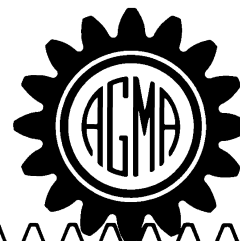


A Model to Predict Friction Losses of Hypoid Gears

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American Gear Manufacturers Association



TECHNICAL PAPER

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

A model to predict friction-related mechanical efficiency losses of hypoid gear pairs is proposed, which combines a commercial available finite element based gear contact analysis model and a friction coefficient model with a mechanical efficiency formulation. The contact analysis model is used to provide contact pressures and other contact parameters required by the friction coefficient model. The instantaneous friction coefficient is computed by using a validated new formula that is developed based on a thermal elastohydrodynamic lubrication (EHL) model considering non-Newtonian fluid. Computed friction coefficient distributions are then used to calculate the friction forces and the resultant instantaneous mechanical efficiency losses of the hypoid gear pair at a given mesh angle. The model is applied to study the influence of speed, load, surface roughness, and lubricant temperature as well as assembly errors on the mechanical efficiency of an example face-hobbed hypoid gear pair.

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American Gear Manufacturers Association
500 Montgomery Street, Suite 350
Alexandria, Virginia, 22314

October, 2005

ISBN: 1-55589-854-8

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1 Introduction

Gear mesh friction has attracted a number of researchers for more than a century [1]. The friction between gear teeth plays an important role in defining the efficiency of the system as well as influencing scoring limits and the dynamical behavior including vibration and noise [2,3]. Both sliding and rolling actions at the gear mesh contact contribute to gear mesh friction. Sliding friction is a direct product of the relative sliding between the two contacting surfaces while rolling friction originates from the resistance to the rolling motion [4]. Coefficient of friction μ that is used widely in the literature usually refers to the coefficient of sliding friction.

A significant number of studies have been published especially within the last forty years on friction and efficiency of gear trains as reviewed by references [5-7]. The first group of studies focused on measuring power losses of gear pair directly [8-17]. Several others measured μ using twin-disk test machines under conditions simulating a gear pair so that this friction coefficient can be used to predict the efficiency of a gear pair [18-30, 38-39]. Some of these studies [18-25] resulted in well-known and widely used empirical formulae for μ . These empirical formulae indicate that μ is a function of a list of parameters such as sliding and rolling velocities, radii of curvature of the surfaces in contact, load or contact pressure, surface roughness, and the lubricant viscosity.

A group of efficiency models [31-33] investigated the efficiency of a spur gear pair by assuming a uniform μ along the entire contact surface. A tangential friction force along the sliding direction was computed by using a given constant friction coefficient μ , and the geometric and kinematic parameters of the spur gears. As a result, the amount of reduction of torque transmitted to the driven gear was used to calculate the mechanical efficiency η of the gear pair. These models were useful in bringing a qualitative understanding to the role of spur gear geometry on efficiency. They fell short in terms of the definition of μ , as a user-defined constant μ value must be used for every

contacting point on the tooth surface. However, the published experiments on sliding/rolling contacts indicate that many parameters might influence μ [18-25]. In addition, these studies were limited to spur gears and many complicating effects of the tooth bending and contact deformations, tooth profile modifications and manufacturing errors were not included.

Another group of efficiency models [34-37, 40] relied on published experimental μ formulae such as those in references [18-21]. The models in this group considered spur [35-37, 40] and helical [34] gear pairs and calculated the parameters required to define μ according to the particular empirical formula adapted. While they are potentially more accurate than the constant μ models, their accuracy is limited to the accuracy of the empirical μ formula used. Each empirical μ formula typically represents a certain type of lubricant, operating temperature, speed and load ranges, and surface roughness conditions of roller specimens that might differ from those of the gear pair that is being modeled.

The models in the last group are more advanced since they use an EHL model to predict μ instead of relying on the user or the empirical formulae [42-54]. Among them, Dowson and Higginson [47], and Martin [48] used a smooth surface EHL model to determine the surface shear stress distribution caused by the fluid film, and hence, the instantaneous friction coefficient at the contact. Adkins and Radzimovsky [49] developed a model for lightly loaded spur gears under hydrodynamic lubrication condition and assumed that the gear tooth is rigid without deflections and local deformations. Simon [50] provided an enhancement by using point contact EHL model for heavily crowned spur gears with smooth surfaces considering the elastic displacement of the surface due to fluid pressure distributions. Larsson [51] and Wang et al [52] analyzed involute spur gear lubrication by using a transient thermal-EHL model with smooth surfaces. Wu and Cheng [53] developed a friction model based on mixed-EHL contacts and applied it to calculate the frictional power losses of

spur gears. The roughness was modeled such that all the asperities have the same radius of curvature whose heights have a Gaussian distribution. Mihalidis et al [54] included the influence of the asperity contacts as well in calculating μ and hence efficiency. These models [47-54] were successful in eliminating to a certain extent the need for prior knowledge of μ , at the expense of significantly more computational effort. While they were relatively enhanced in EHL aspects of the problem, the applications were limited to simple spur gears with ideal load distributions and no tooth bending deformations.

