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Evaluation of Methods for Calculating Effects of Tip Relief on Transmission Error, Noise and Stress in Loaded Spur Gears

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

The connection between transmission error and noise and vibration during operation has long been established. Calculation methods have developed to describe the influence such that it is possible to evaluate the relative effect of applying a specific modification at the design stage. The calculations can allow the designer to minimize the excitation from the gear pair engagement at a specific load. This paper explains the theory behind transmission error and the reasoning behind the method of applying the modifications through mapping the surface profiles and deducing the load sharing. It can be used to explain the results of later experimental validation on various types of tip relief in low contact ratio (LCR) gears, from very long to very short. The paper will also demonstrate that though the effects of modification in any specific case can be modeled with some certainty, the same modifying strategy can not be applied universally but must consider the required operating conditions. It illustrates that the effect of tip relief on transmission error and load sharing is not a black art but can be fully explained by applying existing theory.

A study of high contact ratio (HCR) gears will be presented to demonstrate why it is often necessary to apply different amounts and extents of tip relief in such designs, and how these modifications affect load sharing and highest point of tooth loading. Specific attention will be paid to the phenomenon of extended contact, where if no modification or insufficient tip relief is applied, contact does not stop at the end of active profile but continues beyond this point as the gear rotates resulting in contact on the tip. This effectively increases contact ratio and has implications for the tooth load and in particular how this may affect the loading position, highest point of single tooth contact (HPSTC), which is relevant to both ISO and AGMA standard rating. The paper will consider 3 methods commonly employed in the industry; a simple 2D mapping procedure carried out on graph paper, a 3D linear tooth stiffness computation method, and a 3D finite element analysis (FEA) calculation. The paper will demonstrate that though in some cases these methods can produce similar results, albeit with varying degrees of accuracy, further examples will be presented which demonstrate behavior which can only be detected using some of the more complex analysis methods. The commercial viability of implementing a better quality models against the time constraints in the development process will be discussed and conclusions drawn.

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Introduction

Transmission Error (T.E.) occurs when the driven gear is often momentarily ahead or behind its theoretical position in respect to the constant speed position. Gear design methods assume perfect geometric conditions and alignment between components to maintain constant angular velocity often referred to as conjugate action. This conjugate action is usually achieved in spur gears by using involute profiles on the teeth. However due to their ability to transmit large loads, the elastic deflection of the material from which the gears are made becomes significant. These small deflections of the teeth cause transmission error (the driven gear is often momentarily ahead or behind its theoretical position), and also the possibility of extended tip contact, that can lead to scuffing of the teeth and excessive noise and vibration. Other causes of transmission error are the manufacturing processes often resulting in deviations from the true involute profile, and tooth spacing (pitch) errors. For convenience the transmission error is expressed as a linear value measured at the base radius. This eliminates the need to specify on which gear it is measured as is the case with angular measurements.

To compensate for transmission errors it is well established practice to apply small profile corrections to the gear teeth, often termed tip/root reliefs. The amount of relief (material removed from the flank) is generally agreed upon, that is enough to allow for tooth deflections expected at a given load and also errors due to manufacturing tolerances. The extent of relief (how far down the tooth material is removed) is not so clear, and in spur gears is known to have a significant effect on gear performance. In the past designers tended to use empirical values from previous experience, which may not have been the optimum but due to lack of information in the design standards made them cautious about change.

The theory of profile relief to allow for tooth deflections under load was first proposed by Walker [1]. The suggested amount of relief was equal to the combined tooth pair deflection under load and the suggested extent what we now know as long relief. Harris [2] extended work in this area and covered different types of relief. He also introduced the concept of T.E. Using what have become known as "Harris Maps" he suggested that the T.E. curves for different loads could be used to describe the static and predict the dynamic behavior of a gear pair. Gregory, Harris and Munro [3] confirmed Harris's predictions experimentally.

Munro [4] later explained the fundamental mechanism behind profile relief and established a sound theoretical basis for design. He examined the effects of long and short relief and allowed the extent of relief to be varied at an intermediate position to obtain a low variation of T.E. at the desired design load.

Mapping 2D tooth profiles to calculate transmission error

The deviations from the involute profile of the pinion and wheel are combined from start of active profile (SAP) to the end of active profile (EAP). See Figure 1.

