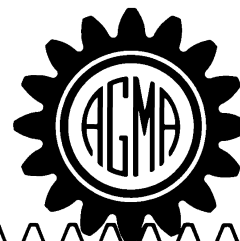


Computerized Design of Face Hobbed Hypoid Gears: Tooth Surfaces Generation, Contact Analysis and Stress Calculation

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American Gear Manufacturers Association



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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

While face milled hypoid gears have been widely studied, about face hobbed ones only very few studies have been developed. Aim of this paper is just to propose an accurate tool for computerized design of face hobbed hypoid gears. Firstly, a mathematical model able to compute detailed gear tooth surface representation will be derived; then, the obtained surfaces will be employed as input for an advanced contact solver that, using a hybrid method combining finite element technique with semianalytical solutions, is able to efficiently carry out contact analysis under light and heavy loads and stress calculation of these gears.

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Nomenclature

N_b	Number of blade groups
R_b	Ratio between N_b and the number of being generated gear teeth
r_b	Blade Radius
ε_b	Eccentric Angle
Γ_b	Hook Angle
h_f	Blade Height
κ_e	Rake Angle
α_e	Blade Angle
r_e	Edge Radius
L_T	Length of Toprem [®]
τ	Angle of Toprem [®]
ρ_t	Blade radius of curvature
i	Tilt Angle
j	Swivel Angle
S_r	Radial Setting
q	Cradle Angle
E_m	Blank Offset
γ_m	Machine Root Angle
ΔX_p	Machine Center to Back
ΔX_b	Sliding Base
R_a	Ratio of Roll

1. Introduction

In the geared transmission design it is very useful to have a numerical tool able to simulate the real behavior of the gear drive, searching for proper contact pattern, low level of transmission error and acceptable fillet stress. In this paper a set of numerical procedure for performance analysis of face hobbed hypoid gear will be proposed.

As well known, this kind of gear is largely applied when it is needed to transfer power and motion between intersecting and crossing axles [1]. Hypoid pairs are mainly manufactured by face hobbing (FH) or face milling (FM) cutting process [2]; while this latter uses a single indexing cutting method, in FH process the being generated gear has continuous rotation and rotates in a timed relationship with the cutter. As the gear is being cut, successive cutter blades groups engage successive tooth slots guaranteeing a continuous indexing. This process is now spreading in automotive industry because of its fast manufacturing time.

Since many decades, a lot of studies about tooth surface representation, contact and stress analysis of FM gears have been carried out [3-6]. On the contrary, about FH process, which is considerably more complex, only a small number of papers have been published. Regarding tooth geometry, Litvin et al. derived a mathematical model able to describe tooth surfaces only of a non-generated Oerlikon gear member [7]; Fong proposed a computerized universal generator able to simulate virtually all primary spiral bevel and hypoid cutting methods without providing a detailed description of the FH case [8]. Referring to performance analysis, the state of art is even worse: in the open literature any work has been found. Consequently, nowadays, it is possible to study this kind of gears only by using, basically as a "black box", proprietary software which has been developed by manufacturing machine and tools suppliers.

Goal of this paper is just to propose an integrated tool for computerized design of FH hypoid gears.

The first step in order to build a reliable numerical model is to get a fine geometrical representation of gear tooth surfaces. With this aim, a series of algorithms able to compute tooth surfaces of FH gears starting from cutting process will be described [9]. The geometry of real FH head cutter (Gleason Tri-Ac[®]) will be firstly analyzed; many kinds of blade configuration (straight and curve blades, with or without Toprem[®]) will be considered. Then, according to the theory of gearing, FH cutting process (with and without generation motion) will be simulated and gear tooth surfaces equations will be computed. The proposed mathematical model is able to provide an accurate description of the whole tooth, including fillet region; it will also take into account undercutting occurrence, which is very common in FH gears due to uniform depth tooth [4]. By means of this model, tooth surfaces of a real gear drive, which is mounted in a truck differential system, will be computed and the results will be validated by comparing them with the ones calculated by a reference proprietary software and with the real surfaces.

