



AGMA Technical Paper

Practical Considerations for the Use of Double Flank Testing for the Manufacturing Control of Gearing

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

The gearing industry has developed many unique measuring techniques for the production control of their products. Each technique has inherent advantages and limitations which should be considered by designers and manufacturers when selecting their use. Double flank composite inspection, (DFCI) is one such technique that can functionally provide quality control results of test gears quickly and easily during manufacturing. The successful use of DFCI requires careful planning from product design, through master gear design and gage control methods in order to achieve the desired result in an application.

This document explains the practical considerations in the use of double flank testing for the manufacturing control of spur, helical, and crossed axis helical gearing including:

- a general description of double flank inspection equipment including an explanation of what can be measured.
- recommendations on practical master gear design.
- the calculation of tight mesh center distance and test radius limits.
- the resulting backlash that can be anticipated in gear meshes based on applying double flank tolerances in a design.
- initial and ongoing statistical techniques in double flank testing and how they can be practically used to improve gear quality.
- double flank gage measurement system analysis including case studies of gage repeatability and reproducibility (R&R) and uncertainty analysis.

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Description of double flank composite inspection

Double flank testing is a technique that has been used in the gear industry to identify potential manufacturing defects in the design intent of the gear. It is a practical, fast and effective screening tool that can identify when the gear manufacturing process has deviated from an ideal condition that could result in a loss of conjugate action, a change in backlash or an unwanted noise in a gear mesh.

The test itself involves an apparatus of general layout as shown in Figure 1 and of actual configuration shown in Figure 2. A master gear of known precision is mounted on a fixed base with only rotational freedom. The test gear is mounted on a floating slide mechanism which allows rotation of the test gear, and movement along an axis between the line of centers of the master and test gear. A spring (with a preset force) pushes the floating slide resulting in zero backlash double flank contact (i.e., on both left and right flanks) on each of the test gear and the master gear.

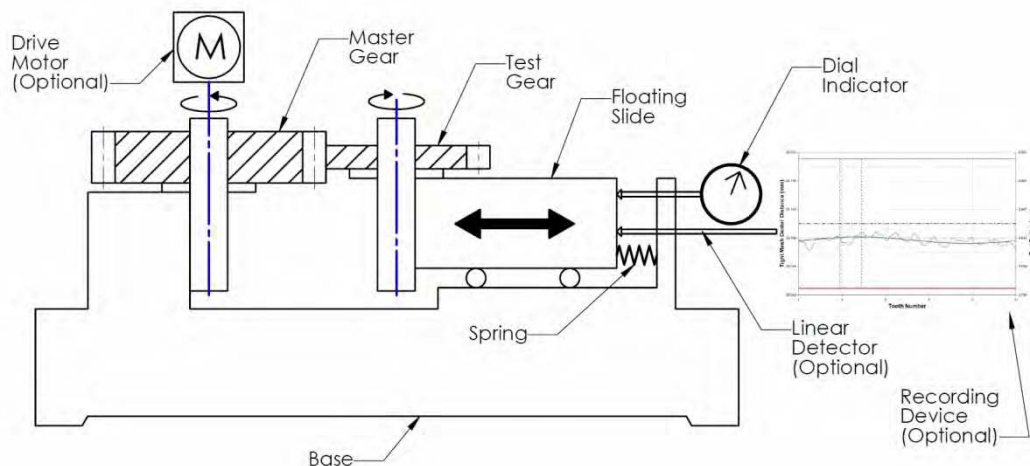


Figure 1. General arrangement of a double flank composite tester



Figure 2. An actual double flank composite tester in tight mesh
(Courtesy of Web Gear Services Ltd.)

