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(Revision of ANSI/AGMA 9004-A99)

American National Standard

Flexible Couplings – Mass Elastic Properties and Other Characteristics

ANSI/AGMA 9004-B08

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[Revision of ANSI/AGMA 9004-A99]

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ABSTRACT

This standard provides information and calculation methods related to mass elastic properties of flexible couplings. Properties discussed are coupling weight, polar weight moment of inertia (WR^2), center of gravity, axial stiffness, axial natural frequency, lateral stiffness, lateral natural frequency and torsional stiffness. Calculation examples are provided in informative annexes.

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Foreword

[The foreword, footnotes and annexes, if any, in this document are provided for informational purposes only and are not to be construed as a part of ANSI/AGMA Standard 9004-B08, *Flexible Couplings – Mass Elastic Properties and Other Characteristics*.]

This standard was developed through intensive study of existing practices, standards, text books and literature. The intent of this standard is to offer to rotating equipment designers, builders and users, a standard for design practice and methods of calculation of certain physical and mass elastic properties of flexible couplings. In general, the information in this standard is a consolidation of the most common practices and calculations currently in use by the flexible coupling manufacturers, rotating equipment designers and users.

This AGMA standard utilizes the physical dimensions and properties of the coupling or coupling components for the calculation methods presented. This standard does not cover coupling characteristics that are covered in other AGMA flexible coupling standards, such as coupling balance, which is covered in ANSI/AGMA 9000-C90, *Flexible Couplings – Potential Unbalance Classification*.

Work was started on this standard in 1989 at the suggestion of the AGMA Flexible Coupling Committee. The purpose was to gain uniformity in the methods of calculation of some of the mass elastic properties of flexible couplings.

This revision of ANSI/AGMA 9004-A99 corrects errors in equation 6 (axial natural frequency) and equation 8 (floating shaft coupling with central weight, lateral natural frequency). It also corrects equations E.3 and E.4 in Annex E; reworks Annex G to reflect the changes made to equation 6 and adds a paragraph to Annex G that discusses the damped response to external forced vibration that was added to ANSI/AGMA 9104-A06 during its development. There were also minor editorial and clarification changes to the annexes.

The first draft of ANSI/AGMA 9004-B08 was made in October, 2004. It was approved by the AGMA Technical Division Executive Committee in August, 2008. ANSI/AGMA 9004-B08 was approved as a standard by the AGMA membership on September 30, 2008, and approved as an American National Standard on December 19, 2008.

Suggestions for improvement of this standard will be welcome. They should be sent to the American Gear Manufacturers Association, 500 Montgomery Street, Suite 350, Alexandria, Virginia 22314.

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American National Standard – Flexible Couplings – Mass Elastic Properties and Other Characteristics

1 Scope

This standard presents information and calculation methods for the mass elastic properties and other characteristics of flexible couplings. This data is of importance to system designers for the selection of system components and natural frequency calculations. Calculation methods of the properties of the coupling flexible elements are not included in this standard. Due to the diversity of coupling types, this standard presents generally accepted practices rather than rigorous engineering analysis. Some characteristics are not covered in this standard, such as coupling balance which is covered in ANSI/AGMA 9000-C90, *Flexible Couplings – Potential Unbalance Classification*.

2 Definitions and symbols

2.1 Definitions

2.1.1 Flexible element

The part of a coupling which provides flexibility. Various flexible element designs utilize a number of operating principles to provide flexibility. The design of this element determines the character of the coupling in terms of reaction forces, dynamics and reliability. For this standard, common flexible element types have been grouped into three major categories which are defined below. Note that the character of a particular flexible element type may cross or fall outside the definitions below. Also note the properties of flexible elements themselves are

not covered in this standard. The reader is directed to the appropriate coupling manufacturers for information on the properties of a particular type of flexible element.

2.1.2 Metallic element

A form of flexible element which accommodates misalignment by material deflection of a metal or composite member. These elements are very much like springs in that they have a free form shape and will resist a change in shape with a reaction force. Examples of metallic elements are metal or composite contoured diaphragm, convoluted diaphragm and disc.

2.1.3 Mechanical element

A form of flexible element which accommodates misalignment by sliding or rolling on mating surfaces. These parts normally require lubrication. These elements do not have a free state position. They can be at rest at any combination of axial and angular positions within their flexible capability. Mechanical elements resist change in axial and angular position mainly as a function of shaft torque and coefficient of friction between the mating surfaces. Examples of mechanical elements are gear, grid and pin-bushing.

