



## **AGMA Technical Paper**

# **Mathematical Modeling for the Design of Spiroid®, Helical, Spiral Bevel and Worm Gears**

By Dr. G. Kazkaz, Gearometry,  
Inc.

# Mathematical Modeling for the Design of Spiroid®, Helical, Spiral Bevel and Worm Gears

Dr. Ghaffar Kazkaz, Gearometry, Inc.

[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

## Abstract

Spiroid® and worm gears have superior advantages for high torque and miniaturization applications, and for this reason, they are particularly preferred in aerospace, robotic and medical applications. They are typically manufactured by hobbing technology, a process that has had an overall lead time of 4 to 14 weeks.

Besides the relatively lengthy lead time, and despite the fact that the tooth profile is defined through its pressure angles, 3D drawings of the gears cannot be produced. This is due to the difficulty of capturing the entire curvature of the gear face from the outside to the inside diameter. Because of this difficulty, 3D quality control and FEA analysis under load are difficult and must be accomplished through classical analysis that incorporate pinion bending stresses, gear tooth shear stresses and compressive stresses between pinion and gear teeth. Due to some of these challenges, these gears have maintained a niche status with the industry.

This paper will present a novel work for Spiroid® and worm gears that mathematically calculates the gear tooth profile in terms of the geometry of the cutting tool (hob) and the machining setup. Because of their similarity, the work was also expanded to spiral bevel gears. We have developed software to plot the gear tooth when the parameters of the geometry of the tool and machining setup are entered. The gear tooth shape can then be altered and optimized by manipulating the input parameters until a desired tooth profile is produced. In effect, the result will be designing the hob and machining setup for best gear tooth profile on the computer. Afterward, the generated gear tooth data are entered into CAD software to generate a true 3D model of the gear. The tool path will also be generated from the data for CNC machining beside hobbing.

This mathematical modeling allows for direct CNC machining rather than hobbing and may reduce the prototype lead time from weeks to hours. The pinion can be designed in a similar process and its tooth can be graphed inside the gear's groove, showing the contact points and the clearance between the two surfaces. This novel work has already resulted in the invention of a new gear type combining Spiroid® and worm gear in a single gear driven by the same pinion. This provides significant increase in torque capability.

Mathematical modeling is presented in this paper as a tool design to reduce the lead time and cost for designing Spiroid®, worm and spiral bevel gears. Mathematical modeling is based on mathematically calculating the 3D gear tooth profile in terms of the cutting tool (hob) geometry, the machining setup and the gear size (inside and outside diameters). Software has been developed to allow designers to enter values of these parameters and observe the resulting 3D gear tooth profile. The designer can observe the resulting tooth profile on the computer and adjust the input parameter values to obtain the desired profile. The software also generates the profile in numerical xyz points which is necessary to produce 3D drawings of the gear, conduct direct quality control and perform FEA analysis under load. It was also demonstrated that mathematical modeling can be a tool for gear innovation. It has already resulted in the invention of a new gear type, Spiroid®/worm hybrid that combines a double Spiroid® gear and a worm gear in one. It more than doubled the gear torque capability in comparison to a single Spiroid® gear with a minimal increase in size or weight.

Copyright © 2014

American Gear Manufacturers Association  
1001 N. Fairfax Street, Suite 500  
Alexandria, Virginia 22314

October 2014

ISBN: 978-1-61481-103-9

# Mathematical Modeling for the Design of Spiroid®<sup>1</sup>, Helical, Spiral Bevel and Worm Gears

Dr. Ghaffar Kazkaz, Gearometry, Inc.

## Spiroid® gears

Oliver Sari invented Spiroid® gears in 1954 while working for ITW and the division ITW Spiroid® was created. The gear set consists of a gear and a helical pinion assembled as shown in Figure 1. It is similar to spiral bevel gear set, except the pinion axis is moved a certain distance off the gear axis. This distance is called offset or center distance. It makes it possible to hold the pinion shaft on both ends to increase leverage and stability. Spiroid® gears are special gears with a wide range of RPM ratio from as little as 4 to as many as a few hundred. This facilitates single stage designs that reduce size and increase efficiency. They can also have a high contact ratio which makes them suitable for high torque and low noise requirement. They are used in robotics, aerospace, medical and industrial applications.

There are three Spiroid® gear types shown in Figure 2. They are: the flat face, the skewed angle with cylindrical pinion and the skewed angle with tapered pinion.

