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Precision Bevel Gears with Low Tooth Count

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

The paper deals with the geometry and kinematics of right-angle bevel gears that feature low tooth count. Gears that have 12 teeth and fewer are referred to as the low-tooth-count-gears (or just gears, for simplicity). It should be stressed from the very beginning that gears are not covered by either AGMA, or by any other national/international standards on gearing. This is mostly because of the kinematics and geometry of gears is not profoundly investigated yet.

When operating, right-angle bevel gears generate vibration and produce an excessive noise. Dynamic loading of the gear teeth can result in the tooth failure. These problems become more severe in bevel gearings with low tooth count.

Variation of the base pitch in bevel gearing is the root cause for the vibration generation, an excessive noise excitation, and unfavorable dynamic loading of the gear teeth. It has been proved [1] that the equality of:

- The base pitch of the gear
- The base pitch of the mating pinion
- The operating base pitch

is the fundamental requirement for the operating of right-angle bevel gears that feature low tooth count.

In this paper, the main features of design and machining of bevel gears are discussed. The proposed approach is based on the triple equality:

Base pitch of the gear = Base pitch of the pinion = The operating base pitch

Once the base pitches are equal, then the root cause for vibration generation, noise excitation, and unfavorable dynamic loading of the gear teeth is eliminated.

The consideration is focused mainly on right-angle bevel gearing. However, the reported results of the research are applicable for bevel gearings with different shaft angles.

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The paper deals with the geometry and the kinematics of right-angle bevel gears that feature low tooth count. Right-angle bevel gears are a particular case of intersected-axis gearing (further I_a -gearing) with an arbitrary value of the shaft angle. Commonly, bevel gears with the base cone angle (Γ_b for the bevel gear and γ_b for the bevel pinion) larger than the root cone angle (Γ_f for the bevel gear and γ_f for the bevel pinion), that is, when the inequalities $\Gamma_b > \Gamma_f$ and $\gamma_b > \gamma_f$ are observed, are referred to as low-tooth-count gears, *LTC-gears*¹.

The geometry and the kinematics of gears that have 12 teeth and fewer are the main focus of this paper². All of the equations derived for *LTC-gears* are valid for gears with an arbitrary tooth count, and not only for gears with a large tooth count.

When operating, right-angle bevel gears often generate vibration and produce an excessive noise. Dynamic loading of the gear teeth can result in the tooth failure. These problems become more severe in bevel gearings with low tooth count. The performed analysis shows that inequality of base pitches of the gear and mating pinion is the root cause for insufficient performance of *LTC-gears*.

In most applications, the main purpose of I_a -gearing is to smoothly transmit a rotation and torque between two intersected axes. Gear pairs that are capable of transmitting a uniform rotation from the driving shaft to the driven shaft are referred to as the geometrically accurate intersected-axis gear pairs (or, in other words, the ideal intersected-axis gear pairs).

Three requirements need to be fulfilled in order to a bevel gear pair can be referred to as the geometrically accurate bevel gear pair:

- The geometry of the tooth flanks of a bevel gear and a mating bevel pinion has to obey the condition of contact. The condition of contact can be analytically represented in the form of dot product $n \cdot V_{\Sigma} = 0$ of the unit vector n of a common perpendicular at point of contact of tooth flanks G and P of the gear and the mating pinion, and the vector of the velocity of the relative motion of the tooth flanks G and P . The equation of contact $n \cdot V_{\Sigma} = 0$ is commonly referred to as Shishkov's equation of contact [1], [2]. This equation was proposed by Shishkov as early as 1948 (or even earlier).
- The geometry of the tooth flanks of a bevel gear and a mating bevel pinion has to obey the condition of conjugacy. To meet this requirement, common perpendicular at every point of contact of the tooth flanks G and P must intersect the axis of instant rotation (the pitch line, in other words). The shaft angle of a bevel gear pair is subdivided by the pitch line in a proportion that corresponds to gear ratio of the bevel gear pair (see equations 4 through 8).
- The geometry of the tooth flanks of a bevel gear and a mating bevel pinion has to ensure equal base pitches of (a) the gear, (b) the pinion, and (c) the operating base pitch of the gear pair, that is, these three base pitches must be equal to one another at every instant of time.

A few comments regarding the aforementioned requirements, which ideal bevel gearing has to obey, immediately follow.

First, the necessity to meet the condition of contact, $n \cdot V_{\Sigma} = 0$, is obvious. If the condition of contact is violated, this immediately results either in the interference of the tooth flanks G and P into each other or in departure of the tooth flanks G and P from one another. None of these two scenarios is valid in gearing.

