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American National Standard

Tooth Proportions for Plastic Gears

ANSI/AGMA 1106-A97



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American National Standard

Tooth Proportions for Plastic Gears

ANSI/AGMA 1106-A97

[Metric edition of ANSI/AGMA 1006-A97]

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ABSTRACT

This standard presents a new basic rack, AGMA PT, which, with its full round fillet, may be preferred in many applications of gears made from plastic materials. It also explains and illustrates the general concept of the basic rack. It contains a description, with equations and sample calculation, of how the proportions of a spur or helical gear may be derived from the design tooth thickness and the basic rack data. These equations and calculations use ISO based symbols and metric units. In several annexes, there are discussions of possible variations from this basic rack and also a procedure for defining tooth proportions without using the basic rack concept.

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Foreword

[The foreword, footnotes, and annexes are provided for informational purposes only and should not be construed as a part of ANSI/AGMA 1106-A97, *Tooth Proportions for Plastic Gears*.]

AGMA has issued standards for gear tooth proportions over a period of many years. The most recent versions have been AGMA 201.02 (withdrawn 1995), *Tooth Proportions for Coarse-Pitch Involute Spur Gears*, and ANSI/AGMA 1003-G93, *Tooth Proportions for Fine-Pitch Involute Spur and Helical Gears*. These standards and their predecessors were prepared in response to the need to standardize gear generating cutting tools such as hobs and shaper cutters. Without such standards, the variety of tools needed by gear shops would have become unlimited.

The manufacture of gears by the molding process is not subject to the same practical constraints as manufacture by the gear cutting process. Every mold is inherently “non-standard”. The geometry of the mold cavity cannot follow a standard because of varying allowances for shrinkage. Furthermore, there are some methods for manufacturing the mold cavity which do not depend on cutting tools and, even for those that do, special tools are generally required. Thus, tooth proportions for molded plastic gears need not follow those established for machined gears.

Some of the special properties of plastic materials influence the selection of gear tooth proportions as the two following examples illustrate:

- The structure and orientation of plastic molecules, regardless of processing method, makes the strengths of the materials particularly sensitive to sharp internal corners. A substantially stronger tooth will result if sharp fillets at the base of the tooth are avoided. The tooth proportions for gears made according to the AGMA fine-pitch standard noted above generally result in relatively sharp fillets.
- In certain applications, the higher expansion properties of plastic materials may create the need for a greater depth of engagement between mating gears than permitted by the other standard tooth forms.

As a result of this preference for a different tooth form, members of the plastic gear molding industry have adopted their own individual sets of tooth proportions. One set that has gained wide usage by plastic gear designers, and is often specified in place of the AGMA Fine-Pitch Standard, has been developed by William McKinley [1]. Because these tooth forms contain the preferential features for molded plastic gears and because they are already well recognized in the industry, they were used, with some changes, as models in the preparation of this standard. The first of the four variations in this set has a depth of engagement, or working depth, that is the same as in the above mentioned AGMA standards. The other three have increased depths of engagement in varying degrees. This standard has selected only the first variation, which is the one in widest use, as the model for the new tooth proportions. However, data similarly based on the other three variations are included in annex B.

The tooth forms in this standard are defined with the use of the basic rack concept. For those that might be unfamiliar with this concept, a detailed description of the basic rack is included as annex A.

Suggestions for the improvement of this standard will be welcome. They should be sent to the American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, Virginia, 22314.

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American National Standard – Tooth Proportions for Plastic Gears

1 Scope

1.1 Basic rack

This standard defines a basic rack for spur and helical plastic gears. However, the specific basic rack can be applied to any type of gear which employs the basic rack concept to help define its geometry.

NOTE: This basic rack is an optional alternative to other AGMA basic racks. It will often be preferred for those applications in which tooth bending strength is a major factor in the design of plastic gears.

A detailed description of the basic rack concept, including its features and its application to various types of gears, is given in annex A.

The data for three other basic racks, differing primarily in depth of engagement, are given in annex B.

1.2 Gear tooth proportions

This standard shows how basic rack dimensions determine the tooth proportions of the gear once the tooth thickness has been established. Specific values of the gear tooth thickness and the resulting gear tooth proportions are not covered by this standard. However, many of the considerations that go into such a selection are discussed in annex C.

The tip relief from a modified basic rack is discussed in annex D.

1.3 Non-traditional practices for defining tooth proportions

Annexes E and F describe some non-traditional practices currently used to help achieve gear designs that are optimum for their applications. Their use with molded plastic gears has been made more convenient by modern methods of mold cavity manufacture. These practices include modifying standard basic racks, creating custom designed basic racks, and designing tooth proportions without reference to basic racks. In the annexes, the description of each method is accompanied by potential advantages and design cautions. An example of one procedure for the last method, designing without reference to basic racks, is presented in annex F.

2 Definitions and symbols

2.1 Definitions

The terms used in this standard conform to ANSI/AGMA 1012-F90, *Gear Nomenclature, Definition of Terms with Symbols* and AGMA 904-B89, *Metric Usage*.

Terms used in this standard which require elaboration and concepts which are specific to plastic gears are covered where they are first used.

2.2 Symbols

The symbols used in the tables and formulas are shown in table 1.

NOTE: The symbols and definitions used in this standard may differ from similar items in other AGMA standards. The user should not assume that familiar symbols can be used without a careful study of these definitions.

