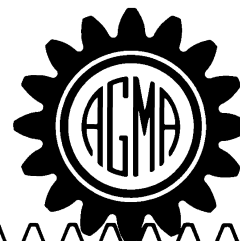


New Developments in Tooth Contact Analysis (TCA) and Loaded TCA for Spiral Bevel and Hypoid Gear Drives

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American Gear Manufacturers Association



TECHNICAL PAPER

New Developments in Tooth Contact Analysis (TCA) and Loaded TCA for Spiral Bevel and Hypoid Gear Drives

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

Tooth Contact Analysis (TCA) and Loaded Tooth Contact Analysis (LTCA) are two powerful tools for the design and analysis of spiral bevel and hypoid gear drives. Typical outputs of TCA and LTCA are the graphs of contact patterns and transmission errors. TCA and LTCA respectively simulate gear meshing contact characteristics under light load and under significant load. TCA and LTCA programs have been widely employed by gear engineers and researchers in their design of high strength and low noise spiral bevel and hypoid gear drives.

Application of modern CNC hypoid gear generators has brought new concepts in design and generation of spiral bevel and hypoid gears with sophisticated modifications. This paper presents new developments in TCA and LTCA of spiral bevel and hypoid gears. The first part of the paper describes a new universal tooth surface generation model which is developed with consideration of the universal motion capabilities of CNC bevel gear generators. The new universal model is based on the kinematical modeling of the basic machine settings and motions of a virtual bevel gear generator which simulates the cradle-style mechanical hypoid gear generators and integrates both face milling and face hobbing processes. The tool geometry is generally represented by four sections, blade tip, Toprem, profile, and Flankrem. Mathematical descriptions of gear tooth surfaces are represented by a series of coordinate transformations in terms of surface point position vector, unit normal, and unit tangent. Accordingly, a new generalized TCA algorithm and program are developed.

In the second part of this paper the development of a finite element analysis (FEA) based LTCA is presented. The LTCA contact model is formulated using TCA generated tooth surface and fillet geometries. The FEA models accommodate multiple pairs of meshing teeth to consider a realistic load distribution among the adjacent teeth. An improved flexibility matrix algorithm is formulated, in which the nonlinear formation of the area of contact common to the gear and pinion teeth is predicted by introducing specialized gap elements with considerations of deflection and deformation due to tooth bending, shearing, local Hertzian contact, and axle stiffness.

The advanced TCA and LTCA programs are integrated into Gleason *CAGE t for Windows* software package. Two numerical examples, a face-hobbing design and a face milling design, are illustrated to verify the developed mathematical models and programs.

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New Developments in Tooth Contact Analysis (TCA) and Loaded TCA for Spiral Bevel and Hypoid Gear Drives

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1. Introduction

In the early 1960's the Tooth Contact Analysis (TCA) technique was introduced for the theoretical analysis of the contact characteristics and running quality of spiral bevel and hypoid gear drives [1]. Application of the TCA program has substantially reduced the lengthy trial-and-error procedure for the design and development of spiral bevel and hypoid gear drives.

In the late 1970's, the tooth contact analysis under loaded conditions, the Loaded TCA (LTCA), was developed [2]. The Loaded Tooth Contact Analysis (LTCA) program provides a more realistic picture of tooth contact characteristics because tooth deformation and shaft deflection due to loading are taken into account for determination of the actual geometry of contacting tooth surfaces.

Both techniques have been accepted and applied throughout the bevel gear industry and have become powerful tools for the design and manufacturing of high quality spiral bevel and hypoid gears.

In the TCA computation, the gear and pinion tooth surface geometries are mathematically represented by the machine settings and cutter specifications. The existing TCA algorithms and programs were developed specifically corresponding to each method of tooth surface generation process and type of bevel gear drives. The application of modern CNC bevel gear generators has provided more motion freedoms for bevel gear generations. Traditional generating methods such as modified roll and helical motion can be considered as special cases of the possible motions of the CNC 6-axis generators. An advanced tooth surface modification approach has been developed by using the Universal Motion Concept (UMC) [3]. The UMC technique

can be applied for a sophisticated modification of spiral bevel and hypoid gear tooth surfaces.

In order to satisfy the need for accurate modeling and analysis of the advanced design and modification of spiral bevel and hypoid gears, a new generalized tooth surface generation algorithm and a TCA approach are developed. This paper comprehensively describes these developments as applied to spiral bevel and hypoid gears produced by both the face-milling and face-hobbing processes.

