{\ell} &= \Lambda_t \left[\frac{2m (x_1 + y_1)}{s} \right] \end{aligned} \quad (14)$$

Substituting Equation (14) into Equations (11) and (12) gives:

$$Q_x = \frac{q K_3}{4} (y_2) \left(\frac{m A_t (x_1 + y_1)}{s} \right)$$

$$= \frac{q K_3}{4 f_y} \left(\frac{y_2}{y_1} \right) m \left(1 + \frac{y_1}{x_1} \right) \left(\frac{x_1 y_1 A_t f_y}{s} \right) \quad (15)$$

Similarly,

$$Q_y = \frac{q K_3}{4 f_y} \left(\frac{x_2}{y_1} \right) m \left(1 + \frac{y_1}{x_1} \right) \left(\frac{x_1 y_1 A_t f_y}{s} \right) \quad (16)$$

Substitution of Eq. (14) into Equations (7) and (8) gives:

$$P_{lc} = \frac{q}{4} m \left(1 + \frac{y_1}{x_1} \right) \left(\frac{x_1 A_t f_y}{s} \right) \quad (17)$$

$$P_{lt} = \frac{q}{4} m \left(1 + \frac{y_1}{x_1} \right) \left(\frac{x_1 A_t f_y}{s} \right) \quad (18)$$

The torsional resistance from reinforcement then is:

$$T_{uT} = P_v \left(\frac{x_1}{2} \right) + P_{lc} \left(\frac{x_2}{2} \right) + P_{lt} \left(\frac{x_2}{2} \right) + 2Q_x \left(\frac{y_2}{2} \right) + 2Q_y \left(\frac{x_2}{2} \right) \quad (19)$$

Substitution of Equations (6), (15), (16), (17) and (18) into Eq. (19) gives:

$$T_{uT} = \frac{x_1 y_1 A_t f_y}{2 s} + \left(\frac{q}{4} + \frac{q}{4} \right) m \left(1 + \frac{y_1}{x_1} \right) \left(\frac{x_1 A_t f_y}{s} \right) \left(\frac{x_2}{2} \right)$$

$$+ \frac{q K_3}{4 f_y} \left(\frac{y_2}{y_1} \right) m \left(1 + \frac{y_1}{x_1} \right) \left(\frac{x_1 y_1 A_t f_y}{s} \right) y_2$$

$$+ \frac{q K_3}{4 f_y} \left(\frac{x_2}{y_1} \right) m \left(1 + \frac{y_1}{x_1} \right) \left(\frac{x_1 y_1 A_t f_y}{s} \right) x_2 \quad (20)$$

Equation (20) may also be written:

$$T_{uT} = \alpha_t \left(\frac{x_1 y_1 A_t f_y}{s} \right)$$

where

$$\alpha_t = \frac{1}{2} + m \left(\frac{q}{4} + \frac{q}{4} \right) \left[1 + \frac{y_1}{x_1} \right] \left(\frac{x_2}{2 y_1} \right)$$

$$+ \frac{q K_3}{4 f_y} m \left(1 + \frac{y_1}{x_1} \right) (x_2^2 + y_2^2) \frac{1}{y_1} \quad (21)$$

or if $x_2 \approx x_1$ and $y_2 \approx y_1$, Eq. (21) becomes:

$$\alpha_t = \frac{1}{2} + m \left(\frac{q}{4} + \frac{q}{4} \right) \left(\frac{1}{2} \right) \left(\frac{x_1}{y_1} + 1 \right)$$

$$+ \frac{q K_3}{4 f_y} m \left(1 + \frac{y_1}{x_1} \right) \left[1 + \left(\frac{x_1}{y_1} \right)^2 \right] y_1 \quad (22)$$

If no vertical component of the $P_{\ell t}$ is considered for the torsional resistant moment, the value of the tension zone longitudinal steel moment goes to zero. This means that the tension and shear forces of the fibers are considered. But if only the tensile shear forces are considered for the effect of the concrete, then Equation (22) will be valid. In this presentation, the tension and shear forces are considered for the concrete tension torsional moment. Thus the torsional moment due to reinforcing will be:

$$T_{uT} = \alpha_t \left(\frac{x_1 y_1 A_t f_y}{s} \right)$$

where

$$\alpha_t = \frac{1}{2} + m \left\{ \frac{1}{2} \left(\frac{x_1}{y_1} + 1 \right) + \frac{qK_3}{4f_y} m \left(1 + \frac{y_1}{x_1} \right) \left[1 + \left(\frac{x_1}{y_1} \right)^2 \right] y_1 \right\} \quad (23)$$

Now, α_t can be written as:

$$\alpha_t = C_0 + C_1 (f_t) + C_2 m + C_3 \left(\frac{y_1}{x_1} \right) \quad (24)$$

Thus, α_t is essentially a function of three parameters: m , y_1/x_1 , and f_t ; and C_0 , C_1 , C_2 , and C_3 may be experimentally determined. The effect of fiber type and fiber percentage is introduced as a function of f_t which has to be considered because of $P_{\ell t}$.

Now consider the torque which is developed by the fibrous concrete. From the forces acting on the compression zone, it is found that the resultant P force is:

$$P = \sqrt{2} P_s + P_{\ell c} \quad (25)$$

The first term of Equation (25) represents the portion which is attributed to the concrete in the compression zone. Thus, the torsional moment capacity of the concrete in compression zone is:

$$T_{cc} = \sqrt{2} P_s (\text{arm}) \quad (26)$$

The shear resistance P_s can be expressed as:

$$P_s = v_{avg_c} \sqrt{2} y a \quad (27)$$

where

v_{avg_c} is the average shear stress acting over the compression zone, $y \sqrt{2}$ is the width of the compression zone and a is the depth of the compression zone. The depth of the compression zone a is:

$$a = K_1 x \quad (28)$$

According to Mohr's failure theory with Timoshenko's straight line simplification (9), v_c' , the pure shear strength of concrete, can be found by the relationship:

$$V'_c = \left(\frac{f'_c}{f'_c + f_t} \right) f_t \quad (29)$$

where f'_c and f_t are the cylinder compressive strength and uniaxial tension strengths, respectively.

$$\text{Thus } V'_c = A f_t \quad (30)$$

$$\text{and } V_{avg_c} = K_c V'_c \quad (31)$$

where K_c is a coefficient relating the shear strength in combined shear and compression to the pure shear strength. The value for A can be taken as .9 for fiber concrete without introducing serious error.⁽⁷⁾ From tests run on fiber concrete, f_t can be taken as

$6 \sqrt{f'_c}$. The shear strength of the concrete can be found from the expression $V'_c = 5.4 \sqrt{f'_c}$. Substituting V'_c into Eq. (27) gives:

$$P_s = 7.64 K_c K_1 x y \sqrt{f'_c} \quad (32)$$

Now $K_c K_1$ must be evaluated. From Eq. (26), the torsional moment due to the concrete alone is:

$$T_{cc} = \sqrt{2} P_s (\text{arm}) \quad (33)$$

$$T_{cc} = \sqrt{2} P_s X_{cT} \quad (34)$$

assuming that the arm is X_{cT} which is $(x-a)/2$.

