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New Methods for the Calculation of the Load Capacity of Bevel and Hypoid Gears

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

A failure mode called “flank breakage” is increasingly observed in different applications of cylindrical and bevel gears. These breakages typically start from the active flank approximately in the middle of the active tooth height and propagate to the tooth root of the unloaded flank side. Crack initiation can be localized below the surface in the region between case and core of surface hardened gears. This failure mode can neither be explained by the known mechanism of tooth root breakage nor by the mechanism of pitting. Even bevel gears in truck and bus applications are at the risk to suffer from subsurface fatigue, if the optimum utilization of the material should be achieved. In this case a balance between the flank breakage and pitting risk has to be found. The purpose of this paper is to describe a new material, physically based calculation method to evaluate the risk of flank breakage versus the risk of pitting. The verification of this new method by experimental tests is exemplarily shown.

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Introduction

A failure mode called “flank breakage” is increasingly observed in different applications of cylindrical and bevel gears. These breakages typically start from the active flank approximately in the middle of the active tooth height and propagate to the tooth root of the unloaded flank side.

Crack initiation can be localized below the surface in the region between case and core of surface hardened gears. This failure mode can neither be explained by the known mechanism of tooth root breakage nor by the mechanism of pitting. Even bevel gears in truck and bus applications are at the risk to suffer from subsurface fatigue, if the optimum utilization of the material should be achieved. In this case a balance between the flank breakage and pitting risk has to be found. The purpose of this paper is to describe a new material-physically based calculation method to evaluate the risk of flank breakage versus the risk of pitting. The method was used to improve the design of a gear set that failed in several cases by flank breakage on the wheel in test vehicles of MAN Truck & Bus (MTB). In Figure 1 a typically damaged tooth on the wheel is shown. In some cases pitting occurred on the coast flanks of the wheel (see Figure 1 right). It was possible to increase the load capacity of the wheel regarding pitting and flank breakage by means of the new method. Previous test runs approved this method.

Flank breakage in bevel gears

Flank breakage often appears without any other surface failures such as scuffing, pitting or micropitting. In some cases only one tooth is affected but usually more than one tooth fail as shown in Figure 2. Non-adequate material properties and heat treatment were expected to increase the risk of flank breakage, especially too low core strength and toughness, or too low or too high case depths [4] [5]. In many American publications flank breakage is also called “subsurface fatigue” or “subcase fatigue”. In these papers the flank pressure is also regarded as the decisive parameter [2] [6] [13] [15] [16].

