

AMERICAN GEAR MANUFACTURERS ASSOCIATION

*Load Distribution Factors - Analytical
Methods for Cylindrical Gears*

AGMA 927-A01



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Load Distribution Factors - Analytical Methods for Cylindrical Gears
AGMA 927-A01

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Approved October 22, 2000

ABSTRACT

This information sheet describes an analytical procedure for the calculation of the face load distribution. The iterative solution that is described is compatible with the definitions of the term face load distribution (K_H) of AGMA standards and longitudinal load distribution ($K_{H\beta}$ and $K_{F\beta}$) of the ISO standards. The procedure is easily programmable and flow charts of the calculation scheme as well as examples from typical software are presented.

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Foreword

[The foreword, footnotes and annexes, if any, in this document are provided for informational purposes only and are not to be construed as a part of AGMA Information Sheet 927-A01, *Load Distribution Factors – Analytical Methods for Cylindrical Gears*.]

This information sheet provides an analytical method to calculate a numeric value for the face load distribution factor for cylindrical gearing.

This is a new document, which provides a description of the analytical procedures that are used in several software programs that have been developed by various gear manufacturing companies. The method provides a significant improvement from the procedures used to define numeric values of face load distribution factor in current AGMA standards. Current AGMA standards utilize either an empirical procedure or a simplified closed form analytical calculation. The empirical procedure which is used in ANSI/AGMA 2101-C95 only allows for a nominal assessment of the influence of many parameters which effect the numeric value of the face load distribution factor. The closed form analytic formulations which have been found in AGMA standards suffer from the limitation that the shape of the load distribution across the face width is limited to a linear form.

The limitations of the previous AGMA procedures are overcome by the method defined in this information sheet. This method allows for including a sufficiently accurate representation of many of the parameters that influence the distribution of load along the face width of cylindrical gears. These parameters include the elastic effects due to deformations under load, and the inelastic effects of geometric errors as well as the tooth modifications which are typically utilized to offset the deleterious effects of the deformations and errors.

The analytical method described in this information sheet is based on a "thin slice" model of a gear mesh. This model treats the distribution of load across the face width of the gear mesh as being independent of the any transverse effects. The method also represents all of the elastic effects of a set of meshing teeth (tooth bending, tooth shear, tooth rotation, Hertzian deflections, etc.) by one constant, i.e., mesh stiffness (C_{ym}). Despite these simplifying assumptions, this method provides numeric values of the face load distribution factor that are sufficiently accurate for industrial applications of gearing which fall within the limitations specified.

The first draft of this information sheet was made in February, 1996. This version was approved by the AGMA membership on October 22, 2000.

Special mention must be made of the devotion of Louis Lloyd of Lufkin for his untiring efforts from the submittal of the original software code through the prodding for progress during the long process of writing this information sheet. Without his foresight and contributions this information sheet may not have been possible.

Suggestions for improvement of this document will be welcome. They should be sent to the American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, Virginia 22314.

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American Gear Manufacturers Association –

Load Distribution Factors – Analytical Methods for Cylindrical Gears

1 Scope

This information sheet covers a method for the evaluation of load distribution across the teeth of parallel axis gears. A general discussion of the design and manufacturing factors which influence load distribution is included.

The load distribution factors for use in AGMA parallel axis gear rating standards are defined, to improve communication between users of those standards.

Historically, analytical methods for evaluating load distribution in both AGMA and ISO standards have been limited by the assumption that load is linearly distributed across the face width of the meshing gear set. The result of this assumption is often overly conservative (high) values of load distribution factors. The method given here is considered more correct.

1.1 Method

A simplified iterative method for calculation of the face load distribution factor, based on combined twisting and bending displacements of a mating gear and pinion, is presented. The transverse load distribution (in the plane of rotation) is not evaluated in this information sheet. This method assumes that the mesh stiffness is a constant through the entire contact roll and across the face. General guidance for design modifications to improve load distribution is also included.

1.2 Limitations of method

This method is intended to be used for general gear design and rating purposes. It is intended to provide a value of load distribution factor and a means by which different gear designs can be analytically compared. It is not intended for rigorous detailed analysis to calculate the actual distribution of load across the face width of gear sets.

The knowledge and judgment required to evaluate the results of this method come from experience in designing, manufacturing and operating gear units. This method is intended for use by the experienced gear designer, capable of understanding its limitations and purposes. It is not intended for use by the engineering public at large.

