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A Practical Approach for Modeling a Bevel Gear

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

The modern bevel gear design engineer is often faced with knowing the basic appearance of the bevel gear tooth that he is designing. The geometry of the bevel gear is quite complicated to describe mathematically, and much of the overall surface topology of the tooth flank is dependent on machine settings and the cutting method employed. AGMA 929-A06, *Calculation of Bevel Gear Top Land and Guidance on Cutter Edge Radius*, lays out a practical approach for predicting the approximate top land thicknesses at certain points of interest regardless of the exact machine settings that will generate the tooth form. The points of interest that AGMA 929-A06 is concerned with consists of toe, mean, heel, and the point of involute lengthwise curvature.

The following method expands upon the concepts described in AGMA 929-A06 to allow the user to calculate not only the top land thickness, but the more general case, the normal tooth thickness anywhere along the face and profile of the bevel gear tooth. This method does not rely on any additional machine settings; only basic geometry of the cutter, blank, and teeth are required to calculate fairly accurate tooth thicknesses. The tooth thicknesses are then transformed into a point cloud describing both the convex and concave flanks in a global Cartesian coordinate system. These points can be utilized in any modern computer aided design software package to assist in the generation of a 3D solid model. All pertinent macro tooth geometry can be closely simulated using this technique. Furthermore, a case study will be presented evaluating the accuracy of the point cloud data to a physical part.

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Introduction

The first question that comes to mind from any engineer presented by this paper is why would anyone be interested in a close approximation to the tooth flanks of a straight or spiral bevel gear? That is a very valid question. A bevel gear designer is typically interested in the minute details of the tooth flank's *surface topology to gain the optimized contact conditions under the loads that the bevel gears will be operated*. The method presented in this paper does not address any of these needs.

Purpose

One of the main purposes of this method is to create a working three dimensional model for visual interrogation of fits and proportions. The bevel designer is typically concerned with the proportions of a myriad of geometric values describing the basic layout of the tooth form. Many of the values like top land balance, slot width tapers, spiral angle adjustments, etc., are hard to visualize for the engineer, and their impact to the tooth form can be drastic.

The gear engineer's tool kit should include a method for generating a three dimensional model of a bevel gear, as the dependency of using CAD for the general engineer is rapidly increasing. Many application engineers are demanded to provide three dimensional models to their customers. These customers utilize their CAD system to validate fits and clearances between components in their systems. Gears should not be left out of this analysis. Advancements in 5 and 6-axis machining of gears are rapidly approaching the precision and capability of dedicated gear generating machines. Most of these machining centers require a three dimensional model for programming. If the application engineer provides a fully developed gear model to their customer, the 5 or 6-axis machining techniques may become very attractive. For this case alone, many gear companies opt out of providing models of their designs.

Background

The method outlined in AGMA 929-A06, *Calculation of Bevel Gear Top Land and Guidance on Cutter Edge Radius*, describes how to calculate the top land thickness for a bevel tooth at specific points of interest. These points of interest are at the toe, mean, heel, and point of involute lengthwise curvature. Reasons regarding the purpose for these calculation points are beyond the scope of this document. The formulas inside AGMA 929 can be generalized so that a set of equations can be devised to calculate the normal circular tooth thickness anywhere along the profile and length of the tooth.

Why is this method approximate

The geometry of a spiral bevel gear is mathematically complicated. The machine settings used to create a spiral bevel gear, whether using face milling or face hobbing, adjust the final flank form of the teeth. A strong understanding of the machine settings and motions are necessary to achieve an accurate tooth model. AGMA 929-A06 uses the technique of a virtual spur gear to approximate the profile of a bevel tooth in the normal plane without knowing the motions of the machine generating the final form. In doing so, the tooth thicknesses can be calculated quite simply using traditional methods for spur gears.

The virtual spur gear technique assumes that the tooth form will follow an involute in the profile direction in the normal plane. This is only a close approximation to the true form of a spiral bevel tooth. Most spiral bevels follow the octoid tooth form, which is similar but not the same as the involute tooth form found on most cylindrical gears.