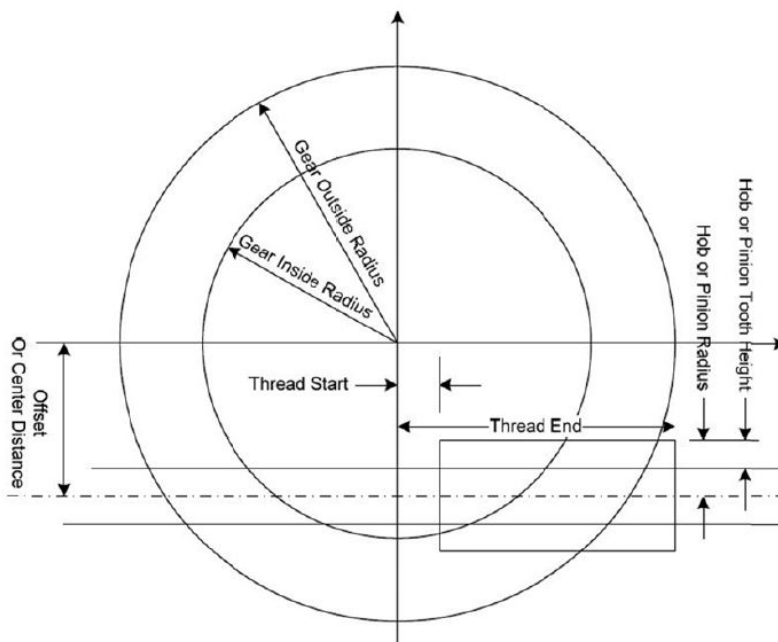


Figure 1. Spiroid® gear/pinion assembly or gear/hob setup



Figure 2. The three types of Spiroid® gear

<sup>1</sup>) Spiroid is a registered trademark of ITW. The views expressed herein are those of the author alone and do not necessarily reflect the views of ITW.

The pinion can be manufactured by any method for making helical gears, including: grinding wheel, shaping or CNC milling but the grinding wheel is the dominant method. The gear is manufactured in a hobbing process using a cutting tool called hob. The hob geometry is similar to the pinion geometry, except its tooth sides in the axial direction are straight as shown in Figure 3. It has a certain number of gashes in the direction perpendicular to the helix that are sharpened for cutting. During machining the gear and the hob will be rotating with an RPM ratio equals to the ratio of their teeth numbers.

A cylindrical hob and a tapered hob are shown in Figure 4. There are a total of nine basic geometry parameters. They are: outside diameter, tooth height, number of teeth (starts), lead (axial pitch), high side pressure angle, low pressure angle, tooth width on top in axial direction and the start and the end of its tooth along the shaft with respect to centerline. The taper angle is an additional parameter for the tapered hob. Other parameters that have to be taken into account when designing a hob are: gear inside radius, gear outside radius, gear number of teeth and the center distance (offset) between the gear axis and the hob axis. Also in the skew angle case the skew angle is another additional parameter. This large number of parameters, 13 - 15, makes it hard to predict the gear tooth shape and profile.

Designing a hob for a new gear set has the potential to be a trial and error process. The design objective is to have a gear tooth that is free of gouging on its sides or clipping of its top. It also should have the desired width and a balance of pressure angles and land area at the root and at the top. After selecting the geometry, the hob is manufactured by an outside machine shop with a lead time of eight to ten weeks. Then the gear is cut in-house. It can be difficult during this process to realize the precise design objectives of the desired gear tooth geometry.

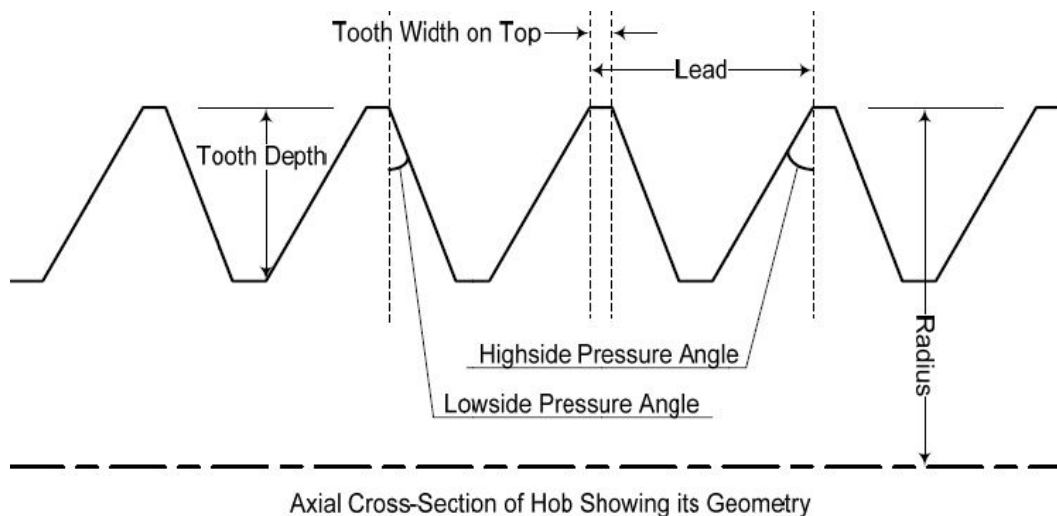


Figure 3. Axial cross-section of a cylindrical hob



Figure 4. Cylindrical and tapered hobs

Initially the hob is placed over the gear blank as shown in the Figure 1. During machining the gear and the hob will be rotating. Machining will be complete when the hob penetrates a distance equal to the tooth height. In the case of the skew angle gear, the hob axis will have an incline angle over the gear plane. After cutting a certain number of gears, the hob becomes dull. The hob will be sharpened and its diameter will be reduced by a few thousands of an inch each time. This will have some effect on the gear tooth profile and its contact with the pinion.

