

Simulation of Meshing, Transmission Errors and Bearing Contact for Single-Enveloping Worm-Gear Drives

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[The Statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

ABSTRACT:

The authors have developed a computerized method for simulation of meshing and bearing contact (TCA) for single-enveloping worm-gear drives. The developed computer programs enable to determine the transmission errors and the shift of bearing contact that are caused by worm and gear misalignment.

An important theorem is proven for determination of the transfer point on the theoretical line of contact where the path of point contact starts for a misaligned worm-gear drive.

A method of assembly for compensation of misalignment is proposed.

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Nomenclature

C - center distance.
 $\mathbf{e}_i$ ($i=f,h,s,q$) - principal direction unit vectors.
 m_{21} - gear ratio
 N_i - number of teeth on the i th gear.
 $\mathbf{\tilde{n}}_i$ - normal to the j th surface represented in coordinate system S_i
 P_d - worm-gear diametral pitch
 q_i ($i=1,\dots,n$) - worm-gear drive parameters which, when deviate from the nominal values, cause misalignment.
 $\mathbf{r}_i$ - position vector to a point on the i th surface.
 r_{b*} - base cylinder radius of the involute worm.
 $\Delta \mathbf{r}_{\sim}$ - displacement of the gear contact point associated with misalignment - the actual displacement due to misalignment plus the displacement due to compensating gear turn to restore contact.
 $\Delta \mathbf{r}_{\sim}^{(i)}$ - displacement of contact point in motion with i th gear due to rotation.
 $\Delta \mathbf{r}_{\sim}^{(i)}$ - displacement of contact point in motion with i th gear due to misalignment.
 $\Delta \mathbf{r}_{\sim}^{(i)}$ - displacement of contact point in motion over the surface of the i th gear.
 (u_i, θ_i) - curvilinear coordinates of the i th surface.
 $\mathbf{v}_i^{(12)}$ - relative velocity of the point on the surface of the worm with respect to that on the surface of the worm-gear.
 α - pressure angle.
 γ - twist angle.
 ψ_c - worm generating tool angle of rotation.
 ϕ_i - angle of rotation of the i th gear during generation.
 $\Delta \phi^*$ - change of orientation of the unit normal associated with misalignment - due to

misalignment itself plus due to compensating gear rotation to restore contact.

ϕ_i - angle of rotation of the i th gear during meshing.

Σ_i - surface of the i th gear ($1=\text{worm}, 2=\text{worm-gear}$).

κ_i ($i=f,h,s,q$) - principal curvatures.

λ_b - lead angle on the base cylinder of the involute worm.

1. Introduction

Worm-gear drives have been a subject of intensive research. Geometry of worm-gear drives with ground worm surfaces and worm surfaces generated by blades have been developed by (Litvin, 1968). (Winter et al, 1981) and (Octrue, 1988) have concentrated their efforts on the determination of contact stresses and fatigue of worm-gear drives that mesh in line contact. The investigation performed by (Bagci, 1987) showed that the real contact stresses for the case of point contact are much higher (up to six times) than for the case of line contact.

Efforts have been made to develop modified methods for generation of worm-gear drives based on worm-hob mismatch (see (Janninck, 1988; Colbourne, 1989)). However, we consider that investigation of modified worm-gear drives must be based on computerized methods of simulation of their meshing and contact - the Tooth Contact Analysis (TCA) programs. It is well known that such an approach applied for spiral bevel and hypoid gears (Gleason Works, 1970; Litvin and Gutman, 1981) is a significant achievement that could improve substantially the technology and the quality of the gears. A similar approach to

the investigation of worm-gear drives is on the agenda and its importance for the theory and practice of worm-gear drives is without doubts.

The TCA program for worm-gear drives must have specific features taking into account the following:

(i) Theoretically, the worm and the worm-gear tooth surfaces are in line contact if the hob is identical to the worm and there is no misalignment. However, the line contact of worm-gear drives is unstable and due to misalignment turns into point contact.

(ii) To localize the bearing contact and fight with the influence of misalignment, it has been proposed: (a) to oversize the hob in comparison with the operating worm, (b) to use a hob with an increased number of threads and to change respectively the crossing angle for cutting, (c) to change the lead angle on the hob and the crossing angle for cutting, and (d) to modify the profile of the hob. To the authors' knowledge, the above-mentioned modifications are based on the experience of manufacture and have not been investigated by a TCA program.