Table 1 - Nomenclature: symbols and terms

Symbol	Term	First Used
C_{BR}	Clearance of basic rack	Figure 1
d	Reference diameter (standard pitch diameter) of gear	Eq 1
d_F	Form diameter of gear	Eq 6
d_{ai}	Inside diameter of internal gear	Eq 9

(continued)



Table 1 (concluded)

Symbol	Term	First Used
d_b	Base circle diameter of gear	Eq 5
d_{ae}	Outside diameter of external gear	Eq 3
d_{aeE}	Effective outside diameter	Eq 29
d_f	Root diameter of gear	Eq 4
e_{BR}	Space width of basic rack	Figure 1
h_{aBR}	Addendum of basic rack	Figure 1
h_{aTBR}	Effective addendum of the basic rack (distance from pitch line to the start of the circular-arc tip modification)	Figure 2
h_{fBR}	Dedendum of basic rack	Figure 1
h_{fBR}	Form dedendum of basic rack (distance from pitch line to start of fillet radius)	Figure 1
h_{kBR}	Working depth of basic rack	Figure 1
h_{tBR}	Whole depth of basic rack	Figure 1
m	Module	Cls 5
m_n	Normal module	Cls 5
p_{BR}	Circular pitch of basic rack	Figure 1
R_{TBR}	Radius of the circular-arc tip modification on the basic rack	Figure 2
r_{fBR}	Fillet radius of basic rack	Figure 1
r_T	Tip radius of gear	Eq 28
s	Tooth thickness at reference diameter	5.1
s_{BR}	Tooth thickness of basic rack	Figure 1
s_{ae}	Tooth thickness at outside diameter of spur external gear; top land	Eq 8
s_{aeE}	Effective top land with tip rounding	Eq 30
s_{aeR}	Remaining top land with tip rounding	Eq 28
s_{ai}	Tooth thickness at inside diameter of spur internal gear; inside tip land	Eq 12
s_n	Normal tooth thickness at reference diameter	5.4
s_{nai}	Normal tooth thickness at inside diameter of helical internal gear; normal inside tip land	Eq 27
s_{nae}	Normal tooth thickness at outside diameter of helical external gear; normal top land	Eq 23
s_{naeE}	Effective normal top land with tip rounding	Eq 34
s_{naeR}	Remaining normal top land with tip rounding	Eq 32
s_{tai}	Transverse tooth thickness at inside diameter of internal helical gear	Eq 25
s_{tae}	Transverse tooth thickness at outside diameter of external helical gear	Eq 21
y	Rack shift	Eq 2
z	Number of teeth of gear	5.1
α	Profile angle of basic rack; pressure angle at reference diameter of spur gear; normal pressure angle at reference diameter of helical gear	Figure 1
α_{ai}	Pressure angle at inside diameter of spur internal gear	Eq 11
α_{nae}	Normal pressure angle at outside diameter of helical external gear	Eq 31
α_{ae}	Pressure angle at outside diameter of spur gear	Eq 7
α_t	Transverse pressure angle at reference diameter of helical gear	Eq 17
α_{tai}	Transverse pressure angle at inside diameter of helical gear	Eq 24
α_{tae}	Transverse pressure angle at outside diameter of helical gear	Eq 20
β	Helix angle at reference diameter	5.4
β_{ai}	Helix angle at inside diameter of internal helical gear	Eq 26
β_{ae}	Helix angle at outside diameter of external helical gear	Eq 22

3 Tooth proportions and basic rack

It is general practice to establish a system of tooth proportions by defining a basic rack. The actual tooth outline for a gear with a specified number of teeth and value of tooth thickness will be generally determined by the generating action of this rack-shaped outline. (Some features of the tooth outline may be determined by additional considerations. See 5.8.1, 5.8.2 and 5.8.3.) A description of this basic rack concept and how it is applied to various types of gears is given in annex A.

4 Standard basic rack for plastic gears

A standard basic rack is established in this AGMA standard. Plastic gears with tooth forms defined by this basic rack will mesh properly with both AGMA fine-pitch and ISO coarse-pitch gears. (Three

experimental basic racks with greater working depths are defined in annex B.)

NOTE: This basic rack is an optional alternative to other AGMA basic racks. It will often be preferred for those applications in which tooth bending strength is a major factor in the design of plastic gears. See 4.4.

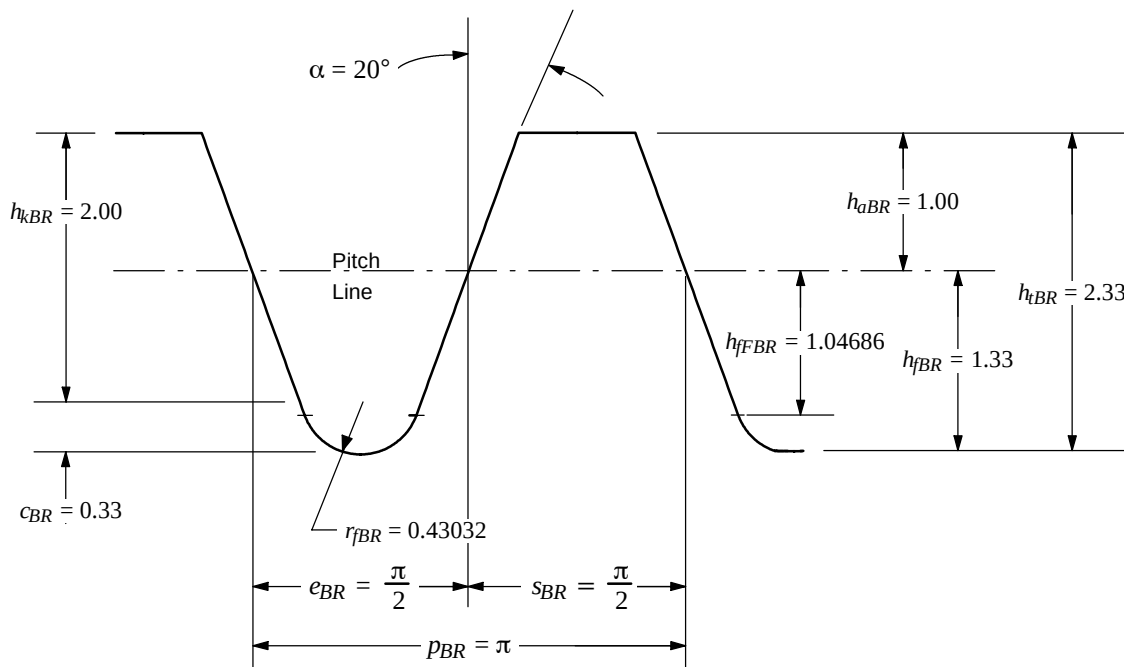
4.1 Designations

This basic rack is designated as:

AGMA Plastic Gearing Toothform, abbreviated as: AGMA PT.