Now, looking at the forces acting on the tension zone, it is found that the resultant P force is:

$$P = Q + P_v + \frac{P_{TF}}{\sqrt{2}} + \frac{P_{SF}}{\sqrt{2}} \quad (35)$$

The last two terms in the Equation (35) represent the portion which is attributed to the fibers in the concrete in the tension zone. Thus the torsional moment capacity of the fibers in the tension zone is:

$$T_{cT} = \left(\frac{P_{TF}}{\sqrt{2}} + \frac{P_{SF}}{\sqrt{2}} \right) (\text{arm}) \quad (36)$$

The tension stresses across the tension zone should be considered to act uniformly over the region. The fiber force in tension at failure according to Williamson (10) is:

$$P_{TF} = f_{tf} (x - a) \sqrt{2} y \quad (37)$$

$$\text{then } P_{TF} = .7 P_f (u(x-a) \sqrt{2} y)$$

if $f_{tf} = .7 P_{fn}$
 f_{tf} is actually a function of fiber distribution and the plane of failure (geometrical and p , dependent).

where:

f_{tf} is the uniform fiber tensile stress at failure of the section,
 P_f is the pullout force of a fiber,
 n is the number of effective fibers in any one direction per unit area:

$$n = \frac{4_p (.41)}{\pi d_f^2 l_f} \quad (36)$$

p is the percentage by volume of fibers,
 d_f is the diameter of the fiber,
 l_f is the length of the fiber,
 $.41$ is the percentage of fibers effective.

There is also a shear resistance across the failure plane which is contributed by the fibers which is P_{SF} . The P_{SF} can be related to the P_{TF} (tension force) in the following manner:

$$P_{SF} = V_{avg_t} \sqrt{2} y (x-a) \quad (39)$$

The V_{avg_t} can be found in a similar way as V_{avg_c} .

$$V_{avg_t} = K_T f_{tf} \quad (40)$$

Where K_T is a coefficient relating the shear strength in combined shear and tension at failure of section.

$$V_{avg_t} = K_T (.7 P_f n) \quad (41)$$

Thus the shear force P_{SF} is:

$$P_{SF} = .7 K_T P_f (n (x-a) \sqrt{2} y) \quad (42)$$

The total torque which is present due to the fibers, T_{CT} (Eq.36), at failure is:

$$T_{CT} = \left[\frac{.7 P_f (n (x-a) \sqrt{2} y)}{\sqrt{2}} + \frac{.7 K_T P_f (n(x-a) \sqrt{2} y)}{\sqrt{2}} \right] (\text{arm}) \quad (43)$$

The moment arm is X_T which is equal to $a/2$. Substituting for moment arm and combining terms in Equation (43), T_{CT} becomes:

$$T_{CT} = .35 (1 + K_T) P_f n(x-a) a y \quad (44)$$

Now the total torque which is carried by the concrete at failure is:

$$T_c = T_{cc} + T_{CT} \quad (45)$$

$$T_c = (7.64 K_c K_{1xy} \sqrt{f'_c}) \left(\frac{x-a}{2} \right) + .35 (1 + K_T) (P_f n(x-a) a y) \quad (46)$$

Simplifying Equation (46) becomes: $T_c = \alpha_c x^2 y \sqrt{f'_c}$



where

$$\alpha_c = 3.82 K_c (K_1 - K_1^2) + .35(1 + K_T)(K_1 - K_1^2) \frac{P_{fn}}{\sqrt{f'_c}} \quad (47)$$

The quantity α_c is essentially a function of one parameter and might be written:

$$\alpha_c = C_4 + C_5 \frac{P_{fn}}{\sqrt{f'_c}} \quad (48)$$

where constants C_4 and C_5 may be experimentally determined.

Now the total ultimate torque will be a combination of the steel and concrete contributions

$$T_u = T_{uT} + T_{uc} \quad (49)$$

and

$$T_u = \alpha_t \left(\frac{x_1 y_1 A_t f_y}{s} \right) + \alpha_c (x^2 y \sqrt{f'_c}) \quad (50)$$

where

$$\alpha_t = C_0 + C_1 (f_t) + C_2 m + C_3 \left(\frac{y_1}{x_1} \right) \quad (24)$$

and

$$\alpha_c = C_4 + C_5 \frac{P_{fn}}{\sqrt{f'_c}} \quad (48)$$

The constants C_0 , C_1 , C_2 , C_3 , C_4 , and C_5 , as mentioned earlier can be experimentally determined. To determine the constants, let us look at Figures 18 and 19. Due to the complexity of determining the constants C_0 , C_1 , C_2 , and C_3 theoretically, the whole term α_t should be determined from test procedures as the slope of the line Torque vs. $(x_1 y_1 f_{sy} A_t)/s$. The value of α_t for these test beams varied from 1.05 to 2.75 as results show in Table 3. The value of α_t varies with the amount and type of hooked end fibers present in the beam. The quantity α_t is a function of: 1) fiber type; 2) fiber percentage; 3) m ;

4) f_y/f_{sy} ; 5) y_1/x_1 ; and 6) y_2 . Because of the small number of variations of fiber type and fiber percentages, the constants C_0 , C_1 , C_2 , and C_3 will not be evaluated. The value of α_t as previously mentioned can be determined by the slope of the curve of T_u vs. $(x_1 y_1 f_{sy} A_t)/s$ which can be determined experimentally. Further testing and evaluation of beams are being conducted at New Jersey Institute of Technology for the value of the constants C_0 , C_1 , C_2 , and C_3 . Some observations should be noted that:

i) The dowel forces Q are dependent on the stirrup spacing, fiber content and fiber type which affect the lateral displacement of the bar.

ii) The longitudinal forces in the compression and tension zones at failure are dependent on ϕ and ξ . The value of ξ is dependent on the effectiveness of the fibers in the tension zone in the concrete as can be realized by the composition of

force P in the tension zone which is shown in Figure 17. The magnitude of P_{TF} and the value of K_T will determine this efficiency factor. With the increase in Q and the presence of P_{TF} and P_{SF} due to the fibers, the value for ϕ will vary from a conventional beam theory, depending on the magnitude of P which is affected by the increase in the compression zone (s will be larger for the beams with fiber) and the presence of fibers. The value for the longitudinal compression force is:

$$P_{LT} = \frac{P_{TF}}{\sqrt{2}} - \sqrt{2} P_{SF} \quad (51)$$

iii) The value of $m = \frac{f_{ly}}{f_{sy}}$ may be substituted for m if the

yield stress for the longitudinal steel and hoops are different. The work here has been derived for $f_{ly} = f_{sy}$. This was shown to be true by other investigators.