2 References

The following documents were used in the development of this information sheet. At the time of publication, the editions were valid. All publications are subject to revision, and the users of this manual are encouraged to investigate the possibility of applying the most recent editions of the publications listed:

AGMA Technical Paper P109.16, *Profile and Longitudinal Corrections on Involute Gears*, 1965

ANSI/AGMA 1012-F90, *Gear Nomenclature, Definitions Of Terms With Symbols*

ANSI/AGMA 2101-C95, *Fundamental Rating Factors And Calculation Methods For Involute Spur And Helical Gear Teeth*

ANSI/AGMA ISO 1328-1, *Cylindrical Gears – ISO System of Accuracy – Part 1: Definitions and Allowable Values of Deviations Relevant to Corresponding Flanks of Gear Teeth*

ISO 6336-1:1996, *Calculation of load capacity of spur and helical gears – Part 1: Basic principles, introduction and general influence factors*

Dudley, D.W., *Handbook of Practical Gear Design*, McGraw-Hill, New York, 1984

Timken Engineering Design Manual, Volume 1



3 Definitions and symbols

The terms used, wherever applicable, conform to ANSI/AGMA 1012-F90.

NOTE: The symbols and definitions used in this standard may differ from other AGMA standards. The user should not assume that familiar symbols can be used without a careful study of their definitions.

The symbols and terms, along with the clause numbers where they are first discussed, are listed in alphabetical order by symbol in table 1.

3.1 Load distribution factor

The load distribution factor, K_H , modifies the rating equations to reflect the non-uniform distribution of load along the gear tooth lines of contact as they rotate through mesh. In past AGMA standards, the variables C_m and K_m have been associated with this factor. In ISO standards, the variables $K_{H\beta}$, $K_{H\alpha}$, $K_{F\beta}$ and $K_{F\alpha}$, have been associated with the factor. In

current AGMA standards the load distribution factor, K_H , is used for both pitting resistance and bending strength calculations. There is no separate value, K_F , for bending strength as found in ISO standards.

The magnitude of K_H is affected by two components, transverse load distribution factor and face load distribution factor.

The transverse load distribution factor pertains to the plane of rotation and is affected primarily by the correctness of the profiles and indexing of the mating teeth. Standard procedures to evaluate it have not been established and it is assumed to be unity in this information sheet.

The face load distribution factor is the focus of this information sheet.

3.2 Target mesh

The target mesh is that mesh for which load distribution is being analyzed. The target mesh includes a target pinion and a target gear.

Table 1 - Symbols and definitions

Symbol	Definition	Units	First referenced
A	Apex factor	--	5.3
BT	Axis in the base tangent plane	--	5.2
BTN	Axis normal to base tangent plane	--	5.2
BTZ	Axis in the base tangent plane perpendicular to BT	--	5.2
C_{ym}	Tooth stiffness constant, for the analysis	N/mm/μm	9.1
b	Helical/bevel gear face width	mm	5.3
D	Drive factor	--	5.3
D_{pG}	Operating pitch diameter, gear	mm	5.3
d	Outside effective twist diameter	mm	7.1
d_{in}	Inside shaft diameter	mm	6.1
d_{sh}	Outside diameter, effect outside diameter of the teeth	mm	6.1
E	Modulus of elasticity	N/mm ²	6.1
F_{aG}	Axial thrust force, gear member	N	5.3
F_{aP}	Axial thrust force, pinion member	N	5.4
F_g	Total load in the plane of action	N	9.2
F_i	Gearing or external force at a distance	N	6.1
F_{sG}	Separating force, gear member	N	5.3
F_{sP}	Separating force, pinion member	N	5.3
F_{tG}	Tangential force, gear member	N	5.3
F_{tP}	Tangential force, pinion member	N	5.3
G	Modulus of elasticity in shear	N/mm ²	7.1
H	Hand factor	--	5.3
I	Moment of inertia	mm ⁴	6.1

(continued)

Table 1 (concluded)

