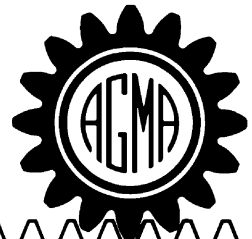


Application of Statistical Stability and Capability for Gear Cutting Machine Acceptance Criteria

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American Gear Manufacturers Association



TECHNICAL PAPER

Application of Statistical Stability and Capability for Gear Cutting Machine Acceptance Criteria

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

Machine tool manufacturers supplying machines to the gearing world have been in existence for many years. The machines have changed and so has the acceptance criteria for the machines. Before the 1980's the criteria for virtually all machine acceptance was either a suppliers standard test job or the supplier producing one or two of the customers' parts to within a print tolerance. It wasn't until about 1984 that we were required to perform a capability analysis for a machine acceptance on a cylindrical hobbing machine. Capability requirements for bevel machine acceptance did not occur until several years after that. Today virtually every customer requires a capability study on at least one parameter for at least one type of part.

Since the introduction of capability requirements for machine acceptance we have seen the goal post move. Initially the requirement was for a C_p or C_{pk} of 1.33 using a 6 sigma analysis. Now we have seen requirements of a 1.67 or 2.0 C_p or C_{pk} with a 6, 8 or 10 sigma analysis on tolerances that have been tightened from the original tolerance! This can cause some real headaches for the machine tool supplier, and that is why it is very important for the supplier to understand the true machine and process capability before agreeing to any capability requirement.

In 1991 the quality and supplier assessment staffs at Chrysler, Ford and General Motors worked under the auspices of the Automotive Division of the American Society for Quality Control (ASQC) Supplier Quality Requirements Task Force in collaboration with the Automotive Industry Action Group (AIAG) to put together the Statistical Process Control (SPC) Reference Manual. The same group developed the Measurement Systems Analysis (MSA) Reference Manual. Many companies, such as Delphi Automotive Systems, have developed their own statistical qualification requirements based on both the SPC and MSA Reference Manual. This paper will be referencing the SPC and MSA Manuals and the Delphi Specification SD-002 for much of the material presented.

With this paper we hope to review some of the basics of Statistical Process Control [SPC] and provide a better understanding of its application as it relates to a machine runoff.

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Customer Agreement, Data Collection & Distributions

Before conducting a machine runoff that has a capability study tied to it, it is very important that both the customer and the machine tool supplier agree on the parameters to be measured, and to what the tolerance and capability requirement is for the machine acceptance. Once this is established, the parts are usually produced on the machine in a continuous run without interruption under the conditions agreed to. The parts are then inspected on equipment that has passed a GR&R study [more on GR&R later] and the resultant data analyzed. There are a number of tools that are used today to analyze the data collected, from the use of spread sheet software that can be tailored using the built in statistical functions, to software designed specifically for statistical analysis such as the popular MINITAB package. Once collected, the data can be organized, analyzed, interpreted and presented. Besides the average, range and other statistics of the data, the standard deviation can be calculated. Think of the standard deviation as the statistical spread or dispersion of the data collected from the mean. There are several methods that can be used to calculate the standard deviation, designated by the Greek letter σ [sigma]. One method is to use the individual values of a process characteristic, and another method estimates the standard deviation using the average range from a subgroup analyses and a factor [designated as d_2]. See below for further discussions on subgroups.

No two parts are exactly alike because every process contains some source of variability. While

individual values may be different, as a group they can form a pattern that can be described as a distribution.

Distributions are characterized by a location [the typical value], by a spread [the span of values] and a shape [the pattern of variation]. See the Figure 2.

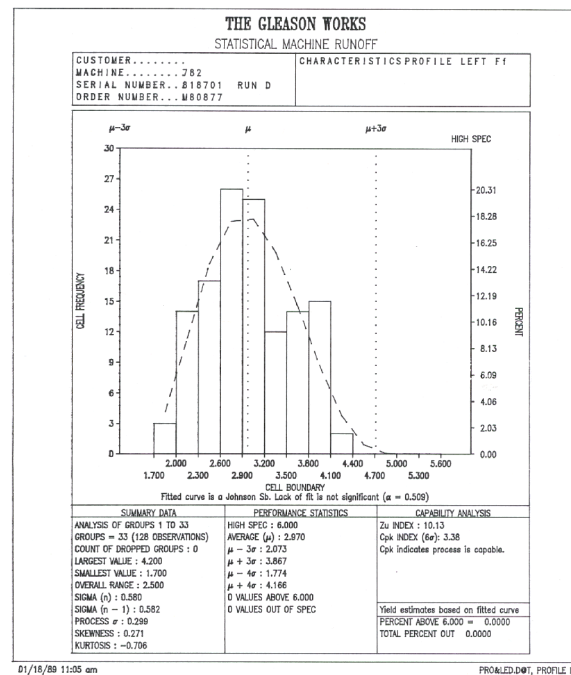


Figure 1. Histogram and Distribution

The causes of variation in a distribution are referred to as either "Common Causes" or "Special Causes". The term "Assignable Causes" is often used in place of "Special Causes".