4.2 Proportions

The basic rack is shown in figure 1. The figure identifies all the features described in A.2. Values of the dimensional features are listed in table 2 along with comparative values for the AGMA fine-pitch standard and the ISO mostly coarse-pitch standard. All of these values are referenced to unit module. Actual values are found by multiplying the table values by the module.



Dimensional features:

c_{BR} = clearance	h_{fBR} = form dedendum	r_{fBR} = fillet radius
e_{BR} = space width	h_{kBR} = working depth	s_{BR} = tooth thickness
h_{aBR} = addendum	h_{tBR} = whole depth	α = profile angle
h_{fBR} = dedendum	p_{BR} = circular pitch	

Figure 1 - AGMA PT basic rack (for m or $m_n = 1$)

Table 2 - Standard basic racks (based on unit module)

Basic rack feature	AGMA PT	ANSI/AGMA 1003-G93 fine-pitch	ISO 53 (1974) coarse pitch
Profile angle, $\alpha^{1)}$	20 deg.	20 deg.	20 deg.
Circular pitch, p_{BR}	3.14160	3.14160	3.14160
Tooth thickness, s_{BR}	1.57080	1.57080	1.57080
Addendum, h_{aBR}	1.00000	1.00000	1.00000
Whole depth, h_{tBR}	2.33000	2.20000 (+0.05000) ²⁾	2.25000
Fillet radius, r_{fBR}	0.43032	0.00000 ³⁾	0.38000
Dedendum, h_{fBR}	1.33000	1.20000 (+0.05000) ²⁾	1.25000
Working depth, h_{kBR}	2.00000	2.00000	2.00000
Clearance, c_{BR}	0.33000	0.20000 (+0.05000) ²⁾	0.25000
Form dedendum, $h_{fFBR}^{4)}$	1.04686	1.20000 (+0.05000) ²⁾	1.05261
Space width, e_{BR}	1.57080	1.57080	1.57080

NOTES:
¹⁾ α is also the pressure angle at reference diameter of spur gear or the normal pressure angle at reference diameter of helical gear.
²⁾ After multiplying the table value by the module, add the value in parentheses.
³⁾ ANSI/AGMA 1003-G93 states: "A zero fillet radius implies a sharp corner on the tip of the generating tool. In actual practice, the corner is made with a small radius."
⁴⁾ h_{fFBR} is the distance from pitch line to start of fillet radius.

4.3 Optional modifications

Certain modifications may be made to these basic racks without deviating from their essential character. When any such change is desired, a description in an appropriate form must be included in the gear specifications.

4.3.1 Profile modification

It is sometimes desirable to make slight modifications to the involute profile of the gear. One such example is tip relief. This is the removal of a small sliver of material along a portion of the involute leading to the tooth tip. Tip relief will often benefit gear action by reducing the abrupt action accompanying load transfer between adjoining teeth, especially when the teeth are deflecting under a heavy load. Such modification to the tooth profile can be introduced by an appropriate modification to the straight-sided flank of the basic rack tooth. When this is desired, a drawing of the modified basic rack is required as part of the gear specifications. An example of such a modification applied to the AGMA PT tooth form is shown in figure 2. Annex D provides equations for calculating the effect of this modification on certain features of the generated tooth.

4.3.2 Tooth-tip diameter modification

It is sometimes necessary to change the outside diameter of external gears or the inside diameter of

internal gears from the diameters defined by the basic rack. Some of these considerations are discussed in 5.8.1 and annex C. Such adjustments do not significantly depart from the standard basic rack and do not require a special basic rack drawing showing the modification. The specification of the outside or inside diameter is, by itself, sufficient to define the modification.

4.4 Comparison to other AGMA basic racks

This new basic rack is similar to the other listed AGMA basic racks. Gears derived from all of them will generally operate together. However, there are differences which, for the new basic rack, may result in plastic gear designs of enhanced load capacity and, particularly in the case of molded plastic gears, also of greater accuracy.

4.4.1 New features

Of all the features compared in table 2, the most significant is the increase in fillet radius. The new radius, corresponding to a full-round radius, is the maximum radius possible while at the same time maintaining a form dedendum value greater than 1.0. This form dedendum greater than 1.0 insures compatibility with the other AGMA basic racks. The remaining new features, that is, the whole depth, dedendum, and clearance, are geometry changes needed to accommodate the larger fillet radius.