The constants C_4 and C_5 can be evaluated by letting the $T_{uc} = T_0$. T_0 is obtained experimentally as the value of the ultimate torque without any reinforcing. The T_0 will be considered to be the torque which causes cracking of the beam (longitudinal steel was present in all beams). The force produced by the fibers in the tension region is not great enough to increase the resisting torsional moment much above the cracking torsional moment, if any. The value α_c has constants C_4 and C_5 which can be evaluated by the experimental tests. The constant C_4 depends on the torsional resisting moment provided by the shear strength of the shear-compression zone. Similarly, the constant C_5 depends on the torsional resisting moment provided by the shear and tension strength of the fibers in the shear-tension zone. The value of C_4 was found by:

$$T_{oc} (0\% \text{ Fiber}) = C_4 x^2 y \sqrt{f'_c} \quad (52)$$

Thus:

$$C_4 = \frac{T_{oc} (0\% \text{ Fiber})}{x^2 y \sqrt{f'_c}} \quad (53)$$

Then C_5 can be evaluated as:

$$T_{oF} (\text{Fiber}) = T_{oc} (\text{Plain}) + \frac{C_5 P_{fn}}{\sqrt{f'_c}} (x^2 y \sqrt{f'_c}) \quad (54)$$

Therefore:

$$C_5 = \frac{T_{oF} (\text{Fiber}) - T_{oc} (\text{Plain})}{P_{fn} x^2 y} \quad (55)$$

Using the results from testing, the calculations are shown in Table 4. The values for C_4 and C_5 were 2. and .12. The value for T_c is:

$$T_c = (2.0 + \frac{.12P_{fn}}{f'_c}) x^2 y \sqrt{f'_c} \quad (56)$$

Thus, the full theoretical capacity of rectangular sections for reinforced fibrous concrete may be written as:

$$T_u = T_{uT} + T_{uc} \quad (57)$$

$$T_u = \alpha_t \left(\frac{x_1 y_1 A_t f_y}{s} \right) + (2.0 + \frac{.12P_{fn}}{\sqrt{f'_c}}) x^2 y \sqrt{f'_c} \quad (58)$$

The values presented here for α_t , C_4 and C_5 are determined from testing of beams presented with this report. The value of C_4 may be taken as similar to Hsu's work (7) as $2.4/\sqrt{x}$, which would be a better representation of beams of different sizes. α_t , as mentioned previously, has to be found by experimentation because of the many variables.

CONCLUSIONS

The ultimate torsional theory presented showed good comparison with the results obtained in the testing program. The theory which was based on the theory for conventional reinforced concrete beams by Hsu (7) was based on a failure plane perpendicular to the wider cross-section face and inclined at 45° to the longitudinal axis of the beam. This failure plane assumption seemed to be valid, observing the cracking behavior of the test beams. The theory, because of the presence of fibers in the concrete, had to account for the tension and shear stresses in the tension zone of the failure plane. The value of α_t in the expression for the ultimate torsional moment is dependent on the fiber content and fiber type (f_t); x_1/y_1 ; m ; f_{ly}/f_{sy} ; y_1/x_1 ; and y_2 . Because of the complexity of finding α_t (fiber being present), more testing is required in order to come up with the constants C_0 , C_1 , C_2 , and C_3 which make up α_t given by:

$$\alpha_t = C_0 + C_1 (f_t) + C_2 m + C_3 \left(\frac{y_1}{x_1} \right) \quad (24)$$

The value of α_t presently can be found experimentally as described in this paper. Equation (50) should be used as the design equation for the ultimate torsion for beams. This expression should be compared to more test results in order to substantiate its validity because there are many different fibers which can be used in constructing reinforced fibrous concrete.

beams. The results of testing showed that the performance of the reinforced fibrous concrete beams is heavily dependent on the amount and type of fibers. The fibers used in the testing program showed good comparisons with the theoretical development presented.

The behavior of the reinforced fibrous concrete beams used in the testing program showed that the addition of fibers in the beams resulted in: 1) an increase in stiffness; 2) increase in the ultimate torsional capacity; 3) increase in ductility and rotational capacity; 4) increase in cracks with smaller crack widths; and 5) increase in confinement and damage tolerance of the concrete section over the conventional reinforced concrete beam.

ACKNOWLEDGEMENT

This investigation was carried out in the Structural Laboratory of the New Jersey Institute of Technology with SBR funding from the school. I would like to thank the following undergraduate students: Mike Armm, James Cowley, John W. Korunow, Jr., and Leonardo A. DePinto, who helped in constructing the test set-up and casting and testing the specimens. The work and assistance by Nashat Dib, a graduate student, is also appreciated. Special recognition should be made regarding Danny DiSerio and Erik Skurbe, the laboratory technicians in the Department of Civil and Environmental Engineering, for their assistance.

BIBLIOGRAPHY

- (1) Edgington, J., "Steel Fibre Reinforced Concrete," Research Report submitted to Department of the Environment, January 1974. Also available as Ph.D. thesis, University of Surrey, 1974.
- (2) Edgington, J., "Steel Fibre Reinforced Concrete," Intermediate Report submitted to the Environmental for the period ending March 1, 1972.
- (3) Hsu, T. T. C., "Torsion of Structural Concrete - Plain Concrete Rectangular Sections," Torsion of Structural Concrete, SP-18, Detroit, Michigan: ACI 1968 (pp. 203-238). Also Portland Cement Association D.D.B. D 134.)
- (4) Paul, S. L. and Sinnamon, G. K., "Concrete Tunnel Liners: Structural Testing of Segmented Liners," Final Report, Department of Transportation, FRA-OR & D-75-93 August, 1975.
- (5) Criswell, M. E., "Shear in Fiber Reinforced Concrete," National Structural Engineering Conference, August 1976, Madison, Wisconsin.

- (6) L. Turner and V. C. Davies, "Plain and Reinforced Concrete in Torsion, with Particular Reference to Reinforced Concrete Beams," Selected Engineering Papers No. 165, London, Institution of Civil Engineers, 1934 (31 pp.)
- (7) Hsu, T. T. C., "Ultimate Torque of Reinforced Rectangular Beams," *Journal of Structural Division, ASCE*, 94 (ST2) February 1968, 485-510. (Also Portland Cement Association Development Department Bulletin D 127).
- (8) Wang, C. K. and Salmon, C. G., "Reinforced Concrete Design," Intext Educational Publisher, New York, N. Y. 1972, 2nd Edition, pp. 774-832.
- (9) Timoshenko, S., "Strength of Materials, 3rd Edition, Part II, D. Van Nostrand Co., Inc., Princeton, N.J., 1956, p.459.
- (10) Williamson, G. R., "Compression Characteristics and Structural Beam Design Analysis of Steel Fiber Reinforced Concrete."
- (11) Henager, C. H. and Doherty, T. J., "Analysis of Reinforced Fibrous Concrete Beams," *Journal of the Structural Division, ASCE*, Vol. 102, No. ST1, Proc. Paper 11847, January 1976, pp. 177-188.
- (12) Whittaker, W., "Flexural Performance of Steel Fiber Reinforced Concrete Beams," 12-648 Senior Project, Department of Civil Engineering, Carnegie-Mellon University, Pittsburgh, Penna., January, 1979, pp. 74.

NOTATIONS

- a - depth of the shear-compression zone.
- A_l - cross sectional area of longitudinal bars.
- A_t - cross sectional area of hoop steel.
- A_{tf} - failure surface area in tension.
- $C_0, C_1, C_2, C_3, C_4, C_5$ - empirical constants.
- d_f - diameter of the fiber.
- f'_c - cylinder compressive strength of concrete.
- f_{ct}, f_{sp}' - splitting tensile strength.
- f_r - modulus of rupture.
- f_{sy} - yield stress of hoop steel.

