

The Finite Strips Methods as an Alternative to the Finite Elements in Gear Tooth Stress and Strain Analysis

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TECHNICAL PAPER

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

The Finite Element Method is widely used in the gear industry to assess gear tooth strength and stiffness. Although automatic meshing software is available, the Finite Element Method still requires a significant amount of manipulation which, though acceptable as a verification tool when the design is completed, may be considered prohibitive at the design stage.

The Finite Strip Method, which may be considered as a subset of the Finite Element Method, is presented as an alternative that requires limited manipulation and computer memory while offering super-fast computation along with precision comparable to that of Finite Elements. The formulation allows for easy integration to a gear tooth geometry simulation software since, besides tooth thickness distribution, several geometrical parameters are derived directly from the gear blank, and most common gear tooth geometries may be accommodated.

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THE FINITE STRIP METHOD AS AN ALTERNATIVE TO FINITE ELEMENTS IN GEAR TOOTH STRESS AND STRAIN ANALYSIS

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1. Introduction

The knowledge of tooth stiffness is fundamental in the calculation of the load sharing between meshing gear teeth, which can then lead to a precise evaluation of the contact and bending stresses.

Several analytical stiffness and stress models have been presented in the past [1, 2, 3, 4, 5] to solve this problem, which is compounded by the complexity of the tooth geometry. For spur and helical gear teeth, where the tooth geometry remains constant in the lengthwise direction, reasonable agreement is obtained using analytical models; for spiral-bevel and hypoid gear teeth, where the tooth is curved and wound on a cone, and tooth thickness and height varies along tooth facewidth, so far only the Finite Element Method (FEM) provides acceptable results.

This paper presents the Finite Strip Method (FSM), an alternative to FEM and analytical formulations, to obtain reliable results with reduced preparation and solution time and that can be integrated to a gear tooth geometry simulation software.

Cheung [6] and, independently, Powell and Ogden [7], introduced the FSM which can be considered as a special case of the FEM: the Finite Strip is a 2-D element for the analysis of plates, based on simple polynomial functions in one direction and continuously differentiable smooth series in the other direction. Mawenya and Davies [8] included the effect of transverse shear to make the FSM applicable to thick, thin and sandwich plates. The use of cubic B-splines, introduced by Cheung, Fan and Wu [9], ensures C_2 -continuity and their local characteristics allow different boundary conditions. FSM thin plates with variable thickness in the longitudinal direction were treated by Uko and Cusens [10].

The analysis of gear teeth by the FSM, considered as thick plates which may show either uni-directional or bi-directional thickness variation and constant or non-constant depth, is the focus of this paper. To extend the application of the FSM from thin to thick plates, Mindlin's theory is used in the variant of the FSM presented in this paper [11]. FEM deflection and stress results for the clamped-free case are compared to those obtained by the FSM and show good agreement.

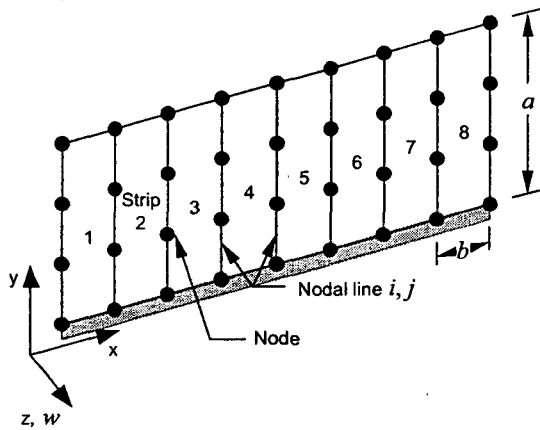
2. Main Nomenclature

a	strip length
a_i^m	vector of nodal parameters of node i and function m
b	strip width
$[B_i^m]$	strain displacement matrix for nodal line i and m th series term
D	elasticity matrix
f_i^m	load vector for nodal line i and m th series term
k_t	stress concentration factor
$k_{i,j}^{m,n}$	finite strip stiffness matrix for terms m, n and nodal lines i, j
q	transverse loading per unit area
w	transverse displacement
δ	displacement function
ϵ	generalized strain vector
Φ_i^m	m th term of displacement function for nodal line i
ν	Poisson ratio
$\theta_{x-y-r-\lambda-s}$	nodal rotation about the x - y - r - λ - s axis
σ_{x-y}	x - y normal stress
$\tau_{xy-yz-xz}$	shear stress in the xy - yz - xz plane

