

On Tooth Failure Analysis in Small-Teeth-Number Gearing: An Analytical Approach

by: Dr. S.P. Radzevich, Eaton | Automotive, Innovation Center

American Gear Manufacturers Association



TECHNICAL PAPER

On Tooth Failure Analysis in Small-Teeth-Number Gearing: An Analytical Approach

Dr. Stephen P. Radzevich, Eaton | Automotive, Innovation Center

[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

The paper pertains to analytical study of the major reasons that cause tooth failure in gearing having small teeth number. Below, gears with small teeth number are referred to *STN*-gears. For the analysis of gear tooth failure, tooth contact stresses, and the combined contact/shear stresses (*CS*-stresses) are investigating. The study is based on the in-depth analysis of the gear tooth loading and on accounting of variation in time of the tooth loading and of other gear parameters in various phases of tooth meshing. The earlier developed by the author *DG/K*-method¹ of surface generation is used in the study. For the computation of the contact loads of parallel-axis gears, a novel load distribution model is proposed in the paper, as well as a novel analytical method for computation (a) of contact stresses, and (b) of combined *CS*-stresses in *STN*-gearing is discussed. The *CS*-stresses are caused by simultaneous impact of (a) contact stresses together with (b) stresses caused by the pinion/gear tooth profile sliding. To use the developed method of computation, minimum of preliminary assumptions are required. While developed for use in *STN*-gearing, the method is also recommended for the accurate computation of stresses in gearing having regular teeth number. Using commercial software MathCAD/Scientific, a computer codes for computation of both of contact stresses, and of the combined *CS*-stresses are worked out. The reported results of the research could be used for extending of the existed standard AGMA 908-B-89 to the area of gear drives comprised of *STN*-gears, e.g. comprised of gears having teeth number less than $N_g = 12$. The developed method for the computation is ready to put in practice.

Copyright © 2006

American Gear Manufacturers Association
500 Montgomery Street, Suite 350
Alexandria, Virginia, 22314

October, 2006

ISBN: 1-55589-893-9

¹ The *DG/K*-method is based on fundamental results obtained in differential geometry of surfaces, and on kinematics of multi-parametric motion of a rigid body in the E_3 space. The interested reader may wish to go for details to the monograph: Radzevich, S.P. *Fundamentals of Surface Generation*, Monograph, Kiev, Rastan, 2001, 592p. [Copy of the monograph is available from The Library of Congress].

On Tooth Failure Analysis in Small-Teeth-Number Gearing: An Analytical Approach

Dr. Stephen P. Radzevich, Eaton | Automotive, Innovation Center

Introduction

Gears fail by pitting and wear as well as by tooth breakage. Gears having small teeth number are considering in this paper.

Often, those involute gears are referred to as *STN*-gears, base diameter $d_{b,g}$ of which exceeds the gear limit diameter $d_{l,g}$. When the inequality $d_{b,g} > d_{l,g}$ observes, then not the entire active tooth profile is shaped properly following the involute profile.

The AGMA standard [1] encompasses involute gears having the minimal teeth number $N_g = 12$. It could be supposed that according to [1], the *STN*-gears are those having teeth number $N_g < 12$ or a bit bigger N_g . No geometrical factors for computation of contact stresses for gears having teeth number $N_g < 12$ are available from the AGMA standard [1].

Those cylindrical gears having the base diameter bigger than the root diameter, are referred to as

STN-gears (small-teeth-number-gears). The inequality $d_b > d_f$ observes for *STN*-gears.

An example of application of small-teeth-number-involute gears (further *STN*-gears) is depicted in Fig. 1. A gear set of the Truetrac differential (Fig. 1) is comprised of two side-gears (having $N_g = 15/28$ teeth number, normal pressure angle $\phi_n = 30^\circ$, and pitch helix angle $\psi = 38^\circ$), which are engaged in mesh with four to six pinions (having $N_p = 5/7$ teeth number) [2], [3].

Gears of the automotive differentials are a good example of *STN*-gears.

Because of small teeth number, many features of the *STN*-gearing observe when operating. This results in that conventional approach for computation of contact stresses is not applicable for computation of *STN*-gearing. This is due to the known approach was developed for gearing with ordinary teeth number.



Figure 1. An example of application of helical *STN*-gears [2], [3].