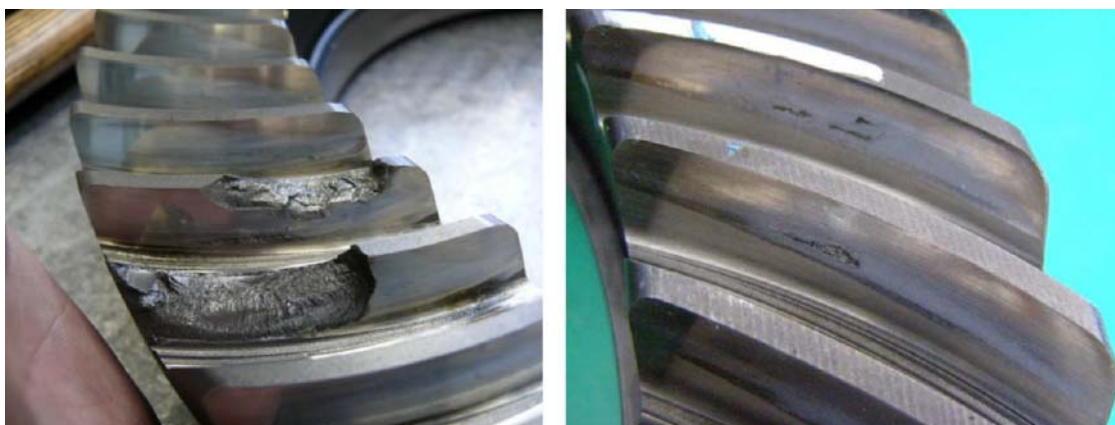


Figure 1. Flank breakage and pitting on the wheel



Figure 2. Flank breakage on two different wheels

Annast [1] investigated on basis of systematic tests the influence parameters on flank breakage on bevel gears. Annast identifies besides the known influence of the load conditions especially the case depth and the core hardness as an important parameter. Analysis of damage patterns of test and practical gears showed that the initiating crack always started below the surface in the region of the transition from case to core. For unidirectional loading the crack propagates to the active flank on the one side and to the tooth root on the other side. Annast analyzed the subsurface stresses with ROSLCOR [9] [17] by using the shear stress intensity hypothesis according to Tobie [18]. Oster [17] defined on basis of [21] a feasibility to consider compressive stresses in the case for the evaluation of the material exposure. Tobie expanded this method with the possibility to regard also tensile residual stresses (in the core). However the magisterial influence of tensile residual stresses pretended by the model according to Tobie, could not be approved by any other investigation (e.g., ANSI/AGMA 2003-B97). Therefore, Annast argued that it is sufficient to analyze only the section between the flank surface and the transient region of case and core. Tensile residual stresses were therefore not examined by Annast. In his proposed standard capable method only the region of the transition is regarded. A critical ratio between the acting maximum shear stress and the core hardness was derived out of the test gears and practical applications. If the ratio exceeds the limit the risk of flank breakage is considered as high.

Because of the uncertainties in the methods described above a new calculation method was proposed by Wirth [20] for the rating of bevel and hypoid gears. The method is based on Oster's and Hertter's [8] work. Hertter proposed for cylindrical gears an enhanced shear stress intensity hypothesis to evaluate the subsurface stresses even under consideration of compressive and tensile residual stresses. A separated examination of dynamic and static exposure allows to regard fatigue failures as well as failures from yielding. Because the local allowable strength values in the considered material element are derived from the hardness by means of material-physical relations the new method is named "material-physically based calculation method". Wirth transferred Hertter's method to bevel and hypoid gears considering their specifics as for example the sliding conditions.

Material-physically based calculation method for bevel and hypoid gears

The material-physically based calculation method allows the consideration of complex stress conditions beneath the flank surface that are caused by the load and the heat treatment process. The permissible stresses are derived from the hardness values and material-physical parameters. The occurring stresses are compared with the permissible stresses in discrete sections in the material. On basis of a shear stress intensity hypothesis (SIH) the material exposure is determined.

General stress conditions in the tooth

Inside the tooth, beneath an ideal smooth flank surface, the total stress conditions are composed of:

- stresses due to normal contact load (Hertzian theory);
- shear stresses on the surface caused by friction;

- thermal stresses caused by the thermal gradient;
- stresses caused by bending mechanism;
- residual stresses.

Figure 3 illustrates the stress components that influence the material exposure in a considered (infinite small) element. In Figure 3a the stress components that result from the normal load on the flank are shown. The stresses according to the Hertzian theory are arising out of the normal force. Due to the sliding components the friction force that is tangential to the flank surface induces shear stresses. Figure 3b demonstrates the effect of bending by a normal force that acts above the considered element (above in profile direction). The components of the normal force cause normal stresses with an approximately linear distribution over the tooth thickness and shear stresses with an approximately parabolic distribution and a maximum in the middle of the tooth. Residual stresses result from the hardening and finishing process. As an example, [20] shows compressive stresses are occurring in the case and are balanced by tensile stresses in the core. Unlike the stresses in Figure 3a and Figure 3b the residual stresses in Figure 3c are load independent.

Oster and Hertter developed the program system "STRORHR" for the calculation of all mentioned stress components on cylindrical gears. With this program it is possible to examine the material exposure in the subsurface below any contact point on the flank surface. Thereby the examination direction is perpendicular to the flank surface.

Stress conditions in the rolling contact

In any contact point on the flank the rolling direction x can also be seen as the time axis. Figure 4 shows in principle the stress components under the surface. All volume elements in the same depth are exposed to equal stresses, but at different times. To evaluate the material exposure in a certain depth beneath the flank surface the corresponding stresses have to be regarded over the whole time axis (x -axis).

However in rolling contacts a turning principal coordinate system complicates the evaluation of the material utilization. A possibility to analyze the dynamic stresses in rolling contacts are the shear stress courses in a sectional plane that is defined on the surface of the base sphere according to Figure 5.

