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American Gear Manufacturers Association



TECHNICAL PAPER

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ABSTRACT:

This paper deals with the design of helical gears that have minimum transmission error and, at the same time, are less sensitive to manufacturing errors. The paper addresses two stages in design: 1) Design generation stage where feasible designs are generated for a given specification and 2) Design of profile and lead modifications that minimize transmission error and its sensitivity to manufacturing errors. The paper presents a brief discussion on how one can effectively minimize transmission error in helical gears by combining both lead and profile modifications.

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American Gear Manufacturers Association
1500 King Street, Suite 201
Alexandria, Virginia, 22314

October, 1990

ISBN: 1-55589-561-1

A PROCEDURE THAT ACCOUNTS FOR MANUFACTURING ERRORS IN THE DESIGN MINIMIZATION OF TRANSMISSION ERROR IN HELICAL GEARS

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1 INTRODUCTION

1.1. Background

This paper focuses on the noise minimization aspect of gear design and takes into account the effect of manufacturing errors on noise and vibration. Gear design is a combination of two processes, namely, synthesis and analysis. The first step generates a set of feasible solutions which satisfy the design requirements and physical constraints. This procedure specifies the geometry of the gear and its strength characteristics, while the second step takes the feasible solutions and evaluates their performance for such issues as durability, lubrication, noise, and vibration.

The prediction of gear noise has always been a major concern in gear design. In recent years, researchers have established a relationship between static transmission error and gear noise. Studies typically employ analytical tools to predict the static and dynamic transmission errors and subsequently, gear noise reduction leads to minimizing both transmission error and mesh stiffness variation.

The static transmission error is the deviation in rotation from its ideal position when a gear pair is rotated with constant torque. Welbourn (1979) defined it as "the difference between the actual position of the output gear and the position it would occupy if the gear drive is perfect (infinite stiffness and conjugate teeth)." This error is usually expressed in terms of the displacement along the line of conjugate gear action. The two main causes of the transmission error at the meshing frequency and its harmonics are: 1) Non-ideal tooth profile and

lead due to designed tooth modifications and manufacturing errors, 2) Elastic deflection of the gear tooth due to the transmitted load.

The explanation for the variation in the transmission error and mesh stiffness lies in the conjugate action of gears. The number of pairs of teeth in contact for a low contact ratio spur gear pair oscillates between one and two. This leads to a sudden increase or decrease in the mesh stiffness and hence, results in the variation of the transmission error. One way to reduce this variation is to modify the profile of the gear tooth (by relieving the tip/root of the gear tooth) such that one tooth starts unloading when the second tooth makes initial contact. This process (called profile modification) lessens the chance of premature gear tooth impact at the initiation of tooth contact.

Profile modification may be characterized by two parameters: 1) the start radius of profile modification and 2) the amount of relief at the tip/root of the gear teeth. The amount of tip relief is usually estimated as the deflection of the gear mesh at the maximum load. In our model, the shape of the modification can be either linear (linear profile modification) or quadratic (parabolic profile modification). However, this design procedure may produce lower contact ratios or substantial shortening of the path of contact when operated at lower loads.

Premature gear tooth contacts in helical gears are reduced by modifying the tooth in both profile and lead directions. We characterize the lead modification by two parameters: 1) the starting position along the face width and 2) the amount of relief at the end faces of the gear

tooth. Lead modifications (also known as crowning) are usually applied to reduce the adverse effects of shaft misalignment.

This paper deals with determining: 1) the basic gear design (numbers of teeth, pressure angle, etc.) and 2) the profile modification that minimizes the transmission error, and at the same time, is least sensitive to manufacturing variance. We apply the concept of **Statistical Optimization** developed by Taguchi (1978) and adapt this methodology to gear design. An existing program called the Load Distribution Program (Houser, 1990), is used to evaluate the transmission error as a function of elastic properties of the gear drive and the initial separation (tooth modifications). The Load Distribution Program takes into account the shaft bending, shaft torsion, gear tooth bending, transverse shear, Hertzian compression, and base rotation/translation of the gear tooth.