Literary Survey

The stresses caused by the pressure between elastic bodies are of importance in connection with the design and/or investigation of *STM*-gears. Hertz [4] was the first who developed the mathematical theory for the surface stresses and deformations produced by pressure between curved bodies, and the results of his analysis are supported by experiment. Formulas based on this theory give the maximum compressive stresses, which occur at the center of the tooth surfaces contact *ellipse*. Unfortunately, they allow computation neither the maximum shear stresses, which occur in the interiors of the compressed teeth, nor the maximum tensile stresses, which occur at the boundary of the contact area and are normal thereto.

Hertz based his work on the assumption that the contact area was small compared with the radii of curvature of the contacting bodies. Because of the very small area involved in what initially approximates a point or line contact, contact stresses for even light loads are very high; but as the formulas show, stresses do not increase in proportion of loading. Furthermore, because of the facts that the stress is highly localized and triaxial, the actual stress intensity can be very high without producing apparent damage. To make use of the Hertz formulas for purposes of design or safe-load determination, it is necessary to know the relationship between theoretical stresses and likelihood of failure, whether from excessive deformation or fracture.

Both surface and subsurface stresses were studied by Belayev [5]. Timoshenko [6] was among the first who used the Hertz's method for investigation of strength of gear teeth.

Tons of research had been carried out in the field since the first publications on the topic [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], and others. Many efforts have been taken aiming use of *FEM* (*Finite Element Method*) for solving the problem of computation of contact stresses in gear tooth. However, most of research pertained to computation of just contact stresses in gearing with relatively big number of teeth. No relative motions (sliding) of the pinion and of the gear teeth are put into account.

If the motion involved is a true rolling without any slip, then under conditions of slow motion the stress conditions are comparable with those produced by static loading. If there is even a slight amount of slip, however, the conditions are very much more severe and failures likely to occur through mechanical wear. The only guide so far to proper design against wear is real or simulated service testing. When the motion involved is at high speed and produces cyclic loading, as in gear train, fatigue is an important consideration.

The practical design of gears that sustain direct bearing is based so far largely on experience since this alone affords a guide as to whether, at any given load and number of stress cycles, there is enough deformation or surface damage to interfere with proper functioning.

Statement of the Problem:

To develop an analytical method for computation (a) of tooth contact stresses, and (b) of tooth combined CS-stresses for gearing having small teeth number.

For computation of the contact loads of parallel-axis *STM*-gears, a novel load distribution model is developed (Fig. 2). It is of critical importance to stress here that the proposed novel model for the computation of the contact loads is valid for the cases when the neighboring contact lines do not overlap each other. The scenario when the neighboring contact lines overlap each other deserves been considered in a separate paper.

The consideration below relates to the computation of contact stresses for *STM*-gearing of all of designs, say for cylindrical gears, bevel and hypoid gears, worm gears *etc.* However, due to limited length of the paper, the computation of contact stresses just in helical *STM*-gearing is considered as an example. The reported results of the research are also recommended for the computation of contact stresses for gearing having regular teeth number.

Major Features of STM-Gearing

Gearing with small number of teeth significantly differs from gearing with regular teeth number. The major differences between the gearing of both kinds are briefly summarized below.

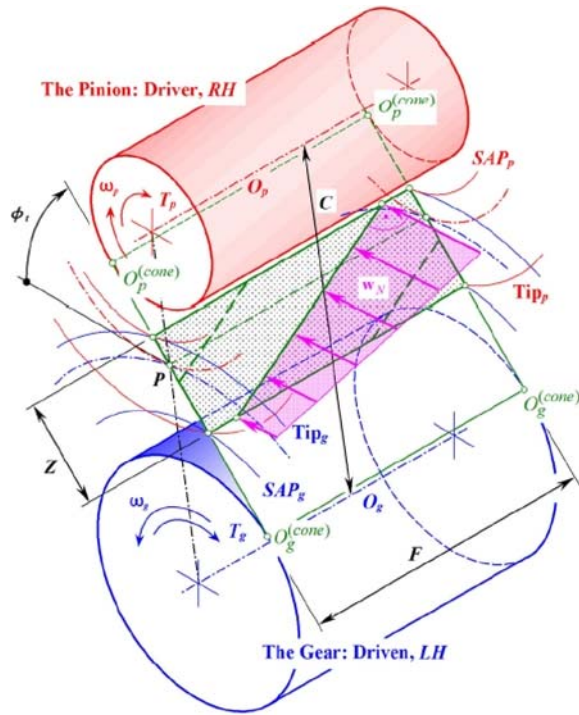


Figure 2. Normal load per unit length w_N of a single line of contact (The load w_N is applied in the direction within the plane of action perpendicularly to the line of contact).