2.1.4 Elastomeric element

These flexible elements are characterized by the use of an elastomer. There are many types of elastomeric elements which accommodate misalignment through varying degrees of material deflection and sliding motion. Reaction forces of these types of flexible elements are determined by element configuration, material stiffness, coefficient of friction and torque. They can be categorized into two general types, compression and shear, based upon the way torque is transmitted through the flexible element. Because of the great variety of designs some actually fit both categories in varying degrees.

2.2 Symbols

The symbols used in this standard are, wherever possible, consistent with other approved AGMA documents. It is known, because of certain limitations, that some symbols, their titles and their definitions, as used in this document, are different than in similar literature.

Table 1 is a list of the symbols used in this standard, along with the associated terms. The “Where first used” column gives the clause or equation number where the particular symbol is first used.

NOTE: Some of the symbols and terminology con-

tained in this document may differ from those used in other documents and AGMA standards. Users of this standard should assure themselves that they are using the symbols, terminology and definitions in the manner indicated herein. The unit “lb” is a unit of force, “lbf”. See annex J.

Table 1 - Symbols

Symbol	Description	Units	First used
ANF	Axial natural frequency	cpm	4.5.1
D_i	Inside diameter	in	Eq 4
D_o	Outside diameter	in	Eq 4
G	Shear modulus	lb/in ²	Eq 4
g	Local acceleration due to gravity	in/s ²	Eq 2
J	Polar mass moment of inertia	lb-in-s ²	Eq 2
K_B	Equipment bearing lateral stiffness	lb/in	Eq 10
K_C	Coupling tubular shaft or spacer tube portion lateral stiffness	lb/in	Eq 10
K_{d1}	Metallic element axial stiffness of end 1 of the coupling	lb/in	Eq 6
K_{d2}	Metallic element axial stiffness of end 2 of the coupling	lb/in	Eq 6
K_E	Equivalent lateral stiffness of the system	lb/in	Eq 9
K_L	Lateral stiffness of coupling flex element	lb/in	Eq 10
K_S	Connected shaft lateral stiffness	lb/in	Eq 10
K_{sw}	Coupling suspended weight axial spring rate	lb/in	Eq 6
K_T	Torsional stiffness for tube section	lb in/rad	Eq 4
K_D	Torsional stiffness for disk section	lb-in/rad	Eq 5
L	Length	in	Eq 4
l	Distance between flex/load points	in	Eq 7
N_c	Coupling lateral natural frequency	cpm	Eq 7
N_s	Coupling system lateral natural frequency	cpm	Eq 9
N_1	Lateral natural frequency of coupling tubular section	cpm	Eq 11
R	Radius of gyration		4.2
T	Torque	lb in	4.2
t	Section thickness	in	Eq 5
V	Volume	in ³	Eq 1
W	Weight	lb	4.1
W_C	Total weight of coupling	lb	Eq 9
W_{CW}	Central weight	lb	Eq 8
W_{cs}	Suspended coupling center weight	lb	Eq 6
W_n	Weight of the n th component or section	lb	Eq 3
WR^2	Polar weight moment of inertia	lb-in ²	4.2
W_t	Weight of the tubular portion of the spacer	lb	Eq 11
W_1, W_2	Weight of the individual components or sections	lb	Eq 3
X_{cg}	Location of the center of gravity of the complex shape (i.e., half couplings, hubs, etc.) from the shaft end	in	4.3
$\bar{x}_{ncg}$	Distance from shaft end to center of gravity location for the n th component or section	in	Eq 3
$\bar{x}_{1cg}, \bar{x}_{2cg}$	Distance from shaft end to center of gravity location for the individual components or sections	in	Eq 3
α	Angular acceleration	in/s ²	4.2
ρ	Density	lb/in ³	Eq 1

3 Responsibility

The coupling manufacturer is responsible for providing accurate coupling data and information for the following calculation methods. The coupling mass elastic properties will affect the system performance. The customer is responsible for system analysis and actual response.

4 Coupling characteristics

4.1 Weight, W

The weight of a flexible coupling or component is used for the calculation of shaft stress, deflection and bearing loading. See annex A for the weight calculation of a flanged coupling hub. See annex D for the equations for the weight calculation for some common shapes. For a discussion on units of mass and weight, see annex J.

$$W = V \rho \quad (1)$$

where

W is weight, lb;
 V is volume, in³;
 ρ is density, lb/in³.

