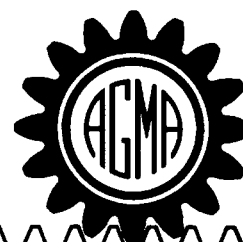


Effects of Helix Slope and Form Deviation on the Contact and Fillet Stresses of Helical Gears

by: R. Guilbault, Laval University

American Gear Manufacturers Association



TECHNICAL PAPER

Effects of Helix Slope and Form Deviation on the Contact and Fillet Stresses of Helical Gears

Raynald Guilbault, Laval University

[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

A cylindrical gear model, called the Displacement-Stress model, is developed to establish the load sharing between meshing gear teeth and along contact lines. The model uses of the Finite Strip Method combined to a model of the tooth base solved with the Finite Differences Method to produce tooth deflexion and fillet stresses. The accuracy of this procedure is established with 3D Finite Element models. The Displacement-Stress model is coupled to a contact cell discretization of the contact areas based on the Boussinesq and Cerruti solution for point normal traction acting on an elastic half-space. An investigation is conducted on the effects of helix slope and form deviations tolerances specified for grades 5 and 7 of the ANSI/AGMA ISO 1328-1 for cylindrical gears. The results show an almost linear correspondence between deviation amplitude and tooth load and fillet stress increases: using grade 7 instead of grade 5 can double the tooth flank load increase and associated fillet stress increase. Results also show that effects are even more significant on the maximum contact pressure.

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1.0 Introduction

The performance of a gear pair depends largely on the manufacturing precision. The production cost consequently increases with accuracy. The gear designer is thus faced with the decision as to the precision level really needed for a given application.

The ANSI/AGMA ISO 1328-1 for Cylindrical Gears [1], which sets 13 gear accuracy levels with allowable lead and form deviations for gearwheels, specifies mandatory inspection items and also contains criteria of quality given as useful information. This paper aims to study the effects of the tolerances on helix form $f_{f\beta}$ and slope $f_{h\beta}$ deviations which, while not individually subject to mandatory tolerances, are recognised to have a significant influence on performance of a gear pair.

The deviations are studied and compared for an helical gear of grades 5 and 7. The investigation is conducted from the perspective of the longitudinal load distribution in view of form and lead deviations, and its influence on tooth fillet bending and surface contact stresses.

2.0 Notation

$\bar{\Sigma}^a$: angular approach of pinion and gear members

$\bar{\Sigma}^c$: angular term related to contact deformation

$\bar{\Sigma}^e$: angular term related to bending and shear deflexion of teeth and their flexible base

$\bar{\Sigma}^i$: angular term related to initial separation of the profiles

$\bar{\Sigma}^g$: angular term related to surface deformation due to frictional traction

$p(\xi)$: load distributions on a contact line

D : number of tooth pairs in mesh

$\vec{n}$: unit normal to the contact plan in the pinion co-ordinates frame of reference

$\|\vec{r}\|$: radius modulus of the meshing position in the pinion co-ordinates frame of reference.

$\vec{t}$: unit vector parallel to the tangential direction of the treated position in the pinion co-ordinates frame of reference.

3.0 Model and solution

The longitudinal load distribution is governed by the stiffness of the tooth pair in contact. If we disregard other components such as shafts, bearings, housing and exclude the torsion and flexion of the gear blanks, the total stiffness is made up of three components: the contact, bending and shear stiffness of the clamped-free teeth and tooth base.

In this paper, the modelling is limited to relatively short facewidth gears. The determination of load distribution along meshing surfaces is obtained by the simultaneous solution of the well-known relations, written as:

$$\bar{\Sigma}_{kc}^i + \bar{\Sigma}_{kc}^c + \bar{\Sigma}_{kc}^e + \bar{\Sigma}_{kc}^g = \bar{\Sigma}^a \quad (1)$$

$$\sum_D \left[\int p(\xi) \|\bar{r}\| (\bar{n} \cdot \bar{t}) d\xi \right] = T \quad (2)$$

Equation (1) expresses the contact condition for a point with co-ordinates kc in the contact plane of the meshing tooth flanks. Angular terms $\bar{\Sigma}^i, \bar{\Sigma}^c, \bar{\Sigma}^e, \bar{\Sigma}^g$ and $\bar{\Sigma}^a$ are combined pinion and gear factors, respectively initial separation of the profiles, contact deformation, surface deformation due to frictional traction, tooth bending and shear deflexion and their flexible base, and angular approach of the mating teeth.

When equation (1) is an inequality, no load is supported by the surfaces at point kc ; otherwise, point kc belongs to the loaded region of the meshing flanks.

Equation (2) gives the static equilibrium condition between the applied torque (T) and the torque produced by the load distributions ($p(\xi)$) along each contact line of the D tooth pairs in mesh. Terms $\bar{n}, \|\bar{r}\|$ and $\bar{t}$ are respectively the unit normal to the contact plane, the radius of the meshing position, and the unit vector parallel to the tangential direction of the treated position in the pinion reference frame.

The formulation presented above leads to the knowledge of the load sharing between meshing gear teeth and along contact lines, when angle $\bar{\Sigma}^a$ is determined. Several analytical and numerical approaches have been proposed to solve the system [2-15] from which the difficulty in obtaining a precise solution with low computation is evidenced. The objective is now to represent helical gear pairs with as high a precision as can be obtained with the Finite Element Method (FEM), but without all the pre and post-processing work required.

In 1996 and 1997 Gagnon and al. [16-17] extended the Finite Strip Method (FSM), introduced earlier for thick plates, to the analysis of spur and helical gear teeth. In the FSM, which can be considered as a particular case of the FEM, the neutral plane of the gear tooth is represented by two-dimensional finite strip based on functions giving the exact tooth shape. The simple formulation involves less calculation without loss of precision. The bending displacement of the tooth is calculated to within 5 % of that with the FEM.