Features of tooth geometry. Because of small teeth number, STM-gearing is featuring a considerable variation of curvature within the active length of the line of action. As an illustrative example, the variation of radii of normal curvatures of a pair of spur involute gears, as well as the variation of the normal curvatures themselves are shown in Fig. 3.

Tooth profile curvature of the gear ρ_g and of the pinion ρ_p change linearly within the active length of the line of action. Changes of normal curvature of the gear k_g and of the pinion k_p follow hyperbolic relationship. Relative normal curvature k_{rel} is minimal at a point that corresponds to the middle of the center distance C , and it increases towards axis of both the pinion and the gear. For further analysis, it is critical to point out here that maximal value of the relative curvature k_{rel}^{max} observes at the point of the line of action that corresponds to outside diameter of the gear $d_{o,g}$, and the limit diameter of the pinion $d_{l,p}$. Similarly, maximum of relative curvature $\tilde{k}_{rel}^{max}$ observes from the opposite side of the active length of the line of action, at the point that corresponds to outside diameter of the pinion $d_{o,p}$, and the limit di-

ameter of the gear $d_{l,g}$. However, the inequality $k_{rel}^{max} > \tilde{k}_{rel}^{max}$ is always valid [the equality $k_{rel}^{max} = \tilde{k}_{rel}^{max}$ observes just in cases when $N_g = N_p$].

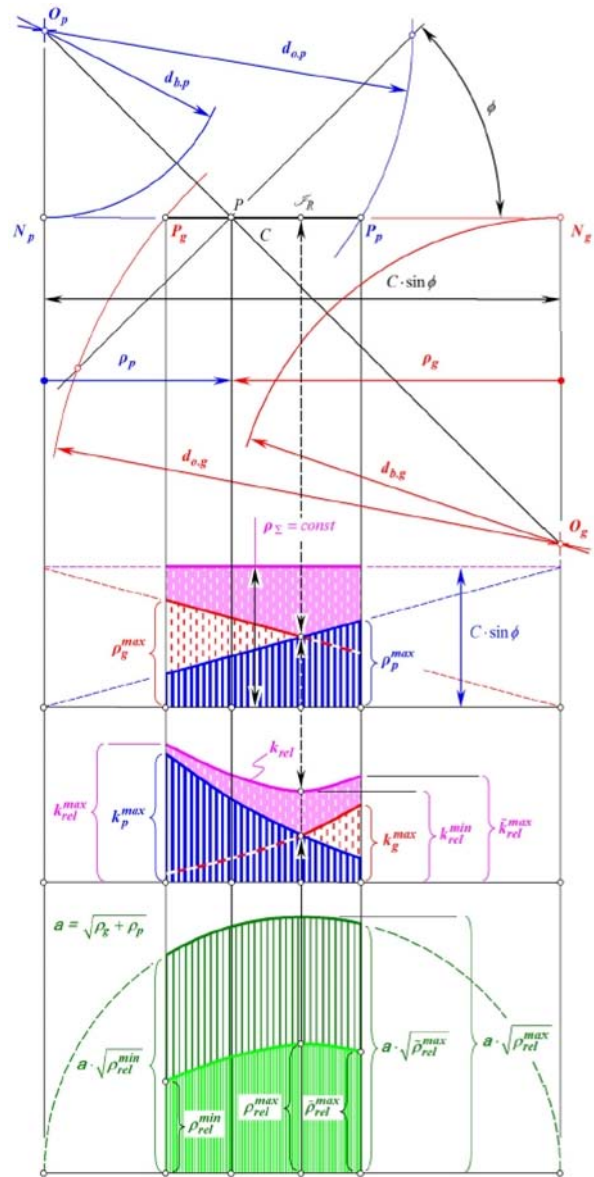


Figure 3. Variation of the major tooth curvatures for spur STM-gearing having involute tooth profile.

As it can be seen from Fig. 3, the teeth flank curvatures change progressively while the point of contact moves from P_g to P_p on the line of action, the relative radius of curvature changing correspondingly. The semi-circle constructed with the line of action $N_g N_p$ as diameter can be shown to represent, to an appropriate scale, the term $\sqrt{(\rho_g + \rho_p) \rho_{rel}}$

[below, the square root $\sqrt{(\rho_g + \rho_p)}$ is designated as a]. This is the term, which determines the change in surface stress at the point of contact whilst it moves from P_g to P_p on the line of action. Near the point N_g , the product $a\sqrt{\rho_{rel}}$ approaches zero and hence the surface stress approaches infinity, and it will be deduced that the surface stress at the tip of the wheel, point P_p , is usually greater than that at the tip of the pinion, point P_g . Variation of the relative curvature ρ_{rel} itself is constructed on the premises of change of $a\sqrt{\rho_{rel}}$ within the segment P_gP_p . The interested reader may wish to go to Fig. 4 for details of construction.