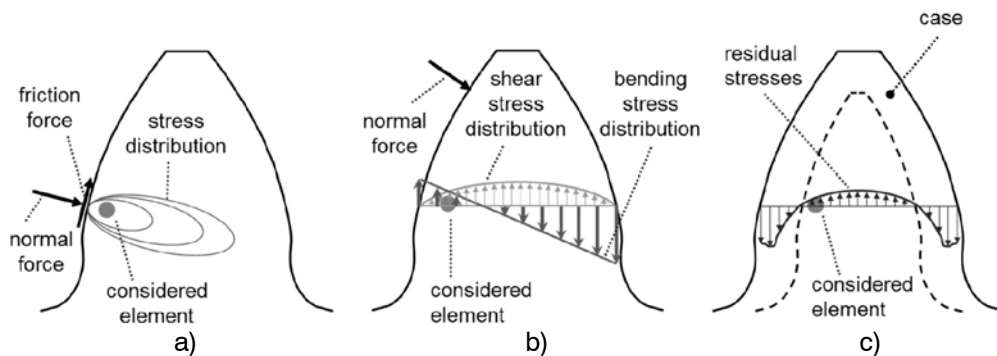


Figure 3. Stress conditions inside the tooth

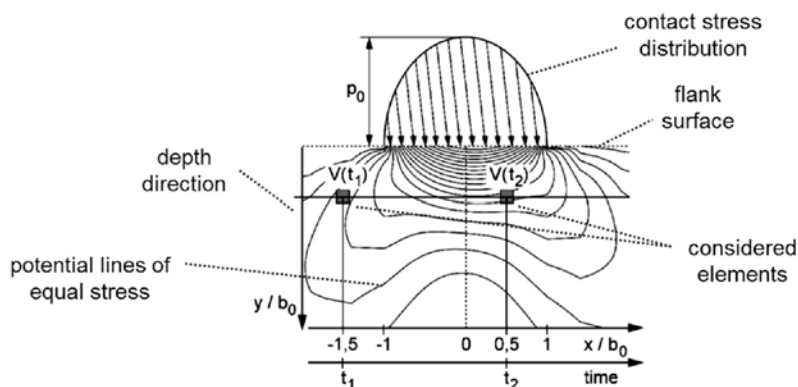


Figure 4. Time-dependent stress components in a rolling contact

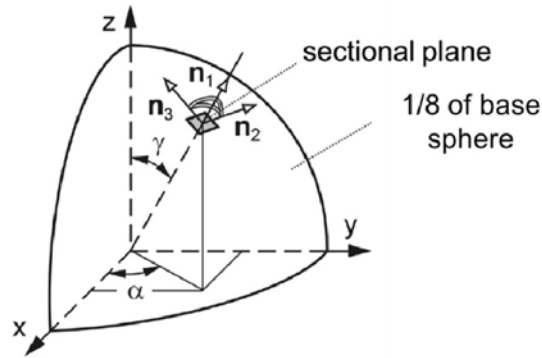


Figure 5. Base sphere with sectional plane [8]

Figure 6 shows for a rolling contact an example of shear stress courses in a certain sectional plane $\gamma\alpha$ and a material depth of $y/b_0 = 0.3$. $\tau_{\gamma\alpha}(t_i)$ is the time dependent graph for the projection of shear stresses in the directions n_2 and n_3 (see Figure 5). In Figure 6a, no residual stresses are considered. As a consequence the point (0/0) is part of the course. At a certain time t_i , when the contact is still unloaded (e.g., the contact point of the flank surfaces is still far away from the regarded volume element) $\tau_2 = \tau_3 = 0$ in the examined sectional plane. During the movement of the contact over the flank surface the stress components τ_2 and τ_3 can be marked in the diagram. Of course if the influence of the moving contact point on the stresses at the examined plane is fading out the course will again reach the point (0/0). As Figure 6a shows, the instantaneous stress vector (τ_2/τ_3) is completely turning during one load cycle what means that it acts as an alternating load. Its maximum length is defined as the “maximum shear stress” $\tau_{\max,a}$, the diameter of the circumscribed circle is $\Delta\tau_{\max,a}$.

Figure 6b shows for the same examined sectional plane an equal load cycle but in consideration of the residual stresses. Unlike before the shear stresses τ_2 and τ_3 have discrete values even if the contact is unloaded. However the course of the pair of values τ_2/τ_3 is similar, what means that $\Delta\tau_{\max,a} = \Delta\tau_{\max,b}$. An important fact is, that the maximum shear stress $\tau_{\max,b}$ is decreasing under the influence of (compressive) residual stresses ($\tau_{\max,b} < \tau_{\max,a}$). In other words the maximum shear stress with superposed compressive residual stresses is smaller than the maximum shear stress without residual stresses. This is also valid for other sectional planes of the base sphere and corresponds with the well-known fact, that compressive residual stresses reduce the maximum material exposure (e.g., [22]).