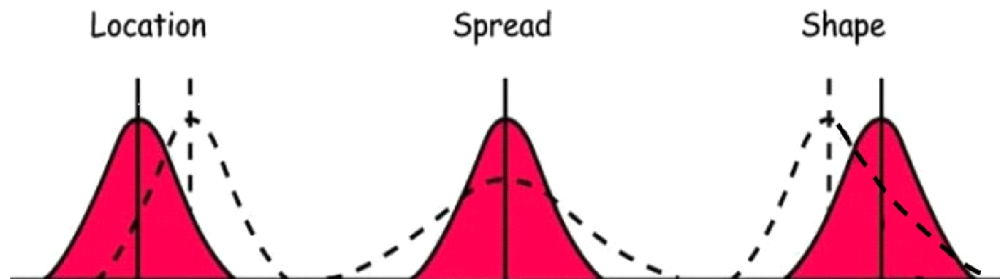


Figure 2. Distributions

The type of variation preferred in any distribution is that of common causes. When common causes exist (also referred to as random variations) the process is said to be “in control” and the output of the process is stable and predictable over time, as depicted in the Figure 3.

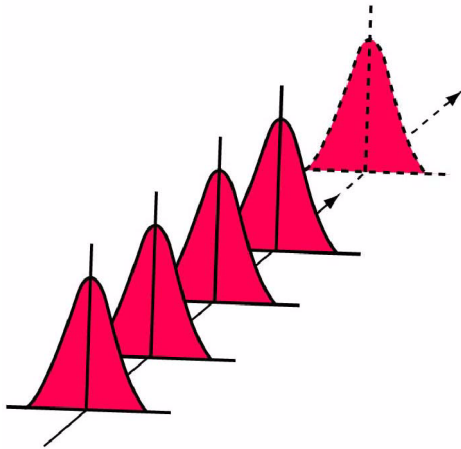


Figure 3. Predictable Distribution

Distributions with special or assignable causes of variation are not stable over time. When present, assignable causes will produce changes in the distribution, and if they are not removed the process output cannot be predicted.

If a process is in control [a predictable distribution] the number of in-specification parts can be estimated. As long as the process remains stable and the distribution does not change in location, spread or shape, it will continue to produce the same distribution of in-specification parts.

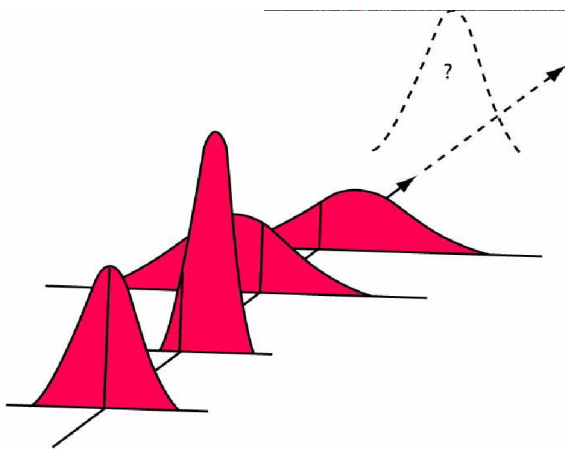


Figure 4. Un-Predictable Distribution

Control Charts: $\bar{X}$ and R Charts:

In the 1920's Dr. Walter Shewhart of Bell Laboratories developed what is known as the control chart to make the distinction between controlled and uncontrolled variation due to common [random] and special [assignable] causes. Control charts for variables are powerful tools that can explain process data in terms of both spread [piece to piece variability] and location [process average]. Control charts should be prepared and analyzed in pairs, most commonly the $\bar{X}$ and R charts. $\bar{X}$ is the average of the values in subgroups, which is the location of the data, and R is the range of the values within each subgroup, which is a measure of the data spread.