A TCA program for worm-gear drives must cover two parts: (1) Determination of the theoretical lines of contact and the curvatures of the worm and the worm-gear surfaces that enable one to compare the conditions of lubrication and load capacity for various types of worm-gear drives; (2) Investigation of the influence of modifications mentioned above as well as misalignment. Such an investigation must cover not only the determination of real bearing contact but also the transmission errors.

A difficult topic of the TCA program is the determination of the transfer point where the theoretical line contact is changed for point contact. The real bearing contact in the case of point contact will be found as a set of contact ellipses.

The authors hope that the approach developed in this paper will become the basis for a TCA program that will cover all the above-mentioned topics for various types of worm-gear drives. It will also give the opportunity to determine the deviations of real worm and worm-gear surfaces by coordinate measurements and minimize those deviations by correction of machine-tool settings. However, the scope of this paper is limited to the discussions of the influence of misalignment but does not cover the investigation of surface modifications. Hopefully, this will be a subject of future research.

The development of TCA programs is a powerful tool that will open the way to the solution of significant technological problems: (i) localization of bearing contact, (ii) predesign of a parabolic function of transmission errors that will reduce the level of vibrations (Litvin et al, 1989b), (iii) generation of modified worm-gear drives, (iv) determination of contact stresses for worm-gear drives, etc.

The authors of this paper have developed an algorithm for computerized simulation of meshing and bearing contact that is applicable to all types of worm-gear drives -- with involute worm, Flender drives, etc. The developed algorithm has been applied to TCA for misaligned involute worm-gear drives.

The developed approach to computerized simulation of meshing and bearing contact of worm-gear drives covers the following topics:

- (1) Determination of worm surface
- (2) Determination of worm-gear surface
- (3) Determination of instantaneous lines of contact for the theoretical conditions of meshing.
- (4) Transition from line to point contact for a misaligned worm-gear drive.
- (5) Determination of the real contact point path, transmission errors, and the bearing contact as a set of instantaneous contact ellipses.

2. Representation of Worm Surface

There are various types of worm surfaces that have found application in the industry (Litvin 1968, 1989): (1) Archimedes' worm surface that is a ruled undeveloped surface. Such a surface is generated by a straight line L that performs screw motion with respect to the worm axis and intersects the worm axis in this motion. Line L is the shape of the blade that is used for manufacturing of the worm. The cross-section of the worm is the Archimedes spiral. The term "undeveloped" means that a small part of the worm surface cannot be developed on a plane because the normals to the worm surface along the generating line are not collinear.

(2) Convolute worm surface that is also a ruled undeveloped surface. The worm surface is generated by a blade that is installed perpendicular to the helix on the pitch cylinder. The cross-section of the worm is an extended involute.

(3) Screw involute surface that is a ruled developed surface that can be generated (ground) by a plane.

(4) Concave-convex worm surface that is generated by a grinding wheel whose axial section is a circular arc (proposed by Flender). The worm surface is the envelope of the family of surfaces of the grinding wheel. A particular case of the concave-convex worm surface represents a set of circular arcs (proposed by Litvin, see (Litvin, 1989a)).

(5) Klingelnberg's type of worm surface that is generated by a peripheral cutter with conical surface. The worm surface is the envelope of the family of cone surfaces that is generated in reference frame rigidly connected to the worm.

In accordance with the above-mentioned methods of worm generation the worm surface can be represented:

(1) In two-parametric form for Archimedes', convolute, involute and the above-mentioned modification of the concave-convex worm

$$\underline{r}_1 = \underline{r}_1(u, \theta), \quad \underline{N}_1^{(1)} = \frac{\partial \underline{r}_1}{\partial u} \times \frac{\partial \underline{r}_1}{\partial \theta} \neq 0 \quad (1)$$

Here (u, θ) are the curvilinear coordinates (surface parameters) and $\underline{N}_1^{(1)}$ is the worm surface normal.