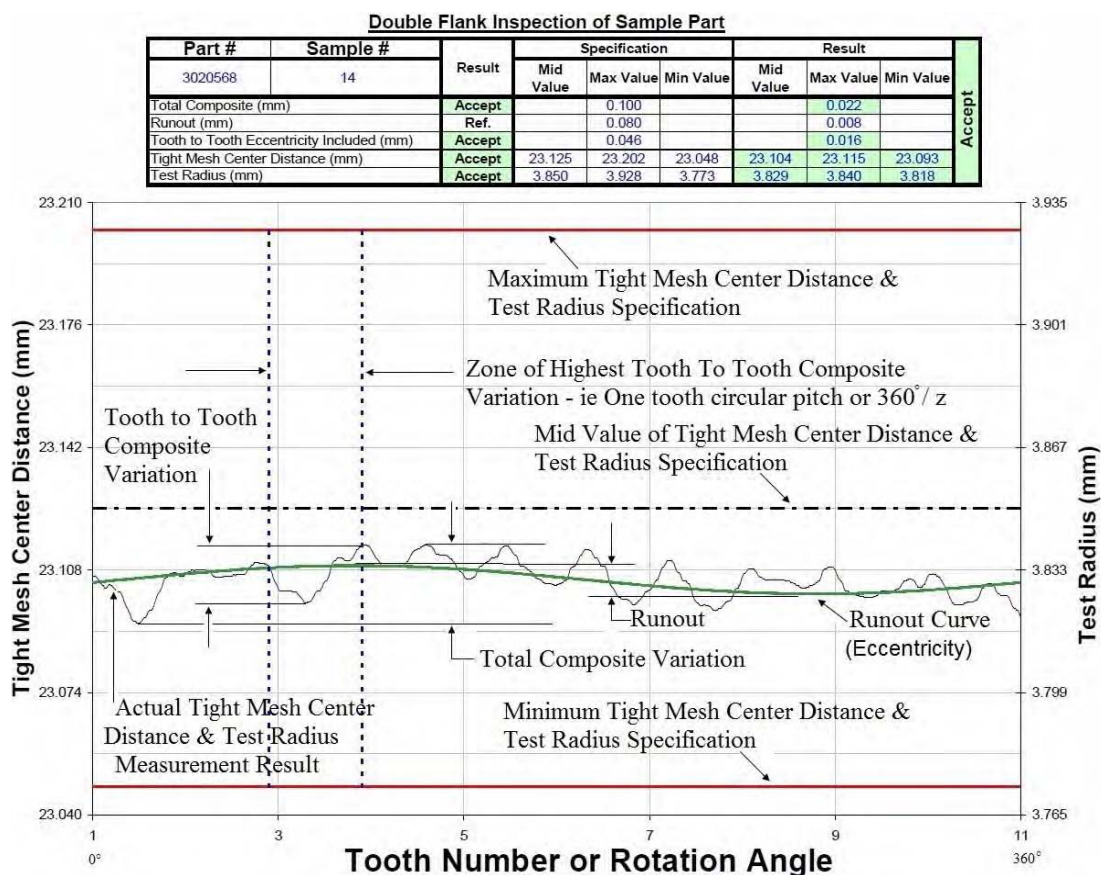
As the master gear is rotated (by hand or by motor), the test gear follows. Involute theory dictates that perfectly formed teeth will result in no movement of the floating slide between the line of centers. However, since no gear can be manufactured in absolutely perfect condition, there will always be some movement of the floating slide as the gears are rotated. The magnitude of this movement is measured with either a mechanical indicator or electronic detector that contacts the slide mechanism. If the measuring instrument is calibrated to an actual distance reading between the centers of the gears, then an actual tight mesh center distance result can be obtained.

In order to maintain accuracy in the measurement, intimate double flank contact must be maintained at all times. Therefore the selection of the preset spring force and the speed of rotation of the gears should be given careful consideration to limit measurement errors.

The preset force may need to be selected specifically for the test gear's design taking into account the material's ability to resist deformation under the load (i.e., plastic gears) where a large preset force may distort the gear into conformity. In addition, if there is excessive resistance coming from the mounting of either the master gear on its mandrel, or more commonly of the test gear on its mandrel, then a low preset spring force will result in separation of the two gears out of double flank contact creating an error in the measured values. The correct preset spring force is the minimum force needed to maintain continuous double flank contact without distorting the test gear.

The speed of rotation of the gears should be selected taking into account the natural response of the mechanical and electrical (if so equipped) elements of the tester. Generally, it is recommended that at least 20 data points per tooth are available in the data set collected to ensure sufficient sensitivity of the results.

The types of measurements that can be made on a double flank tester are shown in Figure 3 and are explained in the following sections.



