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Variation Analysis of Tooth Engagement and Load-Sharing in Involute Splines

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

Involute spline couplings are used to transmit torque from a shaft to a gear hub or other rotating component. External gear teeth on the shaft engage an equal number of internal teeth in the hub. Because multiple teeth engage simultaneously, they can transmit much larger torques than a simple key and keyway assembly. However, due to manufacturing variations, the clearance between each pair of mating teeth varies, resulting in only partial engagement.

A new model for tooth engagement, based on statistics, predicts that the teeth engage in a sequence, determined by the individual clearances. As the shaft load is applied, the tooth pair with the smallest clearance engages first, then deflects as the load increases, until the second pair engage. The two engaged pairs deflect together until the third pair engage, and so on, until the full load is reached. Thus, only a subset of teeth carry the load. In addition, the load is non-uniformly distributed, with the first tooth carrying the biggest share. As a consequence, the load capacity of spline couplings is greatly reduced, though still greater than a single keyway.

The statistical model predicts the average number of teeth which will engage for a specified load, plus or minus the expected variation. It also quantitatively predicts the load and stress in each engaged pair. Critical factors in the model are the stiffness and deflection of a single tooth pair and the characterization of the clearance. Detailed finite element analyses were conducted to verify the tooth deflections and engagement sequence. The closed form statistical results were verified with intensive Monte Carlo simulations.

The more accurate model has lead to increased understanding of the mechanics of involute spline couplings and should permit better prediction tools for designers and improved performance of their designs.

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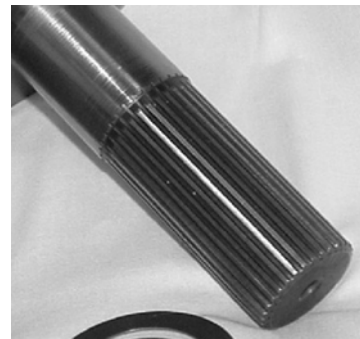
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Introduction

An involute spline coupling consists of a shaft with machined gear teeth on its exterior, mated to a hub with a matching set of interior teeth, as shown in Figure 1. They are found in gear trains, transmissions, pumps, and many other rotating machines. The involute profile makes them self-centering. The transmitted torque is distributed over the teeth, decreasing the load on any one tooth. It is therefore superior to a single key and keyway assembly, and leads to lighter, more efficient shaft designs.

In theory, the full ring of gear teeth on the shaft engage with an equal number of teeth in the hub, resulting in the load equally distributed over all the teeth. In practice, only a fraction actually transmit the load. Due to manufacturing variations, even with precision gear hobbing processes, only a fraction of spline teeth engage. Splines, therefore, perform far below their theoretical capacity. Designers commonly assume only 1/4 to 1/2 of the teeth carry the full load. They also approximate the load as uniformly distributed among the load-bearing pairs of teeth. These assumptions are often satisfactory, but can lead to early failures when applied to high load applications.

Consider an extreme application – the multi-disk brakes on an industrial mining dump truck, shown in Figure 2. The tires on this behemoth are 13 ft. in diameter. The driver climbs two stories to the cab. It hauls loads up to 380 tons over challenging terrain. Stopping at any speed punishes the brakes.



a) External tooth spline



b) Internal tooth spline

Figure 1. Involute splines



Figure 2. Industrial mining truck (courtesy Caterpillar, Inc.)

The multi-disk brake assembly consists of a set of friction plates, separated by pressure plates, as shown in Figure 3a. Friction plates are splined on the inside circumference and engage the shaft spline. Pressure plates are splined on the outside circumference and engage the non-rotating hub. The brake is actuated by a ring of hydraulic pistons, which clamp the pressure plates to provide braking force. The piston cylinders can be seen on the brake assembly in Figure 3b.

The spline couplings must transmit the high braking forces to the hub. Examination of failed plates revealed cracks at the base of the teeth, an indication of bending fatigue failure. Failed tooth fragments can jam between the plates or block cooling passages, which may lead to complete failure. Also, uneven wear was observed, suggesting unequal tooth loads.

A desire for a better understanding of the mechanics of tooth engagement led to the current study. Concerns focus on the effect of variation on the load distribution within an involute spline joint. Insights leading to more reliable designs were sought.

The objectives of the research reported here include:

- Develop a statistical model to predict tooth engagement and loads;
- Investigate the effects of tooth clearance variations on spline performance;
- Estimate tooth load sharing and stresses;

- Verify the tooth engagement model with Monte Carlo simulations;
- Verify the loads with finite element simulations;
- Determine the effects of spline design parameter combinations;
- Develop software for analysis and design of spline couplings.

Previous studies of splines have investigated:

- Spline standards and design; [1], [2], [3], [4], [6], [8], [9], [10]
- Process error sources and resultant tooth errors; [2], [8], [11], [12]
- Tooth stresses and deflections; [3], [7], [11], [13], [17]
- Spline tooth engagement; [2], [8], [11]
- Tooth load distributions. [2], [16]

Many studies of deflection and stress in gear teeth have been published, with valuable results. But spline couplings, although they share involute geometry with gears, are a very different application. Gear designers do not seek simultaneous contact between all the pairs of teeth. For spline design, this is the goal, but it is not possible due to tooth variations. There is, as yet, no quantitative algorithm for predicting tooth engagement in splines resulting from manufacturing process errors. With a realistic analytical tool, which includes all critical spline parameters, as well as realistic estimates of tooth errors, designers may be able to find an optimum combination which significantly improves spline performance.