Loaded Tooth Contact Analysis (LTCA), was first developed assuming that the contact tooth surfaces are subject to Hertzian deformation, and approximating tooth stiffness by a cantilever beam model with modifications to improve deflection calculations at the ends of the teeth. With the advances in computational and memory capacities of modern computers, the Finite Element Analysis (FEA) has been applied to gear tooth strength evaluations. A special FEA software package called T900 was developed for the strength analysis of spiral bevel and hypoid gears [4]. T900 provides an integrated environment with automatic pre-processor, post-processor, and graphic visualization of the FEA results including the behavior of the contact under load, contact stress and bending stress.

Based on the improved FEA models and algorithms a new LTCA program is developed as a subset of T900. LTCA provides the gear designers with detailed information on how the tooth contact pattern and transmission errors behave as a function of torque input, initial contact position and mounting deflections. The LTCA model uses TCA generated tooth surface and fillet coordinates to define the gear and pinion geometry and to identify the instant contact positions. The nonlinear growth and movement of the area of contact common to the gear and pinion teeth is predicted by the use of

specialized gap elements, the applied torque and axle stiffness input data. Multi-tooth finite element models are developed and applied for both the pinion and the gear.

2. Face-Milling and Face-Hobbing

There are two methods for manufacturing the spiral bevel and hypoid gears, face milling and face hobbing, both of which are widely employed by the gear manufacturing industry and can be implemented on modern CNC bevel gear generators [5, 6]. However, the kinematical differences between the two methods are still not clear to some gear engineers and researchers.

The major differences between the face milling process and the face hobbing process are (Fig. 1): (1) in face hobbing process, a timed continuous indexing is provided, while in face milling the indexing is intermittently provided after cutting each tooth side or slot, which is also called single indexing. Similar to face milling, in face hobbing the pinion is cut with generated method while the gear can be cut with either generated method or non-generated (FORMATE) method. The FORMATE method offers higher productivity than the generated method because the generating roll is not applied. However, the generated method offers more freedoms of controlling the tooth surface geometries; (2) the lengthwise tooth curve of face milled bevel gears is a circular arc with a curvature radius equal to the radius of the cutter, while the lengthwise tooth curve of face hobbled gears is an extended epicycloid that is kinematically generated during the relative indexing motion; and (3) face hobbing gear designs use uniform tooth depth system while face milling gear designs may use either uniform depth or various tapered tooth depth systems.

Theoretically, the bevel gear cutting process is based on the generalized concept of bevel gear generation in which the mating gear and the pinion can be considered respectively generated by the complementary virtual generating gears. Fig. 2 shows a configuration of a bevel pinion generation, which consists of a virtual generating gear, a cutter head with blades, and the work (the pinion). The rotational motion of the virtual generating gear is implemented by the cradle mechanism of a bevel gear generator. Generally, the tooth surfaces of the generating gears are kinematically formed by the traces of the cutting edges of the blades. In practice, in order to introduce mismatch of the generated tooth

surfaces, modification is applied on the generating gear tooth surface and on the generating motion.

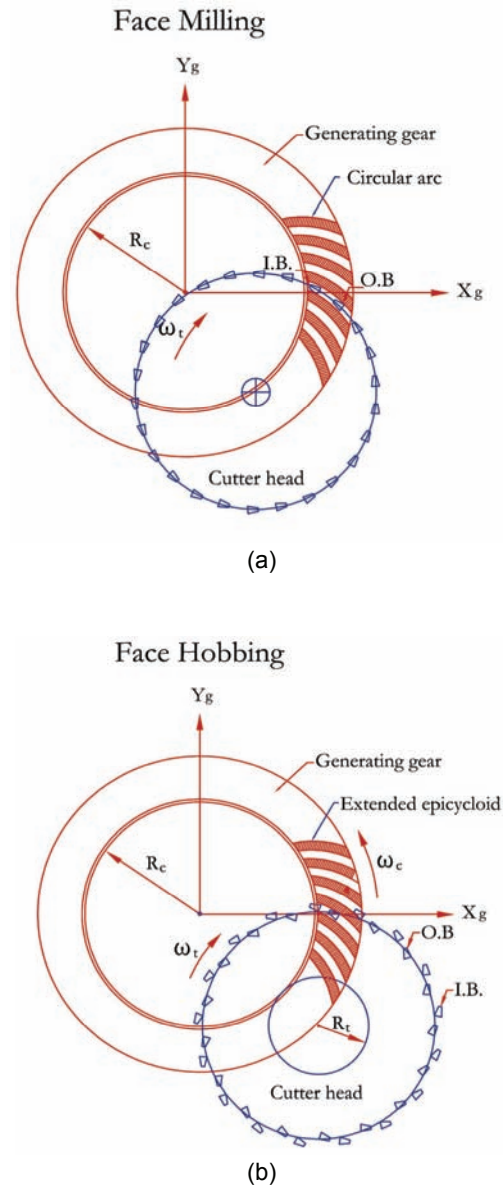


Fig.1: Face Milling vs Face Hobbing

In the spiral bevel and hypoid gear generation process, two sets of related motions are generally defined. The first set of related motion is the rotation of the tool (cutter head) and rotation of the work, namely,