Total composite variation

Measurement of the total composite variation (error) is the difference between the maximum and minimum indicator (or linear detector) readings during one rotation cycle of the test gear as shown in Figure 3. The total composite variation result includes effects of runout in the gear, plus anomalies in the tooth pitches, profiles, and helix and reports the total effect in terms of this linear change as variation in tight mesh center distance. It is not possible to accurately establish the magnitude of each of these individual effects on the total composite variation using the double flank test alone. Hence, the double flank test is very good at screening production quality and flagging potential errors, but the results, may not identify the specific nature of the problem. Other tests, (such as analytical inspections), would need to be performed in order to more closely identify the exact nature of the defect.

Tooth-to-tooth composite variation

The tooth-to-tooth composite variation (error) is defined as the greatest deviation in indicator reading within a single circular tooth pitch. This result is based on the worst tooth on the entire gear. The gear tested in Figure 3 shows this to be in the zone around tooth 3.

As the number of teeth in a gear becomes smaller, the ratio of the tooth-to-tooth error to total composite error generally increases. In the extreme condition, a single start worm (i.e., 1 tooth) will have a tooth-to-tooth composite error equivalent to its total composite error. As the number of teeth increases, the tooth-to-tooth results are considered to be a better indicator of anomalies in the tooth pitch, profile and helix.

Errors in gear pressure angle will result in a repeated pattern of arches similar to that shown between teeth 5 and 7 in Figure 3. Use of tooth-to-tooth test limits also helps to control burrs and nicks in gears which are not always detected by analytical measurement techniques.

Tight mesh center distance

One of the most powerful uses of the double flank test is to measure and control gears not only for total composite variation, but also for tight mesh center distance. The ability to measure this variation gives the necessary insight to control both tooth size and composite parameters simultaneously. Since AGMA and ISO accuracy standards do not include the effect of tooth size, tight mesh center distance is not discussed in those standards. However, the effectiveness of this tooth size measurement should not be overlooked when evaluating backlash in a gear mesh.



Figure 4. Calibration for tight mesh center distance
(Courtesy of Web Gear Services Ltd.)

Tight mesh center distance as shown in Figure 3 can be measured if an additional calibration step is performed during the gage set-up. If the dial indicator or detector is calibrated to a known center distance reading between the spindles prior to measurement as shown in Figure 4, then the actual tight mesh center distance will be the difference between the calibrated value and the rolling variation

Maximum and minimum test limits must be established for tight mesh center distance. These limits are shown by the horizontal red lines on the chart in Figure 3. Every portion of the actual tight mesh center distance measurement must be within the minimum and maximum boundaries for a test gear to be acceptable. When properly calculated through the gear design process, adherence to these boundaries will ensure maximum and minimum operational backlash levels in the gear mesh.

The calculation of tight mesh center distance limits for spur, helical, crossed axis helical and worm gears

In order to establish the maximum and minimum tight mesh center distance limits for external or internal spur, helical, crossed axis helical and worm gears the following gear design data will need to be available;

m_n	normal module of the system, mm;
z_w	number of teeth on the test gear;
NOTE:	For external gears, use a positive value for z_w and for internal gears, use a negative value.
z_3	number of teeth on the master gear;
β_w	helix angle of the test gear, degrees or radians;
β_3	helix angle of the master gear, degrees or radians;
NOTE:	For spur gears $\beta_w = \beta_3 = 0$, degrees or radians
NOTE:	For right hand helical gears, worms, and worm gears use a positive value for the helix angle. For left hand helical gears, worms, and worm gears use a negative value for the helix angle.
α_n	normal pressure angle for the mesh, degrees or radians;
$s_{nw \max}$	maximum normal circular tooth thickness of the test gear, mm;
$s_{nw \min}$	minimum normal circular tooth thickness of the test gear, mm;
s_{n3}	normal circular tooth thickness of the master gear, mm;
F_{idTw}	total composite tolerance for the test gear, mm.

The calculation procedure that follows is sufficiently general to account for gears with non-standard tooth thicknesses and heavily modified profiles.

Step 1. Calculation of the standard center distance, a

The standard center distance, a , of an external or internal test gear when meshed with an external master gear on a double flank tester is:

$$a = \frac{z_w}{|z_w|} \frac{m_n}{2} \left[\frac{z_w}{\cos \beta_w} + \frac{z_3}{\cos \beta_3} \right] \quad (1)$$

where

a is the standard center distance between the test gear and the master gear, mm.

NOTE: These equations are sufficiently general to account for external or internal spur, helical, crossed axis helical and worm gears

Parallel axis double flank tight mesh center distance limits

The following additional steps are needed for the calculation of tight mesh center distance test limits for external and internal parallel axis spur and helical gear meshes.