1.2. Literature Review

Optimization techniques have been used in many areas of gear design. Typical objective criteria include weight, space and load capacity. Some gear designs are limited by bearing size. Gear designers seek to find the optimum helix angle in helical gears such that the supporting bearings can withstand the axial thrust without substantial decrease in the load sharing of the gear mesh. In cases where center-distance is not a constraint, designers aim at the least center-distance to minimize space and pitch-line velocity. The choice of the design variables such as diametral pitch, pressure angle, and number of teeth is usually based on the strength characteristics of the gears. Little research has been reported on the effect of these variables on transmission error.

Modeling of the transmission error started with simple spring models for spur gear teeth. Laskin (1968) approximated a tooth of a spur gear as a cantilever beam with deflections due to bending, shear, and Hertzian stresses at the contact zone. Tavakoli and Houser (1986) used a procedure similar to Box's complex method (Reklaitis, 1983) to determine the profile modification that resulted in minimized transmission error. Conry and Seireg (1972) developed a model for estimating transmission error in helical gears. Their model is now incorporated in the Load Distribution Program. Lin, et al., (1989) studied the effect of profile modification on low contact ratio spur gears. He recommended parabolic tooth profile modification on gears that operate over a range of loads and linear tip modification on gears that operate at a constant load. Biggert and Houser (1988) presented a method for evaluating the sensitivity of transmission error to errors in manufacturing and to operating at off-design loads. They considered four design profiles and studied their sensitivity to deviations in profile and torque.

Umezawa, et al., (1985) studied the effects of profile and lead modifications on helical gear

meshes with different contact ratios. They stated that helical gears with face contact ratio (FCR) greater than 1.0 generally operate with low vibration levels and modifications are very effective for gears with total contact ratio greater than 2.0 and FCR less than 1.0. Though research has been reported on the effect of lead and profile modifications on transmission error and load distribution in helical gears, little work has been reported on the interactions between lead and profile modifications and how they can be combined effectively to reduce transmission error.

In recent years, there have been a growing number of statistical optimization techniques applied to manufacturing quality control. Taguchi (1978) introduced the concept of parameter design as a cost-effective means of improving the quality of a product whose manufacturing process involves significant variability or "noise." He recognized that a process/product can perform its intended function at many settings/values of design parameters. The parameter design concept "reduces variation in performance by reducing the sensitivity of an engineering design to sources of variations rather than controlling the sources." Kackar (1985) presented statistical methods for conducting parameter design experiments.

Hunter (1985) discussed several statistical tools that are used to support Taguchi's approach to product design. d'Entremont and Ragsdell (1988) developed a non-linear programming code which applies Taguchi's quality-loss concepts to design optimization. Sundaresan et al. (1989) introduced a procedure to design spur gears that have minimum transmission error and at the same time, are less sensitive to manufacturing errors. This paper applies the same methodology to helical gears.

1.3. The Approach

Most gear design problems present a list of requirements and constraints. In our case, we will use the following: 1) Requirements: center distance, reduction ratio, and transmitted torque are all fixed, and 2) Constraints: maximum outer diameter, maximum face width and maximum stress. Our proposed procedure generates candidate gear designs which meet these requirements and, in addition, satisfies geometrical constraints such as avoiding undercut, having a minimum root clearance and working depth. Depending on the specifications and the step size of change in design variables, there may be anywhere between 10 to 100 feasible designs. The candidate designs are then analyzed and their tooth profiles are modified to reduce transmission error.

The optimization programs used in the analysis predict the profile modification that results in minimized transmission error. However, manufacturing variance associated with the modification may result in a level of

transmission error far larger than that occurs at the peak optimum. The proposed procedure determines the profile modification that minimizes transmission error and, at the same time, is least sensitive to manufacturing variance. The procedure subjects every candidate design to peak and statistical profile optimization. In effect, an exhaustive search strategy is taken for the "best" design. The procedure thus aids the designer in studying the effect of each design variable on performance issues such as noise and vibration. The work presented in this paper will be the basis of a more sophisticated statistical optimization scheme that encompasses not only the profile modification but also the basic design generation stage that includes discrete design variables such as number of teeth, pressure angle, etc.

