



American
Gear Manufacturers
Association

Technical Resources

10FTM07

AGMA Technical Paper

A New Statistical Model for Predicting Tooth Engagement and Load Sharing in Involute Splines

By J. Silvers, C.D. Sorensen and
K.W. Chase, Brigham Young
University



Documents

Document Sharing Hub

A New Statistical Model for Predicting Tooth Engagement and Load Sharing in Involute Splines

Janene Silvers, Carl D. Sorensen and Kenneth W. Chase, Brigham Young University

[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

Load-sharing among the teeth of involute splines is little understood. Designers typically assume only a fraction of the teeth are engaged and distribute the load uniformly over the assumed number of engaged teeth. This procedure can widely over- or underestimate tooth loads.

A new statistical model for involute spline tooth engagement has been developed and presented earlier, which takes into account the random variation of gear manufacturing processes. It predicts the number of teeth engaged and percent of load carried by each tooth pair. Tooth-to-tooth variations cause the clearance between each pair of mating teeth to vary randomly, resulting in a sequential, rather than simultaneous tooth engagement. The sequence begins with the tooth pair with the smallest clearance and proceeds to pick up additional teeth as the load is increased to the maximum applied load. The new model can predict the number of teeth in contact and the load share for each at any load increment.

This report presents an extension of the new sequential engagement model, which more completely predicts the variations in the engagement sequence for a set of spline assemblies. A statistical distribution is derived for each tooth in the sequence, along with its mean, standard deviation and skewness. Innovative techniques for determining the resulting statistical distributions are described. The results of an in-depth study are also presented, which verify the new statistical model. Monte Carlo Simulation of spline assemblies with random errors was performed and the results compared to the closed-form solution. Extremely close agreement was found. The new approach shows promise for providing keener insights into the performance of spline couplings and will serve as an effective tool in the design of power transmission systems.

Copyright © 2010

American Gear Manufacturers Association
500 Montgomery Street, Suite 350
Alexandria, Virginia, 22314

October 2010

ISBN: 978-1-55589-982-0

A New Statistical Model for Predicting Tooth Engagement and Load Sharing in Involute Splines

Janene Silvers, Carl D. Sorensen and Kenneth W. Chase, Brigham Young University

Introduction

Splined shafts are preferred over keyed shafts for transmitting heavy torque in industrial and automotive applications. The splined shaft and mating hub have matching sets of teeth over the full circumference, as shown in Figure 1. If the tooth loads were distributed uniformly around the circumference, each tooth would carry an equal share. However, due to manufacturing variations, the tooth clearance between each pair of teeth varies, so the teeth do not engage all at the same time. Thus, the load is not shared uniformly.

In practice, as the shaft is turned, the tooth pair with smallest clearance gap will make contact first and begin to carry the load. As the torque increases, the first tooth deflects enough for a second pair, with the next smaller clearance, to engage and begin to share the load. This process of sequential engagement continues with increasing load until the full load is applied. The full load is generally not sufficient to engage all of the teeth, so some teeth will not carry any load.

As a result of this sequential engagement, the first pair of teeth to engage will carry more of the load,

causing the first tooth to be most likely to fail. Each tooth in succession will carry a smaller share. The motivation behind this research is to permit designers to accurately predict the tooth loading and avoid spline failure.

Previous work

Tooth engagement is driven by deflection: as the force increases and the engaged teeth deflect, clearance gaps between other tooth pairs close and additional tooth pairs engage. This deflection can be described using a strength of materials deflection model, as was done by DeCaires [1]. His model encompasses deflection due to shear, bending, and contact forces, and was verified by FEA. Figure 2 shows a force-deflection curve calculated using this model.

Each tooth is modeled as a spring, which acts in parallel with the other teeth. When multiple pairs of teeth are engaged, the stiffness, K_i , of each add together, which can be seen in the figure. At each data point, another pair of teeth engages, changing the slope incrementally.