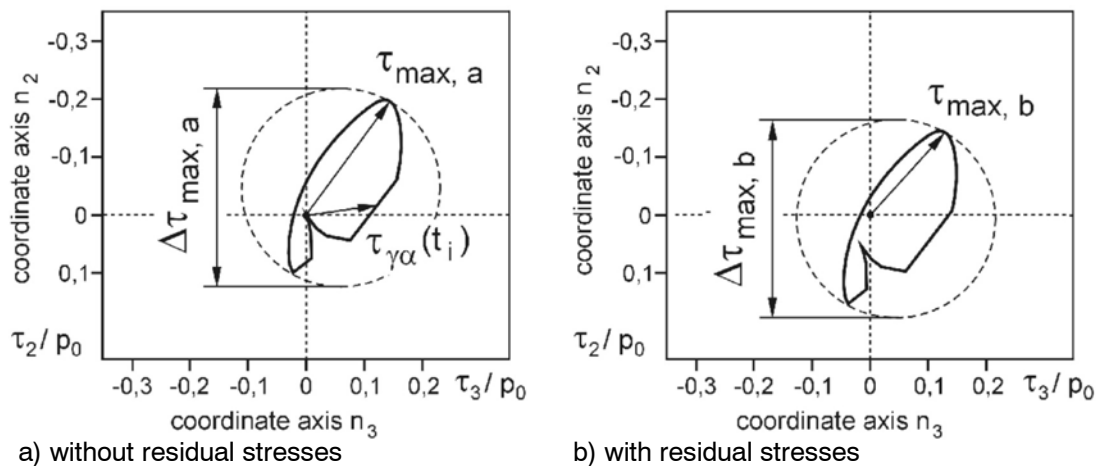


Figure 6. Example of shear stress courses in a sectional plane $\alpha\gamma$ in the depth $y/b_0 = 0.3$ for rolling conditions [8]

The aim of strength hypotheses is the evaluation of the stresses occurring in the examined base sphere (or base element) to get a number for the material exposure that correlates well with the failure mechanism. Many common criteria are not applicable for alternating stresses as described before. Various important hypotheses are discussed to their applicability for the rating of the material exposure in rolling contacts. As a result of the investigation a modified shear stress hypothesis SIH was established [8]. The hypothesis can principally be used for:

- rating the maximum exposure of the material (analysis regarding yielding);
- rating the dynamic exposure of the material (analysis regarding fatigue).

New modified shear stress hypothesis by Hertter [8]

The base for the calculation of the decisive exposure in the examined base sphere are the stress courses in all sectional planes of the base sphere. According to [12], in every sectional plane the normal stresses (orthogonal to sectional plane) can be calculated according equations 3 and 4. The shear stresses $\tau_{\gamma\alpha a}$ and $\tau_{\gamma\alpha m}$ are defined as shown in Figure 7 for the considered sectional plane. The radius of the smallest circumcircle of the stress course is the decisive amplitude of the shear stress. The vector of its center point represents the mean shear stress.

Amplitude of shear stress, according to Figure 7

$$\tau_{\gamma\alpha a} = \tau_a(\gamma, \alpha) \quad (1)$$

Amplitude of shear stress, according to Figure 7

$$\tau_{\gamma\alpha m} = \tau_m(\gamma, \alpha) \quad (2)$$

Amplitude of normal stress, $\sigma_{\gamma\alpha a} = \sigma_a(\gamma, \alpha)$

$$\sigma_a(\gamma, \alpha) = \frac{\sigma_{\max}(\gamma, \alpha) - \sigma_{\min}(\gamma, \alpha)}{2} \quad (3)$$

Mean value of normal stress, $\sigma_{\gamma\alpha m} = \sigma_m(\gamma, \alpha)$

$$\sigma_m(\gamma, \alpha) = \frac{\sigma_{\max}(\gamma, \alpha) + \sigma_{\min}(\gamma, \alpha)}{2} \quad (4)$$

Dynamic exposure in sectional plane, $A(\gamma, \alpha)$

$$A(\gamma, \alpha) = \sqrt[1]{\frac{\alpha \tau_a^1 (1 + m \tau_m^1) + b \sigma_a^1}{\sigma_A^1}} \quad (5)$$

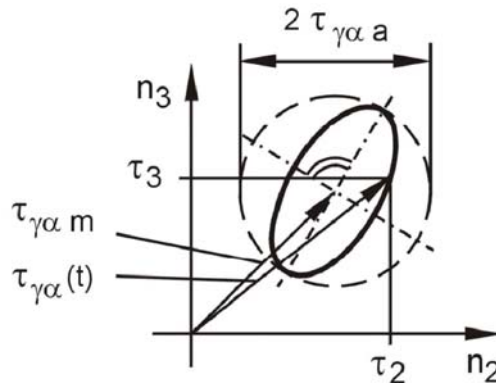


Figure 7. Definition of amplitude and mean value of any shear stress curve plotted over time in a discrete sectional plane [8]