In counterpart of its speed and 2D simplicity, the FSM lacks versatility in modelling the boundary conditions of the tooth base, and the displacement at a given position is constant throughout the tooth thickness. Thus the FSM cannot account for the effects caused by local geometric variations such as the stress concentration in the fillet area.

In this paper, the FSM is combined to a model of the tooth base constructed by considering the part under the tooth as a series segments of equal width. In each segment, the equilibrium for the two-dimensional plane strain problem is written in Navier's form (or in terms of displacements) and solved by the Finite Differences Method [18]. The series of segments is a repetition of an original segment. To avoid calculation redundancy, the solution to Navier's equations is made once for unit loads, and is repeated for all segments in the series. There lies the main advantage of a 2D solution over a 3D in terms of computing time.

A typical discretization of a helical gear tooth is shown in figure 1. Results obtained with the 2D model and FEM for the helical gear tooth are presented in figure 2. The 3D-FEM mesh uses parabolic Hex-20 elements. The geometric parameters of the gear are given in table 1. Two different 1000 N loads are

applied normal to the surface at the tip edge of the pinion tooth: in Load Case A, the load is uniformly distributed; in Load Case B, the load is concentrated at tooth center.

Table 1 Helical gearset geometric parameters.

	<i>Helical Gear</i>
<i>Pressure Angle</i>	20°
<i>Helix Angle</i>	15°
<i>Module</i>	6,00 mm
<i>Tooth Number</i>	20 x 31
<i>Addendum Factor</i>	1
<i>Dedendum Factor</i>	1,25
<i>X Factor</i>	0,163 [-0,163]
<i>Face Width</i>	70,00 mm
<i>D ; M</i>	15,00 mm 30,00 mm
<i>E ; v</i>	200 GPa ; 0,3

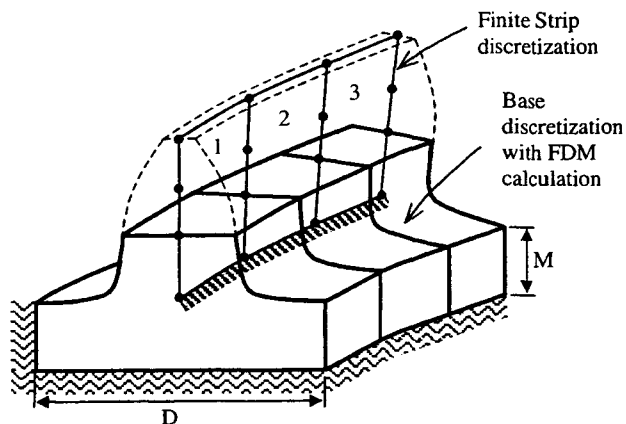
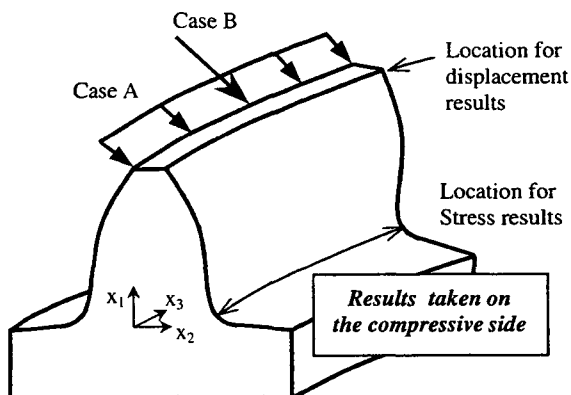
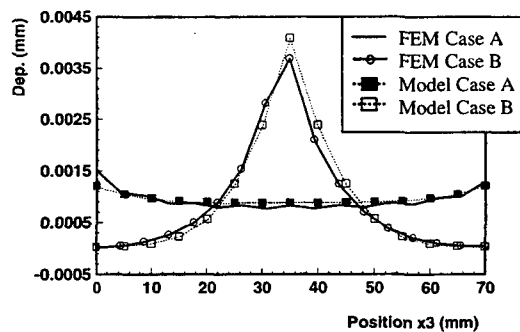


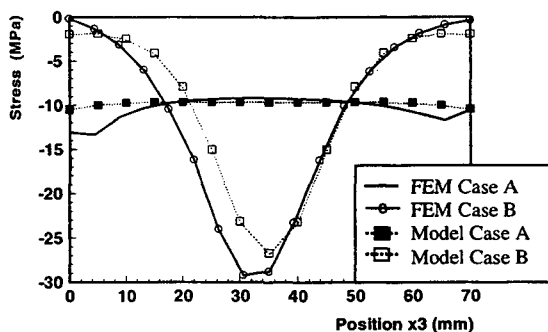
Figure 1 : Typical discretization of a helical gear tooth.



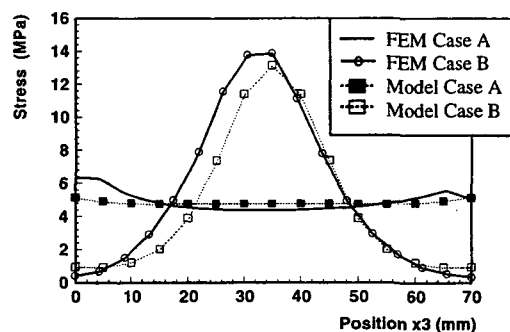
(a) Load cases and results position.



(b) Displacement in the neutral plane.



(c) Minimum principal (compressive side).



(d) Maximum shear (compressive side).

Figure 2 : Displacement and tooth fillet stresses for the pinion