- f_t - uniaxial tensile strength of concrete.
 f_{tf} - uniform fiber tensile stress at failure of the section.
 f_y - yield strength of reinforcing steel.
 K_1 - coefficient relating the shear-compression zone depth to the overall smaller dimension of the beam.
 K_3 - coefficient relating the bar displacements to the longitudinal bar distances.
 K_c - coefficient relating the shear strength in combined shear and compression to the pure shear strength.
 K_T - coefficient relating the shear strength in combined shear and tension to the pure shear strength.
 l_f - length of fiber.
 m - ratio of volume of longitudinal bars to volume of stirrups defined as

$$m = \frac{A_{ls}}{A_t (x_1 + y_1)}$$

- n - number of effective fibers in any one direction per unit area.
 p - percentage by volume of fibers.
 P - vertical force on the shear-compression zone.
 P_c - concrete compressive force perpendicular to the shear-compressive zone.
 P_f - force in the fiber.
 P_{lc} - tensile force of the longitudinal bars in the shear-compression zone.
 P_{lT} - compressive force of the longitudinal bars in the shear-compression zone.
 P_S - shear force on the shear-compression zone.
 P_{SF} - shear force on the tension zone supplied by the fibers.
 P_{TF} - tension force on the tension zone supplied by the fibers.
 P_v - hoop force across failure surface.

- q - dowel stress per unit displacement.
- Q - dowel force in the x and y directions of the longitudinal corner bars outside the shear-compression zone.
- Q_x - dowel force in the x direction of the longitudinal corner bars outside the shear-compression zone.
- Q_y - dowel force in the y direction of the longitudinal corner bars outside the shear-compression zone.
- s - hoop spacing.
- T - externally applied torque.
- T_c - torque resisted by the concrete.
- T_{cc} - torque which is resisted by the shear in the shear-compressive zone.
- T_{cT} - total torque which is resisted by the fibers.
- T_o - ultimate torque of concrete beam without reinforcing.
- T_{oc} - ultimate torque of plain concrete beam.
- T_{of} - ultimate torque of fiber reinforced concrete beam.
- T_u - ultimate torque of reinforced concrete members.
- T_{uc} - ultimate torque contributed by shear-compression zone.
- T_{uT} - ultimate torque resisted by the reinforcing.
- V_{avg_c} - average shear stress of the shear-compression zone.
- V_{avg_t} - average shear stress of the tension zone.
- V'_c - pure shear strength of concrete.
- x - smaller overall dimension of a rectangular cross section.
- x_{cT} - distance from the center of the shear-compression zone to the axis of twist.
- x_T - distance from the center of the tension zone to the axis of twist.
- x_1 - smaller center-to-center dimension of a closed rectangular stirrup.
- x_2 - smaller distance between the centers of longitudinal corner bars.

- y - larger overall dimension of a rectangular cross section.
 y_1 - larger center-to-center dimension of a closed rectangular stirrup.
 y_2 - larger distance between the centers of longitudinal corner bars.
 α_c - coefficient for determining the torque supplied by the fibrous concrete.
 α_t - clope of ultimate torque vs. $\frac{x_1 y_1 A_t f_{sy}}{s}$ curve.
 ξ - efficiency coefficient of longitudinal bars in shear-compressive zone.
 $\bar{\xi}$ - efficiency coefficient of longitudinal bars in tension zone.
 Δ_x, Δ_y - displacement in the x and y directions.

Table 1. Test Results

Specimen Number	Fiber per cent	Cracking Torque T_{cr}	Ultimate Torque T	Compressive Strength f'_c	Splitting Tensile Strength f_{sp}	Hoop Spacing S
1	0	56,700	83,200	4200	481	7"
2	1½ x	83,200	98,000	4670	810	None
3	1½ x	39,100	149,000	4670	810	7"
4	1 *	65,700	125,000	4200	705	7"
5	0	53,800	89,600	4300	513	None
6	1 *	65,700	83,700	4300	705	None
7	2 x	89,600	128,000	4900		None
8	2 x	101,000	179,000	4900		7"
9	1 x	83,700	146,000	4300	705	3.5"

x = Length: 50 mm

* = Length: 30 mm

Diameter: .5 mm

Diameter:  3 mm

Table 2. Comparison of T_u for Fibrous Concrete Beams (Test vs. Theory)

Beam No.	Fiber Per cent	f'_c	f_r	f_t	f_{ct}	T_u Eq. 1	T_u Eq. 2	T_u Eq. 3	T_u Eq. 4	T_u Eq. 5	T_u Test Values
1	0	4200	385	288	481	53,400	45,600	56,600	65,900	46,900	56,700
2	1½ x	4670	975	486	810	55,360	84,700	80,100	111,000	119,000	83,200
3	1½ x	4670	975	486	810	55,360	84,700	80,100	111,000	119,000	89,100
4	1 *	4300	550	423	705	53,400	57,800	72,900	96,800	67,000	65,700
5	0	4200	400	308	513	53,400	46,700	59,100	70,500	48,800	53,800
6	1 *	4300	550	423	705	53,900	57,800	72,900	96,800	67,000	65,700
7	2 x	4900	1104	583	973	56,300	91,900	90,400	133,000	134,000	89,600
8	2 x	4900	1104	583	973	56,300	91,900	90,400	133,000	134,000	101,000
9	1 x	4300	624	423	705	53,900	62,800	72,900	67,000	76,100	83,700

x = small fiber (30-50)

* = large fiber (50-50)

ϕ = Assume $f_t = .6 f_{ct}$ (f_t (max) may not actually be f_t of concrete.)

$$T_u = 6(x^2 + 10) y \sqrt[3]{f'_c} = \text{Eq. 1} \quad T_u = 1.67 (x + 10) y \sqrt[3]{f_{ct}^2} = \text{Eq. 3} \quad T_u = \frac{x^2 y}{3} (.85 f_r) = \text{Eq. 5}$$

$$T_u = 1.56 (x^2 + 10) y \sqrt[3]{f_r^2} = \text{Eq. 2} \quad T_u = \propto x^2 y \left[f_t (\text{max}) \left(\frac{3.24}{1 + x/y} \right) \right] = \text{Eq. 4}$$

Table 3. Evaluation of α_t for Ultimate Torque Equation for Reinforced Fibrous Concrete Beams.

Fiber Type	P %	P' _c	T _o	T _{s=7}	T _{s=3.5}	ΔT	$\frac{x_{ly} A_f}{l_t s_v}$		Δ	α _t
							s			
							s = 7	s = 3.5		
0	0	4200	55.2	83.0	117.	34.	30.2	60.4	30.2	1.12
30-50	1	4200	83.7	179.	262.	83.	30.2	60.4	30.2	2.75
30-50	1.5	4670	86.0	149.	216	67.	30.2	60.4	30.2	2.22
30-50	2	4900	95.3	114.	146	32	30.2	60.4	30.2	2.05
50-50	1	4300	65.7	126	168	62	30.2	60.4	30.2	1.05

Table 4. Evaluation of C_4 and C_5 Constants for α_c for Ultimate Torque:
Equation for Reinforced Fibrous Concrete Beams.