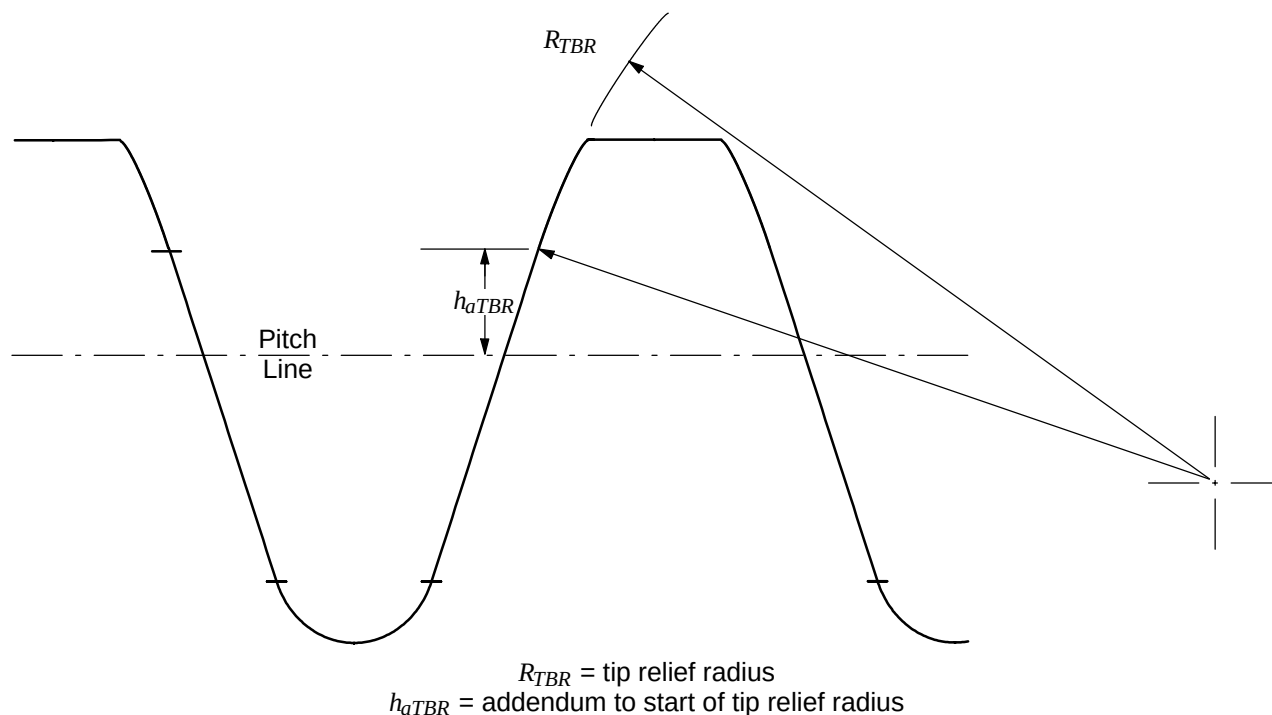


Figure 2 - Example of AGMA PT basic rack modified with tip relief

4.4.2 Generated fillet form and stress concentration

The trochoid-shaped fillet on a generated gear tooth is determined in part by the basic rack fillet radius. This fillet radius is the tip radius of the generating tool as shown in annex A. The larger this basic rack radius, the larger the radii along the trochoid curve and the less sharp the “corner” represented by the fillet.

When load is applied to a gear tooth, it produces maximum bending moments in the fillet area. Stress concentration around relatively sharp fillet corners increases the resulting bending stress. The less sharp this fillet, the lower the maximum local stress and the greater the capacity of the tooth to withstand the applied load. This is especially true under the condition of repeated loads typical of gear applications.

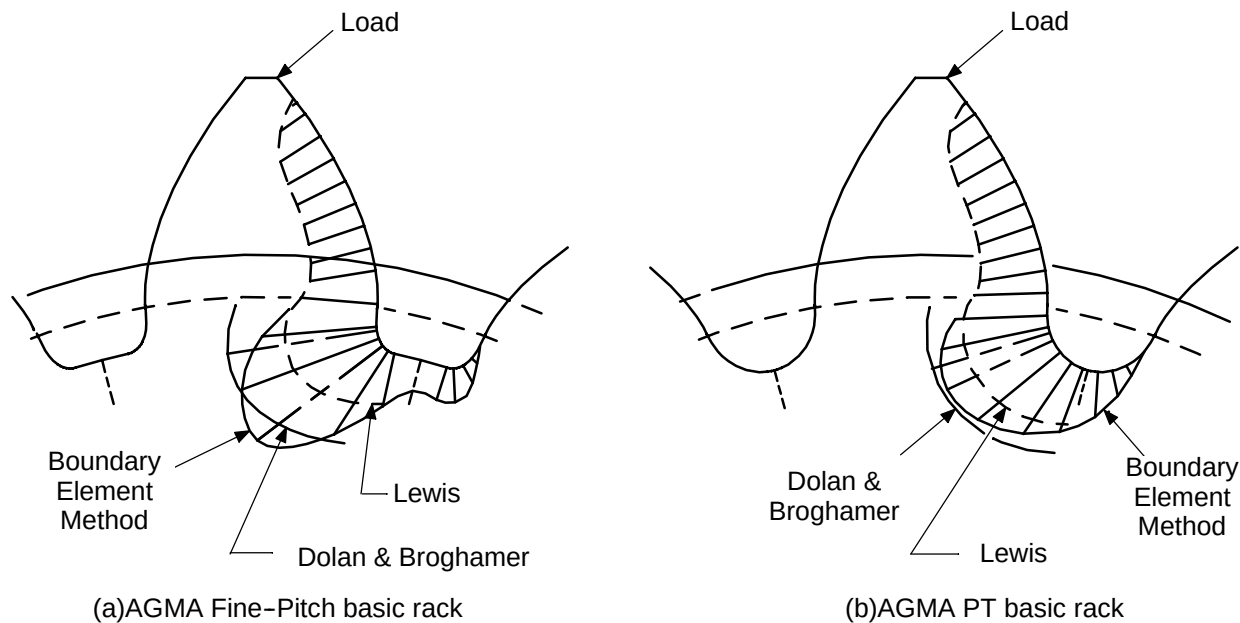
Some features of the new basic rack influence the fillet stresses and, hence, the bending strength of the generated tooth. On one hand, its slightly larger dedendum tends to increase the bending moment at the root of the tooth. However, this slight harmful

effect is generally more than counterbalanced by the combined beneficial effect of the wider tooth base and the reduced stress concentration resulting from the larger fillet radius.

Figure 3, derived from Boundary Element¹⁾ analysis, compares the stress levels at two versions of a pinion fillet, one generated by the AGMA Fine-Pitch basic rack and the other by the AGMA PT basic rack.