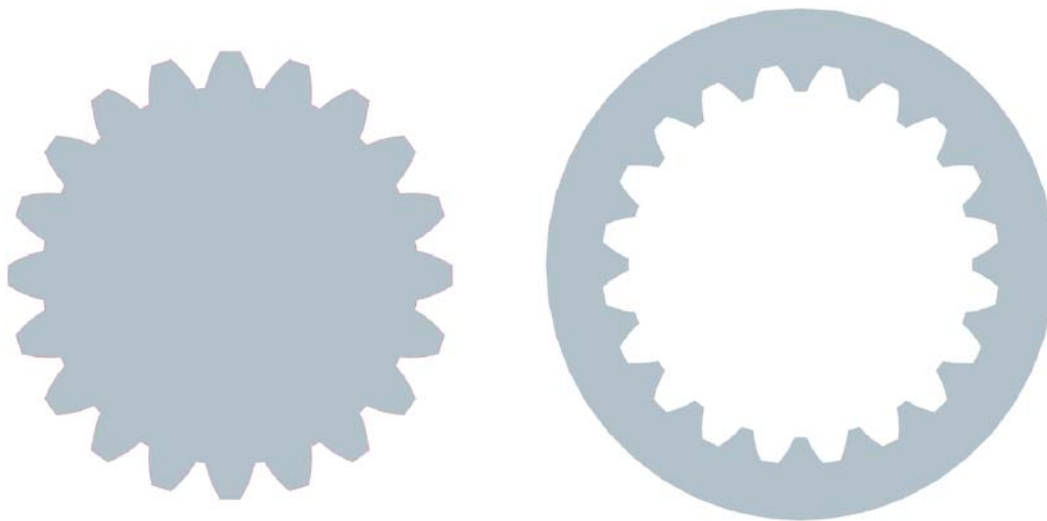


Figure 1. External and internal spline teeth

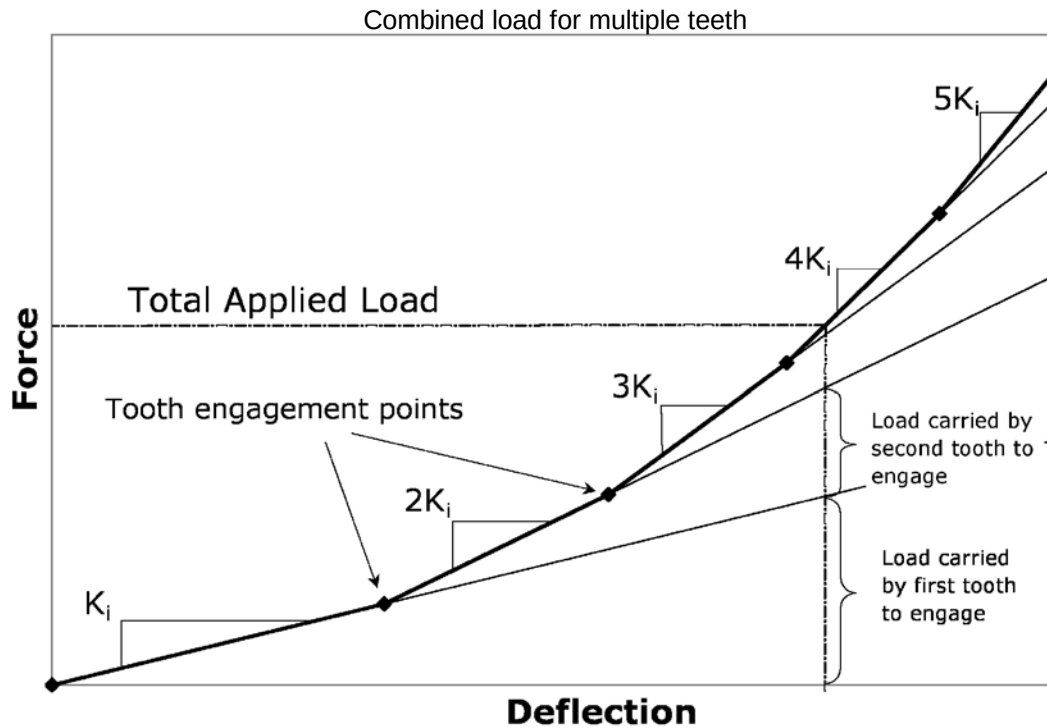


Figure 2. Force vs. deflection curve demonstrating sequential tooth engagement

The amount of load carried by a given tooth can be found by extending the slope of each segment of the graph, as shown, and then measuring the vertical distance between segments at the deflection value corresponding to the applied force. The first tooth to engage, hereafter referred to as Tooth 1, carries the largest load.

DeCaires' model can be used to determine the percentage of the total load carried by Tooth 1. The percentage is compared to the number of teeth engaged in Figure 3. Tooth 1 always carries a larger percentage of the total load than any other teeth. Although the total load on Tooth 1 continues to increase, the percent of the total decreases due to load sharing by an increasing number of teeth.

Models for tooth clearance variation

Multiple sources of error are present in tooth manufacturing for both the internal and external teeth, so the resulting tooth clearance is a combination of several random variables. Therefore, a normal distribution of tooth clearances is a reasonable assumption. This distribution is shown in Figure 4. Note that more clearances are clustered

near the middle of the distribution and spread out near the tails. The teeth are self-sorting in order of increasing clearance—teeth will engage in order, from the smallest clearance to the largest clearance, regardless of their location in the assembly.