What is missing in AGMA 929-A06 to complete a model

A generalized set of equations can be produced from the content of AGMA 929-A06 to calculate the normal tooth thicknesses of a bevel tooth anywhere in the profile or lengthwise direction. The purpose of this document is to fill in the gaps of AGMA 929-A06 so that a model can be generated. The majority of the content of this document is pertaining to how the normal tooth thicknesses are oriented in three dimensions, and resolving these thicknesses into an array of Cartesian coordinates. See Figure 1 for a

visual depiction of the calculation method described within this document. Additional information regarding the terminology described within the figure is in the subsequent sections.

This method's shortcomings

Enough discussion has taken place to illuminate the reader as to why this method creates only an approximate method, but there are additional shortcomings worth mentioning. This method does not currently take into account the root fillet. The coordinates calculated in this method are strictly points following the involute curve that describes the approximate flank form. Coordinates for the root fillet is outside the scope of this document.

The other major shortcoming is that all subsequent formulas are for spiral bevel gears without a hypoid offset. Additional provisions would need to be made to generalize the formulas to account for hypoids.

Coordinate system definition

All bevel gears are designed using a reference right cone called the pitch cone. The pitch cone is used as a basis for describing all other geometric entities of the bevel gear. Since describing the motions of the generating process in three dimensions would be hard to comprehend, the general practice is to unwrap the surface of the pitch cone into a tangent plane, called the pitch plane. The point at the top of the pitch cone is called the pitch apex. The pitch apex is significant because the axes of both gear and pinion intersect at this point. Figure 2 displays the pitch cone, shows the pitch plane unwrapped, and also describes the global Cartesian coordinate system, C_G . In addition, Figure 2 shows the pitch cone sectioned through the YZ plane. This describes the definition of the pitch angle, Γ , and the face width, F , of a part. The global coordinate system follows the right-hand rule and its origin is located at the pitch apex of the member to be modeled.

Basic generation

The majority of all generated spiral bevel gears are manufactured in one of two processes, face milling or face hobbing. When compared, Both manufacturing methods have advantages and disadvantages. For the purposes of this method, a brief understanding of the generating method utilized during these processes is necessary to realize the three dimensional model.