Fiber Type	P* %	l_f Fiber Length (in.)	d_f Fiber Diameter (in.)	f'_c Concrete Strength psi	T_o Torque of Plain Fibrous Beam in K	P_f^+ Pull-out Load lb.	n Fibers inch	$\frac{P_f n}{\sqrt{f'_c}}$ coefficient	f_{tF}^o Fiber Tension psi	C_4	C_5
0	0	---	---	4200	55.2	--	---	---	---	2.0	---
30-50	1	1.18	.197	4200	83.7	21	18.9	6.14	279	2.0	.166
30-50	1.5	1.18	.197	4670	86.0	21	28.5	8.74	416	2.0	.119
30-50	2	1.18	.197	4900	95.3	21	37.9	11.4	557	2.0	.117
50-50	1	1.97	.197	4300	65.7	35	6.83	3.65	167	2.0	.102

* - by volume

+ - determine by reference A

o - $f_t = .7 P_{fn}$

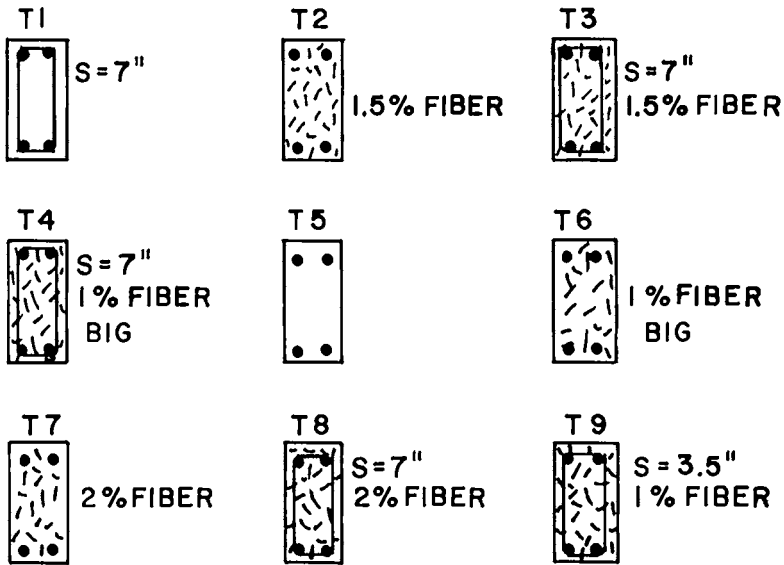


Fig. 1--Test series--Torsion

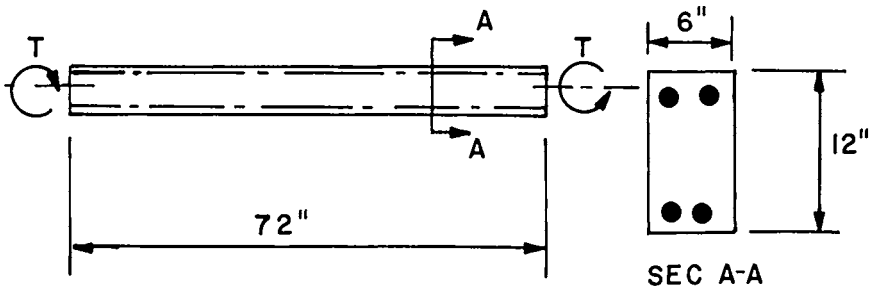


Fig. 2--Test beam section

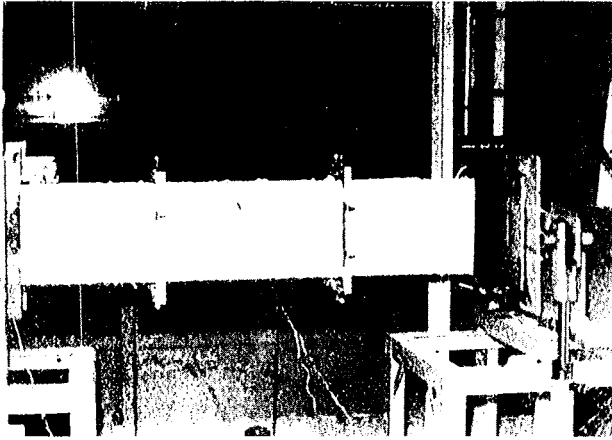


Fig. 3--Torsion test setup

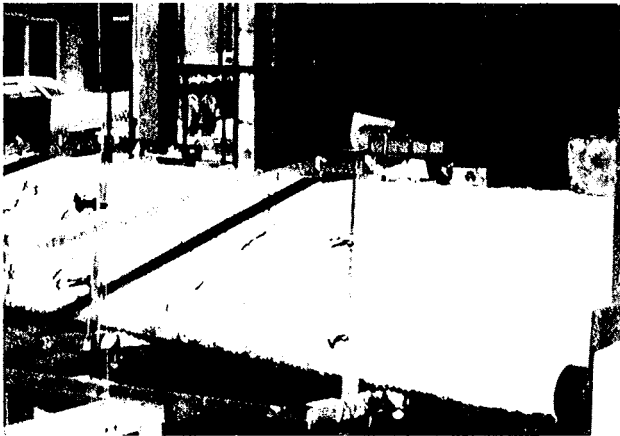


Fig. 4--Beam in torsion at the ultimate torsional moment

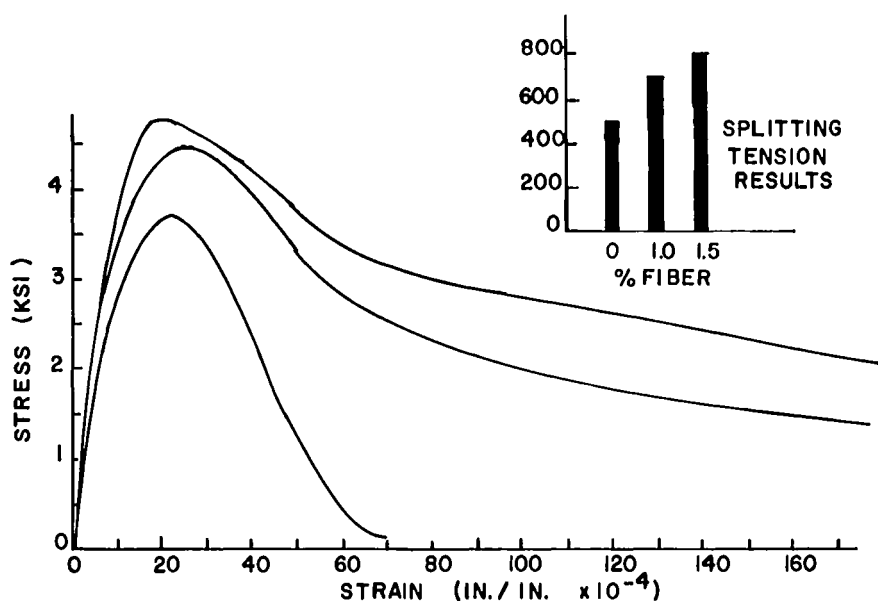


Fig. 5--Typical compressive and tensile stress-strain curves for fibrous concrete used in study