Symbol	Definition	Units	First referenced
IC	Integration constant	--	6.1
i	Station number	--	6.1
K_H	Load distribution factor	--	9.4
L_s	Distance between the supports (reactions)	--	6.1
L_j	Load at station	N	7.1
L_δ	Load intensity	N/mm	9.1
M	Bending moment	N mm	6.1
M_G	Moment due to axial thrust force	N mm	5.4
n	Station number at end support	--	6.1
P	Power transmitted through the mesh	kW	5.3
R	Rotation factor	--	5.3
R_L	Reaction at the left bearing	N	6.1
R_R	Reaction at the right bearing	N	6.1
S	Speed of shaft	rpm	5.3
SL_i	Station slope value	--	6.3
$t_{\delta i}$	Torsional deflection at a station	μm	7.1
V	Shear	N	6.1
x_i	Length of face where point load applied	mm	9.2
X_j	Distance between adjacent stations	mm	7.1
X_{fi}	Distance from left support to load location	mm	6.1
x	Distance between stations	mm	6.1
y	Deflection along the line of action	μm	6.1
δ_{ti}	Tooth deflection at a load point	μm	9.1
γ_G	Bevel pitch angle of gear	degrees	5.3
γ_P	Bevel pitch angle of pinion	degrees	5.3
ψ	Helix angle/spiral angle	degrees	5.3
ϕ	Normal pressure angle	degrees	5.3

4 Iterative analytical method

This information sheet presents an iterative analytical method for determining a value of load distribution factor. The iterative method combines the calculated elastic deflection of the pinion and the gear with other misalignments. The result defines a "mesh gap" in the base tangent plane which is the net mismatch between the gear and the pinion. The teeth in mesh are modeled by an equally spaced series of independent parallel compression springs which represent the mesh stiffness. The mesh gap is then mathematically closed by compressing the springs until the sum of the spring forces equals the total tooth force.

The method has the ability to consider the following influences:

- tooth alignment deviations of pinion and gear;
- tooth alignment and crowning modification;
- alignment of the axes of rotation of the pinion and gear, including bearing clearances and housing bore alignment;
- mesh elastic deflections due to Hertzian contact and tooth bending;
- shaft elastic deflections due to twisting and bending, resulting from the target mesh loads and loads external to the mesh.

Influences that may be accounted for by estimating values and including them as equivalent misalignments of the target shaft axes are:

- elastic deflection of a gear body if it is not a solid disk (such as a spoke gear);

- elastic deflection of the housing and foundations;
- displacements of the gearing due to bearing deflection;
- thermal or centrifugal effects;
- running-in or lapping effects.

The method does not consider the following influences:

- tooth profile, spacing and runout deviations;
- total tooth load including increases due to application influences and tooth dynamics;
- variations of stiffness of the gear teeth;
- double helical gears with one helix overloaded.

4.1 Methodology

The iterative analytical method consists of the following basic steps:

- 1) Calculate the mesh gap resulting from an initial uniform load distribution;
- 2) Calculate a new load distribution by mathematically closing the mesh gap. This is accomplished by compressing the springs until the sum of the spring forces equals the total tooth force;
- 3) Calculate a new mesh gap resulting from the new load distribution;
- 4) Repeat steps 2 and 3 until the change in load distribution from the previous iteration is negligible;
- 5) The load distribution factor is then calculated from this final load distribution.

4.1.1 Calculated elastic deflections

Deflections which are calculated within the iterative method include the elastic deflections of the pinion and gear shafts, plus the mesh. Elastic shaft deflections include shaft twist and bending. Elastic tooth deflections include Hertzian contact and tooth bending.

4.1.2 Equivalent misalignment inputs

Other displacements that are treated by combining them as an equivalent deflection at the target mesh include:

- alignment deviations and modifications of pinion and gear teeth;

- equivalent elastic deflection of non-solid body gears (such as a spoke gear);
- elastic deflection of the housing and foundations;
- displacements due to bearing clearance, alignment and deflection;
- thermal or centrifugal effects;
- running-in or lapping effects.

4.2 Assumptions and simplifications

The following assumptions and simplifications are used:

- the weight of components is ignored;
- effects of uneven distribution of load on meshes other than the target mesh are ignored; load on these meshes is treated as concentrated in the center of the mesh;
- shear coupling between the mesh gap compression springs representing the mesh stiffness is ignored;
- mesh stiffness is a constant across the full width of tooth;
- all shafts are supported on two bearings;
- for double helical gears the net thrust force is zero as the thrust force from each helix cancels each other;
- for double helical gears the tangential and separating force is distributed equally on each hand helix; this is generally true as long as one member can float with respect to the other with no external axial load applied.