Step 2. Calculation of the transverse pressure angle, α_t

The transverse pressure angle, α_t , for the mesh on the double flank tester is:

$$\alpha_t = \tan^{-1} \left(\frac{\tan \alpha_n}{\cos \beta_w} \right) \quad (2)$$

where

α_t is the transverse pressure angle for the mesh in degrees or radians

NOTE: For spur meshes $\alpha_t = \alpha_n$

Step 3. Calculation of the maximum tight mesh center distance limit, $a_{d \max}$

The maximum tight mesh center distance, $a_{d \max}$, of the test gear with the master gear for a spur and parallel axis helical double flank mesh is:

$$a_{d \max} = \frac{a \cos \alpha_t}{\cos \left\{ \text{inv}^{-1} \left[\text{inv} \alpha_t - \frac{z_w}{|z_w|} \left(\frac{\pi m_n - s_{n3} - s_{nw \max}}{2 a \cos \beta_w} \right) \right] \right\}} + \frac{z_w}{|z_w|} \frac{F_{idTw}}{2} \quad (3)$$

where

$a_{d \max}$ is the maximum tight mesh center distance of the test gear with the master gear, mm;

$\text{inv } \phi$ is the involute function and $\text{inv } \phi = \tan \phi - \phi$ with ϕ expressed, radians.

$\text{inv}^{-1} x$ is the inverse involute function where $x = \text{inv } \phi = \tan \phi - \phi$. Therefore, the result of the function $\text{inv}^{-1} x = \phi$, where ϕ is an angle. For more information on the calculation of this function, see AGMA 930-A05, Annex E [1].

Step 4. Calculation of the minimum tight mesh center distance limit, $a_{d \min}$

The minimum tight mesh center distance, $a_{d \min}$, of the test gear with the master gear for a spur and parallel axis helical double flank mesh is:

$$a_{d \min} = \frac{a \cos \alpha_t}{\cos \left\{ \text{inv}^{-1} \left[\text{inv} \alpha_t - \frac{z_w}{|z_w|} \left(\frac{\pi m_n - s_{n3} - s_{nw \min}}{2 a \cos \beta_w} \right) \right] \right\}} - \frac{z_w}{|z_w|} \frac{F_{idTw}}{2} \quad (4)$$

where

$a_{d \min}$ is the minimum tight mesh center distance of the test gear with the master gear, mm.

NOTE: For internal gears, equation 3 will actually give a minimum value result and equation 4 will give the maximum value result.

When specifying tight mesh center distance limits, it is important to also include a definition of the master gear's number of teeth and normal circular tooth thickness upon which the tight mesh center distance limits are based.

Crossed axis helical and worm gear double flank tight mesh center distance limits

The calculation for crossed axis and worm gear double flank meshes differs from other cylindrical gear meshes because the gears see each other in a way which is analogous to two racks in mesh as opposed to two involute gears in mesh. Crossed axis helical gears include the case where the driving member is a master worm as shown in Figure 5 used to measure a helical gear at right angles. The calculations presented here are also sufficiently general to include the case where two helical gears mesh at shaft angles other than ninety degrees as well as the case in Figure 6 where a plastic test worm is meshed against a master spur gear. In the case of worm gears, the master gear would actually be a cylindrical worm mounted at a right angle to the worm gear.

NOTE: The formulas presented here allow for meshing on the double flank tester at any shaft angle.

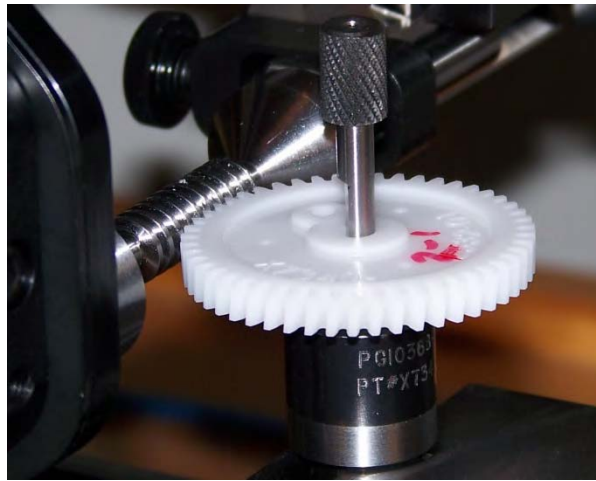


Figure 5. Master worm in double flank mesh with a plastic helical gear
(Courtesy of Web Gear Services Ltd.)