A sample subgroup table or data block is shown in Figure 5. The table consists of a defined number of subgroups, and data from each subgroup. Each subgroup has a Total, an Average [$\bar{X}$] and a Range [R].

Generally, there are three [3] to five [5] individual parts per subgroup and there are generally 25 or more subgroups in the analysis. The frequency of the data collection for the subgroups is determined to detect changes in the process over time. During an initial study, the frequency can be over a short period of time, or even taken with consecutive parts. This is generally the case during a machine qualification study because of the availability of parts and the time allotted to the machine runoff. In a production environment the frequency can be hourly, several times per shift, or any feasible timeframe.

From the data collected, the Average Range [$\bar{R}$] and the Process Average [$\bar{X}$] are calculated simply by averaging the subgroup ranges and averages. The next step in the process of creating the control charts is to calculate the Control Limits. Control limits are used to show the extent by which the subgroup averages and ranges would vary if only common [random] causes of variation were present. They are based on the subgroup sample size and the amount of “within” subgroup variability reflected in the ranges. The formulae for the Upper Control Limit [UCL] and Lower Control Limit [LCL] for the $\bar{X}$ and R charts follow:

$$UCL_R = D_4 \bar{R}$$

$$LCL_R = D_3 \bar{R}$$

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2 \bar{R}$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - A_2 \bar{R}$$

Data and Control Charts for Capability Study

Sub-Group #	1	2	3	4	5	6	7	8	9	10
Data	30.5	30.4	37.3	35.7	37.9	37.0	32.6	35.0	33.5	34.8
From Each	34.3	36.2	36.6	30.3	31.1	30.8	30.6	34.1	31.3	35.7
Sub-Group	36.7	31.3	36.3	37.0	31.2	34.9	34.8	36.1	30.6	37.7
	37.6	35.6	34.3	34.9	32.6	31.5	35.3	31.5	32.9	33.4
	37.5	37.5	37.0	33.1	36.9	33.9	30.8	34.5	34.7	32.9
Total	176.60	171.00	181.50	171.00	169.70	168.10	164.10	171.20	163.00	174.50
Average ($\bar{x}$ -bar)	35.32	34.20	36.30	34.20	33.94	33.62	32.82	34.24	32.60	34.90
Range (R)	7.1	7.1	3	6.7	6.8	6.2	4.7	4.6	4.1	4.8

Figure 5. Data Table for A Control Chart

The factors A_2 , D_3 and D_4 are constants based on subgroup size $[n]$ taken from the chart shown in Table 1. Note that for subgroup sizes below 7 there is no Lower Control Limit for the Range Chart, only an Upper Control Limit. An interesting point to keep in mind is that the control limits have absolutely nothing to do with the tolerance of the parameter

being evaluated.

Now the $\bar{X}$ and R charts can be created with the calculated control limits by plotting the $\bar{X}$ for each subgroup on one chart [Figure 6] and the range R for each subgroup on another chart [Figure 7]. Very often the two charts are combined.

n	2	3	4	5	6	7	8	9	10
D_4	3.27	2.57	2.28	2.11	2.00	1.92	1.86	1.82	1.78
D_3	*	*	*	*	*	0.08	0.14	0.18	0.22
d_2	1.13	1.69	2.06	2.33	2.53	2.7	2.85	2.97	3.08
A_2	1.88	1.02	0.73	0.58	0.48	0.42	0.37	0.34	0.31

