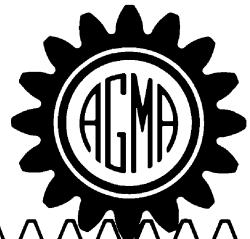


Generalized Excitation of Traveling Wave Vibration in Gears

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American Gear Manufacturers Association



TECHNICAL PAPER

Generalized Excitation of Traveling Wave Vibration in Gears

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

Dynamic loading at tooth mesh frequency and its harmonics excites traveling wave vibration in gears. The associated alternating stress can be large enough to cause high cycle fatigue failure of the gears. In this paper a generalized excitation of traveling wave vibration in gears is developed for both forward and backward waves. Although they do not predict the actual dynamic response of the gear, the developed expressions quantify the relative excitation as a function of nodal diameter allowing the analyst to place modes with constructive excitation outside the gear's operating range. These generalized equations allow the following effects to be addressed: any combination of tooth counts, any number of gears in mesh with the center gear, symmetric or non-symmetric spacing of these surrounding gears, and non-equal power extraction from the surrounding gears. The equations presented are thus applicable for configurations ranging from a single gear set to a sequential, non-symmetric planetary system. Dynamic response from a finite element model is presented to verify the generalized excitation for the various configurations.

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GENERALIZED EXCITATION OF TRAVELING WAVE VIBRATION IN GEARS

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Introduction

As a gear pair rotates through mesh the number of teeth in contact changes. For a spur gear with a contact ratio between one and two, the number of teeth in contact changes from a single pair, to two pairs, back to a single pair, etc. The angular stiffness between the gears is roughly proportional to the number of teeth in contact. This step change in stiffness contributes to the total transmission error and creates dynamic load at mesh frequency and harmonics of mesh frequency.

Flexural vibration of a gear occurs when the alternating load associated with tooth mesh is properly timed to constructively reinforce a traveling wave mode shape. The conditions for constructive reinforcement of the wave are not complicated for a single gear set (a gear meshing with just one other gear). The conditions for constructive reinforcement of the wave become more complicated when the gear meshes with two or more other gears. Stockton (1) identified these conditions for sun gears in both sequential and non-sequential planetary gearboxes, but his work was limited to configurations with symmetric spacing of the planet gears. This excellent work laid the foundation to expand knowledge of traveling wave excitation to more general gearbox configurations. This paper presents generalized equations for constructive reinforcement of traveling waves for any gearbox configuration.

Nomenclature

A	Traveling wave amplitude, inch
D_A	Pitch diameter of gear A , inch
D_i	Pitch diameter of gear i , inch
E_i	Relative excitation of wave from gear i
F_m	Natural frequency of gear A for mode with m waves (in rotating reference frame), Hz
L_i	Tooth mesh lag fraction of gear i
P_i	Relative power transmitted to gear i
X_i, Y_i	Coordinates of center of gear i , inch

f	Natural frequency in fixed frame, Hz
k	Mesh frequency harmonic number
m	Harmonic waves around circumference
n_A	Number of teeth on gear A
n_i	Number of teeth on gear i
θ_i	Angular location of center of gear i , rad
θ_{wave}	Angular position of traveling wave, rad
ω_A	Angular speed of gear A , rad/sec
ω_i	Angular speed of gear i , rad/sec
ω_{wave}	Angular speed of traveling wave in fixed reference frame, rad/sec

Traveling Waves

Figure 1 illustrates the difference between standing and traveling waves. A harmonic excitation at a fixed location on a circular part (i.e. a piezoelectric crystal driver glued a gear) will produce a standing wave with fixed "nodal" locations of zero displacement. Locations between the nodal lines experience harmonic motion undergoing one half a cycle between the blue and red shapes labeled 1 and 3. For harmonic excitation that is not at a fixed location on the part, the mode shape will travel around the part. Traveling wave vibration occurs in gears because the dynamic load occurs at a fixed angular location (the point of mesh) instead of a fixed location on the gear.

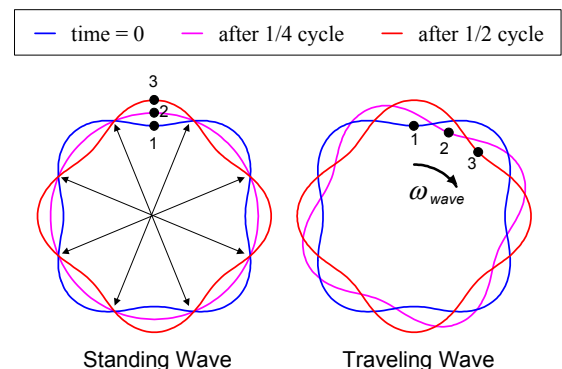


Figure 1. Example of standing and traveling waves

Wave Speed and the Campbell Diagram

The angular speed of the traveling wave is not arbitrary because the natural frequency is the same whether the wave is standing or traveling. For a particle on a non-rotating part to have the same frequency as a standing wave, the angular speed of a forward or backward traveling wave is:

$$\omega_{\text{wave}} = \pm \frac{2\pi F_m}{m} \quad (1)$$

For a part rotating at speed ω , the angular speed of a traveling wave is:

$$\omega_{\text{wave}} = \frac{2\pi \left(m \frac{\omega}{2\pi} \pm F_m \right)}{m} \quad (2)$$

