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Manufacturing Method of Pinion Member of Large-Sized Skew Bevel Gears Using Multi-Axis Control and Multi-Tasking Machine Tool

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

In this paper, a manufacturing method of the pinion member of large-sized skew bevel gears using a multi-axis control and multi-tasking machine tool respecting an existing gear member is proposed. First, the tooth surface forms of skew bevel gears are modeled. Next, the real tooth surfaces of the gear member are measured using a coordinate measuring machine and the deviations between the real and theoretical tooth surface forms are formalized using polynomial equations. It is possible to analyze the tooth contact pattern of the skew bevel gears with respect to the deviations of the real and theoretical tooth surface forms by expressing the deviations as polynomial equations. Further, the deviations of the tooth surface form of the gear member are reflected in the analysis of the tooth contact pattern and transmission errors, and the tooth surface form of the pinion member that has good performance mating with the existing gear member is determined. Finally, the pinion member is manufactured by swarf cutting using a multi-axis control and multi-tasking machine tool. Afterward, the real tooth surfaces of the manufactured pinion member are measured using a coordinate measuring machine and the tooth surface form errors are detected. In addition, the tooth contact pattern of the manufactured pinion member and existing gear member is compared with those of tooth contact analysis. The results show that there is good agreement.

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Introduction

Bevel gears are used to transmit power and motion between the intersecting axes of the two shafts, and are most often mounted on shafts that are 90 degrees apart. They may have straight, Zerol, spiral, and skew teeth [1] [2] [3] [4], and occupy an important place in gear transmissions [5].

The transmission of straight bevel gears is regarded as a particular case of skew bevel gears [6]. The contact ratio of skew gears is larger than that of straight bevel gears because skew bevel gears have oblique teeth. Such skew bevel gears are used at the power generation plants when the gears have large size. In recent years, the renovation of these plants has been active due to the age of the plants. At the same time, it has become necessary to replace the skew bevel gears in the plants. In this situation, there are cases where only the pinion member is changed. It then becomes necessary to manufacture a pinion member that has good performance mating with the existing gear member.

It is now possible to machine the complicated tooth surface due to the development of multi-axis control and multi-tasking machine tools [7] [8]. Therefore, high precision machining of large-sized skew bevel gears has been expected.

In this paper, a manufacturing method of the pinion member of the large-sized skew bevel gears using multi-axis control and multi-tasking machine tool respecting an existing gear member is proposed. The manufacturing method has the advantages of arbitrary modification of the tooth surface and machining of the part without the tooth surface [9].

First, the tooth surface forms of skew bevel gears are modeled mathematically. Next, the real tooth surfaces of the gear member are measured using a coordinate measuring machine (CMM) and the deviations between the real and theoretical tooth surface forms are formalized using the measured coordinates. It is possible to analyze the tooth contact pattern and transmission errors of the skew bevel gears with consideration of the deviations of the real and theoretical tooth surface forms by expressing the deviations as polynomial equations. The components of the deviations of tooth surface forms which correspond to the distortions of heat treatment and lapping, etc., are used because the motion concept may be implemented on the multi-tasking machine. Further, the deviations of the tooth surface forms of the gear member can be reflected in the analysis of the tooth contact pattern and transmission errors, and the tooth surface form of the pinion member that has good performance mating with the existing gear member is determined. Finally, the pinion member is manufactured by a swarf cutting that is machined using the side of the end mill using a multi-axis control and multi-tasking machine tool. Afterward, the real tooth surfaces of the manufactured pinion member were measured using a CMM and the tooth surface form errors were detected. Although the tooth surface form errors were large relatively on the coast side, those were small on the drive side. In addition, the tooth contact pattern of the manufactured pinion member and the provided original gear member was compared with the results from tooth contact analysis and there was good agreement.

Tooth surfaces of skew bevel gears

In this section, the tooth surface forms of skew bevel gears are modeled mathematically. In general, the geometry of the skew bevel gears is achieved by considering the complementary crown gear as the theoretical generating tool. Therefore, first the tooth surface form of the complementary crown gear is considered.

The number of teeth of the complementary crown gear is represented by:

$$z_c = \frac{z_p}{\sin \lambda_{p0}} = \frac{z_g}{\sin \lambda_{g0}} \quad (1)$$

where

- z_c is number of teeth of complementary crown gear;
- z_p is number of teeth of the pinion;
- z_g is number of teeth of the gear;
- λ_{p0} is pitch cone angles of the pinion;
- λ_{g0} is pitch cone angles of the gear.

Figure 1 shows the tooth surface form of the complementary crown gear assuming to be straight bevel gears with depth-wise tooth taper. O - xyz is the coordinate system fixed to the crown gear and z axis is the crown gear axis of rotation. Point P is a reference point at which tooth surfaces mesh with each other and is defined in the center of tooth surface. The circular arcs with large radii of curvatures are defined both in xz and xy planes. xz and xy planes correspond to the sections of the tooth profile and tooth trace of the tooth surface, respectively. This curved surface is defined as the tooth surface of the complementary crown gear. The following equations yield considering the relations between ρ_c , Δc , and M_n in xz , and between ρ_s , Δs , and b in xy planes, respectively [10].