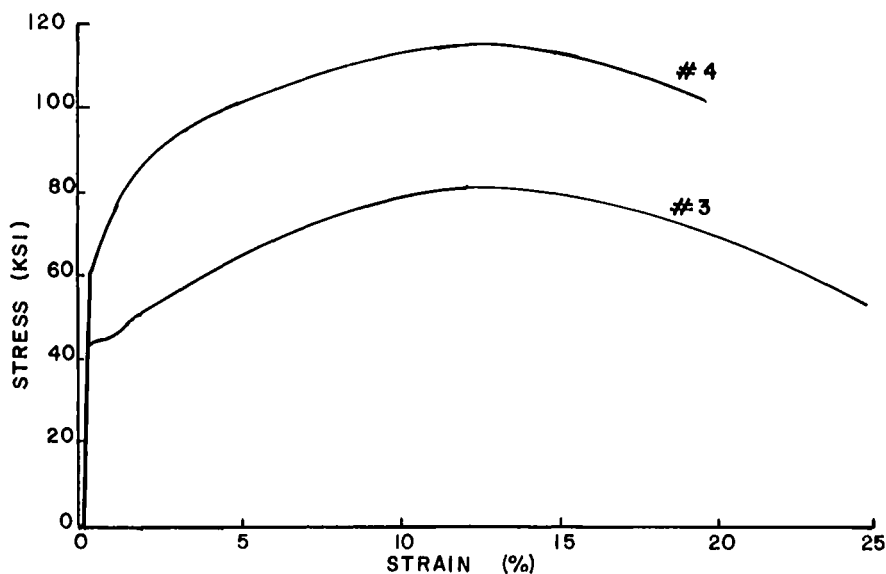


Fig. 6--Typical stress-strain curves for steel in tension

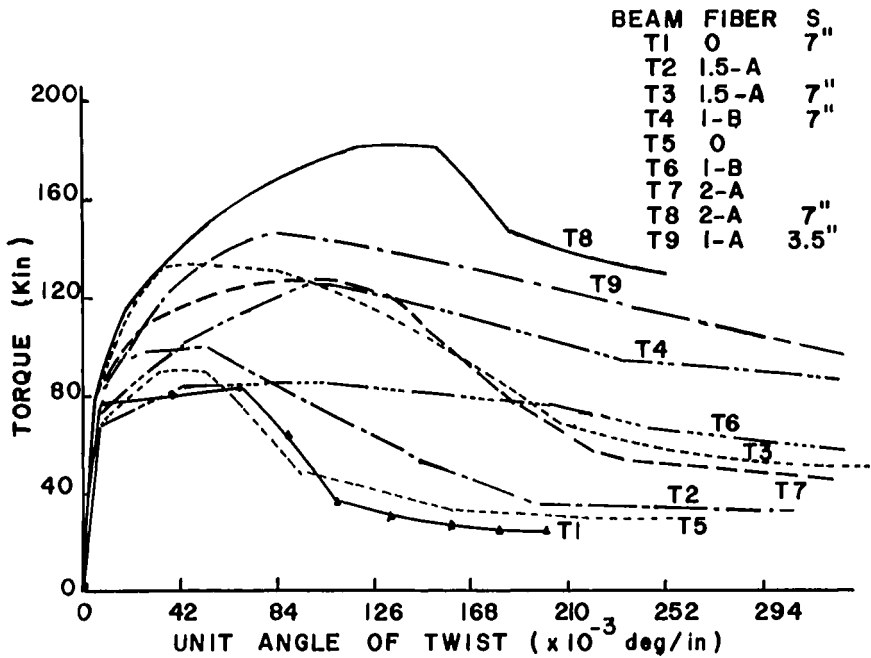


Fig. 7--Results from beam tests on reinforced fibrous concrete members in torsion

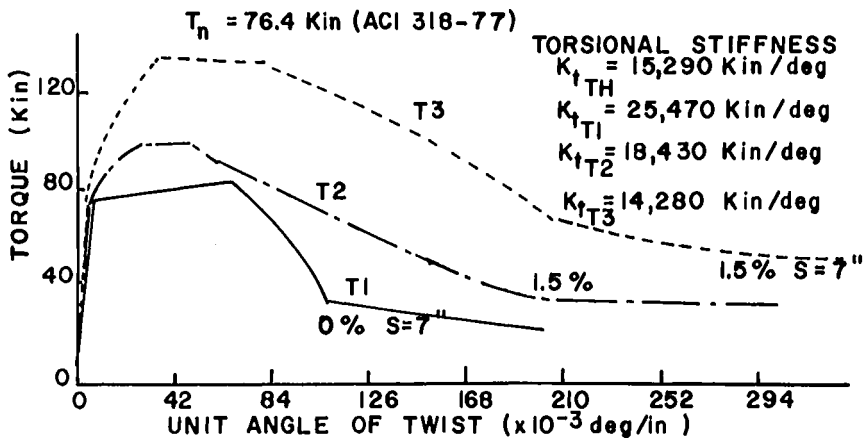


Fig. 8--Behavior of reinforced fiber in torsion

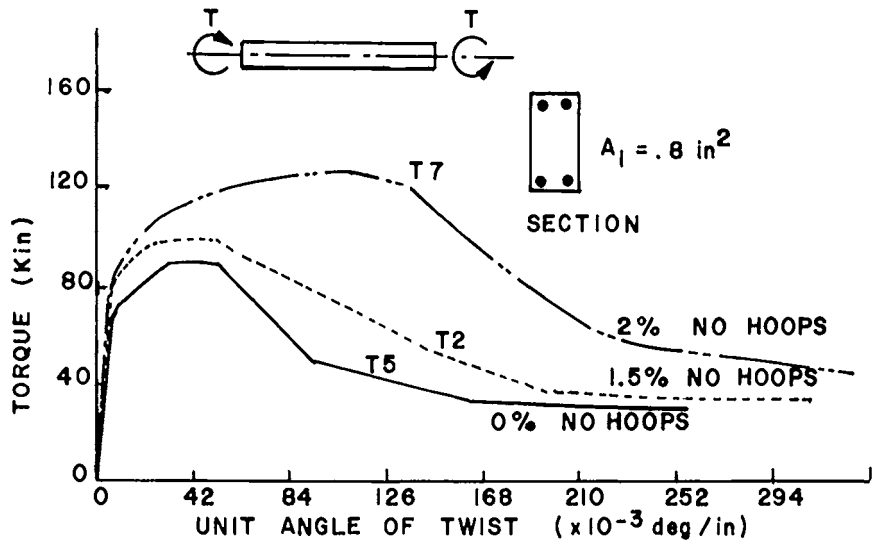


Fig. 9--Influence of percentage of fibers on the behavior of reinforced fibrous concrete beams in torsion