This figure compares the two fillets for a 12 tooth pinion enlarged in accordance with ANSI/AGMA 1003-G93, *Tooth Proportions for Fine-Pitch Gears*. For each fillet, the figure shows three plots of induced stress levels at the base of the tooth. The innermost plot, labelled “Lewis”, gives the stress as calculated by the elementary Lewis equation without consideration of stress concentration. The next plot, labelled “Dolan & Broghamer”, adds the effect of stress concentration as commonly estimated in AGMA gear strength calculations. The third plot shows the stresses as determined by the Boundary Element Method¹⁾. In each case, the fillet form determined by the PT Basic Rack shows lower stress levels than that determined by the AGMA Fine-Pitch Basic Rack.

¹⁾ The Boundary Element Analysis was performed within The External Gear Profile Generation Program, 1987, Ohio State University.



Pinion Data: 1.0 module, 12 teeth, 1.95 mm tooth thickness

Figure 3 - Comparison of calculated bending stresses at fillets from AGMA Fine-Pitch and AGMA PT basic racks

4.4.3 Sharp corners in plastic materials

According to the above explanation, the full-radius feature of a basic rack is to be preferred in gears of any material when bending strength is a major factor in gear design. This is widely recognized and has led to the common practice of using maximum fillet radii in basic racks for heavily loaded gears made from a variety of materials. This approach is especially important for gears made from plastic materials. The relatively low strength of plastics as compared to other engineering materials, their relatively low fracture toughness, and their other notch-sensitive properties make plastics particularly susceptible to stress concentrations.

4.4.3.1 Machined plastic gears

Plastic gears are machined from stock materials generally made by an extrusion or casting process. The machined fillet surface with an unfavorable molecular structure can be especially vulnerable to the higher stress at relatively sharp fillets.

When machined plastic gears are used as prototypes, this can cause part failure that will not fairly represent the performance of the molded part.

4.4.3.2 Molded plastic gears

The plastic molding process can create conditions which reduce material strength at the gear tooth

fillets and which are influenced by the size of the fillet radius. These conditions are described below and shown in figure 4:

- Plastics flowing into a mold develop a flow pattern which is mostly determined by the shear stress induced during the flow process. Sudden transitions, such as flow around sharp corners or abrupt velocity change, can produce flow irregularities (resembling but not identical to turbulent flow) near the part surface. These are quickly frozen in place, see figure 4(a). The result is molded-in stresses which reduce tooth strength. Afterwards, the action of time, temperature, or chemical exposure can lead to relief of these stresses with an accompanying distortion in gear geometry.
- With fiber reinforced plastics, there is an added concern from this type of mold-filling flow. With a smooth, continuous flow process, the fibers in the material near the surface of the part align in the flow direction. However, when the flow encounters a relatively sharp corner, the fibers tend to position themselves in a radial pattern pointing to the corner, see figure 4(b). Thus the fibers do not fully reinforce the critical corner, and may introduce potential failure paths. Furthermore, poorly oriented fibers can produce non-uniform shrinkage and poor tooth geometry.

- Fundamental to molding stable, stress-free parts are designs which encourage uniform cooling in the mold. Sharp corners, such as those at minimal radius root fillets, define a region with a constricted path for heat conduction, see figure 4(c). The result is a region which remains hot well after the adjacent material has cooled. This condition is further intensified by a similar heat conduction problem which raises the temperature in the corresponding sharp external corner of the mold. The non-uniform cooling rates in the plastic generate shrinkage forces which develop voids or high localized stresses near the tooth root. Furthermore, these uncontrolled and unstable forces can distort the tooth profile in an unpredictable manner. A maximum radius at the fillet minimizes the temperature differential and

the resulting shrinkage stresses, profile distortion, and surface material strength loss.

- Another kind of molding problem associated with an undercut tooth profile often appears with gears having a limited number of teeth designed with inadequate basic rack offset. A relatively sharp corner at the fillet emphasizes the undercut condition. During shrinking in the mold, the undercut can cause "hang-up" on the corresponding projections in the mold cavity, restricting the free shrinkage in the combined radial and circumferential directions. Figure 4(d) shows the effect of shrinkage on two identical, except for fillet shape, undercut pinions. This "hang-up" condition is clearly shown on the pinion with the sharper corner and not, in this example, on the other.

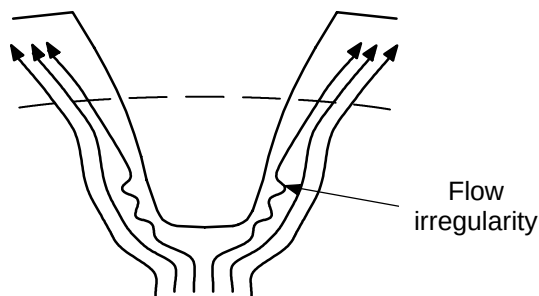


Figure 4(a) - Effect of fillet shape on mold flow

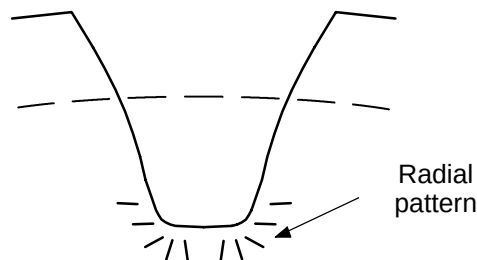


Figure 4(b) - Effect of fillet shape on fiber orientation close to surface (at mid-facewidth location)