Then, the obtained tooth surfaces will be used as fundamental input for a powerful contact solver which is based on a unique semianalytical finite element formulation [10-11]. Firstly, the gear drive it will study under light load by monitoring, for drive and coast side, the contact pattern and transmission error (Tooth Contact Analysis - TCA). After that, with the aim to find out gear drive performance in the real service conditions, a set of torque values will be applied and the influence of the load on contact pattern, on transmission error and on load sharing will be accurately analyzed (Loaded Tooth Contact Analysis - LTCA). Contact pressure and stress distribution will be also evaluated. The obtained results will be compared with the ones calculated by a reference software.

2. Theoretical Background of Face Hobbing Method

As known [2], FH head cutter is provided with a proper number of blade groups N_b , each of them consists in an outer and an inner blade. As reported in Figure 1, in order to accomplish continuous indexing, the head cutter and the being generated gear are rotating in opposite directions and the next group of blades will start to cut the next gear tooth after that the current group of blades has finished cutting the current tooth.

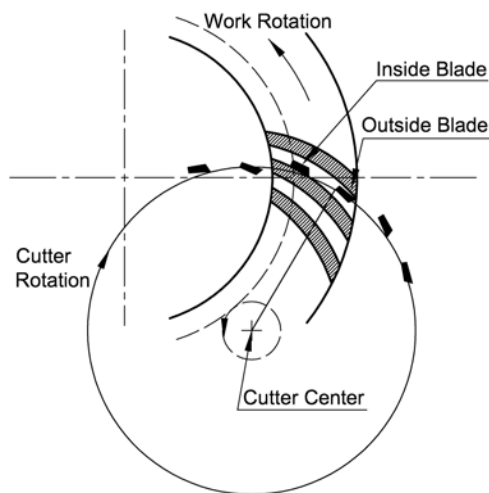


Figure 1. Sketch of FH cutting process

In this way, the angular velocity of the head cutter ω_b is related to the angular velocity of the work-piece ω_w according to the ratio between the number of blade groups N_b and the number of being generated gear teeth N_w :

$$R_b = \frac{N_b}{N_w} = \frac{\omega_w}{\omega_b} \quad (1)$$

It is evident that the edge of the blade, during cutting, tracks an epicycloid curve. In order to accommodate this path, unlike FM method, the effective cutting direction of the blade is not perpendicular to the cutter radius and the blade is moved in the head cutter tangentially to an offset position.

Fig. is referred to the non-generated process (Formate[®]); if a generated tooth is needed, the generation motion, which relates cradle and work-piece rotation, has to be superimposed.

Traditionally, FH gear drive has uniform depth tooth; it follows that FH tooth often shows undercut toe-section with sharp topline. This latter inconvenient can be eliminated by introducing a secondary face angle; undercutting avoidance could be a difficult task and often FH gear drives work affected by it [2, 12].

3. Simulation of Face Hobbing Cutting Process: Tooth Surface Generation

According to the theory of gearing [3-4, 13], in order to get the analytical representation of gear tooth surfaces, firstly cutting process (i.e. head cutter, cutting blades and cutting machine) has to be described. It will be clear that [9], due to the complexity of FH cutting process, FH cutting blades require a more complicated representation than the ones usually illustrated in the literature (typically for FM method). In this paper a real FH process, Gleason Tri-Ac[®], will be studied.

3.1. Cutting Tools: Head Cutter and Cutting Blades

As shown (Figure 1), FH head cutter carries a given number N_b of blade groups; each group contains an outer blade (OB) for cutting concave gear side and an inner one (IB) for convex side. Figure 2 reports, from two different viewing points, one blade group in a Gleason Tri-ac[®] head cutter.

Referring, for example, to the outer blade, in order to correctly locate the blade in the head cutter, the pitch point P of cutting edge (see also Figure 3) has to be defined. The distance from this point to the head cutter center is the equal to r_b ; the angle ε_b is introduced in order to take into account that FH process, unlike FM, shows blades that are not aligned to the cutter radius. It is also evident that the blade is not perpendicular to the head cutter plane, but it is mounted at an angle Γ_b with respect to the head cutter rotation axis. The distance from the pitch point P to the tip of the blade is measured by h_f .