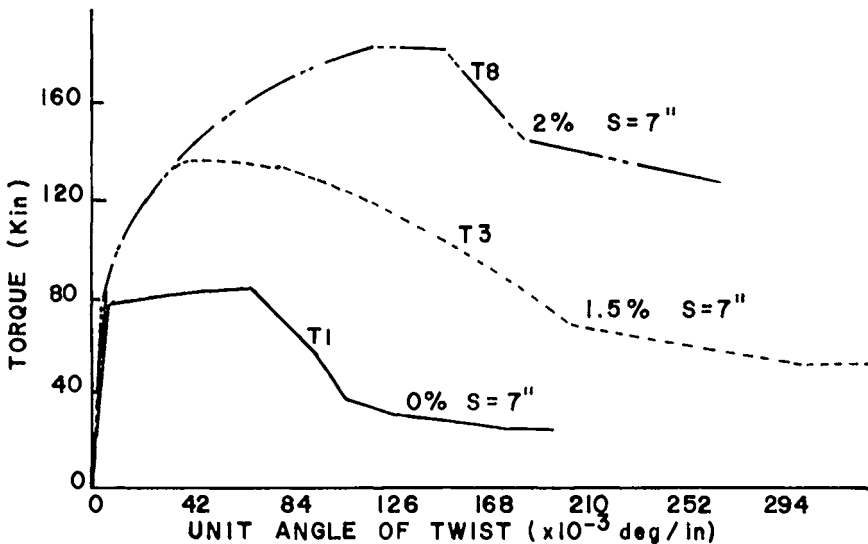


Fig. 10--Influence of percentage of fibers on the behavior of reinforced fibrous concrete beams in torsion

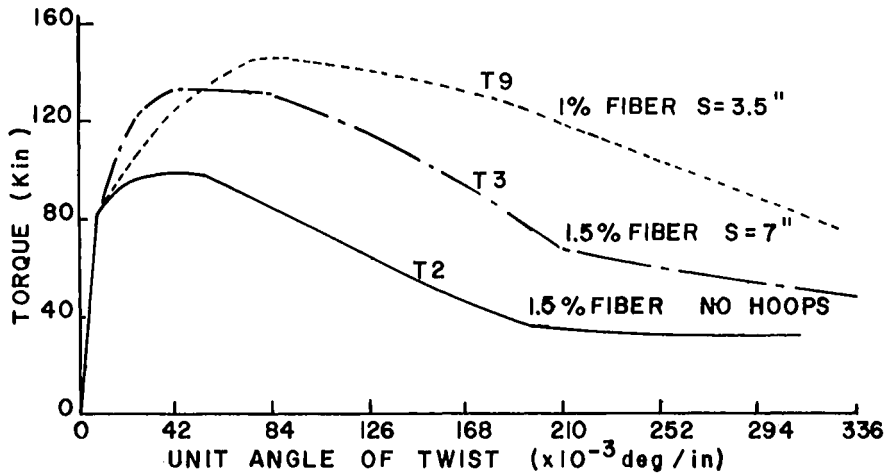


Fig. 11--Influence of hoop spacing on the behavior of reinforced fibrous concrete in torsion

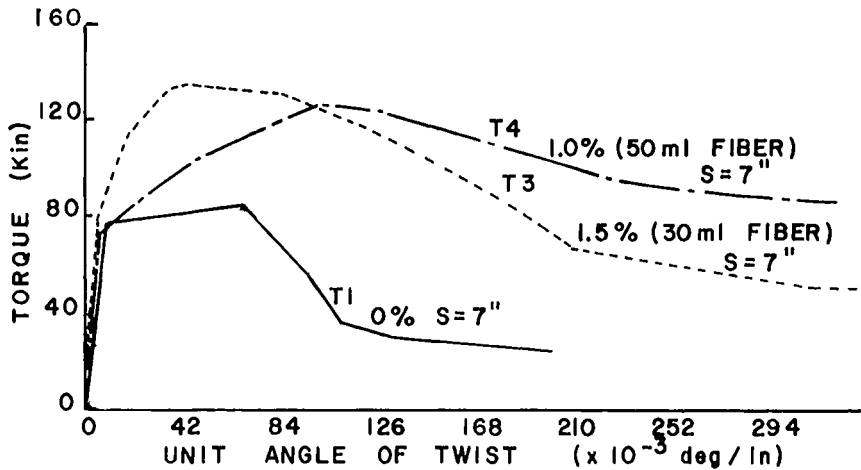


Fig. 12--Influence of fiber type on the behavior of reinforced fibrous concrete in torsion