Figure 6. Plastic test worm in double flank mesh with a master spur gear at an offset shaft angle
(Courtesy of Web Gear Services Ltd.)

Step 2. Calculation of the meshing shaft angle on the double flank tester, ψ

The shaft angle, ψ , on the double flank tester for a given crossed axis helical gear or worm gear mesh is calculated as follows:

$$\psi = \beta_w + \beta_3 \quad (5)$$

where

ψ is the meshing shaft angle on the double flank tester, degrees or radians.

NOTE: Careful adherence to the sign of each of the helix angles (i.e., right and left hand) is crucial in this calculation.

Step 3. Calculation of the maximum tight mesh center distance limit, $a_{d \max}$

The maximum tight mesh center distance, $a_{d \max}$, of the test gear with the master gear for a crossed axis helical or worm gear double flank mesh is:

$$a_{d \max} = \frac{(s_{n3} + s_{nw \max} - \pi m_n)}{2 \tan \alpha_n} + a + \frac{F_{idTw}}{2} \quad (6)$$

Step 4. Calculation of the minimum tight mesh center distance limit, $a_{d \min}$

The minimum tight mesh center distance, $a_{d \min}$, of the test gear with the master gear for a spur and parallel axis helical double flank mesh is:

$$a_{d \min} = \frac{(s_{n3} + s_{nw \min} - \pi m_n)}{2 \tan \alpha_n} + a - \frac{F_{idTw}}{2} \quad (7)$$

Some software programs incorrectly calculate tight mesh center distance for crossed axis helical gears and worm gears using the parallel axis approach in the previous section instead of this method. If the sum of the normal circular tooth thicknesses between the master gear and test gear are close to the normal pitch, the calculation procedure detailed in the previous section may present results that are close to the actual values, however as the sum of these tooth thicknesses deviates from the normal pitch, the calculation error becomes increasingly significant. The method shown in this section is always preferred for crossed axis helical and worm gears.

Test radius

Test radius can be measured on a double flank tester as is demonstrated in Figure 3. Test radius is similar to tight mesh center distance in terms of setup and calibration. However it differs in that it is calculated as the tight mesh center distance of the mesh minus the test radius of the master gear as shown in the following equation:

$$R_{rw} = a_d - \frac{z_w}{|z_w|} R_{r3} \quad (8)$$

where

- R_{rw} is the instantaneous test radius of the external or internal test gear (i.e., working gear), mm;
- a_d is the instantaneous tight mesh center distance of the mesh on the double flank tester, mm;
- R_{r3} is the test radius of the master gear as seen by a rack (see next section for further explanation), mm.

As a result, the scale between the left side vertical axis in Figure 3 and the right side vertical axis is shifted by the magnitude of the master gear test radius.

One of the reasons why test radius is specified instead of tight mesh center distance is due to the common misconception that the master gear's number of teeth and its normal circular tooth thickness have no influence on the limits of a test gear's test radius. The assumption is that regardless of the master used, the test radius limits of a test gear are constant. If this statement were true, then it would obviously be an advantage in circumstances where a manufacturer may have a different master gear than the purchaser. However, in reality, careful analysis of the equations 1 through 8 shows that there is some difference in the test radius results depending on the master gear's number of teeth and normal circular tooth thickness. An illustrative example is shown in Table 1 where master gears A and B have different numbers of teeth and normal circular tooth thicknesses resulting in significantly different test radius limits on the same test gear.

Table 1. Numerical example of the effect of master gear tooth circular tooth thickness and number of teeth on the test radius

	Master gear A	Test gear	Master gear B
Module, mm	1.0	1.0	1.0
Number of teeth	38	20	50
Pressure angle, degrees	20	20	20
Total composite tolerance μm	-	96	-
Normal circular tooth thickness, mm	50% of circular pitch 1.5708 $\pm$ 0.000 mm	40% of circular pitch 1.2566 $\pm$ 0.020 mm	60% of circular pitch 1.8850 $\pm$ 0.000 mm
Test radius limits, mm	9.539 $\pm$ 0.080		
		9.568 $\pm$ 0.075	

Hence, there is no practical advantage in specifying test radius instead of tight mesh center distance. In both cases, the master gear's number of teeth and normal circular tooth thicknesses must be defined to make the specification valid. It is common to report either tight mesh center distance or test radius but not necessarily both. Tight mesh center distance has greater international usage as compared to test radius. Most North American electronic versions of double flank testers available will report tight mesh center distance and test radius, however European or Asian equipment often do not include test radius results with their equipment.