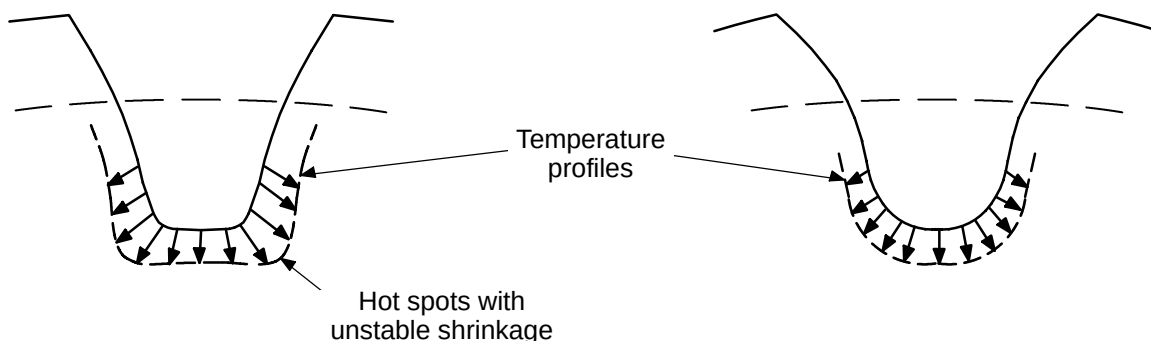
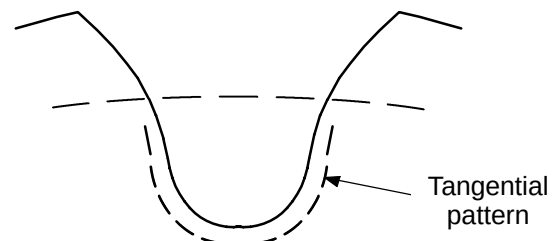


Figure 4(c) - Effect of fillet shape on fillet surface temperature during freezing

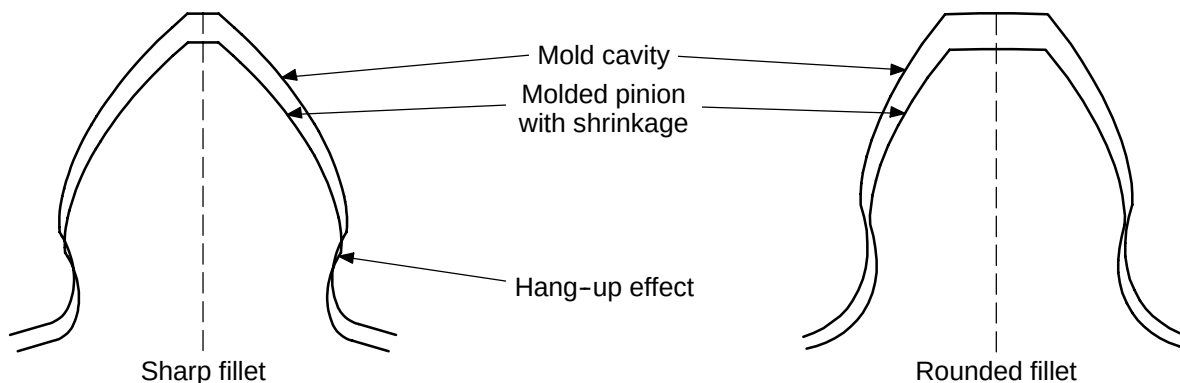


Figure 4(d) - Comparison of shrinkage effect in undercut pinion with sharp and rounded fillets

5 Gear tooth proportions from basic rack data

Almost all of the features defining a spur gear can be derived from the standard basic rack for the specified module, m , and a few items of primary gear data. This is also true for a helical gear, with the specified module of the basic rack becoming the normal module, m_n , of the gear and the profile angle of the basic rack becoming the normal profile angle, α , of the gear. Sample calculations illustrating the use of the equations in this section are supplied in annex G.

5.1 Primary gear data for spur gears

- number of teeth, z ;
- tooth thickness, s .

5.2 Derived gear data for external spur gears

5.2.1 Outside diameter, d_{ae}

Also see 5.8.1.1.

The standard pitch diameter (reference diameter) of the gear, d , is:

$$d = z m \quad \dots(1)$$

where

- z is number of teeth of gear;
- m is module.

Rack shift, y . See A.7.1 and figures A.7 and A.8.

$$y = \frac{s - s_{BR}}{2 \tan \alpha} \quad \dots(2)$$

where

- s is the tooth thickness at the pitch diameter;
- s_{BR} is the tooth thickness of the basic rack;
- α is the profile angle of the basic rack; pressure angle at the pitch diameter of a

spur gear; normal pressure angle at the pitch diameter of a helical gear.

Outside diameter of the external gear, d_{ae} .

$$d_{ae} = d + 2y + 2h_{aBR} \quad \dots(3)$$

where

h_{aBR} is addendum of basic rack.

5.2.2 Root diameter of gear

The root diameter of the gear, d_f , is:

$$d_f = d + 2y - 2h_{fBR} \quad \dots(4)$$

where

h_{fBR} is dedendum of basic rack.

5.2.3 Form diameter, d_F

The base circle diameter of the gear, d_b , is:

$$d_b = d \cos \alpha \quad \dots(5)$$

The form diameter of the gear, d_F , is:

$$d_F = \sqrt{d_b^2 + \frac{[2y + d \sin^2 \alpha - 2h_{fBR}]^2}{\sin^2 \alpha}} \quad \dots(6)$$

where

h_{fBR} is form dedendum of basic rack (distance from pitch line to start of fillet radius).

NOTE: This equation applies only for gears that are not undercut. The condition of no undercut is met if the term in brackets is equal to or greater than zero. The analytical method for finding the form diameter for undercut gears is beyond the scope of this document.

5.2.4 Top land

The involute pressure angle at the outside diameter of a spur gear, α_{ae} , is:

$$\alpha_{ae} = \cos^{-1} \left[\frac{d_b}{d_{ae}} \right] \quad \dots(7)$$

The tooth thickness at the outside diameter of a spur external gear, or top land, s_{ae} , is:

$$s_{ae} = d_{ae} \left[\frac{s}{d} + \text{inv } \alpha - \text{inv } \alpha_{ae} \right] \quad \dots(8)$$

5.2.5 Fillet form

The form of the fillet can be found by graphically reproducing the generating action of the basic rack. The analytical method is beyond the scope of this document. See [2], [3], [4] and [5] in the Bibliography.