4.2 Polar weight moment of inertia, WR^2

The WR^2 of a coupling is a useful characteristic for calculating system torsional natural frequency and rotational energy.

The WR^2 of a coupling is a measure of the weight distribution relative to the rotational axes. The polar mass moment of inertia, J , is a measure of a body's resistance to rotational acceleration or deceleration. The relationship between polar mass moment of inertia and polar weight moment of inertia is given by:

$$J = \frac{WR^2}{g} \quad (2)$$

where

J is polar mass moment of inertia, lb-in-s²;
 WR^2 is polar weight moment of inertia, lb-in²;
 g is local acceleration due to gravity,
 $= 386.4 \text{ in/s}^2$.

Coupling WR^2 is usually provided in (lb-in²). However, the conversion to polar mass moment of inertia, J , is needed so that it can be used in many equations. For instance, the rotational analogy for

Newton's second law relates torque, T , and angular acceleration, α , by: $T = J\alpha$, or $T = (WR^2/g)\alpha$.

The polar mass moment of inertia, J , must not be confused with the moment of inertia, I . The moment of inertia is used in torsional windup and shear stress calculations, and does not consider mass.

The WR^2 can be calculated by multiplying the coupling weight, W , by the square of its radius of gyration, R . The radius of gyration is the radius at which a rotating body mass can be said to be concentrated in a thin ring, and still have the same resistance to rotational acceleration.

Annex B shows a sample calculation for the WR^2 of a flanged coupling hub. This example uses the method of adding or subtracting commonly known shapes to make the hub's WR^2 easy to calculate. Annex D provides the equations for calculating the WR^2 of common shapes.

4.3 Half coupling center of gravity (effective center of gravity) location, X_{cg}

The half coupling center of gravity is the location relative to the shaft end where half the coupling weight can be concentrated and provide equivalent bearing and shaft deflection loads. Knowing this value allows calculation of shaft deflection, bearing loads and system critical speeds.

Calculation of the center of gravity is done as shown in many text books in statics, with one exception. Normally, half the center member weight is lumped at the center of each flex element. Typical locations are given in table 2.

Table 2 - Location of lumped mass

Types	Location
Covers	Centerline of support
Diaphragm	Centerline of diaphragm
Disc	Centerline of disc pack
Gear	Centerline of gear mesh
Jaw/elastomer	Centerline of jaws
Pin/bushing	Centerline of bushing
Shafts (bearing supported)	Centerline of support

For most rotating couplings, the center of gravity location is along the center of rotation. The only thing left to determine is the distance relative to the shaft end, X_{cg} . The basic equation is given as:

$$X_{cg} = \frac{W_1 \bar{x}_{1cg} + W_2 \bar{x}_{2cg} + \dots + W_n \bar{x}_{ncg}}{W_1 + W_2 + \dots + W_n} \quad (3)$$

where

X_{cg}	is location of the center of gravity of the complex shape (i.e., half couplings, hubs, etc.) from the shaft end, in;
W_1, W_2	is weight of the individual components or sections, lb;
W_n	is weight of the n th component or section, lb;
$\bar{x}_{1cg}, \bar{x}_{2cg}$	is distance from shaft end to center of gravity location for the individual components or sections, in;
$\bar{x}_{ncg}$	is distance from shaft end to center of gravity location for the n th component or section, in.

The half coupling center of gravity is referenced from the equipment shaft end. By definition, a positive location is beyond the shaft end and negative location is on the equipment shaft.

Annex C shows a sample calculation for half coupling center of gravity of a hub and center member assembly. This example uses the method of adding or subtracting common shapes to make the coupling half center of gravity easy to calculate. Annex D provides the center of gravity location for common shapes.

4.4 Stiffness

Flexible couplings exhibit stiffness torsionally, laterally (radial stiffness), and in many cases, axially. Each coupling type has distinctly different stiffness characteristics which may be constant or variable. When required, the coupling stiffness information should be obtained from the coupling manufacturer.

4.4.1 Torsional stiffness

Flexible couplings exhibit torsional stiffness. Torsional stiffness relates to the amount of torque required to twist the coupling torsionally, and is also called torsional spring rate. For couplings which exhibit nonlinear torque versus deflection characteristics, the torsional stiffness is defined as the slope of the torque versus deflection curve at the torque under consideration. Care should be taken with elastomeric couplings because the dynamic stiffness of elastomers is higher than the static stiffness. Torsional stiffness is a relatively important factor in determining coupling/system torsional natural frequency. See annex E.

