

Contact Stresses in Gear Teeth

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American Gear Manufacturers Association



TECHNICAL PAPER

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ABSTRACT:

It is shown that neither Hertz's line contact theory nor his point contact theory are entirely adequate for the accurate calculation of contact stresses in gear teeth. A numerical procedure is described, which can be used to find the contact stress in cases where the relative curvatures in the contact region are not constant.

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CONTACT STRESSES IN GEAR TEETH

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Introduction

With the use of mismatch in worm and bevel gears, and crowning in helical gears, there is no longer line contact between the teeth, and it is evident that the contact stress cannot be found accurately by Hertz's line contact theory. His point contact theory is also sometimes unsuitable, because the assumption of constant curvature within the contact area is not realistic, and the

actual contact areas encountered in gearing often differ from the elliptical shapes predicted by conventional point contact theory.

In this paper, we will use examples found in worm gearing, but the method described for calculating the contact stress is equally applicable to bevel gears. It may also be useful for helical gears with crowning, where the real stress distribution is probably similar to line contact near the middle of the teeth, and to point contact near the ends.

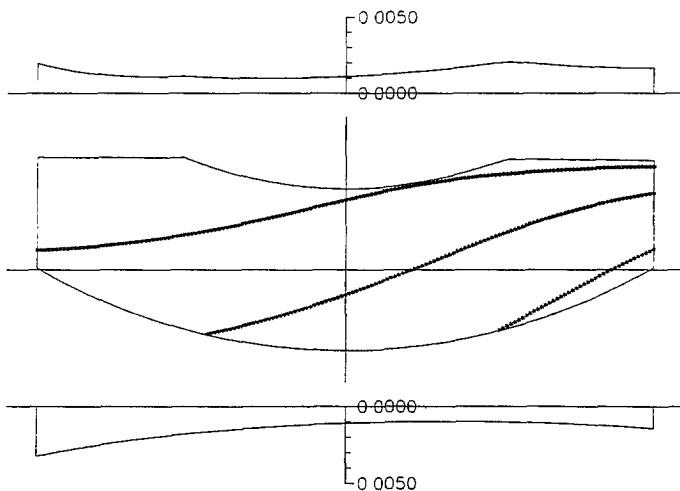


Figure 1.
Gear pair Number 1. Contact Lines.

The Need For a New Contact Stress Theory

A traditional line contact diagram for a worm and worm gear is shown in Figure 1. This is the contact diagram we obtain when there is no mismatch, so that the gear is exactly conjugate with the worm. The three lines shown in the diagram represent the contact lines on three adjacent teeth. To obtain the tooth forces and the corresponding contact stresses, we assume



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that the load intensity is constant along each contact line. From the specified torque, we calculate the total tooth force. We then divide the total tooth force by the total length of the contact lines, to obtain the load intensity, and to find the individual tooth forces we multiply the load intensity by the lengths of the individual contact lines. The contact stress at each point is found by Hertz's line contact theory, using the relative curvature in the direction perpendicular to the contact line.

The assumption of constant load intensity is questionable, even when the worm and the gear are exactly conjugate, and the contact lines extend to the edges of the teeth. If the load intensity is constant along each contact line, it must be independent of the relative curvature, which does vary along the contact lines, and this independence seems unlikely. However, in the case where there is some mismatch the contact areas do not reach the edges of the teeth, and it is quite certain that the load intensity is not uniform. Instead of line contact we have point contact, though the contact areas may be very elongated, and we therefore have to consider the use of Hertz's point contact theory.

Figure 2 shows the contact diagram for

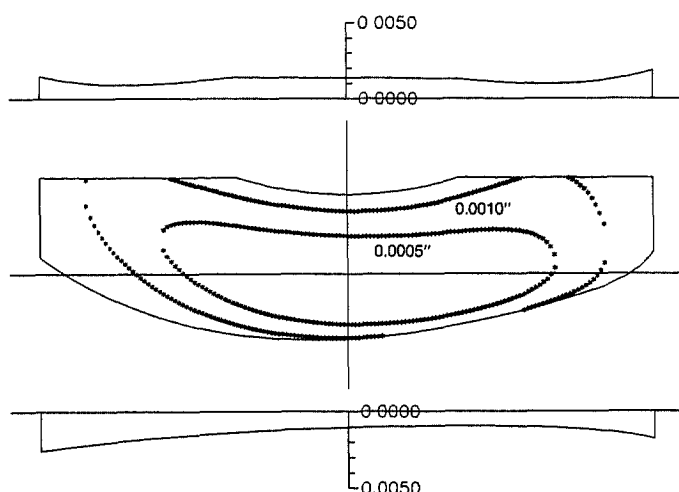


Figure 2.

Gear Pair Number 1. Contact Diagram.

the gear pair of Figure 1, when there is some mismatch between the worm and the gear. The worm is thread-milled with five threads, and has a lead angle of 21.31 degrees. The mismatch is obtained by modifying the profile of the hob that is used to cut the gear, by making its lead slightly shorter than that of the worm, and by swivelling the direction of its axis when the gear is cut, so that it is not exactly perpendicular to the gear axis. The contact diagram shows the contours of constant separation between the worm thread, and the envelope of the gear tooth positions relative to the worm. In other words, it shows how closely the gear teeth approach the worm thread during the entire meshing cycle, assuming that both are rigid and that the angular velocity ratio is exactly constant. The two curves in the diagram show the contours where the separation is 0.0005 inches and 0.0010 inches. If we had included the curve where the separation is 0.00025 inches, this would correspond approximately to the area that would be coloured by the marking compound in a bearing test.