Table 1. Factors For Control Chart Control Limits

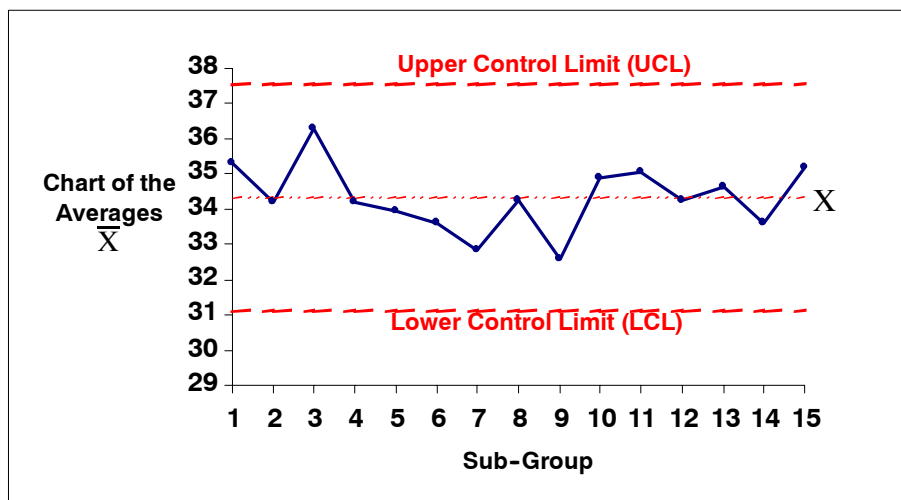


Figure 6. Control Chart for Averages

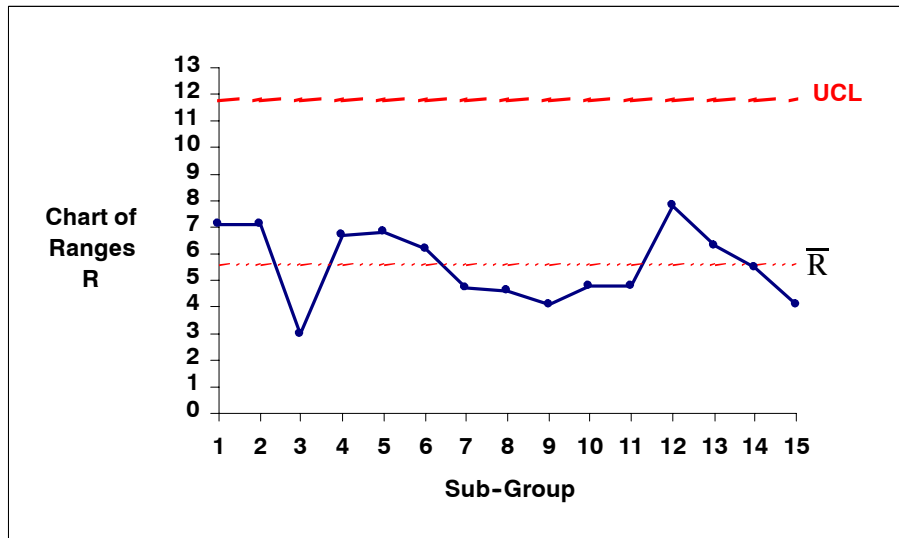


Figure 7. Control Chart for Ranges

Stability

Now that we have the data and charts, what do we do with them? Basically, if the process variability and average were to remain constant, the subgroup ranges and averages would vary by chance only, and would not exceed the control limits. In theory, there would be no runs or trends in the data and the subgroups would be positioned randomly around the centerline. If all of the above were the case, the process would be “in control” and stable.

To summarize, there are several criteria that can be used to determine if a process is “out of control”:

- 1] Data Points Outside the Control Limits [Figure 8]
- 2] Runs Within the Control Limits [Figure 9]
Seven [7] consecutive points above or below the centerline
- 3] Trends [Figure 10]
Seven [7] points consistently increasing or decreasing
- 4] To be stable and in control, 2/3's of the points must be within the middle 1/3 of the chart. The 1/3 chart band is determined by dividing the difference of the UCL – LCL by 3 [Figure 11]