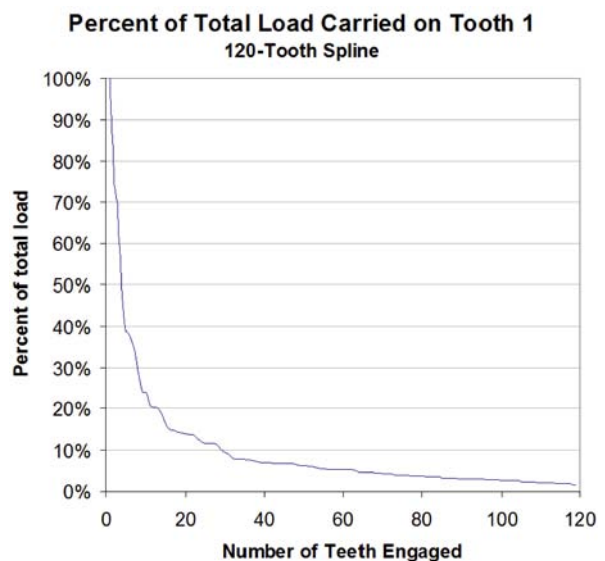


Figure 3. Percent of load carried by Tooth 1 as subsequent teeth engage

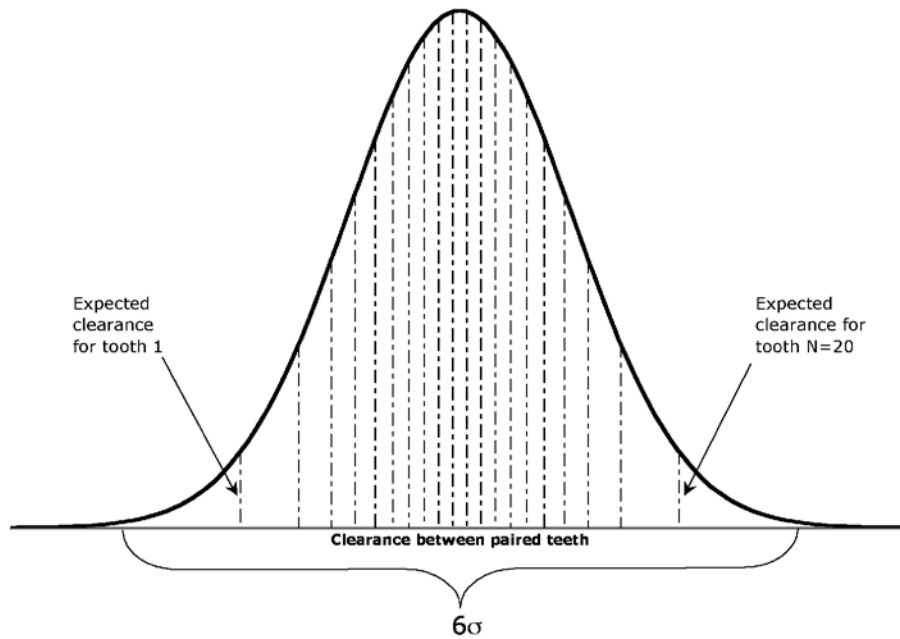


Figure 4. Normal distribution of tooth clearances

Mapping model

One method of predicting the clearance variation is the mapping model, shown in Figure 5 for a 10-tooth spline. A uniform distribution is plotted on the y-axis and divided into 10 equal intervals. The center point of each interval is projected horizontally across to intersect the normal cumulative distribution function (CDF), then vertically down to the x-axis. The resulting distribution on the x-axis is normal. This model predicts the mean, or most likely, clearance values for the first tooth to engage, the second tooth to engage, and so on.

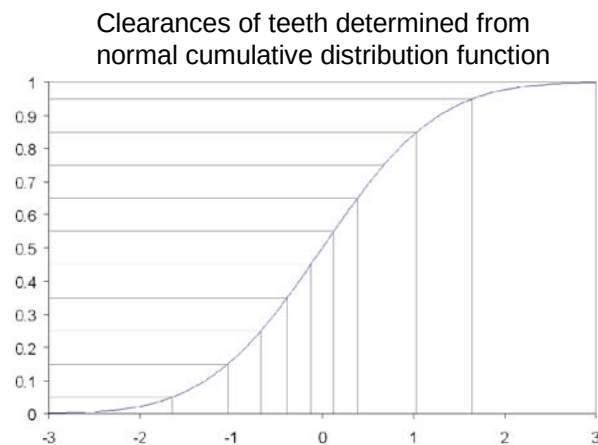


Figure 5. Mapping model to predict tooth clearance for a 10-tooth spline

The horizontal axis has units of standard deviation. If the several process errors are known from inspection data, or estimated from previous experience, their standard deviations may be added by root-sum-squares to estimate the resultant clearance standard deviation. The horizontal axis may thus be scaled to the corresponding dimensional clearance.

Using the CDF to transform one distribution into another is an established procedure [2]. In this case, the center point of each increment, when mapped across and down to the horizontal axis, locates the most likely clearance for each tooth pair. In reality, the clearance of Tooth 1, Tooth 2, etc. is subject to random variation about the most likely value. This is an important aspect of tooth engagement, which is complex statistically. It is dealt with in the next section.