5.3 Derived gear data for internal spur gears

5.3.1 Inside diameter

Also see 5.8.1.2.

The inside diameter of an internal gear, d_{ai} , is:

$$d_{ai} = d - 2y - 2h_{aBR} \quad \dots(9)$$

NOTE: The inside diameter should not be smaller than the base circle diameter.

5.3.2 Root diameter

Also see 5.8.1.3.

The root diameter of an internal gear, d_f , is:

$$d_f = d - 2y + 2h_{fBR} \quad \dots(10)$$

5.3.3 Form diameter, d_F

The form diameter of an internal gear depends on the proportions of its generating gear. The analytical method for determining this diameter is beyond the scope of this document.

5.3.4 Inside tip land

The involute pressure angle at the inside diameter of a spur internal gear, α_{ai} , is:

$$\alpha_{ai} = \cos^{-1} \left[\frac{d_b}{d_{ai}} \right] \quad \dots(11)$$

The tooth thickness at the inside diameter of a spur internal gear, or inside tip land, s_{ai} , is:

$$s_{ai} = d_{ai} \left[\frac{s}{d} - \text{inv } \alpha + \text{inv } \alpha_{ai} \right] \quad \dots(12)$$

5.3.5 Fillet form

The form of the fillet of an internal gear depends in part on the proportions of its generating gear. The exact form can be found by graphically reproducing the generating action. This form can often be closely approximated by a single circular arc. The analytical methods for finding the exact form and its approximation are beyond the scope of this document.

5.4 Primary gear data for helical gears

- number of teeth, z ;
- normal tooth thickness, s_n ;
- helix angle, β .

5.5 Derived gear data for external helical gears

5.5.1 Outside diameter

Also see 5.8.1.1.

The standard pitch diameter (reference diameter), d , of the gear is:

$$d = \frac{z m_n}{\cos \beta} \quad \dots(13)$$

where

m_n is normal module;

β is helix angle at the standard pitch diameter.

Rack shift, y .

$$y = \frac{s_n - s_{BR}}{2 \tan \alpha} \quad \dots(14)$$

where

s_n is normal tooth thickness at the standard pitch diameter.

The outside diameter of the external gear, d_{ae} , is:

$$d_{ae} = d + 2y + 2h_{aBR} \quad \dots(15)$$

5.5.2 Root diameter

The root diameter of the gear, d_f , is:

$$d_f = d + 2y - 2h_{fBR} \quad \dots(16)$$

5.5.3 Form diameter

The transverse pressure angle at the standard pitch diameter of a helical gear, α_t , is:

$$\alpha_t = \tan^{-1} \left[\frac{\tan \alpha}{\cos \beta} \right] \quad \dots(17)$$

The base circle diameter of the gear, d_b , is:

$$d_b = d \cos \alpha_t \quad \dots(18)$$

The form diameter of the gear, d_F , is (see note at equation 6):

$$d_F = \sqrt{d_b^2 + \frac{[2y + d \sin^2 \alpha_t - 2h_{fBR}]^2}{\sin^2 \alpha_t}} \quad \dots(19)$$

5.5.4 Normal top land

The transverse pressure angle at the outside diameter of a helical gear, α_{tae} , is:

$$\alpha_{tae} = \cos^{-1} \left[\frac{d_b}{d_{ae}} \right] \quad \dots(20)$$

The transverse top land tooth thickness at the outside diameter, s_{tae} , is:

$$s_{tae} = d_{ae} \left[\frac{s_n}{d \cos \beta} + \text{inv } \alpha_t - \text{inv } \alpha_{tae} \right] \quad \dots(21)$$

The helix angle at the outside diameter of the gear, β_{ae} , is:

$$\beta_{ae} = \tan^{-1} \left[\frac{d_{ae} \tan \beta}{d} \right] \quad \dots(22)$$

The normal tooth thickness at the outside diameter of a helical external gear, or normal top land, s_{nae} , is:

$$s_{nae} = s_{tae} \cos \beta_{ae} \quad \dots(23)$$

5.5.5 Transverse fillet form

The form of the fillet in the transverse plane can be found by graphically reproducing the generating action of the transverse section of the helical basic rack. The analytical method is beyond the scope of this document.

5.6 Derived gear data for internal helical gears

5.6.1 Inside diameter, d_{ai}

See 5.3.1.

5.6.2 Root diameter, d_f

See 5.3.2.

5.6.3 Form diameter, d_F

See 5.3.3.

5.6.4 Normal inside tip land

The transverse pressure angle at the inside diameter of a helical gear, α_{tai} , is:

$$\alpha_{tai} = \cos^{-1} \left[\frac{d_b}{d_{ai}} \right] \quad \dots(24)$$

The transverse inside tip land at the inside diameter, s_{tai} , is:

$$s_{tai} = d_{ai} \left[\frac{s_n}{d \cos \beta} - \text{inv } \alpha_t + \text{inv } \alpha_{tai} \right] \quad \dots(25)$$

The helix angle at the inside diameter of the gear, β_{ai} , is:

$$\beta_{ai} = \tan^{-1} \left[\frac{d_{ai} \tan \beta}{d} \right] \quad \dots(26)$$

The normal tooth thickness at the inside diameter of a helical internal gear, or normal inside tip land, s_{nai} , is:

$$s_{nai} = s_{tai} \cos \beta_{ai} \quad \dots(27)$$

5.6.5 Fillet form

See 5.3.5.

5.7 Gear data derived from basic rack with tip relief modification

The generated gear tooth outline is changed by the type of basic rack tip relief modification described in 4.3.1 and illustrated in figure 2. Annex D gives the method for calculating the location and size of the outline change caused by such a modified basic rack.

5.8 Gear data not derived from basic rack

There are some details of the gear tooth outline that are not always derived from the basic rack. These require their own separate specifications and are not to be considered as changes in the basic rack.

