

Kinematic Analysis of Transmissions – Based on the Finite Element Method

by: A. L. Sytstra, Delft University of Technology

American Gear Manufacturers Association



TECHNICAL PAPER

Kinematic Analysis of Transmissions – Based on the Finite Element Method

A. L. Sytstra
Delft University of Technology

[The Statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

ABSTRACT:

In order to evaluate the kinematic properties of a design of a transmission in its early stages, a computer program has been developed. By means of geometric reasoning a finite element model is deduced from the conceptual design which has been built using an experimental 3D object editor. Since the interrelations between the objects are not specified by the designer they are found by the computer by supplying a set of rules. The following kinematic analysis uses a mixed Euler/Lagrange description and detects mechanisms (possible infinitesimal displacements of the nodal points without causing strain in the elements) in the transmission. The mechanisms found are visualized by the object editor by means of alternating images of the design on the screen which gives a real idea of motion. A well-founded decision can be made whether the conceptual model has to be changed, rejected, or can be accepted. The method of analysis used gives a good start for a static and dynamic analysis.

Copyright © 1990

American Gear Manufacturers Association
1500 King Street, Suite 201
Alexandria, Virginia, 22314

October, 1990

ISBN: 1-55589-568-9

1. INTRODUCTION.

This chapter describes the reasons underlying the kinematic analysis of transmissions (**why**), the model of the transmissions used (**what**), and the method of analysis (**how**).

WHY: Overdimensioning transmissions as a result of designing for extreme load situations can be avoided at the cost of a complete dynamic analysis. If the response due to the stochastically varying power can be determined, more efficient and lighter constructions with a longer life expectancy can be designed. The Finite Element Method (**F.E.M**) and availability of computers can be successfully applied here.

However, it is possible that dynamic calculations based on the finite element method show unexpected results, or that the programs used fail during the calculation. This can be caused by:

- Model definition errors.
- Seen from a kinematic viewpoint the model is not properly defined. Unforeseen mechanisms (fields of possible infinitesimal displacements without strain in the elements) are present.

These situations result in ill conditioned or even singular matrices, and attempts to invert them result in program failures. Tracing the causes of these errors is a very difficult and time consuming job. This is why the following ideas were proposed:

- use the computer in generating the model of a transmission.
- After definition of the model, its kinematics should first be analyzed before we continue with a dynamic analysis. We will first have to know what the possible mechanisms are, what is going to move, and in what rate these movements may occur.

WHAT:



A transmission can be described as the part between the drive and the driven machine needed to couple two non-fitting characteristics. Since this covers too wide an area a restriction has to be made. A first set up was to build up transmissions with shafts, cones, toothed wheels (gears), fixed bearings in space and beam elements (connecting rods between objects or a model of the gear case).

HOW: The analysis is based on the finite element method because:

- complete automatization of the kinematic analysis is possible;
- a part of the basis for the static and dynamic calculations is obtained, to which the kinematic analysis can be extended.
- there are no restrictions on orientations in space.
- the analysis can be generalized to more element types.

The results of the analysis should contain at least:

- Possible infinitesimal displacements without deformation of the elements ("mechanisms").
- The distribution of the velocities in such a possible motion.

We are interested in which parts are moving and at what rate.

2. MODEL DEFINITION AND GENERATION. (THE USE OF COMPUTER PROGRAMS).

This chapter describes how the computer can aid the non F.E.M. specialist designer in generating a finite element model for the analysis of transmissions.

Initially the computer has to be given information about the construction. A hand made input file containing the finite element model can only be used by specialists, since designers do not think in terms of finite elements (see fig. 1a). In order to meet prevailing standards about user friendliness we made use of an experimental 3 dimensional menu driven object editor (written by K. van der Werff & A.L. Schwab of the T.U. Delft). With the aid of this editor a central model of the transmission is created. By a central model we mean that the same model is used as input for visualization on screen and calculation programs (see fig. 1b). Some explanation about the editor and the design process used is given.

The designer usually starts with nothing else than some ideas in his head and an empty memory in the computer (see fig. 1c). He then wants a quick evaluation of some conceptual designs by means of a user friendly modeling program connected to some powerful tools for analysis. The object editor allows him to place objects in space (shafts, bearings, gears etc. as they can be parts of real structures) without specifying any interrelations and details. In this way his thoughts are shaped and a CONCEPTUAL MODEL is generated which can be analyzed and dimensioned by means of finite element based calculations. These calculations (applications) require an application model which is in our case a finite element model.

In the finite element approach the transmission is modeled by a set of nodal points, interconnected by the finite elements. Contact between the bodies is obtained by allowing the objects to share nodal points. As distinct from the conceptual model, this model contains relations between the elements.

By geometric reasoning the finite element model is deduced from the conceptual model. For this purpose a program is written which can be considered as the coupling between drawing (designing) and calculations and enables the designer to use only one model (fig 1b).

An example of a rule used in the geometric reasoning part:

If a toothed wheel is supported by a shaft (both the same position,orientation and radii)

then the shaft is split into two parts and the three elements are assigned the same nodal number.

The splitting of the shaft is necessary because a nodal point is needed to mount the gear on.

3 KINEMATICS OF RIGID BODIES.

This chapter describes the basics of the theory of kinematics of rigid bodies as it is used in the detection of mechanisms occurring in a transmission.

In the finite element approach a transmission is modeled by a system of nodal points connected by finite elements ([1],[4],[6]). The number of degrees of freedom can be limited by defining deformations for the elements (e.g. bending, torsion, elongation) as function of the nodal displacements and assuming that the strains remain zero (see the example on page 12).

