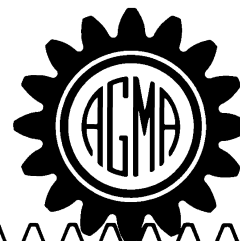


Optimal Tooth Modifications in Spiral Bevel Gears Introduced by Machine Tool Setting Variation

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American Gear Manufacturers Association



TECHNICAL PAPER

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

A method for the determination of optimal tooth modifications in spiral bevel gears based on improved load distribution, minimized tooth root stresses and reduced transmission errors is presented. The modifications are introduced into the pinion tooth surface by the variation of machine tool settings in pinion tooth processing.

The applied load distribution and transmission error calculations are based on a new idea: As the tooth surface modifications are relatively small and the conjugation of the mating surfaces is relatively good, it is assumed that the point contact under load spreads over a surface along the whole or part of the "potential" contact line, which line is made up of the points of the mating teeth surfaces in which the separations of these surfaces are minimal, instead of assuming an elliptical contact pattern. This method includes the bending and shearing deflections of gear teeth, local contact deformations of mating surfaces, gear body bending and torsion, deflections of the supporting shafts, and the manufacturing and alignment errors of mating members. A computer program implements the method.

By using the computer program that was developed the influence of machine tool settings for pinion tooth manufacture on load distribution, stresses and transmission errors is investigated. On the basis of the obtained results the optimal machine tool settings are determined introducing the optimal tooth modifications. By applying this optimal set of machine tool setting parameters the maximum tooth contact pressure is reduced by 5.42%, the tooth fillet stresses in the pinion by 8.07% and the angular position error of the driven gear by 48.43%, in regard to the spiral bevel gear pair manufactured by machine tool settings determined by the commonly used method.

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Introduction

During the last decades many research works have been directed towards the synthesis, tooth contact analysis and manufacture of spiral bevel gears. Handschuh [1] gives a very good review of progress for the analysis of spiral bevel gears. Local synthesis of spiral bevel gears with localized bearing contact and predesigned parabolic function of a controlled level for transmission errors is proposed by Litvin and Zhang [2]. In the paper published by Argyris et al. [3] a computerized method of local synthesis and simulation of meshing of spiral bevel gears with pinion tooth surface generated by applying modified roll is presented. By Huston and Coy [4] an analysis of the surface geometry of spiral bevel gears formed by a circular cutter with involute, straight and hyperbolic profile is presented. Kawasaki et al. [5] deal with spiral bevel gears in Klingelnberg cyclo-paloid system. The paper contains the design method, calculation of paths of contact and transmission errors and the investigation of influence of assembly errors on meshing characteristics. Based on grinding mechanism and machine-tool settings for the Gleason modified roll hypoid grinder, a mathematical model for the tooth geometry of spiral bevel and hypoid gears is developed by Lin and Tsay [6]. The computer algorithm, presented by Gosselin et al. [7], uses the sensitivity of the "error surface" to selected machine setting changes to calculate new machine settings to match a theoretical tooth surface to measurement data, within specified tolerances. In the paper published by Lin et al. [8] the sensitivities of tooth surface due to the variations of machine settings is investigated. The corrective machine-tool settings, calculated by using the sensitivity matrix and the linear regression method, are used to minimize the tooth-surface deviations. By Stadtfeld [9] the use of a six-axis free-form machine for processing spiral bevel gears is discussed. A numerical method for the tooth contact analysis of uniform tooth height epicyclic spiral bevel gears stemming from the Klingelnbergs cyclo-paloid system is proposed by Lelkes et al. [10]. Longitudinal settings of contact patterns or contact across the surfaces from tooth

root to tooth top were obtained as a function of machine-settings and the influence of each cutting parameter was isolated and discussed. Gosselin et al. [11] proposed an algorithm which allows the use of a different cutter, either in diameter and/or pressure angle, to obtain the same tooth flank surface. Significant cost reductions may be obtained with the application of the method. In Ref. [12], published by Litvin et al., different cutter profiles were used to introduce the optimal tooth modifications in order to reduce the noise and vibration levels and to increase endurance.

Papers [13–19] deal with loaded tooth contact analysis and stress analysis in spiral bevel gears. Wilcox [13] in his paper outlines the general theory for calculating stresses in bevel and hypoid gears using flexibility matrix method in combination with the finite element method. The loaded tooth contact analysis predicting the motion error of spiral bevel gear sets, by applying influence matrices, is presented by Gosselin et al [14]. Handschuh and Bibel [15] analytically and experimentally rolled through mesh a spiral bevel gearset to investigate the tooth bending stress by finite element method. Paper published by Falah et al. [16] summarizes the experimental and numerical results of the meshing of spiral bevel gears under load, from which the actual contact ratio is evaluated. Linke et al. [17] calculate the load distribution based on a method for using influence coefficients and also calculate the corresponding tooth root and contact stresses. By Fuentes et al. [18] the FEM was used for stress analysis in spiral bevel gears. Fang and Wei [19] consider the edge contact in loaded tooth contact analysis.