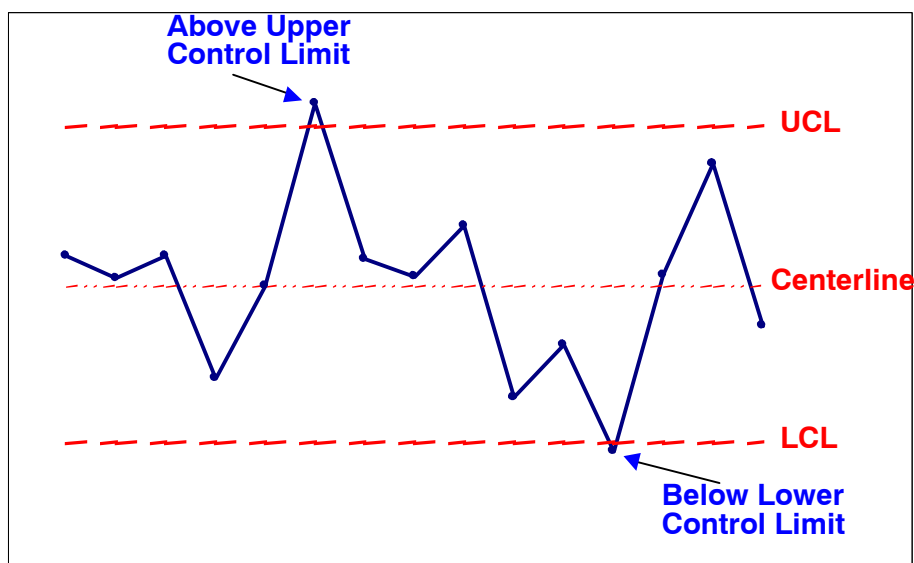


Figure 8. Subgroups Outside the Control Limits

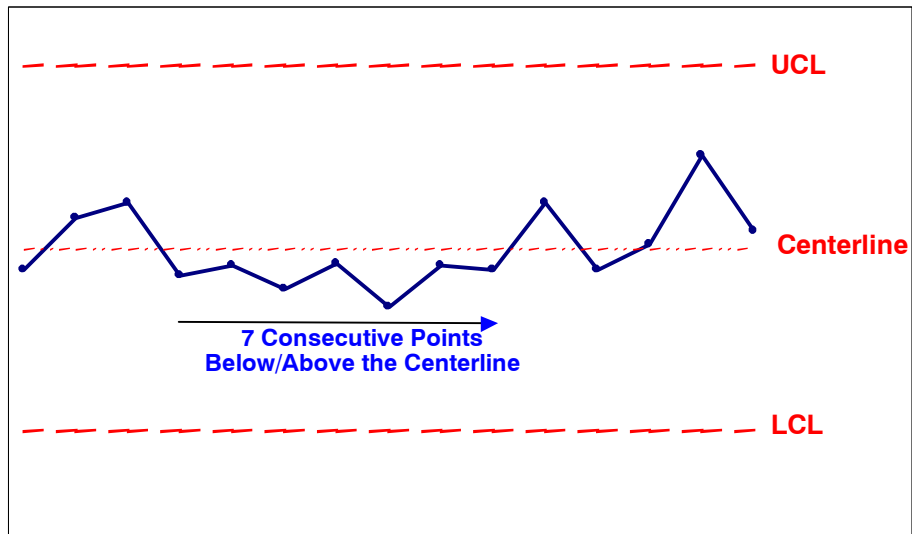


Figure 9. Consecutive Points Above or Below Centerline

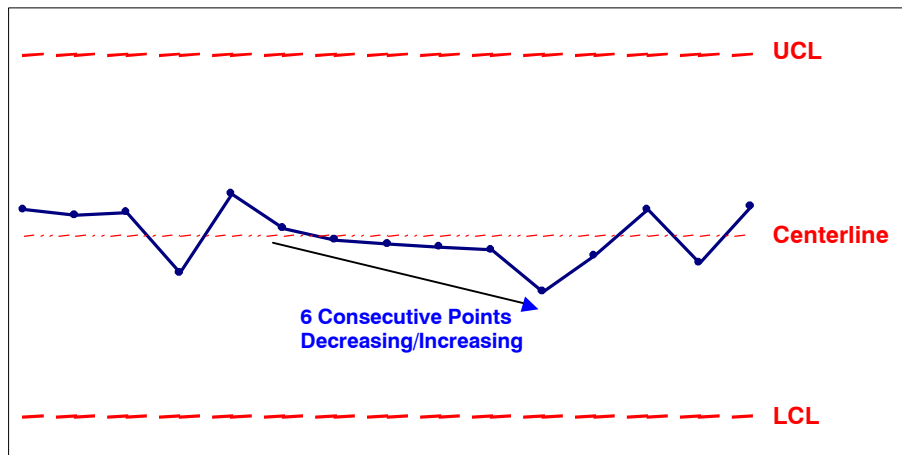


Figure 10. Consecutive Points Moving Up or Down

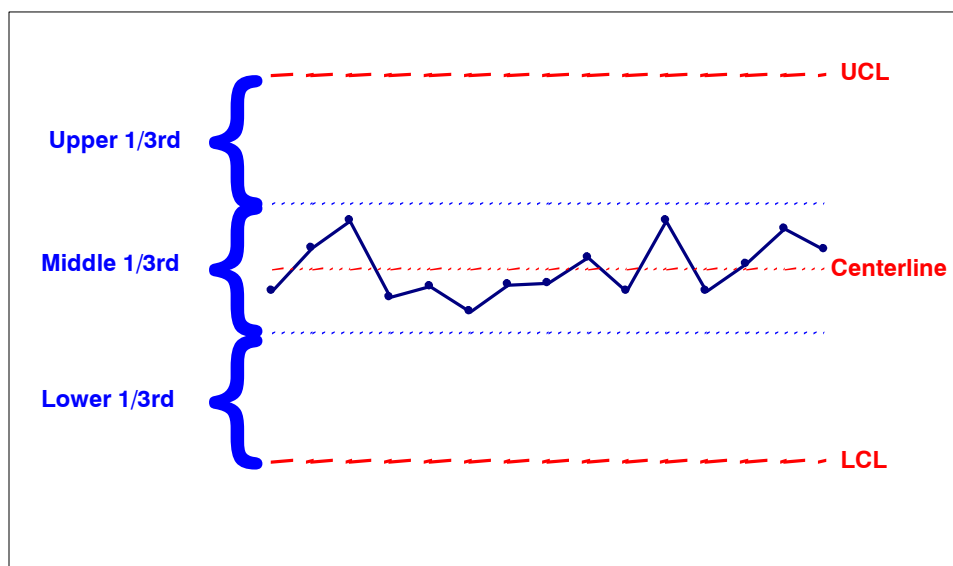


Figure 11. Subgroups Within the Middle 1/3 of the Chart

The criteria for stability may vary depending on the customers' specifications. There may be changes in the number of points that determine a trend or a run, i.e six [6] instead of seven [7], or the number of subgroups specified to be within the middle third of the chart, etc. As stated earlier, agreement with the customer should be reached and understood before any trial begins.

If the process is shown to be out of control, the assignable cause or causes must be identified and eliminated. The process for troubleshooting and correcting the assignable causes in the gear manufacturing process is a subject in itself and will not be addressed in this paper.

The important point is that the process must be in control and stable before determining the process capability.

Capability

Process capability is a measure of how well the process output meets the specified requirements [tolerance]. Every process can be classified as falling into one of four cases as shown in Table 2.

The preferred situation is to have a Case 1 condition where the process is both in control and capable. As often occurs in the case of machine qualifications, a customer will allow a Case 3 condition where the process may not be in control, but capable. For the most part, it is not a case of the process being out of control, but the fact that you do not know if the process is in control or out of control. This is generally due to the fact that not enough parts are available from the customer to perform a true stability study. If this is the case, only a test for capability is conducted at the supplier facility.

Meeting Requirements	In Control	Not in Control
Acceptable (Capable)	Case 1	Case 3
Not Acceptable (Not Capable)	Case 2	Case 4

Table 2