$$\frac{\omega_t}{\omega_w} = \frac{N_w}{N_t} \quad (1)$$

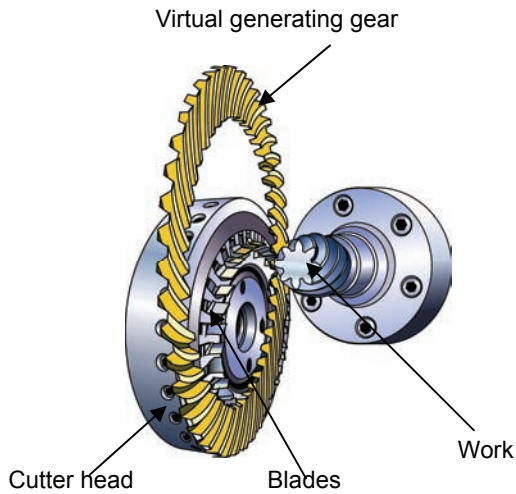


Fig.2: Generalized Bevel Gear Generation Configuration

here, ω_t and ω_w denote the angular velocities of the tool and the work; N_t and N_w denote the number of the blade groups and the tooth number of the work respectively. This related motion provides the continuous indexing between the tool and the work for the face hobbing process. The indexing relationship also exists between the rotation of the tool and the generating gear as,

$$\frac{\omega_t}{\omega_c} = \frac{N_c}{N_t} \quad (2)$$

where ω_c and N_c denote the angular velocity and the tooth number of the generating gear respectively. In the face hobbing process, the indexing motion between the tool and the generating gear kinematically forms the tooth surface of the generating gear with an extended epicycloid lengthwise tooth curve. However, Eq. (1) and (2) are not applicable for the face milling process where the cutter rotates independently at its selected cutting speed and forms a surface of revolution for the generating gear teeth with a circular lengthwise curve Fig. 1(a).

The second set of related motion is the rotation of the generating gear and rotation of the work. Such a related motion is called rolling or generating motion and is represented as,

$$\frac{\omega_w}{\omega_c} = \frac{N_c}{N_w} = R_a \quad (3)$$

where R_a is called the ratio of roll. The generating motion is provided for both face milling and face hobbing processes when the gear or pinion is cut in the generated method. In the non-generated (FORMATE) process, which is usually applied to the gear, both the generating gear and the work are held at rest and only the cutter rotation is provided. Therefore, the gear tooth surfaces are actually the complementary copy of the generating tooth surfaces which are formed by the cutter motion described above.

3. A Generalized Spiral Bevel and Hypoid Gear Tooth Surface Generation Model

Both face milling and face hobbing processes can be implemented on the CNC hypoid gear generating machines. Fig. 4 shows a Phoenix[®] II 275HC Hypoid Gear Generator. A new kinematical model of tooth surface generation is developed based on the traditional cradle-style mechanical machine. The purpose of developing this model is to virtually represent each machine tool setting element as an individual motion unit and implement a given function of motion. These virtual motions can be realized by the CNC Hypoid Gear Generator through the computer translation codes on the machine. The six axes of the CNC Hypoid Gear Generator move together in a numerically controlled relationship with changes in displacements, velocities, and accelerations to implement the prescribed motions and produce the target tooth surface geometry.

The generalized tooth surface generation model consists of eleven motion elements shown in Fig. 4. The cradle represents the generating gear, which provides generating roll motion between the generating gear and the generated work. In the non-generated process, the cradle is held stationary. Basically, these motion elements represent the machine tool settings in a dynamic manner in terms of the cradle rotation parameter.

The generalized machine settings are: (1) ratio of roll R_a ; (2) sliding base X_b ; (3) radial setting s ; (4) offset E_m ; (5) work head setting X_p ; (6) root angle γ_m ; (7) swivel j ; and (8) tool tilt i . The Universal Motion Concept (UMC) represents these machine settings in higher order