Section 2 gives the problem statement. Section 3 describes the proposed procedure for the robust design, starting with the basic design generation (section 3.1), followed by a discussion on the effect of tooth modifications on transmission error (3.2), peak optimization (3.3) and statistical optimization (3.4) of transmission error. Section 4 illustrates the procedure with two examples. Conclusions and future directions of research comprise section 5.

1.4 Nomenclature

a_t	Addendum coefficient of the tool (hob)
a_p	Addendum coefficient of the pinion.
a_g	Addendum coefficient of the gear.
C_d	Center distance.
D_{op}	Outer diameter of the pinion.
D_{og}	Outer diameter of the gear
D_p	Search domain in peak optimization
D_s	Search domain in statistical optimization.
f_c	Transmission error at the center node.
f_i	Transmission error at the corner nodes.
f^*	Optimal performance in Ragsdell (1988) criterion.
F	Objective function during statistical optimization.
L_{min}	Minimum theoretical total length of line of contact.
n	Reduction ratio
N_1, N_2	Number of teeth on the pinion and the gear respectively.
P_d	Generating diametral pitch
R	Resultant vector used for the movement of point C.
r_t	Tool tip radius.
S/N	Taguchi's signal to noise ratio.
s^2	Sample variance of performance
t_{pp}	Pinion tooth thickness at the operating pitch circle.
t_{pg}	Gear tooth thickness at the operating pitch circle.
x_1	Start roll angle of modification,
x_2	Amount of tip relief

x^*	Peak optimum.
x^{**}	Statistical optimum.
$\bar{y}$	Performance sample mean.
α	Weighting constant in the objective function (F)
σ^2	Variance
ϕ	Generating pressure angle of the gear
FCR	Face contact ratio
PCR	Profile contact ratio
LDP	Load Distribution Program.
PPTE	Peak to peak transmission error.
SAP	Start of active profile
SFTFH	Sum of the first three fourier amplitudes of the transmission error
SI	Sensitivity index.
TIF	True involute form

2. PROBLEM DEFINITION

While gear design problems come in various forms, this paper uses the following problem formulation to illustrate the proposed procedure.

1) Specifications

- 1) Fixed center distance between shafts (C_d).
- 2) Speed reduction ratio (n)
- 3) Design torque (T)

2) Design variables

- 1) Discrete geometric variables (Number of teeth, etc.)
- 2) Profile modification variables
 - a) Start roll angle of modification (point P)
 - b) Amount of tip modification (distance BQ)

Fig. 1 shows the profile of a gear tooth with radius scaled in terms of the tooth's roll angle. Line AB represents the true involute form of a gear tooth from its root to tooth tip.

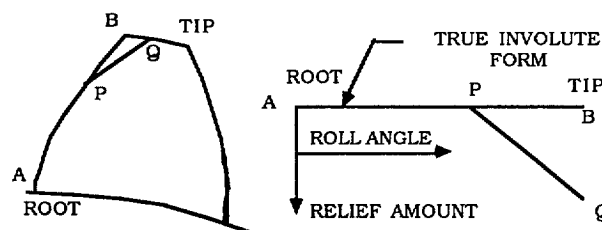


Fig.1 Profile Modification

3) Constraints

- 1) Physical constraints (eg. minimum clearance)
- 2) Stress constraints (eg. maximum bending and contact stresses)

4) Inputs

Variation in profile modification variables.

5) Objective function

Minimize transmission error and its variation due to errors in profile modification

Fig. 2 shows the steps in the procedure for solving the problem.

