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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

A design tool for the contact pattern analysis, based both on analytical algorithms and on FEM models generated automatically, has been developed for Gleason Spiral Bevels gears, that are widely used in aerospace transmissions. The conjugate surfaces theory is applied at the generation of the tooth flank surfaces, by simulating the manufacturing process of the pinion and the gear defined by a Gleason summary. Then an analytical Tooth Contact Analysis is performed in order to determine the theoretical contact points on the flank surfaces versus the meshing position, taking into account misalignments. The Hertz theory is applied to calculate the axes of the theoretical contact ellipses without considering load sharing. Finite Elements models of the gear pair are automatically generated, in order to consider load sharing. The FEM models include the gear blanks and take into account misalignments.

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AN ANALYTICAL – FEM TOOL FOR THE DESIGN AND OPTIMISATION OF AEROSPACE GLEASON SPIRAL BEVEL GEARS

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1. Introduction

Due to the weight reduction requirements, aerospace transmissions structures generally show high deflection under the applied loads, and such deflection, which influence the contact pattern of mating gear teeth, must be taken into account in the design and optimization of gears.

A tool that analyses the contact at the beginning of the design process can be time and money saving, with respect to the traditional development procedures based on tests.

A design tool based both on analytical algorithms and on FEM models generated automatically has been developed for Gleason Spiral Bevels gears, that are widely used in aerospace transmissions.

As a first step, the conjugate surfaces theory is applied at the generation of the tooth flank surfaces, by simulating the manufacturing process of the pinion and the gear defined by a Gleason summary. The simulation considers the real tool data and the machine settings. Then an analytical Tooth Contact Analysis is performed in order to determine the theoretical contact points on the flank surfaces versus the meshing position. The misalignments can be introduced in the model as given data and are therefore taken into account. The Hertz theory is applied to calculate the axes of the theoretical contact ellipses without considering load sharing. In the following step, the information derived by the contact analysis is used in order to automatically generate finite elements models of the gear pair: in particular, on the basis of the theoretical contact pattern, the tooth flank surface mesh can be adapted to the simulation of tooth contact. The automatically generated FEM models also include the main geometric parameters of the gear blank (web and rim thickness, hub diameter) and misalignments that must be calculated separately, on the basis of the characteristics of the supporting structure.

The final simulation by means of FEM models takes into account of the load sharing between tooth pairs.

2. Cutting method

The spiral bevel gear sets considered in this paper are cut with the Spread Blade / Fixed Setting Method. During the generation of the gear, the concave and

the convex tooth flanks are generated simultaneously by means of a tool with alternate inside and outside cutting blades (Spread-blade method).

The pinion is generated following the Fixed Setting method. A tool with internal blades is used to simulate the generation of the convex flank, and another tool with external blades is used to generate the driving concave flank. In this way the two flanks are generated independently with two different machine settings. The space width is usually controlled by means of the stock allowance indexing.

3. Input data

The input data for the developed software comes directly from the Gleason summary. Table 1 reports the needed gear set data.

n, N	Number of teeth
F	Face width
A_0	Outside cone distance
a_P, a_G	Addendum
b_P, b_G	Dedendum
γ, Γ	Pitch cone angle
γ_O, Γ_O	Face angle of blank
γ_R, Γ_R	Root angle
ψ	Mean spiral angle
H	Hand of spiral (LH o RH)
t_{nc}	Pinion normal chordal thickness (due to pinion generation method)

Table 1. Gear set data.

Another set of data is used to define the tool geometry. The gear tool is defined by the cutter diameter, the outside and inside pressure angle, the point width and the blade edge radius. Two tools are used to generate the pinion; the first tool is similar to the gear tool, so the two data set are analogous, while the other tool with only external blades is defined by the cutter point diameter, the outside blade angle, the point width and the blade edge radius.

The last set of data contains the machine settings, listed in Table 2.

	Notes
Machine number	
Machine root angle	Usually zero for the gear, while the pinion has a little offset in order to optimize the bearing contact.
Machine center to back	
Sliding base	Distance between the machine center and the pitch cone apex.
Eccentric angle	This angle and the machine constant (k_2) are related to the radial (distance between the cardle center and the tool center, called b), which define the tool position on the cardle.
Cradle angle	
Ratio of roll	Cradle test roll / Work test roll

Table 2. Machine settings.

The pinion machine settings are defined by two of this set of data, one for each cut.

4. Generation method

Reference frames

In order to establish the kinematical relationship between the tool and the generated wheel (gear or pinion), the following main reference frames are used ($j=F$ and $i=1$ refer to the pinion, $j=P$ and $i=2$ refer to the gear):

S_i	Rigidly connected to the gear being generated
S_{mi}	Rigidly connected to the machine frame
S_{si}	Rigidly connected to the generating surface

Theoretical basis for the generation process

The method used to simulate the generation of the gear is based on the work of Litvin [8]. The tool surface in S_{si} is described by means of the following equation:

$$\begin{bmatrix} r_{si}^{tj}(u'_j, \theta'_j) \end{bmatrix} = \begin{bmatrix} x_{si}^{tj}(u'_j, \theta'_j) \\ y_{si}^{tj}(u'_j, \theta'_j) \\ z_{si}^{tj}(u'_j, \theta'_j) \\ 1 \end{bmatrix} \quad (1)$$

where (u'_j, θ'_j) are the surface variables. During its motion, the tool surface generates a family of surfaces:

$$\begin{bmatrix} r_m^{\Sigma j}(u'_j, \theta'_j, \varphi_j) \end{bmatrix} = \begin{bmatrix} x_m^{\Sigma j}(u'_j, \theta'_j, \varphi_j) \\ y_m^{\Sigma j}(u'_j, \theta'_j, \varphi_j) \\ z_m^{\Sigma j}(u'_j, \theta'_j, \varphi_j) \\ 1 \end{bmatrix} \quad (2)$$

where φ_j is the motion parameter.