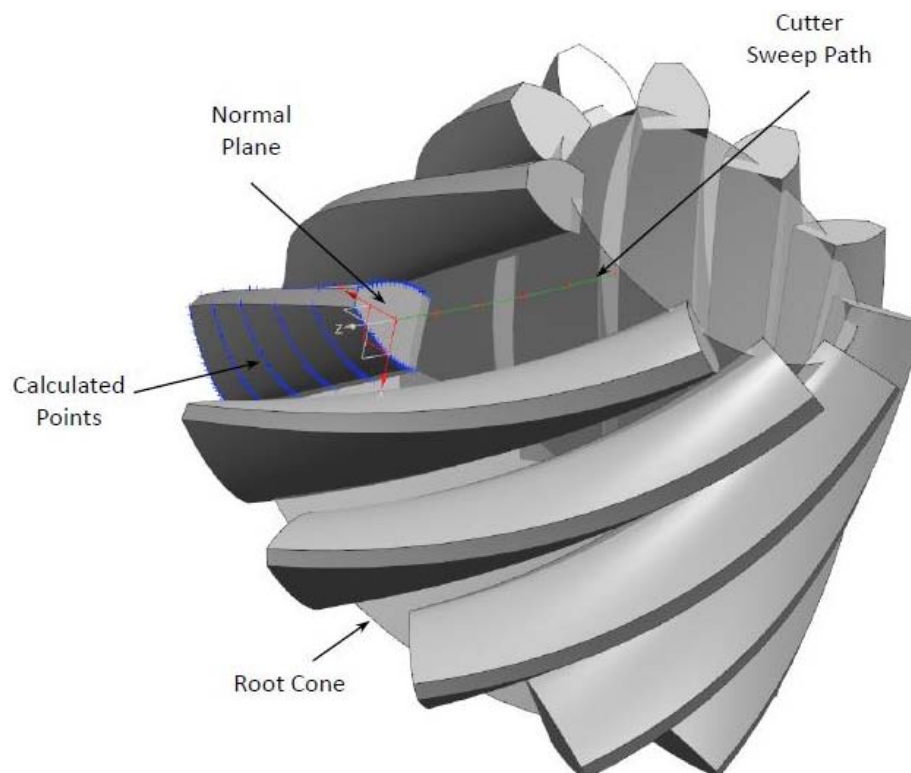


Figure 1. Visual depiction of calculation method

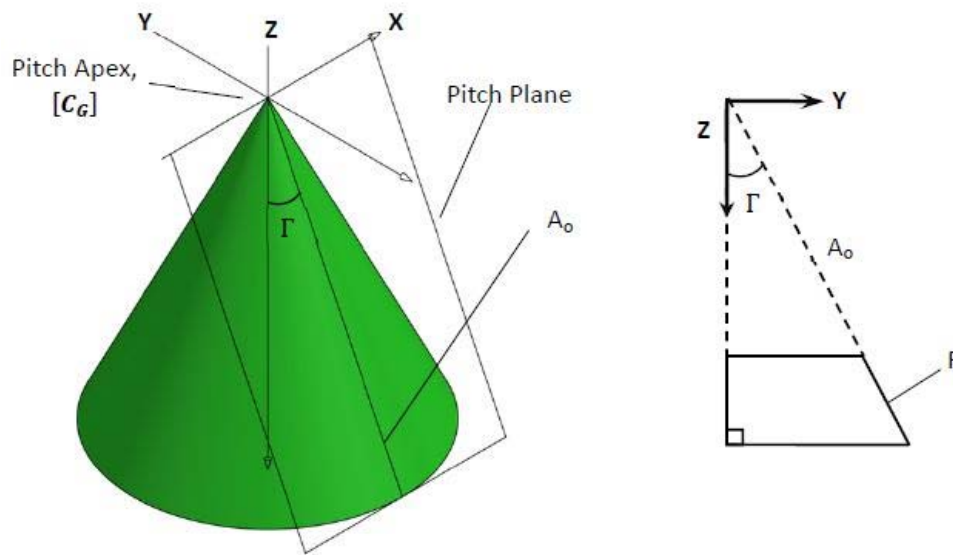


Figure 2. Pitch cone

Face milling

The face milling manufacturing method employs a circular cup shaped cutting tool moving in a timed relationship with the work piece to roll through the gear blank and generate an individual slot. The cutter is then withdrawn and the work is indexed to the location of the next slot and the process repeated. See **Error! Reference source not found..**

Figure 3 displays three instances of the cutter as it passes through the work piece; these points are at the toe, mean, and heel of the crown gear. The path of the cutter sweeps a circular arc in the lengthwise direction with the same radius as the radius of the cutter, R_c . The axis of the crown gear, C_g , is known as the machine center. The local coordinate system for the pitch plane is located at the machine center. Xpp correlates to X and Ypp lies along the line describing A_o in **Error! Reference source not found..**