An intermediate remark. It is of importance to point out the readers' attention here to the point $\mathcal{T}_R$ at the line of action at which the normal relative curvature k_{rel} reaches its minimal value ($k_{rel} = k_{rel}^{\min}$), and, correspondingly, the normal relative radius of curvature ρ_{rel} reaches its maximum ($\rho_{rel} = \rho_{rel}^{\max}$) (Fig. 3). The point $\mathcal{T}_R$ divides the entire line of action N_gN_p on two equal portions $\mathcal{T}_RN_g$ and $\mathcal{T}_RN_p$, and therefore, the equality $\mathcal{T}_RN_g = \mathcal{T}_RN_p$ is valid. The following equations for computation of coordinates of the $\mathcal{T}_R$ are derived

$$\begin{aligned} r_{\mathcal{T}.g} &= \frac{1}{2} \sqrt{d_{b.g}^2 + C^2 \sin^2 \phi_t} \quad \text{and} \\ r_{\mathcal{T}.p} &= \frac{1}{2} \sqrt{d_{b.p}^2 + C^2 \sin^2 \phi_t} \\ \cos v_g &= \frac{r_{\mathcal{T}.g}^2 + C^2 - r_{\mathcal{T}.p}^2}{2 r_{\mathcal{T}.g} C} \quad \text{and} \\ \cos v_p &= \frac{r_{\mathcal{T}.p}^2 + C^2 - r_{\mathcal{T}.g}^2}{2 r_{\mathcal{T}.p} C} \\ E_g &= r_{\mathcal{T}.g} \cos v_g \quad \text{and} \quad E_p = r_{\mathcal{T}.p} \cos v_p \end{aligned} \quad (1)$$

If two identical gears are in mesh, then the point $\mathcal{T}_R$ coincides with the pitch point P .

Equations (1) could be expressed in terms of parameters of design of the gear and of the pinion. For the readers' convenience, derivation of Eqs. (1) is presented in Appendix A.

When the tooth ratio significantly exceeds the unity, then the point $\mathcal{T}_R$ is located out of the active part of the line of action. This might be the major reason for

why this point has not attracted attention of gear professionals earlier.

Actual location of the point $\mathcal{T}_R$ could be of critical importance for *STN*-gearing. Because of relative curvature of the gear and of the pinion tooth surfaces is minimal exactly at the point $\mathcal{T}_R$, it is highly desired this point being located within the active profiles of the tooth surfaces. More over, its location close to the pitch point is desired. This would result in reduction of contact stress.

The desired location of the point $\mathcal{T}_R$ is within the active portion of the line of action (Fig. 3). Equations (1) yield analytical determinations of conditions under which this requirement is satisfied. Inequalities ($r_{\mathcal{T}.g} \leq 0.5 d_{o.g}$ and $r_{\mathcal{T}.p} \leq 0.5 d_{o.p}$) describes the desired location of the point $\mathcal{T}_R$ within the active portion of the line of action. These inequalities could be expressed in terms of parameters of design of the *STN*-gear drive.

Equations (1) reveal that two points $\mathcal{T}_R$ exist for a given gear train. One of them is for a certain direction of rotation of the gear and of the pinion, and another one is for the opposite rotation of the bodies.

Evidently, the point $\mathcal{T}_R$ exists for any and all designs of gear drives: For (a) helical gears, (b) bevel gears, (c) hypoid gears, (d) spiroid gears, etc. For spatial gearing, a three-dimensional $\mathcal{T}_R$ -curve observes instead of the $\mathcal{T}_R$ point. Generally speaking, an analytical representation of the $\mathcal{T}_R$ -curve could be derived on the basis of equality that is equivalent to $\mathcal{T}_RN_g = \mathcal{T}_RN_p$.

Coordinates of the $\mathcal{T}_R$ points could be employed for the purpose of derivation of an analytical criterion to distinguish *STN*-gearing from ordinary gearing.

Similarly, a graphical interpretation of variation of other geometrical parameters of the gear and of the pinion within the active length of the line of action can be constructed.

The plane of action for a *STN* helical gear drive is shown in Fig. 5. Here, L_{pc} designates length of the path of contact in transverse section, $p_{b,x}$ is the axial base pitch, $p_{b,t}$ is the transverse base pitch, and F is face width.



Figure 4. For the determining of the function ρ_{rel} .