A small number of efficiency studies on helical gears were found [34,40,55-58]. Literature on hypoid gear efficiency is even sparser. Buckingham [59] proposed an approximated formula for the power loss of hypoid gears, which is the sum of the losses of a spiral bevel gear and a worm gear. Naruse et al [8,10] conducted several tests on scoring and frictional losses of hypoid gears of Klingelnberg type. Coleman [60] used a simple formula to calculate hypoid gear efficiency with a constant μ or a μ formula with a very limited number of parameters included [61]. Smooth-surface EHL formulations were found applied to hypoid gears by Simon [62] and Jia et al [63].

1.1 Objectives and Scope

Efficiency losses in a gearbox are originated from several sources including gear mesh sliding and rolling friction, windage, oil churning, and bearing friction [34]. When gears are loaded, a gear contact under load experiences combined sliding and rolling, both of which result in frictional losses. The amount of sliding frictional loss is directly related to the coefficient of friction, normal tooth load and relative sliding velocity of the surfaces while the rolling friction occurs due to the deformation of the two contacting surfaces. When the contact is lubricated, rolling frictional losses are originated from the formation of the EHL film [35]. Efficiency can be improved by reducing the coefficient of friction via precision manufacturing and smoothing the contact surfaces and enhancement of lubricant properties. Existing approaches of improving efficiency are based mostly on experimental trial-and-error type procedures focusing on such parameters, while the predictive capabilities have been limited.

The main objective of this study is to develop a mechanical efficiency model for hypoid gears. The model allows an analysis of both face-hobbed and face-milled hypoid gears. The efficiency model will allow two methods of calculating μ , i.e. published

empirical formulae and a thermal EHL formulation. The differences amongst these approaches will be described. Parametric studies will be performed to investigate the influence of several relevant parameters such as speed, load, surface roughness, lubricant temperature as well as the assembly errors on the mechanical efficiency of hypoid gears. This study is focused primarily on the mechanical efficiency losses related to tooth friction, including sliding and rolling friction, while it relies on the published studies in terms of losses associated with windage, oil churning and bearings [34,35,64-69] when necessary.

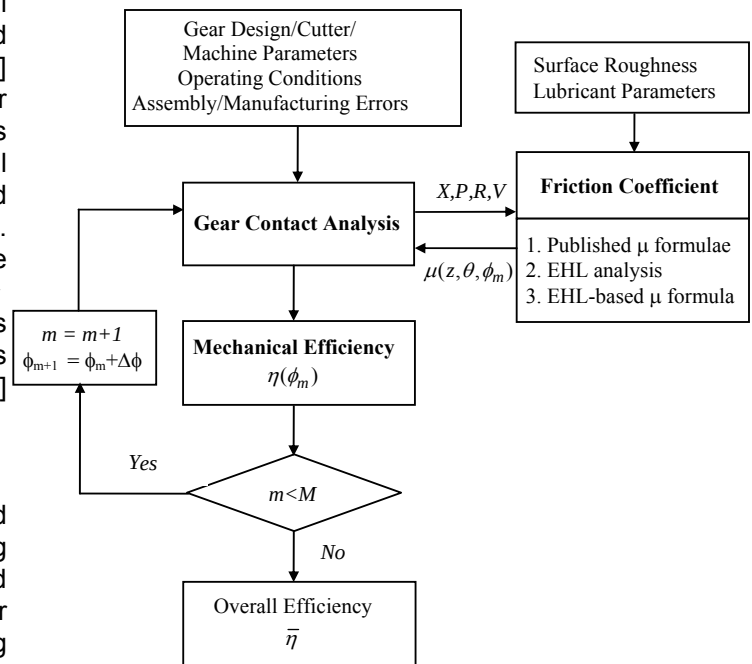


Figure 1. Flowchart for the efficiency prediction.