From Eq. 2 it's possible to compute the normal vector for each surface of the family:

$$\vec{N}_m^{\Sigma j} = \frac{\partial \vec{r}_m^{\Sigma j}}{\partial u'_j} \times \frac{\partial \vec{r}_m^{\Sigma j}}{\partial \theta'_j} \quad (3)$$

and its unit normal vector:

$$\vec{n}_m^{\Sigma j} = \frac{\vec{N}_m^{\Sigma j}}{|\vec{N}_m^{\Sigma j}|} \quad (4)$$

Since the generated wheel surface is the envelope to the family of the tool surfaces, it must be tangent to each surface of this family; analytically this condition is expressed by means of the Equation of Meshing:

$$\vec{n}_m^{\Sigma j} \cdot \vec{v}_m^{tji} = 0 \quad (5)$$

where $\vec{v}_m^{tji}$ is the sliding velocity of the tool with respect to the wheel being generated. Equations 5 and 2 (projected in S_i) define the gear tooth geometry.

Pinion Equation of Meshing calculation

On the basis of the briefly summarized theory, in this paragraph the procedure followed to determine the equation of the active portion of the pinion tooth flank surface is shown.

Step 1. First of all, the equation of the family of the tool surface (Σ_F) will be derived. This family is generated during the generation motion of the tool and its equation in the reference frame $S_{m(1)}$ is:

$$\begin{bmatrix} x_{m(1)}^a \\ y_{m(1)}^a \\ z_{m(1)}^a \\ 1 \end{bmatrix} = \begin{bmatrix} r_c^{(F)} \cdot \cot(\psi^{(F)}) + \dots \\ -u_F^a \cdot \cos(\psi^{(F)}) \\ u_F^a \cdot \sin(\psi^{(F)}) \cdot \sin(\vartheta_F^a) + \dots \\ -q_F + \varphi_F - b_F \cdot \sin(q_F - \varphi_F) \\ u_F^a \cdot \sin(\psi^{(F)}) \cdot \cos(\vartheta_F^a) + \dots \\ -q_F + \varphi_F + b_F \cdot \cos(q_F - \varphi_F) \\ 1 \end{bmatrix} \quad (6)$$

where φ_F is the tool rotation and $r_c^{(F)}$ is the average cutter radius.

Step 2. The normal vector of the surface is derived by means of the following expression:

$$\vec{N}_{m(1)}^a = \frac{\partial \vec{r}_{m(1)}^a}{\partial \vartheta_F^a} \times \frac{\partial \vec{r}_{m(1)}^a}{\partial u_F^a} \quad (7)$$

Starting from this expression the unit normal vector can be derived:

$$\begin{bmatrix} n_{m(1)}^a \end{bmatrix} = \frac{\vec{N}_{m(1)}^a}{|\vec{N}_{m(1)}^a|} = \begin{bmatrix} \sin(\psi^{(F)}) \\ \cos(\psi^{(F)}) \cdot \sin(\vartheta_F^a - q_F + \varphi_F) \\ \cos(\psi^{(F)}) \cdot \cos(\vartheta_F^a - q_F + \varphi_F) \end{bmatrix} \quad (8)$$

Step 3. The sliding velocity can be computed as:

$$\vec{v}_{m(1)}^{aF1} = \vec{v}_{m(1)}^{aF} - \vec{v}_{m(1)}^{a1} \quad (9)$$

where $\vec{v}_{m(1)}^{aF}$ is the velocity of a point M considered rigidly connected to the tool surface Σ_F and $\vec{v}_{m(1)}^{a1}$ is the velocity of the same point M considered rigidly connected to the tooth flank surface Σ_1 .

The vector $\vec{v}_{m(1)}^{aF}$ is computed by means of the following expression

$$\vec{v}_{m(1)}^{aF} = \vec{\Omega}_{m(1)}^F \times \vec{r}_{m(1)}^a \quad (10)$$

where $[\Omega_{m(1)}^F]$ is the generating surface angular

velocity and the vector $\vec{r}_{m(1)}^a$ is expressed by equation (6). This angular velocity is related to the pinion blank angular velocity by means of a polynomial of the 4th order, in order to consider the Modified Roll effects in further developments.

Since axes $x_{m(1)}$ and z_1 do not intersect but cross each other, the velocity of the pinion point M can be computed as:

$$\vec{v}_{m(1)}^{a1} = \vec{\Omega}_{m(1)}^1 \times \vec{r}_{m(1)}^a + \overline{O_{m(1)}O_1} \times \vec{\Omega}_{m(1)}^1 \quad (11)$$

where $\overline{O_{m(1)}O_1}$ is the distance between the pitch cone apex and the machine center (calculated considering the corrections of the machine tool setting), and $\vec{\Omega}_{m(1)}^1$ is the pinion angular velocity projected in reference frame $S_{m(1)}$.

The expression of the sliding velocity $\vec{v}_{m(1)}^{aF1}$ can now be calculated.

Step 4. The Equation of Meshing can now be derived by means of its definition:

$$f(u_F^a, \vartheta_F^a, \varphi_F) = \vec{n}_{m(1)}^a \cdot \vec{v}_{m(1)}^{aF1} = 0 \quad (12)$$

Equation 12 can be solved analytically with respect to u_F^a , but it is not printed here due to its length.

This equation can be used for the concave and the convex flank, using the correspondent parameter.

The Equation of Meshing for the other tooth flank portions (root) has been derived following analogous procedures.

The system of equations composed by the Equation of Meshing and the family of the tool surface in the reference frame S_1 define the pinion tooth active portion.