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Calculation of the Tooth Root Load Carrying Capacity of Beveloid Gears

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

In this paper, two developed methods of tooth root load carrying capacity calculations for beveloid gears with parallel axes are presented. The first method calculates the tooth root load carrying capacity in an FE-based approach. The initial step of the method is the manufacturing simulation in the WZL software GearGenerator. The manufacturing simulation calculates the 3D geometry of the beveloid gears by simulating the generating grinding process. The next step is an FE-based (finite element) tooth contact analysis with the WZL software ZaKo3D which is able to calculate the tooth root stresses of several gear types during the meshing. From these stresses and further parameters (e.g., local material properties) the tooth root load carrying capacity is calculated in an approach which is based on the weakest link model of Weibull.

The second method uses analytic formulas to calculate the tooth root load carrying capacity of beveloid gears. In this method the tooth root load carrying capacity of beveloid gears is compared to the tooth root load carrying capacity of cylindrical gears. The effects which are observed during this comparison are described and formulas are derived to take these effects into account.

Finally both methods are applied to a test gear. The methods are compared to each other and to tests on beveloids gears with parallel axes in test bench trials.

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Introduction and challenge

A particular gear type which becomes more and more important is the beveloid gear, also known as conical involute gear. This is mainly due to their ability to realize small crossing angles between shafts and they can be produced economically on conventional gear grinding machines [1], [2]. Beveloid gears have been used in marine applications, for example, for many years [3], [4], [5]. In recent years the use of beveloid gears in the automotive sector has increased [6], [7]. Here the beveloid gear is used, for example in four wheel drives to transmit torque and rotation from the output of the gearbox to a front axle that may not be parallel.

Geometrical characteristics of beveloid gears

Beveloid gears are used to transmit torque and rotation between elements of crossing, skew or parallel axes [6]. The geometry of beveloid gears is derived from cylindrical spur or helical gears. The base circle and the pitch circle of beveloids are cylindrical, as presented in the middle section Figure 1. The pitch and the module are constant along the tooth width. The difference between beveloid gears and cylindrical gears is the varying profile shift along the tooth width to realize crossed or skew axes. For realizing the varying profile shift, the root cone angle θ_r is defined which is generated during gear cutting and gear grinding by a change of the feed during the process. The form of the tip of a beveloid gear is usually conical. The tip cone angle θ_a is determined by the geometry of the work piece.

A special use of beveloid gears is the arrangement with parallel axes. This is realized by two meshing beveloids which have a cone angle θ of the same absolute value but with opposite orientation [2].

Another gear type which is used for realizing crossing or skew axes is the bevel gear. Bevel gears have a conical pitch and base envelope. This results in a varying module m along the tooth width [2]. Beveloids are usually preferred to bevel gears when small crossing angles must be realized due to manufacturing limits of bevel gears. This is related to the long cone distances of gears with small cone angles which require substantial dimensions of the bevel gear cutting machine. [8]