5.8.1 Diameter modifications

5.8.1.1 Outside diameter

There are some design reasons for making the outside diameter of a gear slightly different from the value determined by the design tooth thickness and the basic rack. For example, a small increase in outside diameter might significantly improve the gear mesh contact ratio without causing interference at the root of the mating gear. Also, a small decrease might be necessary to permit part assembly through a restricted opening. Sometimes a substantial decrease is necessary in the case of pinions with small numbers of teeth and enlarged tooth thicknesses. In these pinions, the outside diameter determined by the basic rack would correspond to top lands that are too small, or even non-existent. This condition is discussed more fully in C.2.

5.8.1.2 Inside diameter

There are also design reasons for making the inside diameters of internal gears different from the basic rack defined values. Some are similar in nature to those that apply to outside diameters. A very important additional reason relates to the possibility of interference between the tooth tips of the mating pinion and the tooth tips of the internal gear. This interference would appear at the teeth adjacent to those in full engagement, either during radial assembly of the pinion into the internal gear or, in some cases, even when rotation is attempted in a fully assembled gear set. This condition is most likely when the pinion and gear are not sufficiently different in numbers of teeth. An increase in the inside diameter is often enough to eliminate the interference.

5.8.1.3 Root diameter

A change in root diameter from the value defined by the design tooth thickness and the basic rack is generally considered as a change in the basic rack. The one possible exception arises in some internal gears. In this exception, at the root (major) diameter calculated using the basic rack data, the width of the space between the teeth is too small or nonexistent. This condition can generally be corrected by a slight reduction in root diameter. The root clearance in the mesh will also be reduced, but generally not enough to require a change in the outside diameter of the mating pinion.

5.8.2 Tip relief

The tip relief, or other involute profile modification, is often not defined by specifying modifications to the basic rack. Instead, a separate profile specification is used. See ANSI/AGMA 2000-A88, *Gear Classification and Inspection Handbook*.

5.8.3 Tip rounding

The round at the tip of the teeth is independent of the basic rack and requires a separate specification. This is usually in the form of a maximum permitted tip radius. An overly generous radius, introduced into molded gears by the cavity manufacturing process, can significantly reduce the effective engagement in very small module gears.

Tooth outline features introduced by tip rounding on external gears are shown in figure 5 and described below:

- The top land, s_{ae} , is reduced to the “remaining top land”, s_{aeR} ;
- The outside diameter, d_{ae} , is replaced by the “effective outside diameter”, d_{aeE} , in determining the extent of engagement with a mating gear;
- The tooth thickness at the outermost part of the involute profile is increased to the “effective top land”, s_{aeE} .

Equations for the calculation of these outline features are provided below. These relatively simple equations give approximate results sufficiently accurate for typical applications of tip rounding.

5.8.3.1 Remaining top land for spur gears

See 5.2.4 for s_{ae} and α_{ae} .

The remaining top land with tip rounding, s_{aeR} , is:

$$s_{aeR} \approx s_{ae} - 2r_T \tan [0.5 (90^\circ - \alpha_{ae})] \quad \dots(28)$$

where

r_T is tip radius.

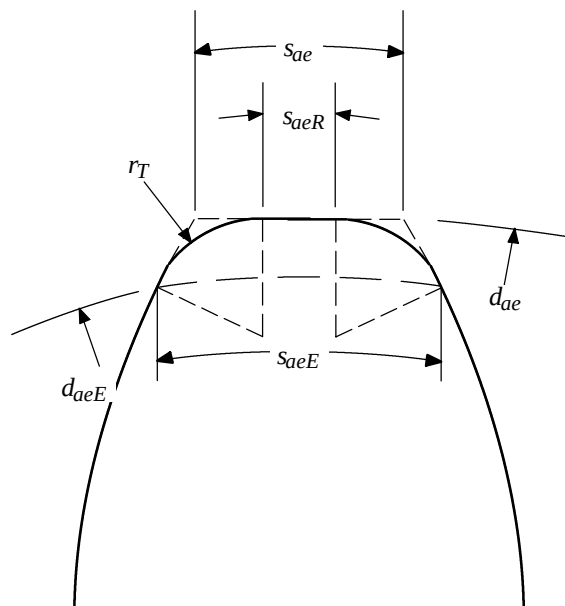


Figure 5 - Tooth outline features introduced by tip rounding on external gears

5.8.3.2 Effective outside diameter for spur gears

The effective outside diameter for spur gears, d_{aeE} , is:

$$d_{aeE} \approx d_{ae} - 2r_T (1 - \sin \alpha_{ae}) \quad \dots(29)$$

5.8.3.3 Effective top land for spur gears

The effective top land for spur gears, s_{aeE} , is:

$$s_{aeE} \approx s_{aeR} + 2r_T \cos \alpha_{ae} \quad \dots(30)$$

5.8.3.4 Remaining top land for helical gears

See 5.5.4 for α_{tae} , β_{ae} and s_{nae} .

The normal pressure angle at the outside diameter of a helical external gear, α_{nae} , is:

$$\alpha_{nae} = \tan^{-1} [\tan \alpha_{tae} \cos \beta_{ae}] \quad \dots(31)$$

The remaining normal top land with tip rounding, s_{naeR} , is:

$$s_{naeR} \approx s_{nae} - 2r_T \tan [0.5(90^\circ - \alpha_{nae})] \quad \dots(32)$$

5.8.3.5 Effective outside diameter for helical gears

The effective outside diameter for helical gears, d_{aeE} , is:

$$d_{aeE} \approx d_{ae} - 2r_T (1 - \sin \alpha_{nae}) \quad \dots(33)$$

5.8.3.6 Effective normal top land for helical gears

The effective normal top land for helical gears, s_{naeE} , is:

$$s_{naeE} \approx s_{naeR} + 2r_T \cos \alpha_{nae} \quad \dots(34)$$