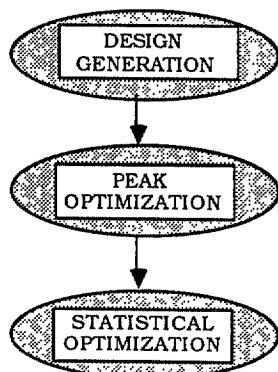


Fig. 2 Stages in the procedure

3. STATISTICAL DESIGN FOR NOISE

3.1 Design Generation

For a given center distance (C_d), torque (T), and reduction ratio (n), the candidate designs are generated by discretely varying the following parameters:

- 1) Generating diametral pitch (P_d)
- 2) Generating pressure angle (ϕ)
- 3) Numbers of teeth on the pinion and the gear (N_1 and N_2)
- 4) Helix angle (ψ)
- 5) Tooth thickness at the operating pitch diameters (t_{pp} and t_{pg})
- 6) Addendum coefficient of the hob (a_t)
- 7) Outer diameters of the pinion (D_{op}) and the gear (D_{og}) or the addendum coefficient of the pinion (a_p) and the gear (a_g).
- 8) Tool tip radius (r_t)

The number of design variables are reduced to five (ψ , P_d , N_1 , a_t and a_p) by making the following assumptions:

- 1) The tool tip radius coefficient (r_t/P_d) is the same for all designs.
- 2) All gear pairs are assumed to possess backlash which is shared equally between the pinion and the gear.
- 3) The addendum of the pinion is equal to the addendum of the gear.
- 4) The face width is assumed to be the same for all gears. This indirectly implies that the weight of each gear pair is approximately the same.
- 5) In the examples, the gear designs had a generating pressure angle of 20° . This assumption was made to reduce the number of candidate designs.
- 6) The generating diametral pitch was varied in the neighborhood of the operating diametral pitch (which is determined once the number of teeth on the pinion and the gear are selected).

The procedure generates the candidate designs by discretely changing the values of the five design variables subject to the following constraints:

- 1) *Avoiding undercut* : This can be done by either limiting the minimum number of pinion teeth or by implementing hob profile shift on the pinion.
- 2) *Minimum clearance* at the root of the gear teeth, ie, addendum of one element should be less than the dedendum of the other.
- 3) Minimum working depth.
- 4) Minimum top land thickness of $(0.3/P_d)$.
- 5) Minimum face and profile contact ratio.
- 6) Minimum roll angle of 15° at the pitch circle to avoid tight and close meshes.
- 7) Bending strength and surface durability.

When a range is specified for gear reduction ratio (eg. 6.23 ± 0.05), not all pinion tooth numbers may be feasible. This limits the number of possible combinations of design variables.

The bending stress is evaluated only for those gears that satisfy the first six constraints. Fig. 3 illustrates the design generation process. The importance of the sixth constraint lies in the fact that not all existing spur gears can be converted to helical gears by introducing a helix angle. As one increases the helix angle, the base radius moves closer to the operating pitch diameter and hence the roll angle at the pitch circle becomes very small.

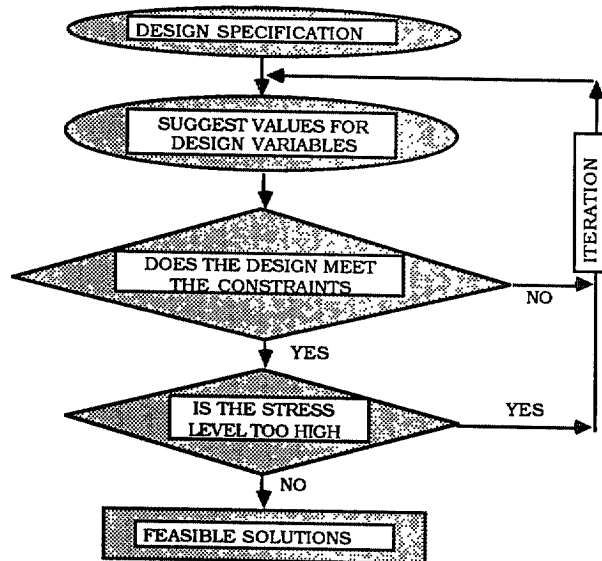


Fig. 3. Design Generation

The procedure uses the iterative technique presented by Errichello (1981) to calculate the bending geometry factor of the gears. This approach inscribes a parabola inside the gear tooth and determines the critical point on the tooth profile that is tangential to the parabola. Contact stress is evaluated using the method presented by Colbourne (1987).

The solutions that satisfy all of these constraints become *candidate designs* of the problem under consideration. The number of candidate designs depends on 1) the