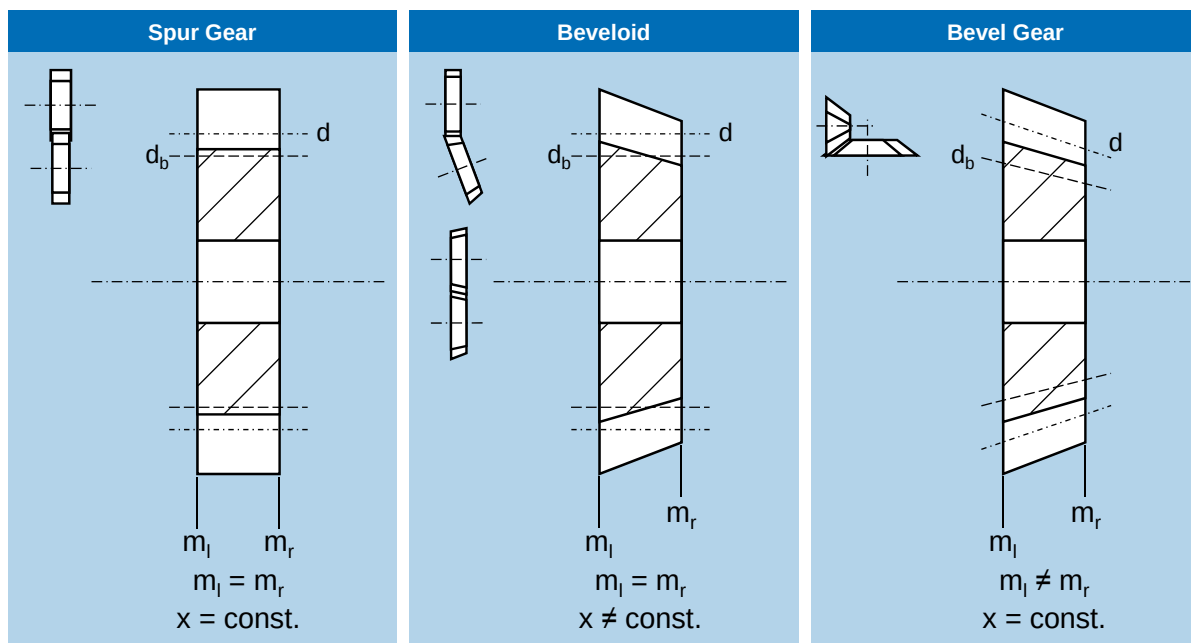


Figure 1. Geometrical characteristics of beveloid gears [2] (© WZL)

Contact behavior of beveloid gears

Beveloid gears can be mounted with parallel, crossed or skew axes. The axis orientation has substantial influence on the gear mesh. This influence is presented in Figure 2. In the left part of Figure 2 a typical contact pattern of beveloid with parallel axes is presented. The contact pattern is spread over the whole flank. On the right part of Figure 2 a typical contact pattern of beveloid gears with crossed axes is illustrated. Two involute beveloid gears with crossed axes have point contact. The resulting contact pattern is narrower than the contact pattern of beveloid gears with parallel axes. To achieve a full contact pattern of beveloid gears with crossed or skew axes at least one gear has to be designed with non-involute flanks. In this case, the manufacturing with standard methods like generating grinding is no longer possible. For some applications beveloid gears with conjugated flanks are manufactured by topological grinding to achieve nearly full contact [6] but for most applications this manufacturing method is avoided for economic reasons.

Challenge

To achieve a high power/weight ratio, a precise calculation of the gear load and load carrying capacity is necessary to design gears in an economical way. At the state of the art, no approved method for the tooth root load carrying capacity calculation for beveloid gears exists. Therefore the beveloid gear is approximated by a substitute spur gear with the gear data of the middle transverse section of the beveloid gear. The inaccuracy of this method is shown in Figure 3.

In the diagrams the tooth root stresses of a beveloid gear and a substitute spur gear are compared. The beveloid gear has an axis angle of 7.2° . The substitute spur gear is derived from the gear data of the middle transverse section of the beveloid gear. It can be seen that the stresses of the beveloid are significant higher. Reasons for this are the different root fillet geometry and the different contact behavior. Thus the calculation of the tooth root load carrying capacity of beveloid gears with a substitute spur gear according to existing standards for cylindrical gears is not possible without further ado.

A more precise calculation method can lead to a better design of beveloid gears with a higher power/weight ratio. Furthermore no simulation method for the running behavior of beveloid gears with and without load exists. Such a method could determine the tooth root load carrying capacity for a large number of variants in a short time. Therefore the project "Development and Verification of a Method to Calculate the Tooth Root Load Carrying Capacity of Beveloid Gears", which is sponsored by the German research funding organization DFG (Deutsche Forschungsgemeinschaft) has been initialized.