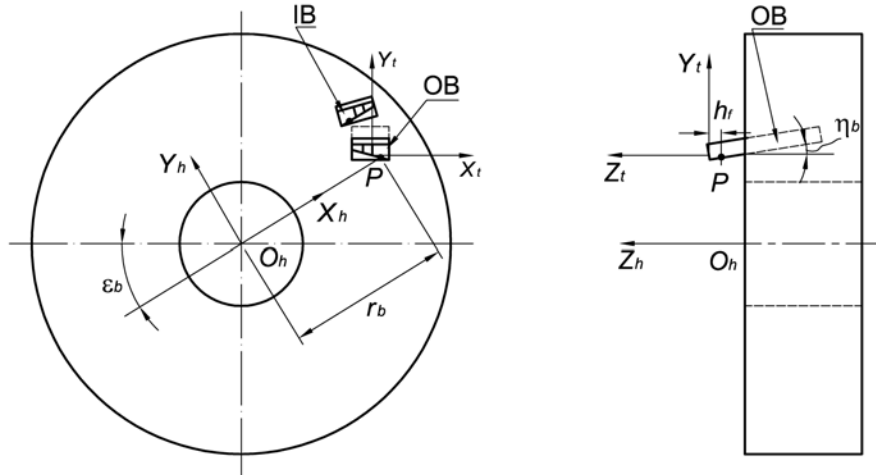


Figure 2. Sketch of a blade group mounted in the FH head cutter.

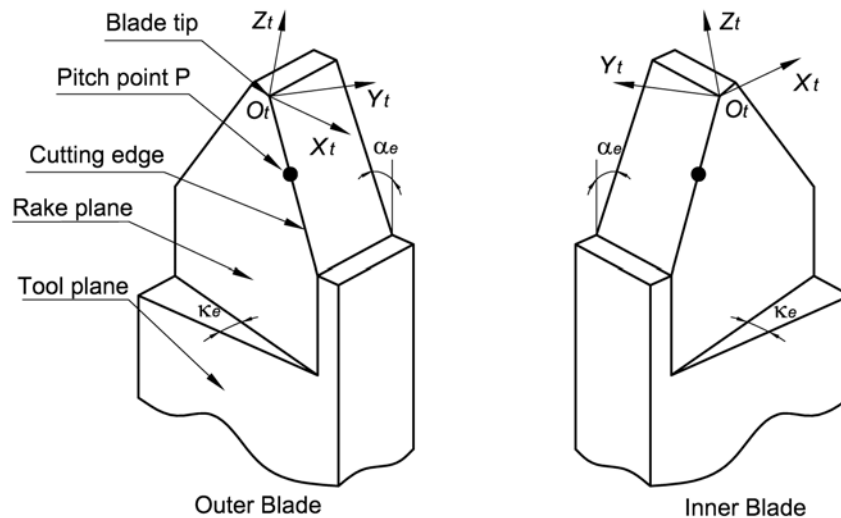


Figure 3. Details of the FH blades.

Figure 3 shows the details of outer and inner blades. It is evident that the cutting edge lies entirely on a plane, called Rake plane, which forms an angle κ_e with tool plane. It is also introduced the angle α_e as the angle between the vertical axis of the blade and the projection of the cutting edge on the tool plane.

Once the blade geometry has been introduced, it is possible to compute the analytical formulation of the cutting edge. Many blade profiles are available in commerce. In this paper, a complex blade shape - curved with Toprem[®] - will be analyzed; simpler configurations as straight blade with and without Toprem[®] or curved blade without Toprem[®] can be easily derived starting from the following discussion. The particular disposition of the cutting edge on the Rake plane requires to set

up the reference frame S_t (Figure 2 and 3) and to search firstly for the analytical description of projection on $X_t Z_t$ plane of the cutting edge. Figure 4 shows the projection on this plane of a curved blade with Toprem[®]; the following sections, which are function of the curvilinear coordinate s , have been defined:

- I) Bottom: straight horizontal segment;
- II) Fillet: circular arc of radius r_e and center at point $R(x_R, z_R)$;
- III) Toprem[®]: inclined straight segment characterized by the length L_T and the angle τ ;
- IV) Curved blade: circular arc with radius of curvature ρ_t and center at point $O(x_O, z_O)$. At pitch point P the segment tangent to the blade is inclined by an angle equal to α_t .