We now consider the gear pair in the same position as in Figure 1, and we calculate the magnitude of the separation between the teeth in each of the three tooth pairs. Figure 3 shows the contours where the separation between the teeth is 0.0010 inches. It is obvious that we have point contact instead of line contact, which is what we would expect when we introduce the mismatch. It also appears that any stress calculation based on line contact will be extremely inaccurate, particularly in the case of the upper tooth pair, where the possible contact area is now much shorter than the line in Figure 1.

There are certain difficulties encountered when we try to use Hertz's point theory. First, the theory was developed to calculate the contact stress when the contact force is known. In our case, we know

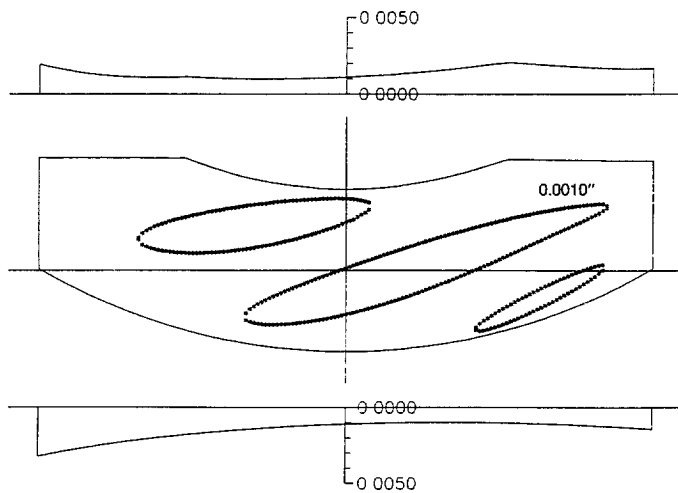


Figure 3.

Gear Pair Number 1. Separation Contours.

the total contact force, but we do not know how it is shared between the tooth pairs. This problem is made more difficult by the fact that there is initially no contact at the upper and lower tooth pairs, and that this contact only occurs when there is some flexibility in the middle tooth pair. The minimum separations in the direction of the worm axis between the three tooth pairs were found to be 0.00033, 0.00003 and 0.00083 inches. These were calculated assuming the teeth are rigid, and that the angular velocity ratio is constant. To understand the effect of these separations we will consider the gear as fixed. First, the worm can make a small rotation, corresponding to an axial displacement of 0.00003 inches, before there is any contact whatsoever. This rotation represents the transmission error under no load, for the current position of the gear pair. Once contact is obtained at the central tooth pair the worm may continue to rotate, due to the contact stress indentation and the bending flexibility of the teeth. As the rotation continues, the upper and lower tooth pairs will eventually come into contact, and the rotation will cease when the three contact forces are in equilibrium with the applied torque. The problem of finding the individual tooth pair contact forces can be represented by the spring system shown in Figure 4. Two of the

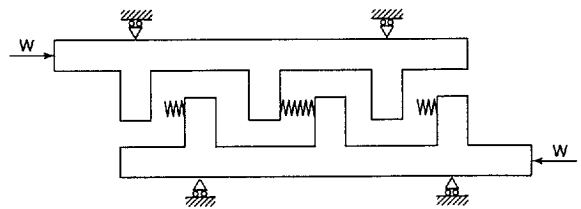


Figure 4.

Equivalent Spring System.

three springs have initial gaps, and the relation between force and compression for each spring is non-linear, as in all contact problems.

The second difficulty we find in using Hertz's point contact theory is that Hertz dealt only with contacting bodies which have constant curvatures in the contact region. This condition implies that the body surfaces near the initial point of contact can be represented by ellipsoids, and that contours of constant separation are then concentric ellipses. The contours in Figure 3 are almost elliptical, so it appears that Hertz's condition may be satisfied for this gear pair. However, Figure 5 shows two separation contours for the upper tooth pair, the inner one at 0.0010 inches and the outer one at 0.0020 inches. Under light load the contact will look like Hertzian point contact, within the elliptical contour, but as the load is increased the contact area

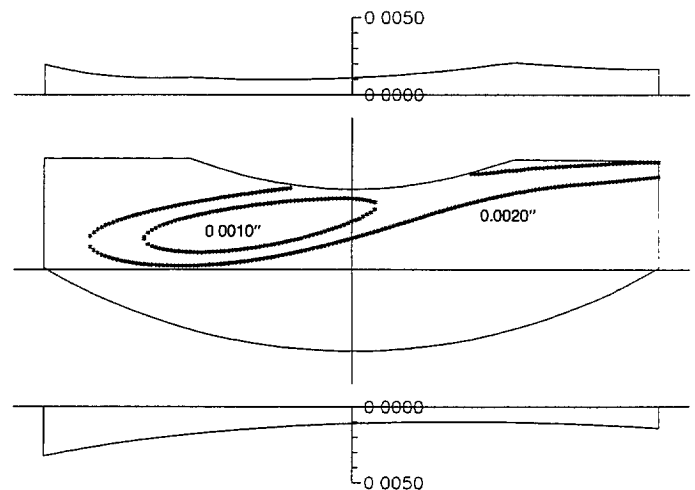


Figure 5.

Gear Pair Number 1. Separation Contours.