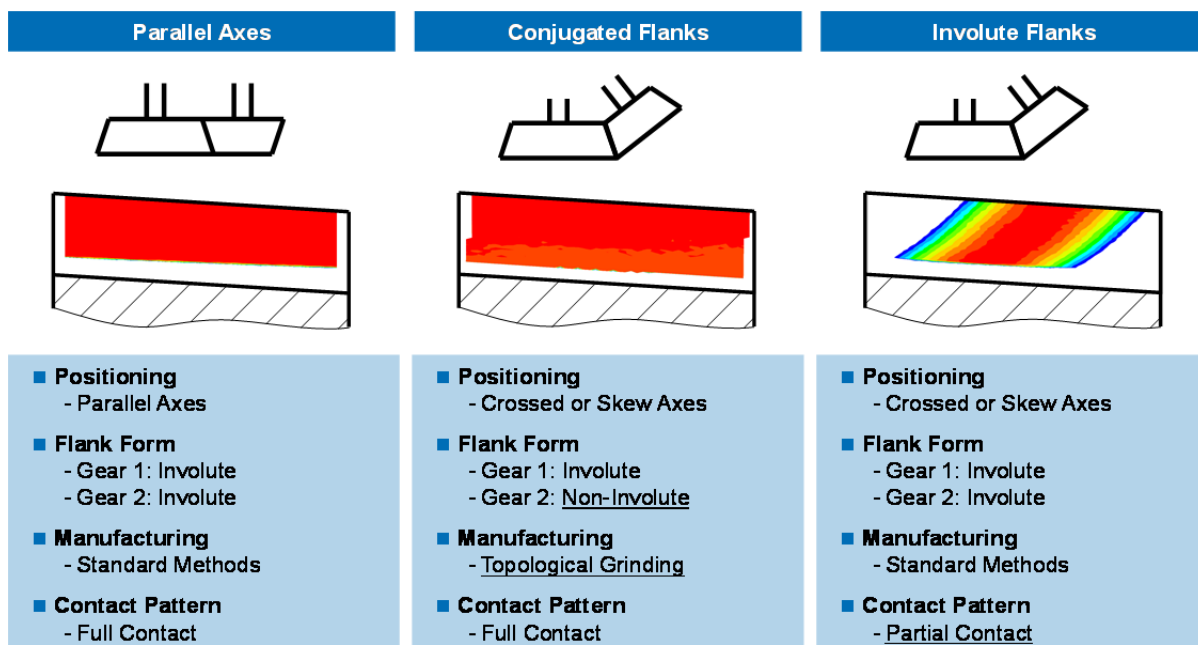


Figure 2. Contact characteristics of beveloid gears (© WZL)

- Gear Data
 - $m = 2,5 \text{ mm}$
 - $a = 0 \text{ mm}$
 - $\Sigma = 7,2^\circ$
 - $b = 35 \text{ mm}$
 - $z_1 = 31$
 - $z_2 = 35$

- Torque
 - $M_2 = 200 \text{ Nm}$

- Beveloid / Substitute Spur Gear
- Mating Gear

Conclusion:

The calculation of the tooth root load carrying capacity of beveloid gears with a substitute spur gear according to existing standards for cylindrical gears is not possible without further ado.

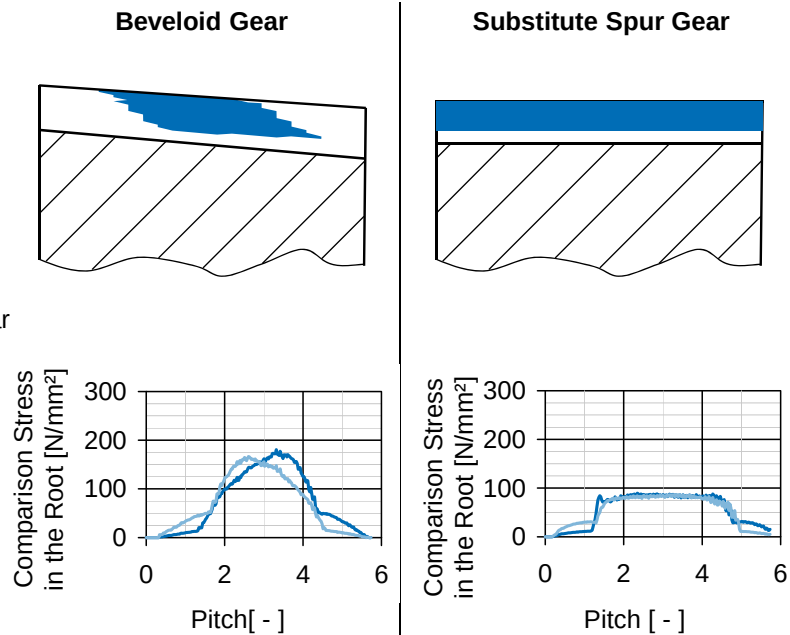


Figure 3. Comparison of the tooth root stresses of a beveloid gear and its substitute cylindrical gear (© WZL)

Objective and approach

In this paper the development of two calculation methods for the tooth root load carrying capacity of beveloid gears with parallel axes is described. In **Error! Reference source not found.** the approach for the development of these methods is illustrated. The initial point is the determination of the tooth root fatigue strength of beveloid gears on a test rig. The results are used to validate a local based calculation method to calculate the tooth root load carrying capacity of beveloid gears.