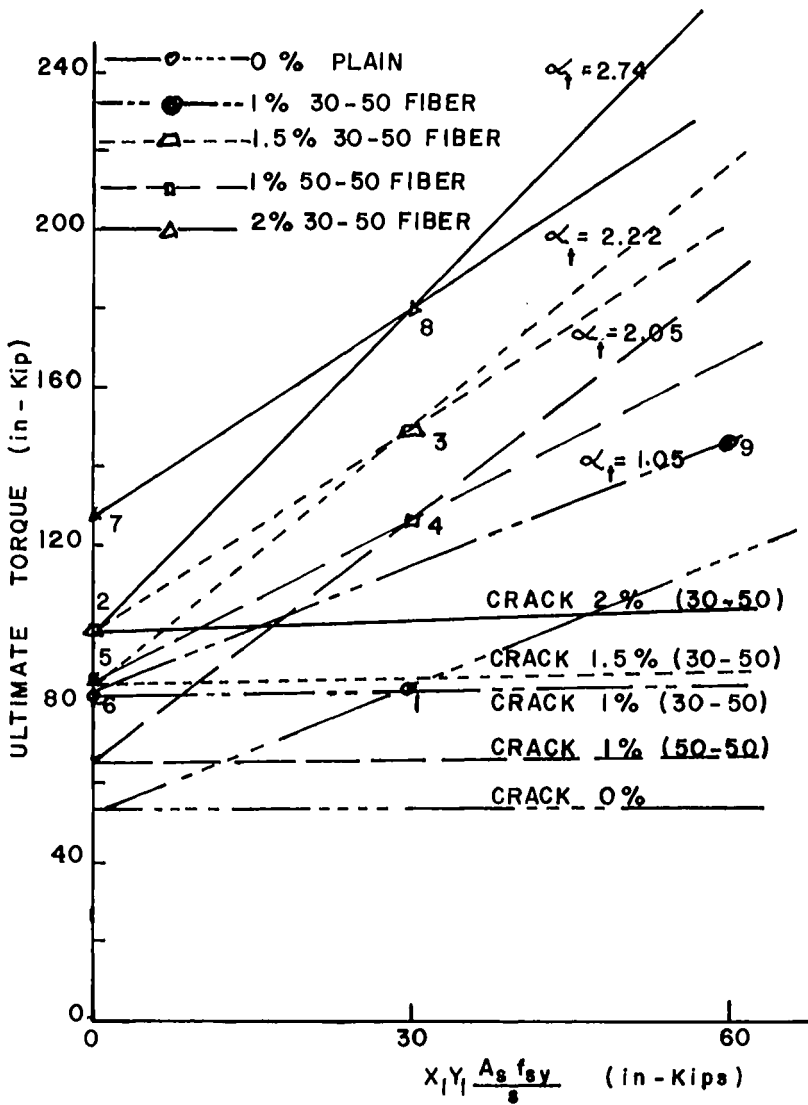


Fig. 13--Results of beam tests in torsion for reinforced fibrous concrete beams

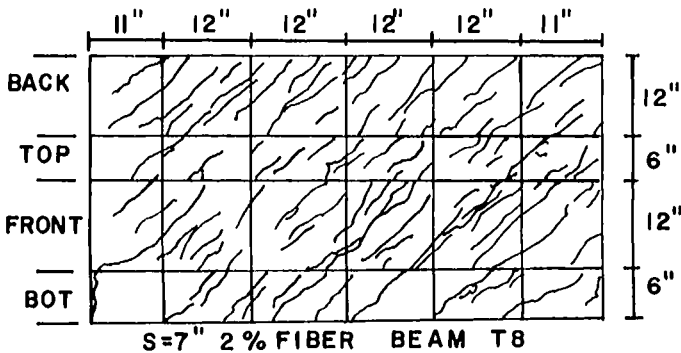
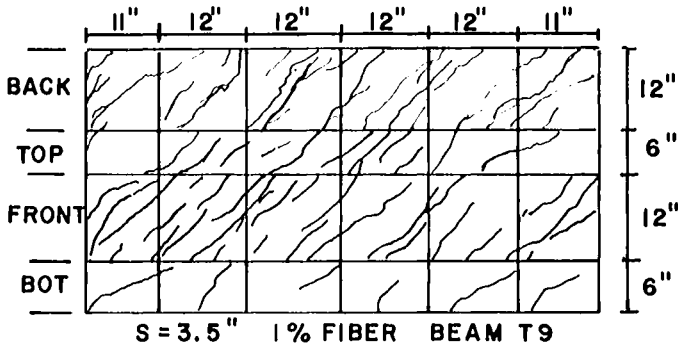
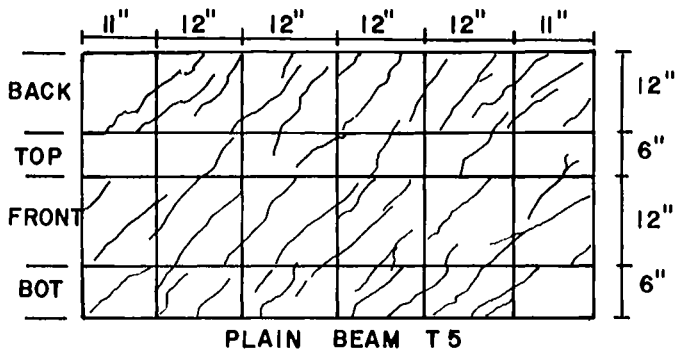


Fig. 14--Cracking patterns for reinforced fibrous concrete beams T5, T9, and T8

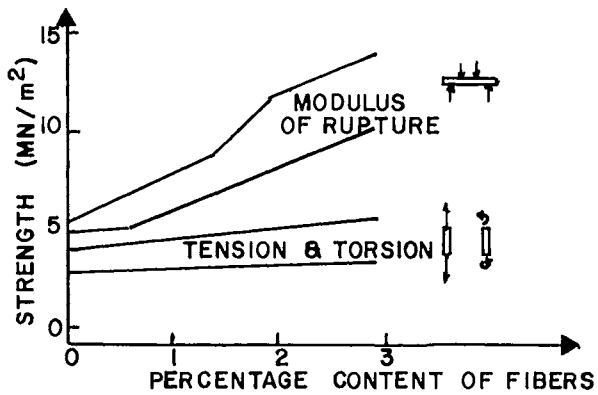


Fig. 15--Flexural, direct tensile and torsional strengths of fibrous reinforced mortar and concrete (1 and 2)

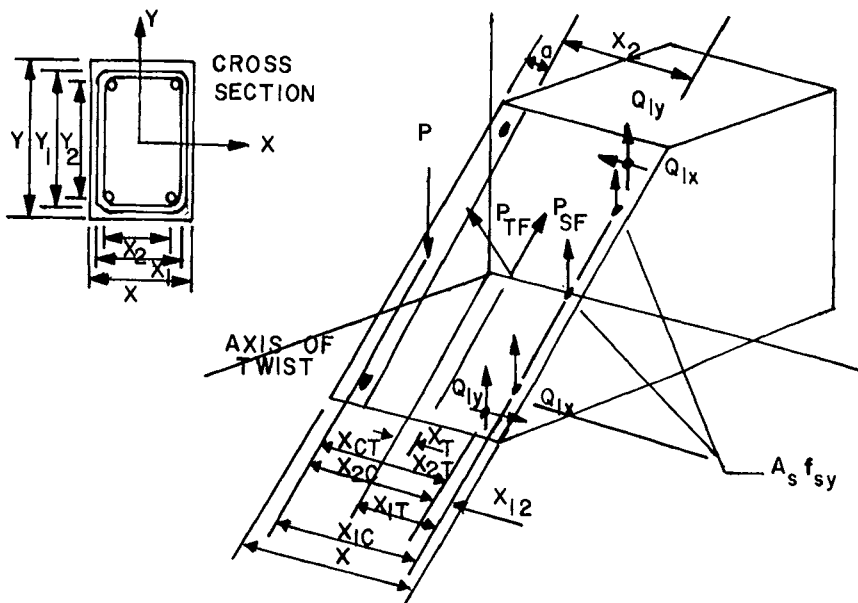


Fig. 16--Proposed failure surface

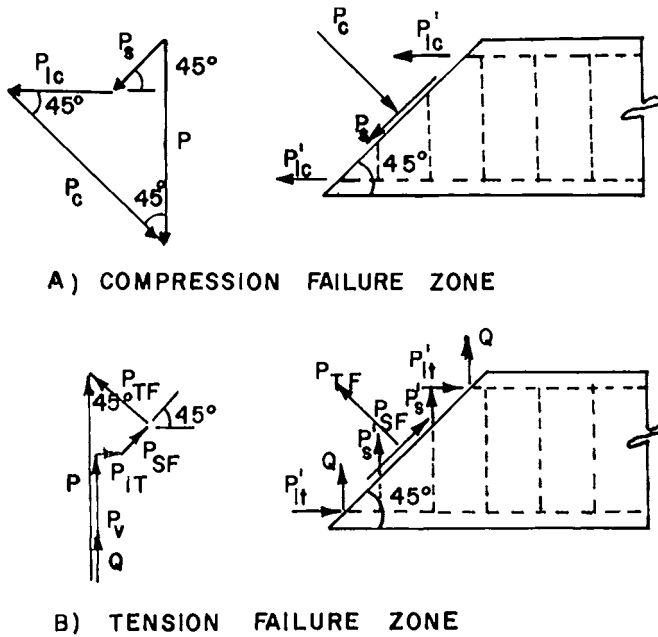


Fig. 17--Components of resultant force P acting on compression and tension zone of failure plane

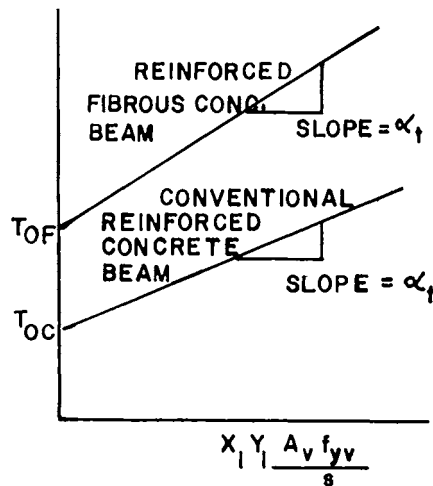


Fig. 18--Behavior of reinforced fibrous concrete beams in torsion

