

Optimal Design of Straight Bevel Gears

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ABSTRACT:

This paper describes the design of a straight bevel gearset with the objective of minimizing the enclosed volume. The specifications for the design are the power requirements, the gear ratio, pinion speed and the material properties. The design variables are the number of pinion teeth, the diametral pitch, and the face width. Constraints are set on facewidth, minimum number of pinion teeth, and the safety factors for bending and pitting strength. The complete analysis for the gearset is based on the rating procedure described in the ANSI/AGMA 2003-A86 standard. The optimization procedure is illustrated through a numerical example and the design is also compared with a spur gear optimization method using Tregold's approximation.

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OPTIMAL DESIGN OF STRAIGHT BEVEL GEARS

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1. Introduction

Traditionally, the design of bevel gears has been based on charts and formulae from handbooks [1, 2, 3]. However, with the growth of Computer-Aided Design (CAD), the designer can now look for light weight gears and compact configuration by formulating the problem as a nonlinear optimization problem. A general non-linear programming problem can be stated as follows :

$$\min f(\mathbf{x}) , \mathbf{x} \in \mathcal{R}^n$$

subject to

$$g_j(\mathbf{x}) = 0 \quad \text{for } j = 1, 2, \dots, m_e$$

$$g_j(\mathbf{x}) \geq 0 \quad \text{for } j = m_e+1, \dots, m$$

$$\mathbf{x}_l \leq \mathbf{x} \leq \mathbf{x}_u$$

Here $f(\mathbf{x})$ is the objective function, $g_j(\mathbf{x})$ are the constraint functions, the first m_e functions defining the equality constraints. $\mathbf{x}_l$ and $\mathbf{x}_u$ are the upper and lower bounds on the variables, and $\mathcal{R}^n$ is the n -dimensional space of real numbers.

In the case of bevel gear optimization, the designer sets a goal to optimize an objective function such as the weight, the enclosed volume,

or the life of the gearing. In doing so, the unknowns such as the number of pinion teeth, diametral pitch, and facewidth are selected as design variables. The given specifications include the gear ratio, power input, pinion speed, and the materials properties. The constraints based on bending strength, pitting failure, and undercutting are also imposed.

Previous research in the field of bevel gear optimization is scarce. Kaixiu [4] describes optimization of straight bevel gears with the objective to minimize the volume of the gearset. He also considers maximum power transmitted as an objective function when the cone pitch is specified. Vanderplaats et al. [5] have developed a spiral bevel gear design program which uses a commercial optimization package. This program handles a wide variety of objective functions like the weight, dynamic load factor or life. But the program can only handle shaft angle 90° , pressure angle 20° and spiral angle 35° because the I and J factor information from the AGMA standards [6] has been curve fitted.

Agrawal [7] has developed a computer aid for the analysis of straight bevel gears based on ANSI/AGMA 2003-A86 standards. This analysis program can generate I and J factors for arbitrary

pressure angles and shaft angles. The optimization procedure described in this paper uses this analysis program repeatedly to iterate to the optimum solution. Thus there is flexibility in the algorithm to accommodate different values of shaft and pressure angles. There are two objective functions considered here, namely the total volume and the enclosed volume of the gear set. The total volume is used to design light weight gears, while the enclosed volume is to ensure a compact design.

For specific cases, the bevel gear optimization is quite similar to the spur gear optimization procedure [8, 9] adopted for minimizing the center distance. This similarity can be derived using Tregold's approximation [10] to convert the bevel gear geometry into an equivalent spur gear. Section 8 of this paper describes how bevel and spur gearing are equivalent to each other with respect to this type of optimization problem.

2. Nomenclature

A_g = gear pitch cone distance.
 A_p = pinion pitch cone distance.
 C_p = elastic coefficient.
 F = facewidth of gear and pinion.
 I = geometry factor for durability.
 J = geometry factor for bending.
 K_a = external dynamic factor.
 K_m = load distribution factor.
 K_s = size factor.
 K_v = internal dynamic factor.
 m_a = facewidth to pitch diameter aspect ratio.
 m_G = gear ratio.
 N_g = number of gear teeth.
 N_p = number of pinion teeth.
 N_{vp} = virtual number of spur gear teeth.
 n_p = pinion speed.
 P = power transmitted.
 P_d = diametral pitch at the large end.
 r_T = cutter edge radius.
 R_g = gear pitch radius at the large end.
 R_p = pinion pitch radius at the large end.
 S_{ac} = allowable contact stress.
 S_{at} = allowable bending stress.
 T = normal operating torque.
 T_c = maximum allowable torque using contact

strength.

T_d = design torque.

T_c = maximum allowable torque using tensile strength.

V = total volume of the gearset.

V_e = enclosed volume of the gearset.

ϕ = pressure angle.

Σ = shaft angle.

γ = pinion pitch cone angle.

Γ = gear pitch cone angle.

3. Problem formulation

In the formulation used for this paper, the specified quantities are ϕ , Σ , m_G , P , n_p , S_{ac} , S_{at} , r_T . The design variables are taken as P_d , N_p and F . The following assumptions are made in the bevel gear analysis:

- (i) The pitch diameter of the heel (large end) of gears is taken to be the representative diameter in calculating the total and enclosed volume. The gears are treated as frustra of cones having the apex at the crossing point of shafts.
- (ii) All the AGMA correction factors, except the geometry factors I and J , are assumed to be unity.
- (iii) For the J -factor calculation, the load is assumed to be at the highest point of single tooth contact.
- (iv) In calculating the geometry factors, the circular tooth thickness factor is assumed to be zero.
- (v) r_T , the cutter edge radius, is normalized with respect to P_d , i.e. $r_T = (\text{actual cutter radius} \times P_d)$. In doing the iterations for optimization, P_d changes continuously, thus changing r_T for a given cutter edge radius. In the present formulation, r_T is kept constant and it is assumed that the actual cutter edge radius is selected from the standard cutters once P_d is determined.

3.1 Objective function

The objective function for the present optimization problem is either the total volume