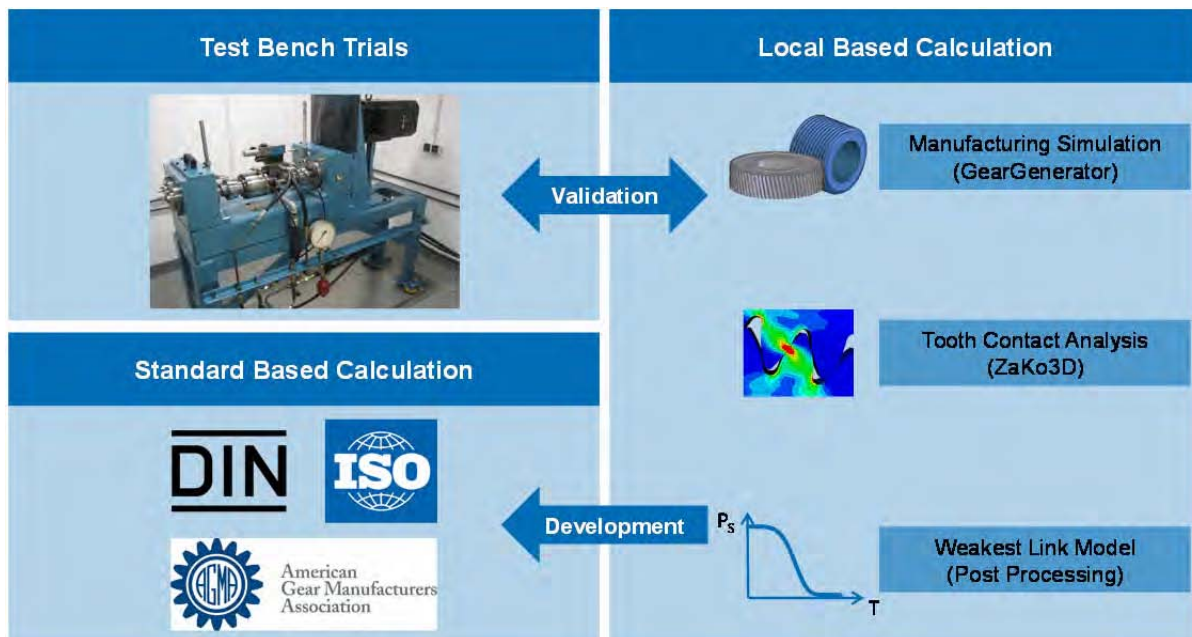


Figure 4. Approach for developing calculation methods for tooth root load carrying capacity of beveloid gears (© WZL)

The initial step of the local based calculation method is the manufacturing simulation with the WZL software GearGenerator. In the manufacturing simulation, a 3D geometry of the beveloid gears is created by simulating the generating grinding process [1], [9]. The resulting beveloid geometries are used in an FE-based tooth contact analysis with the WZL software ZaKo3D which is able to calculate the tooth root stresses of several gear types during the meshing [10]. From these stresses and further parameters (e.g., local material properties) the tooth root load carrying capacity is calculated in an approach which is based on the weakest link model of Weibull [11], [12], [13].

After the local based calculation method is validated, this method is used to derive a standard based calculation method for beveloid gears. The standard based calculation method uses analytic formulas to calculate the tooth root load carrying capacity of beveloid gears. In this method the tooth root stresses of beveloid gears are compared to the tooth root stresses of cylindrical gears. The effects which, observed during this comparison, are described and formulas are derived to take these effects into account.