The capability indices that are used today are Cp, Pp and Cpk, Ppk. Cp and Pp are indices of process variation that are only relative to a specification. Cpk and Ppk are indices that combine process variation and process centering [location] relative to a specification. As you will see below, the equation

for Cp and Pp is the same, except for the method of calculation used for the standard deviation in the equation. As with Cp and Pp, Cpk and Ppk also have the same equation except for the method of calculation for the standard deviation. Cp and Cpk use the standard deviation ($\hat{\sigma}_{\bar{R}/d_2}$) estimated from subgroups using the average range [$\bar{R}$] and the d_2 factor from the chart in Table 1:

$$\hat{\sigma}_{\bar{R}/d_2} = \bar{R}/d_2$$

Pp and Ppk use the sample standard deviation ($\hat{\sigma}_s$) calculated from the individual values of the characteristic:

$$\hat{\sigma}_s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

where x_i is the individual value, $\bar{x}$ is the average, and n is the total number of individuals sampled.

Cp is the capability index defined as the tolerance width divided by the process capability, irrespective of process centering [location]:

$$Cp = \frac{(USL - LSL)}{6\hat{\sigma}_{\bar{R}/d_2}}$$

where USL is the Upper Specification Limit and LSL is the Lower Specification Limit.

Pp is the performance index defined as the tolerance width divided by the process performance, irrespective of process centering [location]. Pp should be used only to compare to or with Cp and Cpk and to measure and prioritize improvement over time.

$$Pp = \frac{(USL - LSL)}{6\hat{\sigma}_s}$$

CPU is the upper capability index and is defined as the upper tolerance spread divided by the actual upper process spread.

$$CPU = \frac{(USL - \bar{\bar{X}})}{3\hat{\sigma}_{\bar{R}/d_2}}$$

CPL is the lower capability index and is defined as the lower tolerance spread divided by the actual lower process spread.

$$CPL = \frac{(\bar{\bar{X}} - LSL)}{\hat{\sigma}_{\bar{R}/d_2}}$$

Cpk is the capability index that accounts for process centering and is defined as the minimum of CPU or CPL. Cpk relates the distance between the process