The frequency of the traveling wave as observed from the fixed reference frame is:

$$f = F_m \pm m \frac{\omega}{2\pi} \quad (3)$$

Campbell (2) identified the phenomena of traveling waves in his investigation of steam turbine disk vibration failures. He graphically presented traveling waves in his classic frequency-speed diagram, now commonly known as a Campbell Diagram (Figure 2).

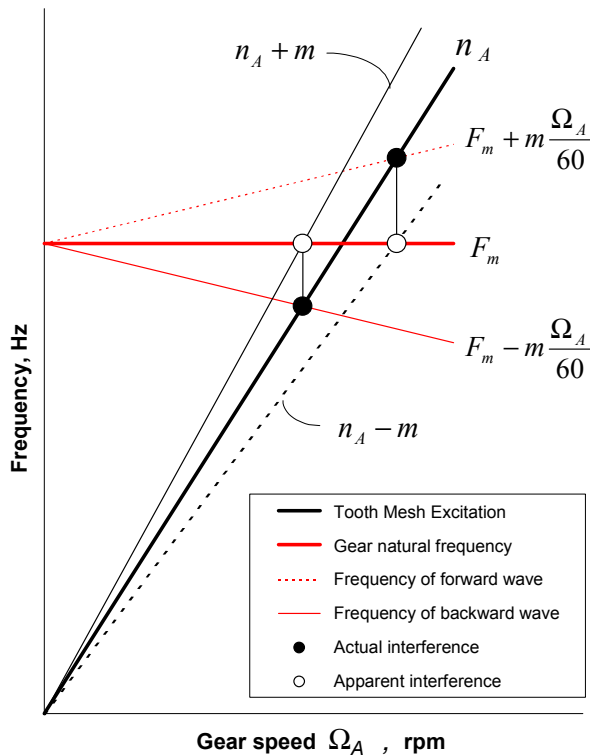


Figure 2. Campbell Diagram for single gear set

Single Mesh Excitation

For a single gear set, traveling wave resonance will occur when the tooth mesh frequency, $n_A \omega_A$, coincides with the frequency of either a forward or backward traveling wave.

$$n_A \omega_A = 2\pi \left(F_m \pm m \frac{\omega_A}{2\pi} \right) \quad (4)$$

The Campbell Diagram shown in Figure 2 illustrates the speeds at which the tooth mesh excitation frequency coincides with the frequency of either the forward or backward traveling wave. As noted by Stockton (1), the “actual interference” between the tooth mesh excitation and the traveling wave occurs in the fixed reference frame. A strain gage on the gear observes the “apparent interference” at the natural frequency. Note that the abscissa in Figure 2 has units of rpm, thus Eq. 3 has a different conversion factor to achieve cycles per second.

For a single gear set, mesh excitation is perfectly timed at some speed to reinforce traveling wave vibration for any value of m . All modes of the gear, both forward and backward traveling, will be excited provided that the resonance defined by Eq. 4 is within the operating range and the rim displacement of the mode shape creates a non-zero modal force.

Multiple Mesh Excitation

Traveling wave excitation becomes more complex when a gear meshes with more than one other gear. Figure 3 illustrates the geometry of a general, multiple mesh gear set. Gear A is the center gear experiencing traveling wave excitation from the surrounding gears B, C, D, E, etc (gear i in general). The center of gear A is located at coordinates (0,0). Gear B is located at 12 o'clock above gear A.

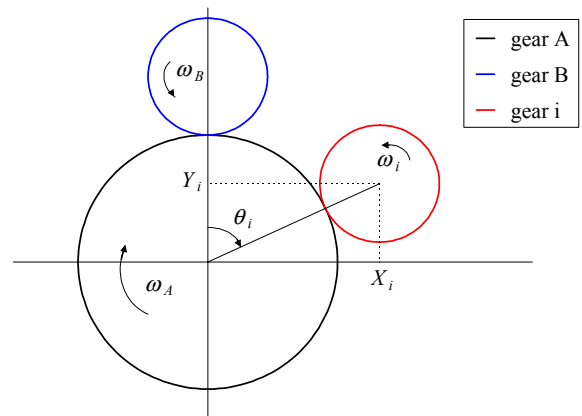


Figure 3. Multiple mesh gear set geometry definition