Conjugated spiral bevel gears are theoretically in line contact. In order to decrease the sensitivity of the gear pair to errors in tooth surfaces and to the mutual position of the mating members, carefully chosen modifications are usually introduced into the teeth of one or both members. As a result of these modifications, the spiral bevel gear pair becomes "mismatched," and a point contact of the meshed

The surface of the tool used for the generation of pinion teeth is in the coordinate system K_{T1} (attached to tool T_1) defined by the following equation (based on Fig. 1)

$$\vec{r}_{T1}^{(T1)} = \begin{bmatrix} -u \\ (r_{T1} + u \tan \alpha_1) \cos \theta \\ (r_{T1} + u \tan \alpha_1) \sin \theta \\ 1 \end{bmatrix} \quad (1)$$

The generation of the pinion tooth surface is described mathematically by the equation

$$\vec{v}_{m1}^{(T1,1)} \cdot \vec{e}_{m1}^{(T1)} = 0 \quad (2)$$

where

$$\vec{v}_{m1}^{(T1,1)} = \text{relative velocity vector of tool } T_1 \text{ to the pinion}$$

$$\vec{e}_{m1}^{(T1)} = \text{unit normal vector of the tool surface}$$

On the basis of Fig. 1 and Eq. (1), for the relative velocity vector and for the unit normal vector of the tool surface, it follows

$$\vec{v}_{m1}^{(T1,1)} = \omega^{(c)} \begin{bmatrix} i_{gp} (z_{m1}^{(T1)} + g) \cos \gamma_1 \\ z_{m1}^{(T1)} - i_{gp} (z_{m1}^{(T1)} + g) \sin \gamma_1 \\ i_{gp} [y_{m1}^{(T1)} \sin \gamma_1 - (x_{m1}^{(T1)} - c) \cos \gamma_1] - y_{m1}^{(T1)} \end{bmatrix} \quad (3)$$

$$\vec{d}_{m1}^{(T1)} = M_{1p} \vec{e}_{T1}^{(T1)} = M_{1p} \begin{bmatrix} \sin \alpha_1 \\ \cos \alpha_1 \cos \theta \\ \cos \alpha_1 \sin \theta \\ 0 \end{bmatrix} \quad (4)$$

where

$$\vec{r}_{m1}^{(T1)} = M_{1p} \vec{r}_{T1}^{(T1)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sin \psi_{cp} & -\cos \psi_{cp} & e_p \cos \psi_{cp} \\ 0 & \cos \psi_{cp} & \sin \psi_{cp} & -e_p \sin \psi_{cp} \\ 0 & 0 & 0 & 1 \end{bmatrix} \vec{r}_{T1}^{(T1)} \quad (5)$$

Matrix M_{1p} provides the coordinate transformations from the movable coordinate system K_{T1} (rigidly connected to the cradle and head-cutter T_1) into the fixed coordinate system K_{m1} rigidly connected to the cutting machine.

The concave side of pinion teeth is in the coordinate system K_1 (attached to the pinion, Fig. 1) defined by the following system of equations

$$\vec{r}_1^{(1)} = M_{3p} M_{2p} M_{1p} \vec{r}_{T1}^{(T1)}$$

$$\vec{v}_{m1}^{(T1,1)} \cdot \vec{e}_{m1}^{(T1)} = 0 \quad (6)$$

where matrix M_{2p} describes the installment of the pinion at the cutting machine and provides the coordinate transformation from system K_{m1} into the stationary system K_{10} rigidly connected to K_{m1} ; matrix M_{3p} defines the transition from the stationary system K_{10} into the system K_1 rigidly connected to the being generated pinion. Stated mathematically (based on Fig. 1)

$$\vec{r}_{10} = M_{2p} \vec{r}_{m1} = \begin{bmatrix} \cos \gamma_1 & -\sin \gamma_1 & 0 & -c \cos \gamma_1 \\ \sin \gamma_1 & \cos \gamma_1 & 0 & f - c \sin \gamma_1 \\ 0 & 0 & 1 & g \\ 0 & 0 & 0 & 1 \end{bmatrix} \vec{r}_{m1} \quad (7)$$

$$\vec{r}_1 = M_{3p} \vec{r}_{10} = \begin{bmatrix} \cos \psi_1 & 0 & \sin \psi_1 & 0 \\ 0 & 1 & 0 & -p \\ -\sin \psi_1 & 0 & \cos \psi_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \vec{r}_{10} \quad (8)$$

$$\text{while } \psi_1 = i_{gp} (\psi_{cp} - \psi_{cp0})$$

Tooth Contact of Mismatched Spiral Bevel Gears

In this paper a modified spiral bevel gear pair with generated pinion and gear is treated whose geometry is fully described in [22]. As the result of the modifications introduced into the pinion teeth, theoretical point contact appears between the mating tooth surfaces. In this contact point the common normal vector of the pinion and gear tooth surfaces exists and the generation equations of the pinion and gear tooth surfaces are satisfied [22, 23]. Therefore, for a particular position of the pinion and the gear, the instantaneous contact point is defined by the following system of equations

$$\begin{aligned} \vec{r}_{02}^{(1)} &= \vec{r}_{02}^{(2)} \\ \vec{e}_{02}^{(1)} &= \vec{e}_{02}^{(2)} \end{aligned} \quad (9)$$

$$\vec{v}_{m1}^{(T1,1)} \cdot \vec{e}_{m1}^{(T1)} = 0$$

$$\vec{v}_{m2}^{(T2,2)} \cdot \vec{e}_{m2}^{(T2)} = 0$$

where $\vec{r}_{02}^{(1)}$ and $\vec{r}_{02}^{(2)}$ are the position vectors of tooth surface points, $\vec{e}_{02}^{(1)}$ and $\vec{e}_{02}^{(2)}$ are the unit normal vectors in the same surface points for the pinion and the