Probabilistic tooth engagement model

If two splined shafts are assembled to two hubs, the resulting clearances will be similar, but not exactly the same. Since the process variations are random, each assembly will have slightly different clearances, just as no two snowflakes are exactly alike. Furthermore, if a spline is disassembled, the shaft is rotated a couple of teeth, and reassembled, all the clearances are rearranged. This means that both the force-deflection curve and the load carried by

Tooth 1 vary for each spline assembly and for each alternate meshing of the same spline-hub pair. Consequently, neither can be defined for all cases, but they can be predicted by statistically modeling the expected clearance variation.

For a given spline assembly, the clearance for every tooth pair could be measured in theory, although this would be difficult in practice. As a large number of spline assemblies is measured, a histogram for the clearance of each tooth pair can be developed, and will approach a continuous distribution.

In this paper, a more complete statistical model to predict tooth clearances is developed and validated. This model is called the Probabilistic Tooth Engagement Model, or ProTEM. While the mapping model predicts the mean or average clearance for each tooth pair, ProTEM also predicts the complete probability distribution of clearances for each pair of teeth. The ranges of predicted clearance values are shown for a 10-tooth spline in Figure 6. Each tooth in the sequence has its own distribution which is different from the normal distribution.

Objective

The objective of this paper is to evaluate the Probabilistic Tooth Engagement Model (ProTEM). A Monte Carlo Simulation was used to model several thousand spline assemblies and generate a distribution of the tooth clearance for each tooth in the sequence. The results were compared to the mapping model and to ProTEM.

Methods

Overview of ProTEM

ProTEM combines three related probabilities for tooth clearance. To calculate the probability of Tooth j on an N -tooth spline having clearance x , it combines the probabilities that there are:

- $j-1$ teeth with a clearance less than x ,
- one tooth with a clearance equal to x , and
- $N-j$ teeth with a clearance greater than x .

Using these probabilities, a probability density function, PDF, is developed for each tooth as a function of x . The PDF for the j^{th} tooth to engage on an N -tooth spline, $h(x, j, N)$, is defined in Equation 1.

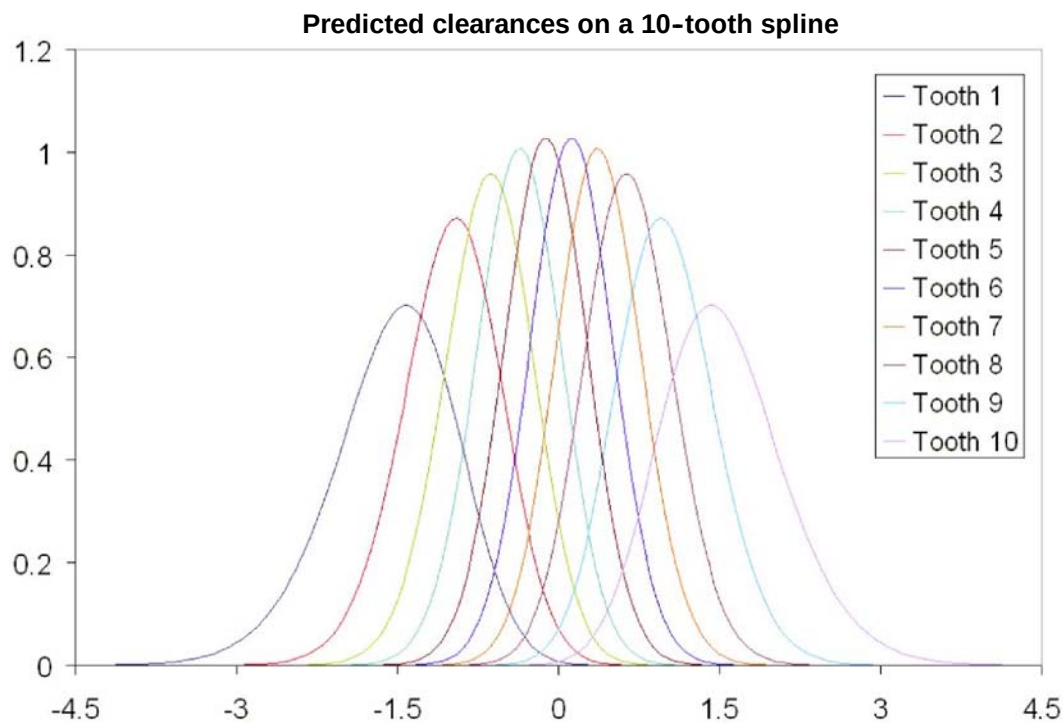


Figure 6. Tooth clearance predicted by ProTEM for a 10-tooth spline