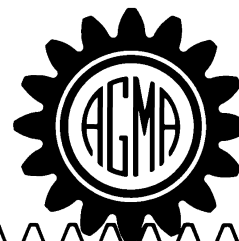


Finite Element Study of the Ikona Gear Tooth Profile

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American Gear Manufacturers Association



TECHNICAL PAPER

Finite Element Study of the Ikona Gear Tooth Profile

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

The Ikona gear tooth profile is a patented non-involute tooth profile for internal gear pairs. Gears with this profile have the following properties: the teeth are conjugate; the contact ratio is very high; there is no tip interference, even when only a one-tooth difference between the pinion and internal gear; there is minimal backlash; and the gears can be cut on conventional gear-cutting machines. Large reduction ratios can be achieved by a single gear pair and a high contact ratio results in lower tooth stresses than for a similar involute gear. Plus, minimal backlash makes the Ikona profile ideal for many applications, such as servo-drives, medical prostheses, and robots. Stress analysis of these gears assumes that the contact force is equal at each contacting tooth pair. Finite element results demonstrate how the number of tooth pairs in contact may increase under load. Finally, an estimate will be presented, showing the variation of tooth force between the contacting teeth.

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Tip Interference

The Ikona tooth profile was originally developed to eliminate tip interference. For involute internal gear pairs, there is generally tip interference if the difference in the tooth numbers is less than six. The purpose of the Ikona design was to create tooth profiles in which there would be no tip interference, even when the tooth number difference was as low as one.

Consider a gear pair in which the internal gear has one tooth more than the pinion. The internal gear is fixed, while the pinion is mounted on an eccentric. In a conventional design, either involute or non-involute, the eccentricity (ie the center distance) would be about 0.5 modules. During one revolution of the eccentric, the axis of the pinion would move up and down about 1.0 modules. A tooth of the pinion, initially in contact with a tooth of the internal gear, would be required to move from one tooth space of the internal gear, around one of the teeth, and into the adjacent tooth space. However, in a conventional design the working depth of the teeth would be at least 1.6 modules. Thus the pinion tooth would inevitably collide with the internal gear tooth, and tip interference would occur.

In the Ikona gear pair with a one-tooth difference, the center distance is increased to about 1.0 modules, so that the pinion tooth moves from one tooth space to the next without hitting the internal gear tooth. The pitch circle radii R_{p1} and R_{p2} of the pinion and the internal gear are given by

$$R_{p1} = N_1 C / (N_2 - N_1) \quad (1)$$

$$R_{p2} = N_2 C / (N_2 - N_1) \quad (2)$$

where N_1 and N_2 are the tooth numbers and C is the center distance. With a one-tooth difference, and a center distance of 1.0 modules, the pitch circle radii are equal to $N_1 m$ and $N_2 m$, where m is the module. For a conventional pinion, the radius of the tip circle is

approximately $(0.5N_1+1)m$, so the pitch circle now lies entirely outside the pinion.

It is advantageous to redefine the module, as applied to the Ikona tooth profile. The module is generally defined for involute gears as the reference circle diameter of either gear, divided by the number of teeth. For non-involute gears, where only one pitch circle exists, the module would be the diameter of the pitch circle divided by the number of teeth. In either case, the module is then used as a measure of the tooth size.

In the case of the Ikona profile, the pitch circle diameter is now much larger than the tip circle diameter. If the conventional definition of the module is used, the module would be unrelated to the tooth size. For this reason, the module is now defined in terms of the pinion tip circle diameter D_{tip1} :

$$m = D_{tip1} / (N_1 + 2) \quad (3)$$

This definition means that an Ikona gear of a given module will have teeth of approximately the same pitch as an involute gear of the same module.

Path of Contact and Tooth Profiles

Figure 1 shows the tip circles of an Ikona gear pair, with the line of centers vertical, and the gear centers at fixed points C_1 and C_2 . Any contact between the teeth must take place in the crescent-shaped area where the tip circles overlap. The pitch point lies on the line of centers, but well outside the tip circles.

The Law of Gearing states that at any contact point, the common normal to the gear teeth must pass through the pitch point. This implies that the path of contact crosses the line of centers at the pitch point. Since the path of contact must lie within the crescent-shaped area, and the pitch point lies outside it, it follows that the path of contact does not intersect the line of centers, but lies entirely on one side.