2 Efficiency Model

2.1 Technical Approach

Figure 1 illustrates a flowchart of the efficiency computation methodology used in this study. Three main components are the gear contact analysis model, the friction coefficient computation model, and the gear pair mechanical efficiency computation formulation. The same methodology was applied by these authors earlier to spur and helical gears successfully [70]. It was also shown the parallel-axis efficiency model compares well with the gear pair efficiency experiments [77]. The gear contact analysis model uses the gear design parameters, operating conditions and errors associated with assembly, mounting and manufacturing of the tooth profile to predict load and contact pressure distributions at every contact point during each mesh position. Predicted load distribution or contact

pressure together with other geometric and kinematic parameters are input to the friction coefficient model to determine the instantaneous friction coefficient $\mu(z, \theta, \phi_m)$ of every contact point (z, θ) on the gear tooth surface. $\mu(z, \theta, \phi_m)$ is then used by the mechanical efficiency computation module to determine the instantaneous efficiency $\eta(\phi_m)$ of the gear pair at the m -th incremental rotational position defined by angle ϕ_m . The above sequential procedure is repeated for an M number of discrete positions ($m = 1, 2, \dots, M$) spaced at an increment of $\Delta\phi$ ($\phi_m = m \Delta\phi$) to cover an entire mesh cycle. These instantaneous mechanical efficiency values $\eta(\phi_m)$ are then averaged over a complete mesh cycle to obtain the average mechanical efficiency loss of the gear pair due to tooth friction. In the following sections, main components of this methodology as shown in Fig. 1 are described in detail.

2.2 Contact Analysis of Hypoid Gears

A commercial available finite element (FE) based hypoid gear analysis package CALYX [71] is

used as the contact analysis tool. Both face-hobbed and face-milled versions of this model are available. The model combines FE method away from the contact zone with a surface integral formulation applied at and near the contact zone [72]. The contact analysis model used in this study has a special setup for the finite element grids inside the instantaneous contact zone. As shown in Fig. 2(a), a set of very fine contact grid is defined automatically on hypoid gear teeth to capture the entire contact zone. These grid cells are much finer than the regular size of finite element meshes elsewhere on the tooth surfaces and they are attached to the contact zones that result in more accurate contact analysis. A schematic view of these grid cells is shown in Fig. 2(b). Along the face width, there are $2n+1$ divisions, and at each division, there is a principal contact point (shown in dot) if contact occurs. In the profile direction, there are $2m+1$ grid cells within each division for capturing potential contact points, which would be in contact due to tooth deflections and local surface deformations.