¹ It is instructive to note here that in case of P_a gearing, e.g., spur gearing, an equation $d_{bg} = d_{tg}$ can be composed. After the base diameter, d_{bg} , and the root diameter, d_{tg} , of a gear are expressed in terms of the module, m , (or diametral pitch, P_d), the tooth count, N_g , and the transverse profile angle, ϕ_t , the solution to the equation $d_{bg} = d_{tg}$ with respect to N_g returns $N_g = 41.6$. Therefore, P_a -gears with the standard tooth profile and the tooth count $N_g \leq 41$ are referred to as *LTC-gears*. In general sense, a similar is valid with respect to I_a -gearing with low tooth count. In a narrower sense, *LTC-gears* are viewed as those with the tooth count $N_g \leq 12$.

² It should be stressed here that *LTC-gears* are covered by neither AGMA, nor by any other national/international standards on gearing. This is mostly because the kinematics and the geometry of *LTC-gears* are not profoundly investigated yet.

Second, the condition of conjugacy of the tooth flanks G and P of the bevel gear and pinion is an equivalent to the well-known Willis' theorem [3]. The Willis' theorem relates to parallel-axis gears (P_a -gears, for simplicity). No condition of conjugacy of the tooth flanks G and P in the cases of I_a -gearing as well as C_a -gearing (that is, for the case crossed-axis gearing) is known so far. Below, the condition of conjugacy is enhanced to the case of I_a -gearing.

Third, it should be noted here that the cycle of meshing of only one pair of gear teeth is covered by the condition of conjugacy of the tooth flanks G and P of the bevel gear and pinion. Contact ratio in all gearings is always greater than one. Therefore, at a certain instant of time, two or more pairs of teeth are engaged in mesh simultaneously. To make the multiple contacts feasible, the equality of base pitches of (a) the gear, (b) the pinion, and (c) the operating base pitch is a must.

Here and below the angular distance between each two adjacent desirable lines of contact LC_{des}^i and LC_{des}^{i+1} is specified by the operating base pitch φ_{bop} (equation 19); here i is an integer number. The angle φ_{bop} is measured within the plane of action, PA , of the gear pair. This angle centered at the common apex $A_g = A_p = A_{pa}$ [4]. The operating base pitch, φ_{bop} , of a bevel gear pair is illustrated in Figure 5.

Equality of three base pitches: $\varphi_{bg} = \varphi_{bp} = \varphi_{bop}$ (see items a) through c)) is referred to as the fundamental law of gearing. Determination of the design parameters for ideal intersected-axis gearings is considered below.

Elements of the kinematics in I_a -gearing

Consider an I_a -gear pair shown in Figure 1. The driving pinion is rotated about its axis, O_p , with a certain angular velocity, ω_p . The driven gear is rotated about its axis, O_g , with a certain angular velocity, ω_g . The axes of rotation, O_g and O_p , intersect at point A_{pa} . These axes form an angle, Σ . Commonly, this angle is equal to a right angle (that is, $\Sigma = 90^\circ$); however, I_a -gearings either with an angle, $\Sigma > 90^\circ$, or with an angle $\Sigma < 90^\circ$ are also known. I_a -gearings can be internal and external. In a particular case, I_a -gearing degenerates to a so-called crown-gear-to-bevel-pinion gearing.

Rotations ω_g and ω_p can be interpreted as vectors³, ω_g and ω_p correspondingly. The rotation vectors ω_g and ω_p are along the axes of rotation, O_g and O_p . The actual directions of vectors ω_g and ω_p depend on the actual direction of the corresponding rotations, ω_g and ω_p . The angle Σ is specified as the angle that is formed by the rotation vectors ω_g and ω_p , that is:

$$\Sigma = \angle(\omega_g, \omega_p) \quad (1)$$

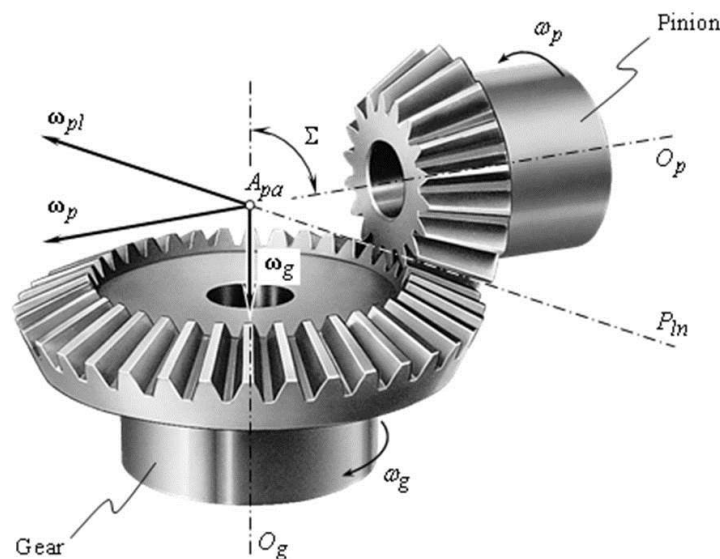


Figure 1. Schematic of right-angle intersected-axis gearing (I_a -gear pair) [4]

³ Note that rotations are not vectors in nature. Therefore, special care is required when treating rotations as vectors.