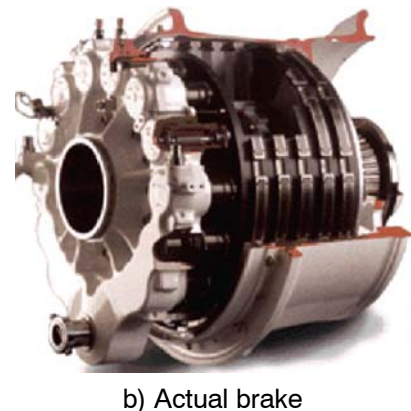
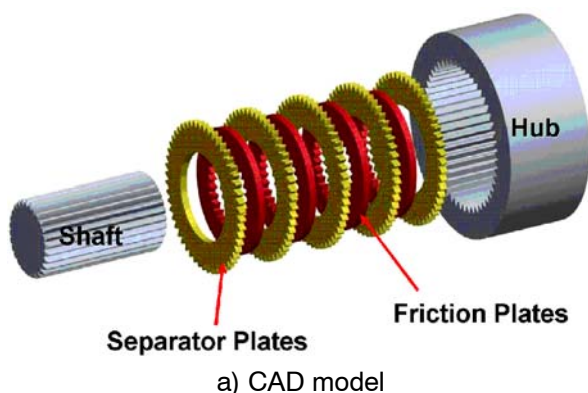


Figure 3. Multi-disk brake assembly (courtesy Caterpillar, Inc.)

The results of this study include:

- The effects of tooth errors on mating tooth clearances;
- Statistical characterization of clearance variations;
- Prediction of tooth engagement;
- Sequence of tooth engagement vs. load;
- Stress in each tooth resulting from non-uniform tooth loads.

Analytical model

Tooth errors/tooth clearance

The natural variation in the spline manufacturing process leads to non-uniform clearance between pairs of mating teeth. Three common sources of variation for this study were suggested by a gear manufacturing expert: index error, profile error and tooth thickness error [12], [15]. Both the internal and external teeth are subject to all three. With six sources of error, it may be assumed that the clearance variation would approximate a normal or Gaussian distribution, as shown in Figure 4. [14] Note that the majority of teeth are clustered close to the mean clearance, and spread out in the tails.

A requirement for smooth braking action of the truck brakes is that the clearance be sufficient to allow the pressure plates to slide axially. Extra clearance and a decreased pressure angle are provided for this application.

Lead error is another source for long spline teeth. It did not contribute in this application because the brake plates are thin and adjust independently. The shaft and hub centerlines were also assumed to remain in alignment due to their stiffness and self-centering geometry.

Tooth engagement

The realization that tooth clearance varies from tooth to tooth, led to a new statistical model for tooth engagement [5]. Consider a shaft and hub assembly. As the shaft rotates to engage the mating internal teeth, the clearance is reduced to zero. However, clearance between all tooth pairs does not go to zero simultaneously. The pair of teeth with the smallest clearance engages first and begins to transmit the torque load. As the load increases, the tooth pair deflect, until the second pair engage. The load continues to increase, causing both to deflect until a third pair engage, and so on, until the full applied load is reached.

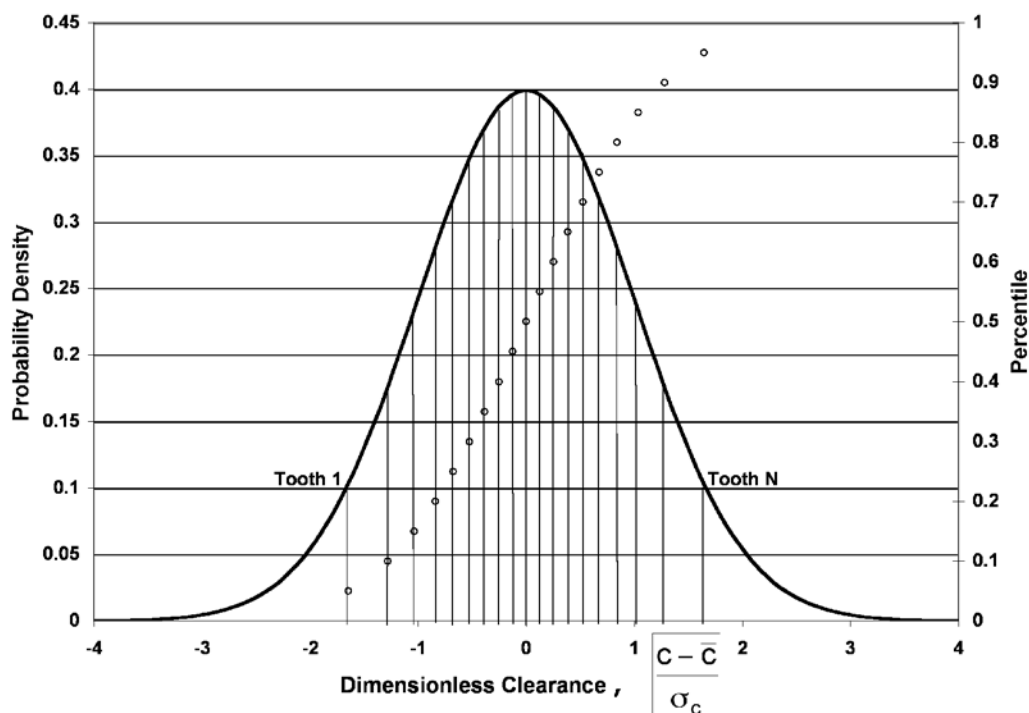


Figure 4. Normal distribution of tooth clearance