The torsional stiffness of an entire coupling can be obtained by test or by calculation. Calculation of

coupling stiffness is done by calculating each component's stiffness in the torque path and summing them as springs in series. Refer to annex F for an example calculation. This section will look at the calculating approaches for components made up of tubular, or disk sections, or both.

4.4.1.1 Tubular sections

A tubular section is defined as a solid or hollow shaft with a finite length through which the torque is transmitted from one end of the shaft to the other. The torsional stiffness can be calculated as follows:

$$K_T = \frac{\pi G (D_o^4 - D_i^4)}{32 L} \quad (4)$$

where

K_T	is torsional stiffness for tube section, lb in/rad;
G	is shear modulus, lb/in ² ;
D_o	is outside diameter, in;
D_i	is inside diameter, in;
L	is length, in.

Where the torque path changes from a tube section to a disk section, the tubular length is taken from the center of the disk section.

4.4.1.2 Disk sections

A disk section is defined as a circular section with a finite thickness through which the torque is transmitted between the outside diameter and the inside diameter of the section. The torsional stiffness of a disk section can be calculated as follows:

$$K_D = \frac{\pi G D_o^2 D_i^2 t}{(D_o^2 - D_i^2)} \quad (5)$$

where

K_D	is torsional stiffness for disk section, lb-in/rad;
t	is section thickness, in.

4.4.1.3 Hub to shaft connection

Some of the parameters that affect torsional stiffness of a hub to shaft connection include: type of fit (clearance, interference, keyed or non-keyed), magnitude of torque, rotational speed and hub flange location.

A simplified method commonly known as "1/3 shaft penetration" was introduced in [2]. When applicable, the 1/3 shaft penetration should be included in the

coupling stiffness. Other method(s) such as those found in ASME 72-PTG-37 [10] or the vendor's standard method can also be used upon mutual agreement.

The 1/3 shaft penetration method assumes, through experience, that the shaft is regarded as unrestrained for one third the length of the hub. The torque travels through only the shaft for the first one-third of the hub shaft engagement, and through only the hub and flange for the remaining two-thirds.

The 1/3 shaft penetration method for a hub to shaft connection can be modeled using the tubular and disk sections from the previous discussion.

4.4.1.4 Other torsional stiffness considerations

Other torsional components of a coupling include stiffness of bolting connections, flexible elements and any other components in the torsional path. The torsional stiffness of a coupling should include all significant stiffness components on the torsional path.

Refer to annex F for examples on modeling and calculating the coupling torsional stiffness.

4.4.2 Axial stiffness

Axial stiffness is a characteristic of flexible couplings where material deflection occurs to accommodate axial shaft displacements. It is commonly used in the calculation of axial natural frequency and axial reaction force. Couplings exhibiting this characteristic include diaphragm, disc and some elastomeric type couplings. Couplings which slide when accommodating axial deflections do not possess axial stiffness, since material deflection does not occur. Chain, gear, grid and some pin-bushing type couplings are the most common type of couplings without axial stiffness factors. However, these types of couplings resist axial motion due to sliding friction characteristics.

The axial stiffness is different for each type of coupling. The coupling manufacturer should supply the coupling axial stiffness to the system designer for the specific coupling application. The coupling axial stiffness may be constant or variable for a specific axial displacement range.

4.4.3 Lateral stiffness

Lateral (radial) stiffness is the ratio of the force to displacement while deflecting the coupling metallic or elastomeric element in a plane perpendicular to

the coupling axis of rotation. The lateral stiffness for those couplings which accept misalignment in two planes by way of angular deflection of the metallic or elastomeric element is determined based on a single metallic or elastomeric element.

Lateral stiffness may be of importance in two areas: the determination of the coupling or system lateral natural frequency, and the lateral loading on system components. Lateral stiffness can be constant or variable depending on torque and deflection range. When required, lateral stiffness should be obtained from the coupling manufacturer.

When the metallic or elastomeric element lateral stiffness is low in comparison to other components, it should be included in the calculation of lateral natural frequency.