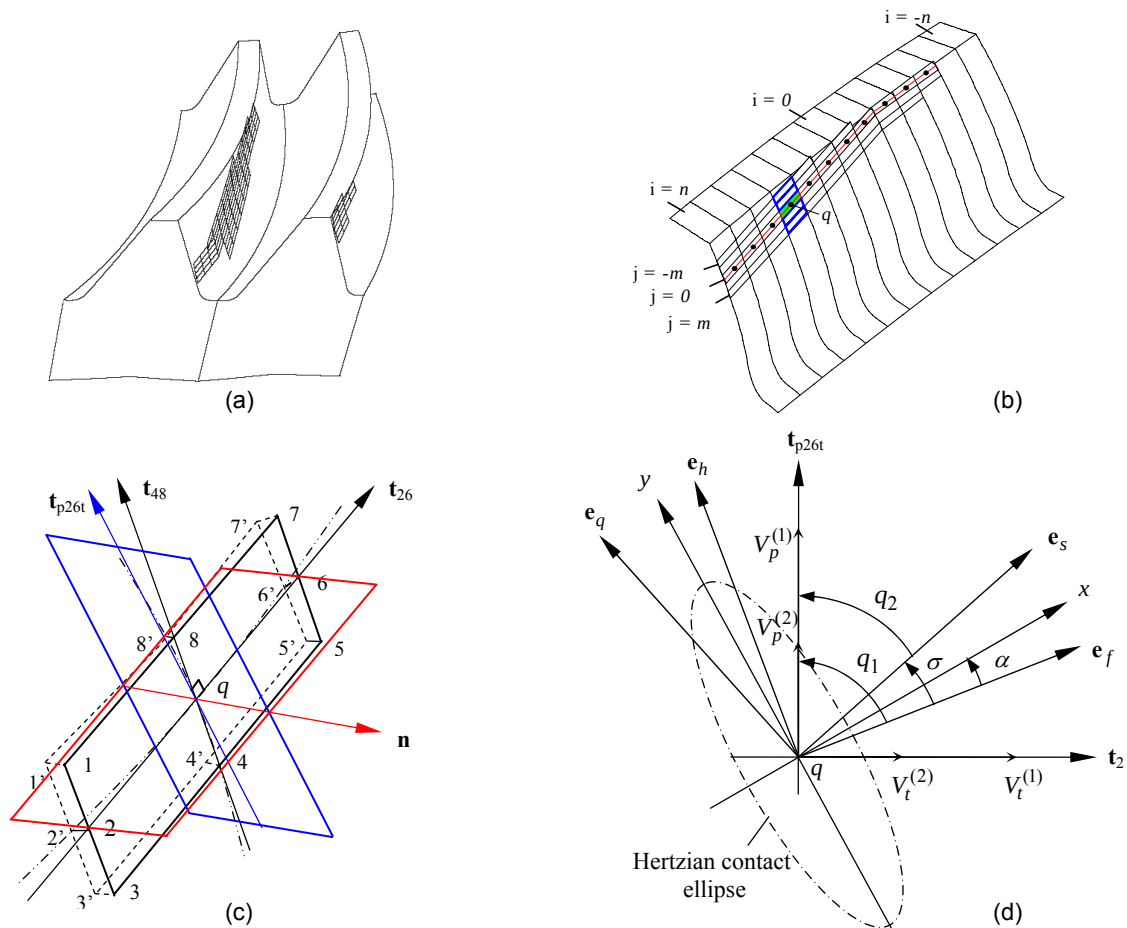


Figure 2. (a) Moving grids for contact zones, (b) moving grid setup, (c) grid in tangent plane for μ calculation, and (d) principal directions and contact ellipse.

The μ calculation is carried out in the grid of principal contact point in the tangent plane as shown in Fig. 2(c), which is a magnified grid cell for the principal contact point q . The surface formed by dotted lines, with points 1' to 8' along the edges, is the grid on the real tooth surface and the plane formed by solid lines, with points 1 to 8 along the edges, is the grid in the tangent plane for this particular contact point q . Points 2(2'), 4(4'), 6(6') and 8(8') are the mid point at each side of the grids. Vector $\mathbf{t}_{26}$ that connects point 2 and 6 is approximated as the instant line of contact and vector $\mathbf{t}_{p26t}$ is normal to $\mathbf{t}_{26}$ in the tangent plane. $\mathbf{n}$ is the surface normal vector at the contact point q . When μ is obtained for this principal contact point, same μ value will be assigned to the potential contact point within the same face width division. While the load distribution and contact pressure at each grid is provided by the contact analysis model, surface velocities and curvatures are calculated in the following sections.

Definition of Principal Contact Points. Assume pinion surface and gear surface are defined by $\mathbf{r}_1(s_1, t_1)$ and $\mathbf{r}_2(s_2, t_2)$ respectively, where s_1, t_1 and s_2, t_2 are the surface curvilinear parameters. The principal contact point is determined and located when $\mathbf{r}_1$ and $\mathbf{r}_2$ become the closest to each other [72]. Surface $\mathbf{r}_1(s_1, t_1)$ was discretized into a grid of points $\mathbf{r}_{1ij} = \mathbf{r}_1(s_{1i}, t_{1j})$ and for each of these grid points, an effort was made to locate $\mathbf{r}_{2ij} = \mathbf{r}_2(s_{2i}, t_{2j})$ such that $\|\mathbf{r}_1(s_1, t_1) - \mathbf{r}_2(s_2, t_2)\|$ is minimized with respect to the variable s_2 and t_2 . This extremization is equivalent to solving the following system of nonlinear equations [72]

$$\begin{cases} [\mathbf{r}_{1ij} - \mathbf{r}_2(s_{2i}, t_{2j})] \partial \mathbf{r}_2(s_{2i}, t_{2j}) / \partial s_{2i} = 0 \\ [\mathbf{r}_{1ij} - \mathbf{r}_2(s_{2i}, t_{2j})] \partial \mathbf{r}_2(s_{2i}, t_{2j}) / \partial t_{2j} = 0 \end{cases} \quad (1)$$

First, solutions from Eq. (1) for each of the grid point $\mathbf{r}_{1ij}$ were obtained by the Newton-Raphson method. Then a new grid, which is finer than the original grid, was set up around the point $\mathbf{r}_{1ij}$ for which the separation $\|\mathbf{r}_{1ij} - \mathbf{r}_{2ij}\|$ was the smallest. This search process is repeated several times with progressively smaller grids to locate the principal contact point [72]. The pinion and gear surfaces were divided in several parts along the face width. In each face width division, if there is contact, the principal contact is determined and then a set of grids is laid out above and below the principal contact point. These grids are added for candidate contact points that will likely be in contact due to elastic deformations of the two contacting surfaces.