It can be seen that we have two important spaces

X = space of position parameters of the nodal points. (1)

E = space of deformation parameters of the elements. (2)

The non-linear relation between the X and E space is given by:

$$\vec{e} = \mathbf{D}(\vec{x}) ; \quad \mathbf{D}: X \rightarrow E ; \quad \vec{e} \in E ; \quad \vec{x} \in X ; \quad (3)$$

This relation is called THE 0th ORDER CONTINUITY EQUATION.

Since $\mathbf{D}$ is non-linear, system behavior is approximated by linearization for simplicity.

$$\dot{\vec{e}} = \mathbf{DD}(\vec{x}) \dot{\vec{x}} ; \quad \mathbf{DD}: \dot{X} \rightarrow \dot{E} ; \quad \mathbf{DD} \text{ is linear} ; \quad (4)$$

$\mathbf{D}$ is the differentiation operator and by a "." (dot) we mean differentiation with respect to time.

This relation is called THE 1th ORDER CONTINUITY EQUATION which expresses the rate of change of deformations in terms of the rate of change of position parameters in a current position.

The problem we want to solve is : what rate of change of position parameters $\dot{x}$ do not result in a rate of change of deformations $\dot{e}$ of the elements. This amounts to determining the kernel of the first order continuity equation and can be achieved by solving the following equation:

$$\vec{0} = \mathbf{DD}(x) \dot{\vec{x}} \quad (5)$$

Before solving this system, different X spaces and the concepts 'kinematically indeterminates' and 'first order transfer function' are introduced (see[1],[4],[6]).

X can be subdivided into three different subspaces: (6)

X^0 :the space of constant position parameters; These are given and constant in time.

X^m :the space of prescribed position parameters; These can be arbitrarily prescribed by the designer. (e.g. the input velocity of a transmission)

X^c :the space of remaining position parameters;

The three subspaces do not overlap; X is the Cartesian product of X^0 , X^m and X^c .

The kinematically indeterminates are: the set of position parameters of which the velocities have to be prescribed in order to eliminate

all unwanted degrees of freedom (they then become the X^m space). The rate of change of all position parameters can be calculated in terms of the rate of changes of the kinematically indeterminates (the inputs): This is called the first order transfer function (DF).

$$\dot{\vec{x}} = DF \dot{\vec{x}}^m ; \quad DF: \dot{\vec{x}}^m \rightarrow \dot{\vec{x}} ; \quad DF \text{ is linear.} \quad (7)$$

The first order transfer function is deduced from the first order continuity equation by two successive steps:

- subdivide the $DD(\vec{x})$ matrix into three submatrices ($D^o D(\vec{x})$, $D^m D(\vec{x})$, $D^c D(\vec{x})$) corresponding to the subdivision of the X space into X^o , X^c and X^m .

$$\vec{0} = DD(\vec{x}) \dot{\vec{x}} = D^c D(\vec{x}) \dot{\vec{x}}^c + D^m D(\vec{x}) \dot{\vec{x}}^m + D^o D(\vec{x}) \dot{\vec{x}}^o \quad (8a)$$

- substitute $\dot{\vec{x}}^o = \vec{0}$ in (8a) (constant position parameters)

$$\text{this leads to : } \dot{\vec{x}}^c = -[D^c D(\vec{x})]^{-1} [D^m D(\vec{x})] \dot{\vec{x}}^m = DF \dot{\vec{x}}^m \quad (9)$$

For the sake of clearness we rewrite (8a) and (9) into (8b) and (10) using a new notation in which the vectors $\dot{\vec{x}}^c, \dot{\vec{x}}^m, \dot{\vec{x}}^o$ are placed above their corresponding matrices.

$$\vec{0} = [DD(\vec{x})] \dot{\vec{x}} = [D^c D(\vec{x}) \quad D^m D(\vec{x}) \quad D^o D(\vec{x})] \begin{matrix} \dot{\vec{x}}^c \\ \dot{\vec{x}}^m \\ \dot{\vec{x}}^o \end{matrix} \quad (8b)$$

Inverting the $D^o D(\vec{x})$ submatrix in the left part of (10) results in the right part which is similar to equation (9).

$$\begin{bmatrix} \dot{\vec{x}}^c & \dot{\vec{x}}^m & \dot{\vec{x}}^o \\ D^c D(\vec{x}) & D^m D(\vec{x}) & D^o D(\vec{x}) \end{bmatrix} \vec{0} \implies \begin{bmatrix} \dot{\vec{x}}^c & \dot{\vec{x}}^m & \dot{\vec{x}}^o \\ I & -DF & * \end{bmatrix} \vec{0} \quad (10)$$

The * means : "is not of interest here".

The remaining problem is that the X^m space has to be detected by the computer itself since we are in search for it. This can be achieved by decomposing the original X space of the system in a X^o space and the remaining part called X^{remain} . By a 'sweeping' matrix-procedure the X^{remain} space can be decomposed into a X^c space and the required X^m space. By sweeping a matrix we mean adding rows and multiplying them by a scalar until unit columns appear, as many as possible. To obtain the identity matrix I in (11) we may have to interchange columns of the $D^{\text{remain}} D(\vec{x})$ and the corresponding variables with them.

Solving the system $\vec{0} = DD(\vec{x}) \dot{\vec{x}}$ can now be represented with the aid of the consecutive matrices:

$$\vec{0} = \begin{bmatrix} \dot{\vec{x}}^{\text{remain}} & \dot{\vec{x}}^o \\ D^{\text{remain}} D(x) & D^o D(x) \end{bmatrix} \xrightarrow{\text{sweeping}} \vec{0} = \begin{bmatrix} \dot{\vec{x}}^c & A & \dot{\vec{x}}^o \\ I & B & F \\ E & D & C \end{bmatrix} \quad (11)$$