Test radius of the master gear, R_{r3}

In order to calculate the result in equation 8, the test radius of the master gear, R_{r3} , must be determined. Unfortunately, there are several methods by which the test radius of a master gear is defined, all having potentially different results, creating even more confusion in the industry.

The practical issue in the definition is that the test radius may be different depending on whether the master gear is defined by its action against a rack or itself (see Figure 7), or against another cylindrical gear. Furthermore, if using a definition based on its action against another cylindrical gear, a single master gear may have many different test radius values depending on the cylindrical gear it mates with. Hence, based on equation 8 the test radius of a test gear will change depending on the test radius of the master. Therefore if a master gear has an ambiguous test radius definition, the part gear test radius will also be inherently ambiguous. Specifying tight mesh center distance as opposed to test radius will remove this ambiguity.

However, if test radius must be used, the most common practice to avoid ambiguity is to define the test radius of a master gear by the tight mesh radial distance between the master gear center and the pitch line of a mating rack whose pitch line is defined as the location where the tooth thickness is equal to its space width. In making such a standardized definition, a master gear will have a single test radius regardless of the gear it mates with.

The equation for the test radius of a master gear, R_{r3} , as seen by a rack is as follows:

$$R_{r3} = \frac{m_n z_3}{2 \cos \beta_3} + \frac{(s_{n3} - 0.5 \pi m_n)}{2 \tan \alpha_n} \quad (9)$$

where

R_{r3} is the test radius of the master gear as seen by a rack, mm.

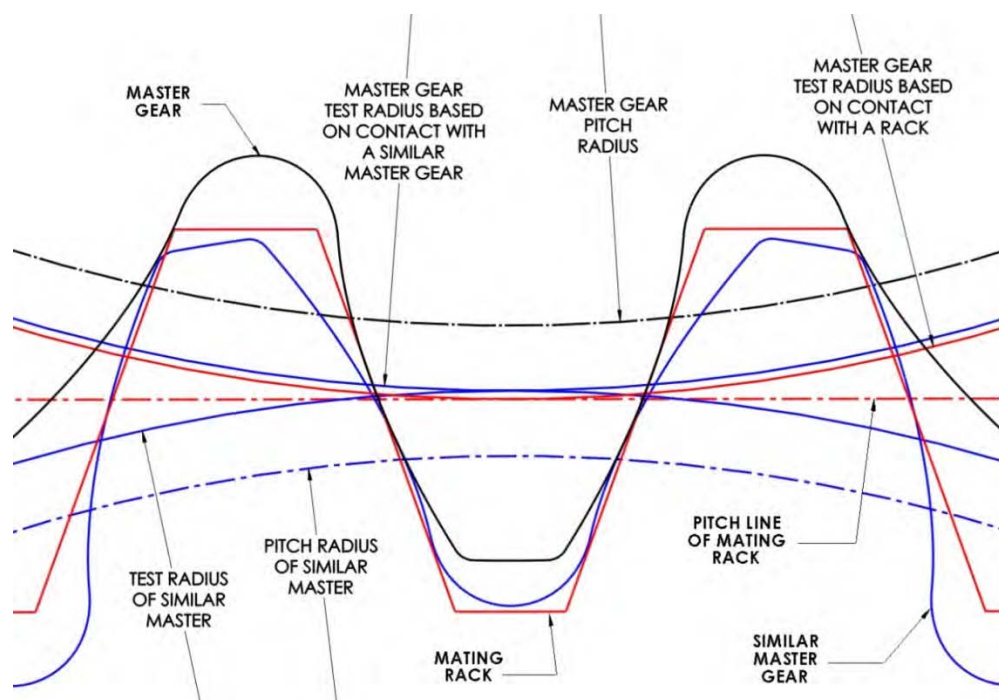


Figure 7. Test radius of a master gear (in black) against a rack (in red) or a similar master (in blue)