Since skew bevel gears have teeth that are straight and oblique, the skew bevel gears have the skew angle as described in Figure 2. Therefore, the complementary crown gear also has the skew angle that is defined as β . The tooth surface of the complementary crown gear is expressed in O - xyz using ρ_c and ρ_s :

$$\rho_c = \frac{\Delta c^2 + \left(\frac{M_n}{\cos \alpha} \right)^2}{2 \Delta c} \quad (2)$$

$$\rho_s = \frac{\Delta s^2 + \frac{b^2}{4}}{2 \Delta s}$$

where

- ρ_c is radius of the curvature of the circular arcs in the xz plane, and has influence on Δc ;
- Δc is amount of tooth profile modification;
- M_n is the normal module;
- α is pressure angle;
- ρ_s is radius of curvature of the circular arcs in the xy plane, and influence on Δs ;
- Δs is amount of tooth profile crowning;
- b is face width.

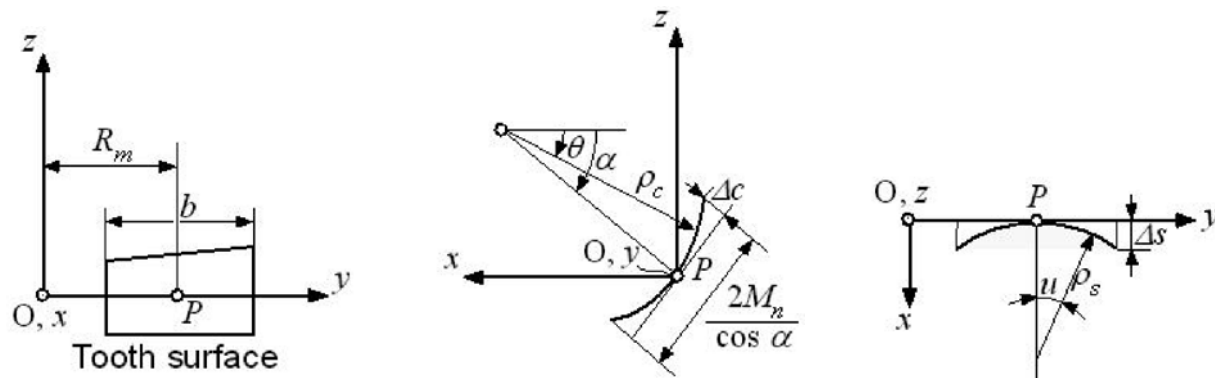


Figure 1. Tooth surface form of complementary crown gear

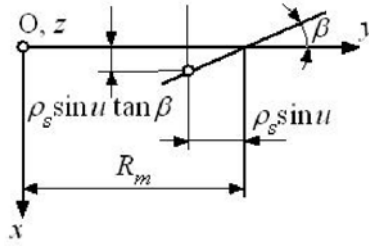


Figure 2. Skew angle of complementary crown gear

$$X(u, \theta) = \begin{bmatrix} -\rho_c (\cos \theta - \cos \alpha) - \rho_s (1 - \cos u) + \rho_s \sin u \tan \beta \\ \rho_s \sin u + R_m \\ \rho_c (\sin \alpha - \sin \theta) \end{bmatrix} \quad (3)$$

where

- X is position vector of tooth surface of complementary crown gear in $O-xyz$;
- u is parameter which represents curved lines;
- θ is parameter which represents curved lines;
- R_m is mean cone distance.

The unit normal of X is expressed by N .

X expresses the equation of the tooth surface of the complementary crown gear. The complementary crown gear is rotated about z axis by angle ψ and generates the tooth surface of the skew bevel gear. This rotation angle, ψ , of the crown gear, is the generating angle. When the generating angle is ψ , X and N are rewritten as X_ψ and N_ψ in $O-x_\psi y_\psi z_\psi$ assuming that the coordinate system $O-xyz$ is rotated about z axis by ψ in the coordinate system $O-x_\psi y_\psi z_\psi$ fixed in space. When ψ is zero, $O-x_\psi y_\psi z_\psi$ coincides with $O-xyz$.

Assuming the relative velocity $W(X_\psi)$ between the crown gear and the generated gear at the moment when generating angle is ψ , the equation of meshing between the two gears is as follows [11] [12]:

$$N_\psi(u, \theta; \psi) W(u, \theta; \psi) = 0 \quad (4)$$

where

- N_ψ is unit normal vector of X_ψ in $O-x_\psi y_\psi z_\psi$;
- X_ψ is position vector of tooth surface of complementary crown gear in $O-x_\psi y_\psi z_\psi$;
- ψ is parameter representing rotation angle of complementary crown gear about z axis;

From equation 4, we have $\theta = \theta(u, \psi)$. Substituting $\theta(u, \psi)$ into X_ψ and N_ψ , any point on the tooth surface of the crown gear and its unit normal are defined by a combination of (u, ψ) , respectively. When the tooth surface of the complementary crown gear in $O-x_\psi y_\psi z_\psi$ is transformed into the coordinate system fixed to the generated gear, the tooth surface of the skew bevel gear is expressed. The tooth surfaces of the pinion and gear are expressed as x_p and x_g , respectively. Moreover, the unit normals of x_p and x_g are expressed as n_p and n_g , respectively. Henceforth, the subscripts "p" and "g" indicate that each is related to the pinion and gear, respectively.

Measurement of gear member

Manufacturing errors occur in bevel gear cutting. Whether the mathematical model as mentioned earlier fits the real tooth surface of the existing gear member or not is not obvious. Therefore, the tooth surfaces of the gear member are measured using a CMM and the deviations between the real and theoretical tooth surface forms are formalized.

Coordinate measurement of real tooth surface

The theoretical tooth surfaces of the gear member is expressed as $x_g(u_g, \psi_g)$ as mentioned earlier. A grid of n lines and m columns is defined and a point called the reference point is specified on the tooth surfaces of both