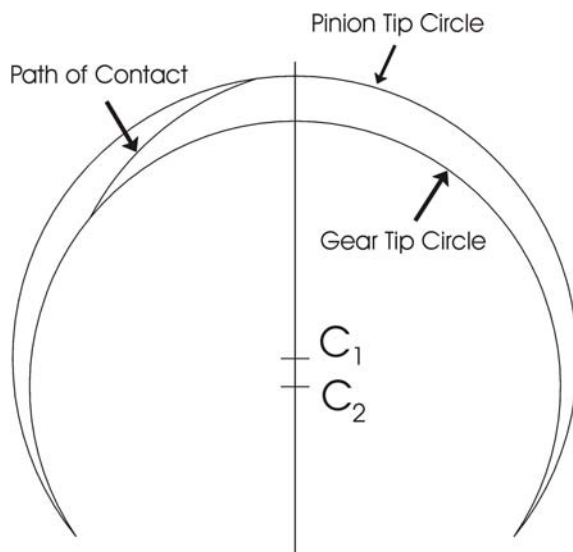


Figure 1 Crescent-shaped area formed by the pinion and gear tip circles, and the path of contact.

The path of contact will start at a point on the internal gear tip circle, and will end at a point on the pinion tip circle. At any point on the path of contact, the common normal to the tooth profiles must pass through the pitch point. Therefore, we know the direction of the common tangent, and hence we can calculate the pressure angles of the tooth profiles at that point. For the Ikona gear, the initial point of the path of contact is generally chosen as the point on the internal gear tip circle giving a pressure angle of 8 degrees. The final point is generally chosen as the point on the pinion tip circle giving a pressure angle of 35 degrees.

For the path of contact between these two points, we can choose any curve whose radius of curvature is approximately equal to the tip circle radius of the pinion, so that the curve remains within the crescent-shaped area. The curve chosen for the Ikona gear is an Archimedes spiral. This has the advantage that the radius, measured from either gear center, increases monotonically as we move along the curve. Hence the contact point moves continuously along the tooth profiles, so that there is no possibility of interference, or of undercutting when the gears are cut.

Since the radius and the pressure angle of each profile are known at every point of the path of contact, the tooth profiles can be constructed using finite differences. The pinion teeth can be hobbled by any hobbing machine, using a specially shaped hob, and the internal gear teeth can be cut using a shaper cutter.

Contact Ratio and Backlash

The pinion position is known at the start and end of the path of contact. The difference between the angular positions at these two points is divided by the angular pitch, to give the contact ratio. Because of the curved path of contact, the contact ratios are very much higher than those of involute gear pairs. For example, the contact ratio of a 45-46 tooth gear pair is 3.24, and the contact ratio of a 78-79 tooth gear pair is 5.21.

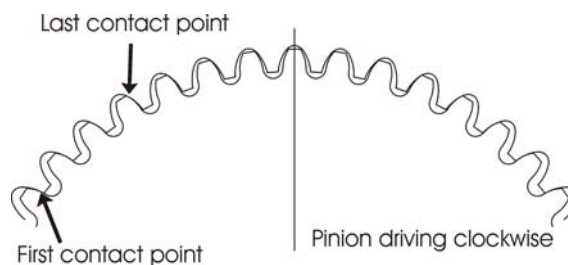


Figure 2 An Ikona gear pair with 45-46 Teeth.

Figure 2 shows a gear pair with the center-lines of a pinion tooth and a tooth space of the internal gear coinciding with the line of centers. To the left of the line of centers, the right-hand faces of several pinion teeth are in contact with the internal gear teeth, each of the contact points lying on the path of contact. However, since both gears are symmetric about the line of centers, there is another path of contact lying to the right of the line of centers, where the left-hand faces of the pinion teeth are in contact with the internal gear teeth. If the pinion is driving and turning clockwise, the left-hand path of contact is active, while small gaps occur at the contact points to the right of the line of centers, due to the tooth flexibility. If the pinion turns counter-clockwise, the situation is reversed.

If there is no backlash in an involute gear pair, at least one of the pinion teeth will make contact on both faces with the internal gear.