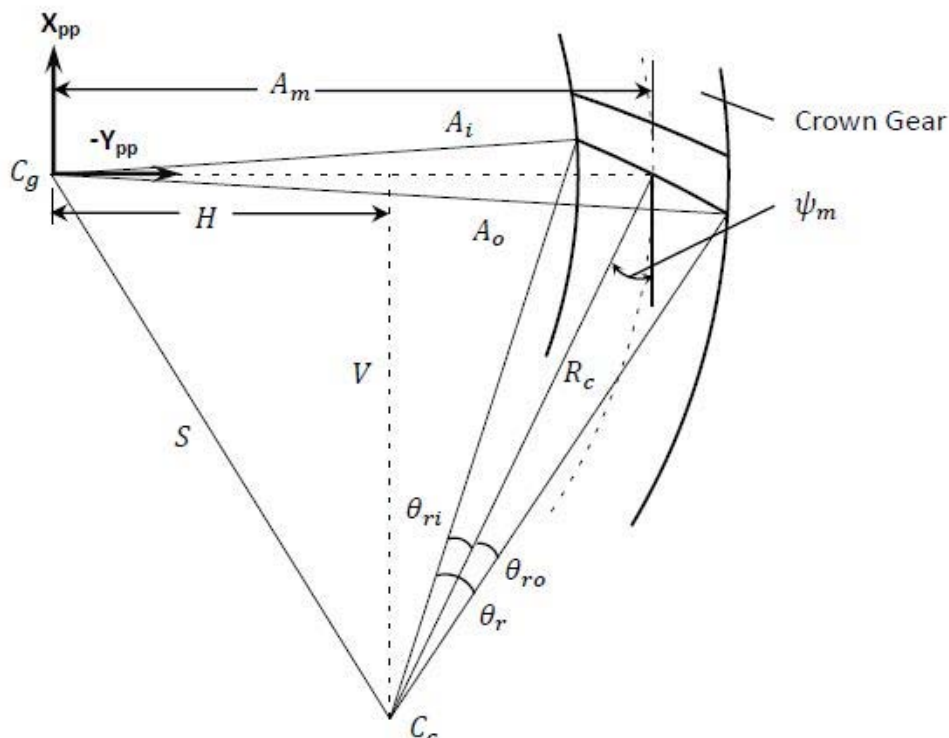


Figure 3. Generating triangle in pitch plane for face milling

The cutter axis, C_c . This location can be found by:

$$V = R_c \cos \psi_m \quad (1)$$

$$H = A_m - R_c \sin \psi_m \quad (2)$$

$$C_c = (V, H) \quad (3)$$

The sign of the vertical term, V , may either be positive or negative depending on the hand of spiral to be modeled. Figure 3 shows a right hand member (use negative value for V when calculating left hand members). The cutter sweep angle, θ_r , is necessary to determine how much rotation is used during the cutting process. When calculating points to model the gear tooth, these will be the endpoints of the working portion of the cutter path.

$$S = \sqrt{V^2 + H^2} \quad (4)$$

$$\theta_r = \cos^{-1} \left[\frac{S^2 + R_c^2 - A_o^2}{2S R_c} \right] - \cos^{-1} \left[\frac{S^2 + R_c^2 - A_i^2}{2S R_c} \right] \quad (5)$$

$$\theta_{ro} = \cos^{-1} \left[\frac{S^2 + R_c^2 - A_o^2}{2S R_c} \right] - \cos^{-1} \left[\frac{S^2 + R_c^2 - A_m^2}{2S R_c} \right] \quad (6)$$

$$\theta_{ri} = \theta_r - \theta_{ro} \quad (7)$$

Discrete points can be calculated along the cutter sweep on the pitch plane. Using the general parametric formulas for a circle, the solution for a point, p , along the cutter path is:

$$p_x(t) = V + R_c \cos t \quad (8)$$

$$p_y(t) = H + R_c \sin t \quad (9)$$

Where t is a parameter that has the following ranges:

For left hand members,

$$\pi + \psi_m - \theta_{ri} \leq t \leq \pi + \psi_m + \theta_{ro} \quad (10)$$

For right hand members,

$$2\pi - \psi_m + \theta_{ri} \leq t \leq 2\pi - \psi_m - \theta_{ro} \quad (11)$$

This cutter path can be broken into as many discrete sections as desired.

Face hobbing

The face hobbing manufacturing method is a continuously indexing process. The cutting tool has groups of staggered blades. The work piece moves in a timed relationship with the cutter so that a group of blades in the cutter passes through a slot of the work piece. The face hobbing method generates an extended epicycloidal shape in the lengthwise direction. See **Error! Reference source not found.** for a detailed layout of the face hobbing generating triangle.

Since the lengthwise shape of a face hobbled part creates an extended epicycloidal shape, a little trigonometry is necessary to calculate the discrete points along the path created by the cutter.

The crown gear tooth count,

$$N_c = \frac{N}{\sin \Gamma} \quad (12)$$

The lead angle of the cutter,

$$\nu = \sin^{-1} \left[\frac{A_m N_s}{R_c N_c} \cos \psi_m \right] \quad (13)$$