Test bench trials

To detect the tooth root bending strength of a beveloid gear, a back-to-back test rig is used according to DIN 51354 Part 1, which uses the power circuit principle. The setup is illustrated in Figure 5. The tested beveloid gears are mounted in a gear box. They are connected by shafts to a transmission gear box. This setup is called power circuit. The test gear box is equipped with cylindrical gears. The cylindrical gears have the same gear data as the beveloids, but the cone angle is $\theta = 0^\circ$. The profile shift of the cylindrical gears is taken from the middle section of the beveloid gears. To avoid damage, the test gears are designed significantly wider than the beveloid gears. It is possible to include a torque into the power circuit at the coupling which is mounted at one of the shafts. The other shaft is designed as torque shaft. An electric motor is used to drive the gears. Since the torque is realized by the power circuit, the motor only needs to apply power into the system which corresponds to the power losses due to e.g., friction.

The gear data of the test gears is presented on the left side of Figure 6. To use the test principle of the back-to-back test rig according to DIN 51354 Part 1 parallel axes are used with a center distance of $a = 91.5 \text{ mm}$. The module of the gears is $m_n = 2 \text{ mm}$, the helix angle is $\beta_{1/2} = 23.024^\circ$ and the number of teeth are $z_{1/2} = 45/39$. Furthermore the cone angle of gear 1 is $\theta_1 = 3.6^\circ$. To realize parallel axes, the cone angle of gear 2 is $\theta_2 = -3.6^\circ$.

Goal of the tests is to determine the fatigue limit of the test gears for a probability of survival of $P_S = 50\%$. The principle used is the staircase method according to Hück. In this method the test load is dependent on the result of the previous test run. The load is reduced at breakage and increased at run-out. In these tests the load step is fixed at $\Delta T_2 = 25 \text{ Nm}$. A complete test run is reached, if gear 2 experiences $n_2 = 3\,000\,000$ load cycles without root breakage.

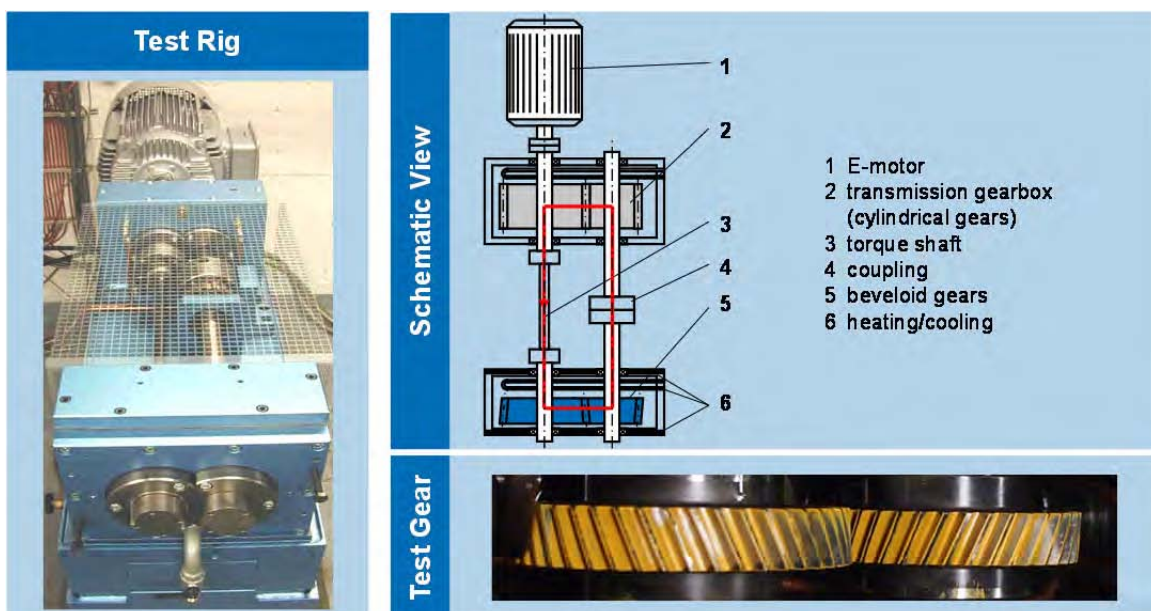


Figure 5. Back-to-back gear test rig according to DIN 51354 Part 1 (© WZL)