If the combined deviations of all the pairs of teeth for a pair of gears in mesh are superimposed, the pattern of T.E. as the gears are rotated can be created. This is achieved by spacing the tooth pair profile deviations one base pitch apart. See Figure 2.

The uppermost point on the curve from all the overlapped tooth pairs gives the transmission error at zero load. This is similar to that obtained from the single flank tester. When the load is applied from the torque acting on the gear, the analysis of the transmission error in one full tooth length region from SAP to EAP allows us to form a model of the contact in 2 dimensions. See Figure 3.

Assuming constant tooth pair stiffness:

At position a

$$\text{Total load} = (\text{stiffness of pair 1})(x_1 + x_2) + (\text{stiffness of pair 2})(x_2)$$

At position b

$$\text{Total load} = (\text{stiffness of pair 1})(x_4) + (\text{stiffness of pair 2})(x_3 + x_4)$$

When a whole series of these loaded curves is plotted we have produced the Harris map. It displays the quasi-static transmission error for

gears under a range of loads. Using this method shows the regions of single and dual pair contact, and allows the effect of different amounts and extents of tip relief to be examined. Each curve under load shows a different deflection from the nominal zero load position and changes in form due to changes in tooth load share during engagement. The changes in quasi-static curve form represent changes in displacement in a dynamic system which will ultimately be the source of excitation for noise and vibration in the system. A designer should look for a reduction in the amplitude of this curve form to reduce excitation.

