

Finite Element Stress Analysis of a Generic Spur Gear Tooth

by: Eugene A. Tennyson, The University of Tennessee

American Gear Manufacturers Association



TECHNICAL PAPER

AVA Documents
Document Sharing Hub

Finite Element Stress Analysis of a Generic Spur Gear

Eugene A. Tennyson
The University of Tennessee, Space Institute

[The Statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

ABSTRACT:

The prediction of bending stresses in a gear tooth, resulting from an externally applied torque, requires special consideration when designing spur gear systems. The tooth geometry is such that excess risers exist which must be accounted for. In addition, variables affecting the exact load point on the tooth and the direction of the applied load are critical. An interactive preprocessor is developed which generates all the information, including a detailed tooth profile, necessary to perform a finite element bending stress analysis of the gear system. To validate the procedure, a test group of spur gears is identified and analyzed. The results are then compared to those obtained via the American Gear Manufacturers Association (AGMA) standards. The comparison revealed the finite element stresses to be slightly more conservative than the corresponding AGMA standard stresses. A generalized stress equation and geometry factor, based on the finite element approach, are also introduced. This paper is intended only as a proof of concept.

Copyright © 1991

American Gear Manufacturers Association
1500 King Street, Suite 201
Alexandria, Virginia, 22314

October, 1991

ISBN: 1-55589-615-4

Finite Element Stress Analysis of a Generic Spur Gear Tooth

Eugene A. Tennyson*

The University of Tennessee Space Institute, Tullahoma, Tennessee 37388

The prediction of bending stresses in a gear tooth, resulting from an externally applied torque, requires special consideration when designing spur gear systems. The tooth geometry is such that stress risers exist which must be accounted for. In addition, variables affecting the exact load point on the tooth and the direction of the applied load are critical. An interactive preprocessor is developed which generates all the information, including a detailed tooth profile, necessary to perform a finite element bending stress analysis of the gear system. To validate the procedure, a test group of spur gears is identified and analyzed. The results are then compared to those obtained via the American Gear Manufacturers Association (AGMA) standards. The comparison revealed the finite element stresses to be slightly more conservative than the corresponding AGMA standard stresses. A generalized stress equation and geometry factor, based on the finite element approach, are also introduced. This paper is intended only as a proof of concept.

Introduction

The selection and design of spur gear systems is primarily guided by the anticipated bending stresses in the loaded tooth. Indeed, excessive bending stress in the fillet region is often the cause of gear failure. It is therefore important to have the means to reliably predict these stresses.

Wilfred Lewis¹, in 1893, proposed a bending stress formula, which for the first time, took into account the form of the tooth. To this day, this formula remains the basis for most gear design. The gear tooth is assumed to be a cantilever beam of uniform cross section, rigidly fixed at the base as shown in Figure 1. The "theoretical weakest section" AB is then located by inscribing a parabola within the tooth outline. The parabola should be tangent to the fillets on either side, and its vertex is at the point where the line of action crosses the center line. The bending stress σ_L at AB is then determined by the following Lewis equation

$$\sigma_L = \frac{6W_t}{hFt} \quad (1)$$

in which t is the length of AB and h is the distance from C to AB in Figure 1. Furthermore, W_t

represents the transmitted tangential load and F the tooth face width. Although Eq. (1) represented significant progress at the time, it is not very accurate in view of today's computational capabilities.