In spite of their torque capability advantages, Spiroid® gears have traditionally been a niche industry. Because of the lead time and costs associated with making the hob, their use is sometimes reserved for cases where high torque is a must. Many manufactured gears are smaller than 5 inches in diameter. Since the full-length tooth profile of the manufactured gear remains unknown, true 3D models of the gear cannot be produced, and it can be difficult to determine FEA and tooth strength analysis under load outside of classical analytical methods.

## Mathematical modeling solution

Gearometry has developed math equations and processes to accurately calculate the gear 3D,  $xyz$ , tooth profile in terms of the hob geometry, the machining setup and the gear parameters. We also developed software in the form of Excel spread sheets where values of the parameters are entered and the 3D profile of the gear tooth or gear groove is produced in the form of sketches, as in Figures 5 through 7, and in the form of  $xyz$  point data organized data arrays.

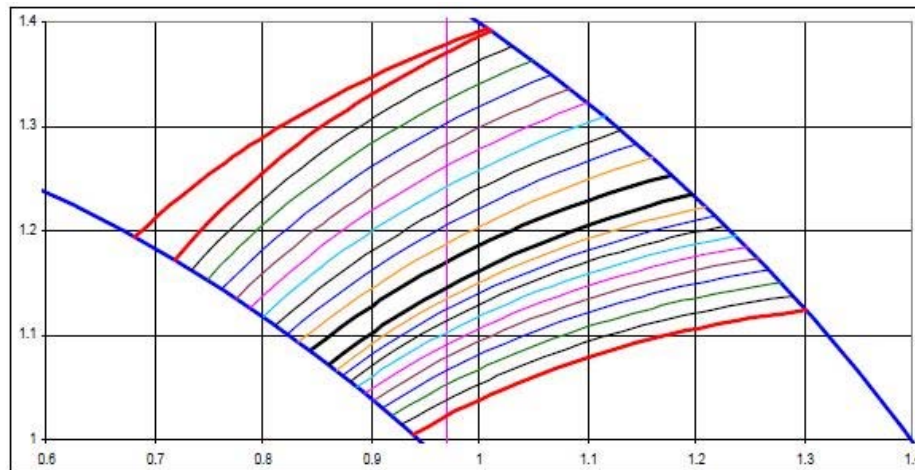


Figure 5. 3D sketch of an optimized flat face Spiroid® gear groove showing the high and low sides

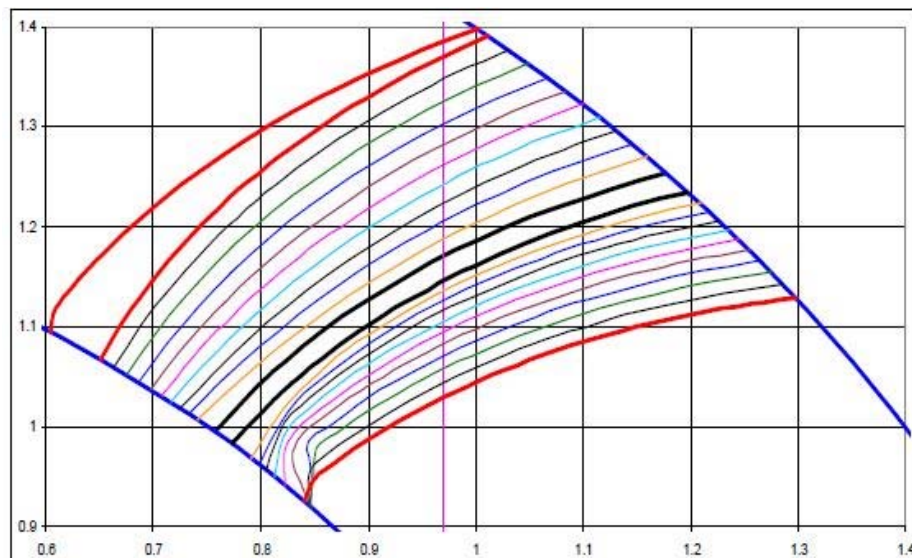


Figure 6. Gear groove profile for a gear inside radius of 1.25"