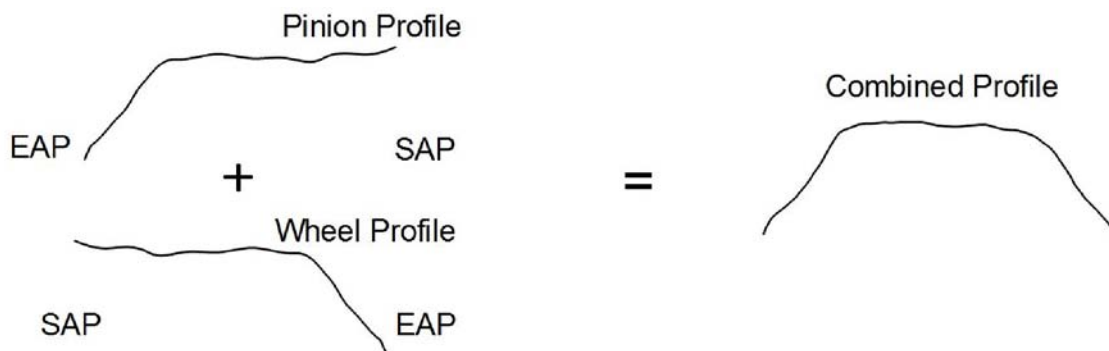


Figure 1. Combining the tooth profiles

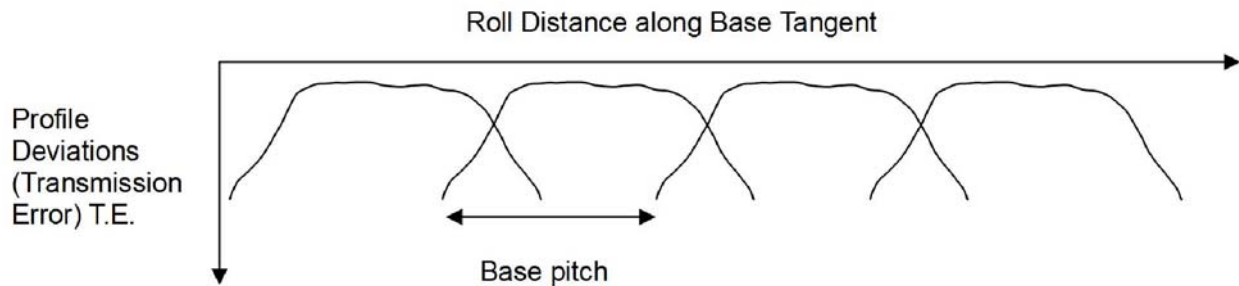


Figure 2. Tooth pair profiles offset one base pitch apart

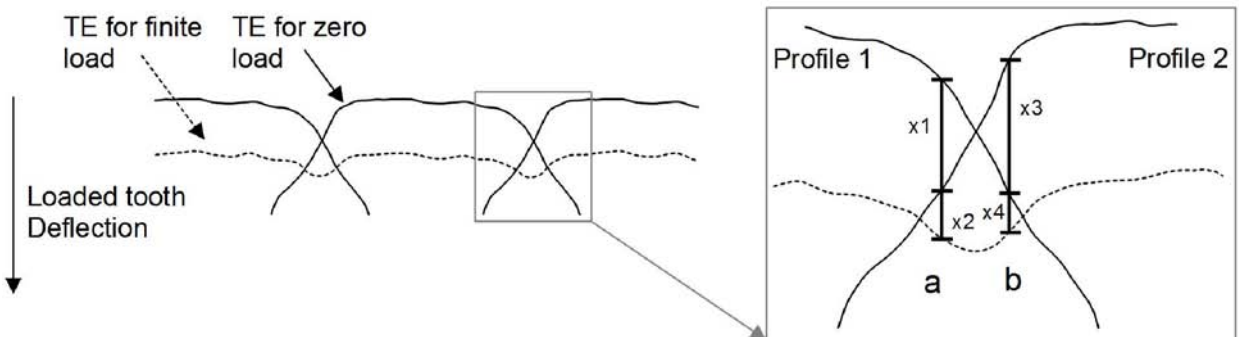


Figure 3. Calculating the loaded tooth deflection

Calculating the amount and extent of tip relief

The 2D mapping technique was used by Munro [4] to establish the theoretical basis for spur gear profile relief design. See Figure 4 and equations 1 and 2.

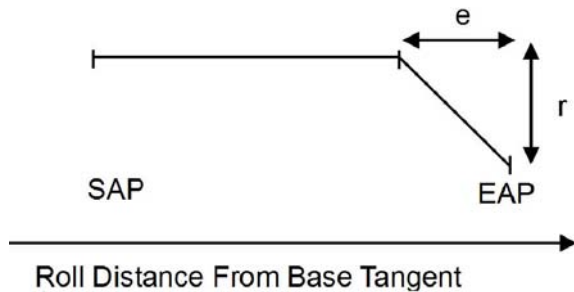


Figure 4. Calculating the amount and extent of tip relief in low contact ratio gears

$$\text{Extent } e = \frac{(EAP - SAP - p_{tb})}{\left(2 - \frac{P_o}{rc}\right)} \quad (1)$$

$$\text{Amount } r = \frac{P_{\max}}{c} + f_p \quad (2)$$

where

$P_{\max}$ is maximum load per unit face width (N/mm);

P_o is design load per unit face width (N/mm);

SAP is start of active profile roll distance (mm);

EAP is end of active profile roll distance (mm);

c is tooth pair stiffness (N/mm/ μm);

f_p is adjacent pitch error (μm);

r is extent of profile relief from tip;

p_{tb} is transverse base pitch.

Where the extent occurs at one base pitch from the start of active profile is termed long relief, and where the extent occurs at half the remaining distance

from the long position to the end of active profile is termed short relief. The loaded transmission errors of these 2 types of relief have very different characteristics as will be described in the following pages. See Figure 5.

The effect of any linear tip relief can be shown on graph paper. The examples for the case of an intermediate relief at varying loads using linear tip relief are shown in the following diagrams. What the theory allows is the adjustment of the extent of relief to obtain low variation in transmission error at a specific design load.

The charts in Figure 6 show that the amount and extent can also be adjusted to account for adjacent pitch errors. Additional relief is applied to compensate for the pitch error and the extent of relief made slightly shorter to maintain the design load low T.E., in this case load 2.

Experimental validation of Munro's theoretical basis for the application of tip relief

Munro's theory of transmission error was experimentally proven during the 1990's ref. [5], where a series of 6 low contact ratio spur gears with the same amount of tip relief but different extents were tested. These results are displayed in Figure 7.

The difference in the overall T.E. level of the curves in the measured data in Figure 7 is due to the bearing deflections that are not considered in the 2D mapping.

The peak to peak T.E. and the measured sound pressure level for the long, short and intermediate tip relief cases are shown in Figure 8.



Figure 5. The extent of long and short tip relief