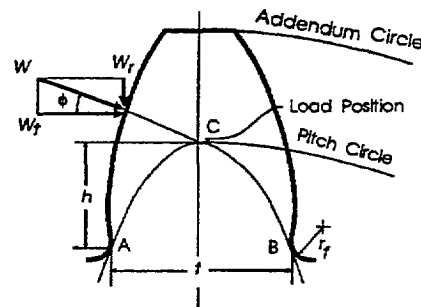


Fig. 1 Lewis' Parabola of Uniform Strength

In 1942, Dolan and Broghamer² conducted a photoelastic study of stresses in gear teeth which provided much detailed information about the nature of stress distributions in the neighborhood of fillets. This study also yielded a new stress concentration factor K which, when applied to the Lewis equation, results in a more realistic value for the bending stress at the tooth fillet. The factor K reflects the combined effect of the tangential component W_t and the radial component W_r of the external load W on the tooth. As an example, the stress concentration factor K for a 20° pressure angle stub tooth is given by

*Graduate Student, Dept. of Mechanical Engineering.

$$K = 0.18 + \left(\frac{t}{r_f}\right)^{0.15} \left(\frac{t}{h}\right)^{0.45} \quad (2)$$

where r_f represents the fillet radius of the tooth as shown in Figure 1.

The American Gear Manufacturers Association (AGMA) incorporated both the Lewis equation and the Dolan and Broghamer stress concentration factor K into its standard³. The AGMA bending stress for a statically loaded standard addendum spur gear with 0° helix angle is given by

$$\sigma_{AGMA} = \frac{W_t P_d}{F J} \quad (3)$$

where σ_{AGMA} is the bending stress number, R_p the pitch radius of the gear, and P_d the diametral pitch. The Geometry Factor J depends on the Lewis Form Factor Y as well as on the stress concentration factor K . In practice then, Eq. (3) is often used to determine the maximum static bending stress in a spur gear tooth subjected to the usual operating loads.

An alternative to the AGMA standard approach to computing bending stresses in gears is provided by the finite element method. The possibility of using the finite element method is very appealing, especially to the designer who is only occasionally responsible for gear selection and application. In addition, it is expected that, when properly implemented, the finite element approach will yield highly accurate results.

In the course of the present study an interactive preprocessor was developed which queries the user for minimal information concerning the spur gear system at hand. The preprocessor outputs all the information needed by a standard finite element program to perform a stress analysis of the gear tooth. In order to validate the procedure, a test group of spur gear systems was identified and analyzed using the finite element method. The resulting bending stresses are compared with those obtained via the AGMA standard Eq. (3). In addition, a new generalized bending stress equation and geometry factor are proposed, based on the finite element results. In the next section, the model definition for a generic spur gear tooth will be discussed.

Model Definition

Tooth Profile

One of the most widely accepted gear tooth profiles is defined by the involute curve which provides a natural line of contact for two mating gears. Before a finite element grid can be generated, it is necessary to derive a set of cartesian coordinates (x_{fi}, y_{fi}) for a generic point on the tooth profile.

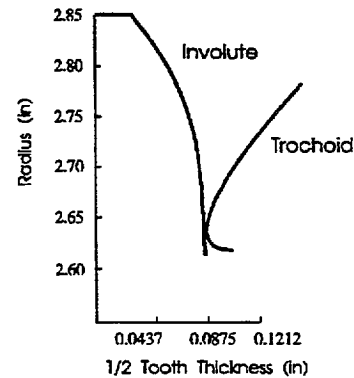


Fig. 2 Involute and Trochoid Curves

One of the more popular methods for cutting involute teeth is the hobbing process^{4,5}. This process consists of traversing a straight sided rack across a rotating gear billet. As the rack moves across the rotating gear an involute is generated on the gear tooth blank. In addition, a fillet is cut in the root of the tooth in the form of a trochoid curve as depicted in Figure 2. Equations for both these curves can be derived. Assuming a base radius R_b is given, it can be seen from Figure 3a that the profile angle ϕ_{pi} corresponding to a generic point on the involute at radius r_i is given by

$$\phi_{pi} = \sin^{-1} (R_b/r_i) \quad (4)$$

The involute angle θ_i is then defined as

$$\theta_i = \tan \phi_{pi} - \phi_{pi} \quad (5)$$

Also, from Figure 3b it can be seen that

$$\theta_a = \theta_i + \frac{t_i}{2r_i} \quad (6)$$