Calculation of Principal Curvatures and Principal Directions. According to differential geometry [73], for two surfaces in contact at a common point, the principal curvatures and principal directions at the common point can be determined if the coefficients of the first and second fundamental form of the surface are given. Let $\kappa^{(1)}$ and $\kappa^{(2)}$ be the two eigenvalues of the eigen problem

$$\mathbf{N}\boldsymbol{\lambda} = \kappa \mathbf{M}\boldsymbol{\lambda} \quad (2)$$

where

$$\boldsymbol{\lambda} = \begin{Bmatrix} \lambda_1 \\ \lambda_2 \end{Bmatrix}, \quad \mathbf{M} = \begin{bmatrix} \frac{\partial \mathbf{r}}{\partial s} \cdot \frac{\partial \mathbf{r}}{\partial s} & \frac{\partial \mathbf{r}}{\partial s} \cdot \frac{\partial \mathbf{r}}{\partial t} \\ \frac{\partial \mathbf{r}}{\partial t} \cdot \frac{\partial \mathbf{r}}{\partial s} & \frac{\partial \mathbf{r}}{\partial t} \cdot \frac{\partial \mathbf{r}}{\partial t} \end{bmatrix},$$

$$\mathbf{N} = \begin{bmatrix} \mathbf{n} \cdot \frac{\partial^2 \mathbf{r}}{\partial s^2} & \mathbf{n} \cdot \frac{\partial^2 \mathbf{r}}{\partial s \partial t} \\ \mathbf{n} \cdot \frac{\partial^2 \mathbf{r}}{\partial t \partial s} & \mathbf{n} \cdot \frac{\partial^2 \mathbf{r}}{\partial t^2} \end{bmatrix}, \quad \mathbf{n} = \frac{\frac{\partial \mathbf{r}}{\partial s} \times \frac{\partial \mathbf{r}}{\partial t}}{\left\| \frac{\partial \mathbf{r}}{\partial s} \times \frac{\partial \mathbf{r}}{\partial t} \right\|}.$$

Here, $\mathbf{n}$ is the unit normal vector, and s and t are the surface curvilinear parameters at the common contact point. Matrices $\mathbf{M}$ and $\mathbf{N}$ contain the coefficients of the first and second fundamental form of the surface, respectively. Then $\kappa^{(1)}$ and $\kappa^{(2)}$ are the principal normal curvatures. If the two corresponding eigenvectors $\boldsymbol{\lambda}^{(1)}$ and $\boldsymbol{\lambda}^{(2)}$ are normalized such that $\{\boldsymbol{\lambda}^{(i)}\}^T \mathbf{M} \boldsymbol{\lambda}^{(i)} = 1$, $i = 1, 2$, then the two unit vectors in the principal directions corresponding to the principal curvatures are defined as [72]

$$\mathbf{e}^{(1)} = \{\boldsymbol{\lambda}^{(1)}\}^T \begin{Bmatrix} \partial \mathbf{r} / \partial s \\ \partial \mathbf{r} / \partial t \end{Bmatrix}, \quad (3a)$$

$$\mathbf{e}^{(2)} = \{\boldsymbol{\lambda}^{(2)}\}^T \begin{Bmatrix} \partial \mathbf{r} / \partial s \\ \partial \mathbf{r} / \partial t \end{Bmatrix}. \quad (3b)$$

Determination of The Orientation of the Contact Ellipse. Assume q is the contact point of surface Σ_1 and surface Σ_2 . Let $\mathbf{e}_f$ and $\mathbf{e}_h$ be unit vectors of principal directions, and κ_f and κ_h be principal curvatures of surface Σ_1 at point q . Let $\mathbf{e}_s$ and $\mathbf{e}_q$ be unit vectors of principal directions, and κ_s and κ_q be principal curvatures of surface Σ_2 at point q . These values of principal directions and curvatures can be solved by Eq. (2). The angle σ , formed by $\mathbf{e}_f$ and $\mathbf{e}_s$, and the angle α that determines the orientation of the coordinate axes x and y with respect to $\mathbf{e}_f$ as shown in Fig. 2(d) can be obtained as [72]