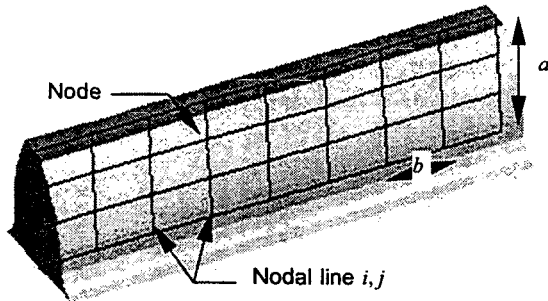
2. Finite Strip Formulation

2.1 Strip of constant height (rectangular)

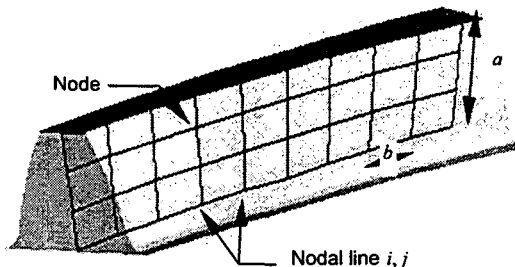
A typical Finite Strip discretisation of a rectangular plate, such as the tooth of a spur, helical or face gear, is presented in Figure 1. Strips of width b and length a are parallel to the y -axis and connected by nodal lines that lead to the definition of the displacement function [12]. Strip widths are normally kept constant.



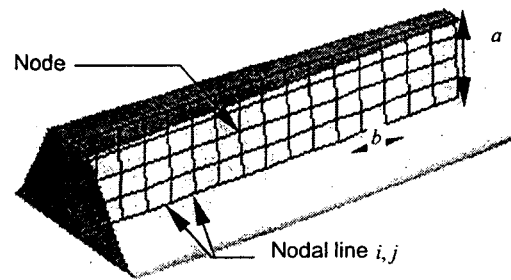
a) Finite Strip of a rectangular plate



b) spur gear tooth



c) helical gear tooth



d) Face gear tooth

Figure 1

The displacement function δ for a linear strip is the sum of a series of l terms:

$$\delta = \sum_{m=1}^l \sum_{i=1}^2 \Phi_i^m a_i^m \quad (1)$$

where Φ_i^m is a combination of polynomial N_i and series ϕ_m^w , $\phi_m^{\theta_x}$ and $\phi_m^{\theta_y}$:

$$\Phi_i^m = \begin{bmatrix} N_i \phi_m^w & 0 & 0 \\ 0 & N_i \phi_m^{\theta_x} & 0 \\ 0 & 0 & N_i \phi_m^{\theta_y} \end{bmatrix} \quad (2)$$

Using Mindlin's plate theory, the mid-plane displacement vector is (figure 2):

$$\delta = [w, \theta_x, \theta_y]^T \quad (3)$$

and the vector of nodal parameters of node i for the m th function:

$$a_i^m = [w_i^m, \theta_{xi}^m, \theta_{yi}^m]^T \quad (4)$$

The generalized strain vector ϵ can be expressed as:

$$\epsilon = \left[\frac{\partial \theta_x}{\partial x}, \frac{\partial \theta_y}{\partial y}, -\left(\frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \right), \frac{\partial w}{\partial x} - \theta_y, \frac{\partial w}{\partial y} - \theta_x \right]^T \quad (5)$$

where w is the transverse displacement and θ_x and θ_y , the section rotations. β denotes an additional rotation due to transverse shear deformation.