(ii) In three-parameteric form for a regular concave-convex form and Klingenberg's type of worm

$$\underline{N}_c^{(c)} \cdot \underline{v}_c^{(c1)} = f_c(u_c, \theta_c, \psi_c) = 0$$

$$\underline{r}_1 = \underline{r}_1(u_c, \theta_c, \psi_c) = [M_{1c}] \underline{r}_c \quad (2)$$

Here:

$$\underline{r}_c = \underline{r}_c(u_c, \theta_c)$$

is the vector equation that represents the generating surface -- the surface of the peripheral cutter (grinding wheel) that generates the worm and

$$\underline{N}_c^{(c)} = \frac{\partial \underline{r}_c}{\partial u_c} \times \frac{\partial \underline{r}_c}{\partial \theta_c}$$

is the normal to the generating surface of the tool that is represented in coordinate system S_c . Vector $\underline{v}_c^{(c1)}$ represents the sliding velocity in relative motion (it is a screw motion) with respect to the being generated worm. Designation ψ_c indicates the motion parameter in the generating process. Matrix $[M_{1c}]$ describes the coordinate transformation in transition from S_c to S_1 .

The worm surface Σ_1 and the generating surface Σ_c contact each other along a line and the normal to the worm surface coincides with that to the generating surface at each point of that line. This normal can be represented by

$$\underline{N}_1 = [L_{1c}] \underline{N}_c \quad (3)$$

where $[L_{1c}]$ is obtained from $[M_{1c}]$ by eliminating the fourth row and column.

It is obvious that the representation of the worm surface by

$$\underline{r}_1 = \underline{r}_1(u_c, \theta_c, \psi_c), \quad f_c(u_c, \theta_c, \psi_c) = 0 \quad (4)$$

is equivalent to the direct representation of the worm surface in two-parameteric form

$$\underline{r}_1 = \underline{r}_1(u, \theta)$$

Henceforth, for the purpose of simplification of notation we will consider that the worm surfaces of all types are represented by equations (1).

3. Representation of Worm-Gear Surface

The worm-gear surface Σ_2 is generated by a hob whose surface is identical to worm surface Σ_1 . The meshing of the worm with the being generated worm-gear is simulated in the process of generation. The worm-gear surface Σ_2 is represented as an envelope of the family of surfaces Σ_1 that is generated in S_2 by the

following equations:

$$\underline{r}_1 = \underline{r}_1(u, \theta), \quad \underline{r}_2 = [M_{21}] \underline{r}_1,$$

$$\underline{N}_1^{(1)} \cdot \underline{v}_1^{(12)} = f(u, \theta, \phi) = 0 \quad (5)$$

Equations

$$\underline{r}_2 = \underline{r}_2(u, \theta, \phi), \quad f(u, \theta, \phi) = 0 \quad (6)$$

represent the worm-gear surface in three-parameteric form, but with related parameters. The angles of rotation of the worm and the gear are related in the process of generation and ϕ designates the generalized parameter of motion. Vector $\underline{v}_1^{(12)}$ is represented in S_1 and designates the relative (sliding) velocity of the worm with respect to the worm-gear.

4. Representation of Lines of Contact.

The lines of contact on the worm surface are determined by the equations

$$\underline{r}_1 = \underline{r}_1(u, \theta), \quad f(u, \theta, \phi) = 0 \quad (7)$$

where the value of ϕ is fixed for each contact line. Similarly, the lines of contact on the worm-gear surface are determined by the equations

$$\underline{r}_2(u, \theta, \phi_2) = [M_{21}(\phi_2)] \underline{r}_1(u, \theta), \quad f(u, \theta, \phi) = 0 \quad (8)$$

where again the value of ϕ is fixed for each contact line. The lines of contact in the fixed coordinate system are determined by the equations

$$\underline{r}_f(u, \theta, \phi) = [M_{f1}(\phi_1)] \underline{r}_1(u, \theta), \quad f(u, \theta, \phi) = 0 \quad (9)$$

where $[M_{f1}]$ describes the coordinate transformation from S_1 to S_f . The totality of the contact lines in S_f represents the so-called surface of action.

5. Principal Directions and Curvatures on Worm-Gear Surface. Instantaneous Contact Ellipse.

The determination of the principal curvatures and directions on the worm-gear surface is a difficult problem due to the extreme complexity of the worm-gear surface. We recall that these equations represent the surface in three-parameteric form. The solution to this problem can be substantially simplified using the approach proposed in (Litvin, 1989a) that enables to express the principal curvatures and directions of the envelope in terms of those of the generating surface (see Appendix A). Knowing the principal curvatures and directions of the two contacting surfaces, it becomes possible to determine the dimensions and orientation of the instantaneous contact ellipse (Appendix B). The dimensions of the contact ellipse are proportional to the elastic approach of the contact surfaces.

6. Algorithm for Simulation of Meshing and Bearing Contact for a Misaligned Worm-Gear Drive.

The misalignment of a worm-gear drive is caused by a change of center distance, the twist