4.4.4 Bending stiffness

Metallic element and elastomeric element couplings produce a restoring moment when the flexing element is angularly deflected. Bending (angular) stiffness is the ratio of restoring moment to angular deflection. Couplings which slide while accommodating angular misalignment do not possess angular stiffness since material deflection does not occur. Chain, gear and grid couplings are the most common types of couplings without angular stiffness. However, they do produce a bending moment which is a function of the shaft torque, misalignment and sliding coefficient of friction.

4.5 Natural frequencies

Flexible couplings exhibit natural frequencies depending on the type of coupling being used. The coupling natural frequencies being considered in this standard are axial natural frequency and lateral natural frequencies. A generally accepted simplified method for calculating the torsional natural frequencies of two and three mass systems is contained in annex E.

4.5.1 Axial natural frequency

Couplings which have axial stiffness may exhibit axial natural frequencies. The axial natural frequency may exist because the metallic elements are essentially springs which suspend the mass of the spacer. Figure 1 depicts a model of this system.

Couplings without an axial spring back restoring force generally do not have axial natural frequencies.

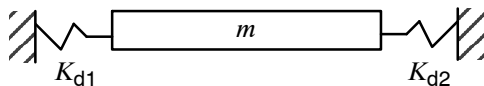


Figure 1 - Coupling axial natural frequency model

It is generally accepted that gear, grid, pin and bushing, chain and most free axial movement elastomer couplings do not have axial natural frequencies.

4.5.1.1 Coupling axial natural frequency calculation

The axial natural frequency of a coupling is calculated by:

$$ANF = 187.7 \sqrt{\frac{K_{sw}}{W_{cs}}} \quad (6)$$

where

ANF is axial natural frequency, cpm;

K_{sw} is coupling suspended weight axial spring rate, lb/in;
 $= K_{d1} + K_{d2}$

K_{d1} is metallic element axial stiffness of end 1 of the coupling, lb/in;

K_{d2} is metallic element axial stiffness of end 2 of the coupling, lb/in;

W_{cs} is suspended coupling center weight, lb.

See annex G for derivation and example calculation of the coupling axial natural frequency.

4.5.1.2 Coupling axial natural frequency evaluation

The axial natural frequency characteristics of a metallic element coupling are determined by the axial stiffness properties of the metallic element and whether it exhibits a damped or undamped response. Single element convoluted styles exhibit an undamped response and will have an axial natural frequency that must be considered. Multiple discs, multiple diaphragm and non-convoluted single element flexible elements exhibit a damped response and axial natural frequency does not have to be considered.

For those couplings that exhibit an axial natural frequency response, the system designer and the coupling manufacturer should agree upon the operational frequencies and the range of values around those frequencies that the coupling axial natural frequency should not fall within. It is

acceptable to operate through the axial natural frequency during start-up.

4.5.2 Coupling lateral natural frequency

The lateral natural frequency of a rotating shaft system can be expressed as that speed such that any change in speed results in a reduction in the lateral displacement. This discussion focuses on the first bending mode of a coupling with two separate flex elements (floating shaft). Cardan shafts and spacer couplings are considered to be floating shafts for this analysis.

Floating shaft coupling systems will have a lateral natural frequency. In general the coupling(s) will have a very high lateral natural frequency and is ignored in the analysis. However, the shafting must be evaluated. Operation of the system at or near the primary lateral natural frequency may cause damage to the system. Therefore, the system should be designed to work significantly below the first critical speed, or to go through the first critical speed quickly and operate significantly above the critical speed.

Calculation methods for the determination of lateral natural frequency can be found in many text books such as [6]. The following methods are given as a guide for initial design purposes; see annex H for the derivation of equations and annex I for the example calculations. The stiffness required for lateral natural frequency calculations are based on the lateral or radial stiffness of the connecting member at the plane of connection. For diaphragm, disc and gear couplings, the value is high and can normally be ignored. For elastomer couplings, the value may be of importance and should be obtained from the coupling manufacturer. It is suggested that the customer consult with the coupling manufacturer for additional assistance and recommendations.

Lateral natural frequency is generally of importance in applications with long shafts or applications operating at high speeds.

The actual operating speed is generally held below 0.7 of the calculated lateral natural frequency for couplings that provide uniform motion under misaligned conditions.

4.5.2.1 Lateral natural frequency calculation for coupling floating shafts

The following simplistic method calculates the lateral natural frequency of a steel floating shaft coupling assembly, which is considered to be simply supported on rigid supports at the flex/load points, see [6], [7] and figure 2.