5 Coordinate system, sign convention, gearing forces and moments

5.1 Rules

The rules that govern the coordinate system, sign convention, gearing forces and moments are:

- the target mesh shafts are mutually parallel;
- the coordinate system for all calculations lies in the base tangent plane;
- the base tangent plane is a plane tangent to the base circles of the target mesh;
- the driving element is the element for which contact first occurs in the root of the tooth and traverses to the tip of the tooth;

- a modified Timken sign convention is followed;
- each analysis includes only the two shafts under consideration;
- the origin of the shaft is the bearing or point of application of a force or moment on the target pinion shaft which is most remote from the target mesh toward the reference end of the shaft (see 5.2);
- the input torque to the driving element enters the shaft from one side only and is fully balanced by torque in the target mesh.

5.2 Coordinate system and sign convention

The coordinate system is aligned with the base tangent plane, BTP, of the target mesh and is defined as the base tangent coordinate system, BTCS. The BTCS is comprised of three orthogonal axes: BT, BTN (base tangent normal), and BTZ.

The BTZ axis is parallel to the axes of the target mesh shafts. The BT axis lies in the BTP and is perpendicular to the BTZ axis. The BTN axis is perpendicular to both the BT and the BTZ axes (normal to the base tangent plane). The origin of the BTCS lies at the intersection of the base tangent

plane and the edge of the target mesh face closest to the reference end (see figures 1 and 2).

For consistency in defining the positive direction of the BTCS axes and in calculating the mesh loads, a "reference end" needs to be identified. For purposes of this information sheet, the reference end is the end of the driving element shaft opposite the torque input end.

Using this definition of the reference end, the positive directions of the BTCS axes are determined as follows:

- + BTZ: away from the reference end;
- + BTN: toward the driven element;
- + BT: obtained by right hand rule; BTN to BTZ.

Figures 1 and 2 illustrate the base tangent plane and the base tangent coordinate system for a typical target mesh. In figure 1, the input torque is clockwise when viewed from the reference end. In figure 2, the input torque is counterclockwise when viewed from the reference end.

The force, moment and deflection along the positive direction of BT, BTN and BTZ are assigned positive values. Along the negative direction of BT, BTN and BTZ, they are assigned negative values.

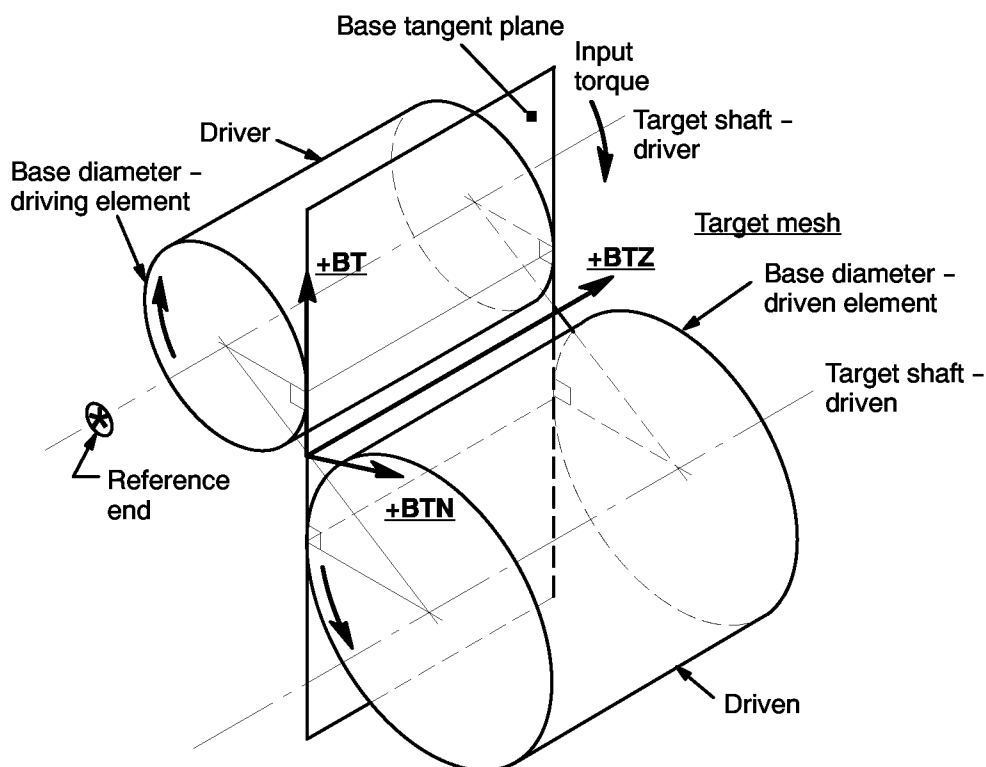


Figure 1 - Base tangent coordinate system for CW driven rotation from reference end

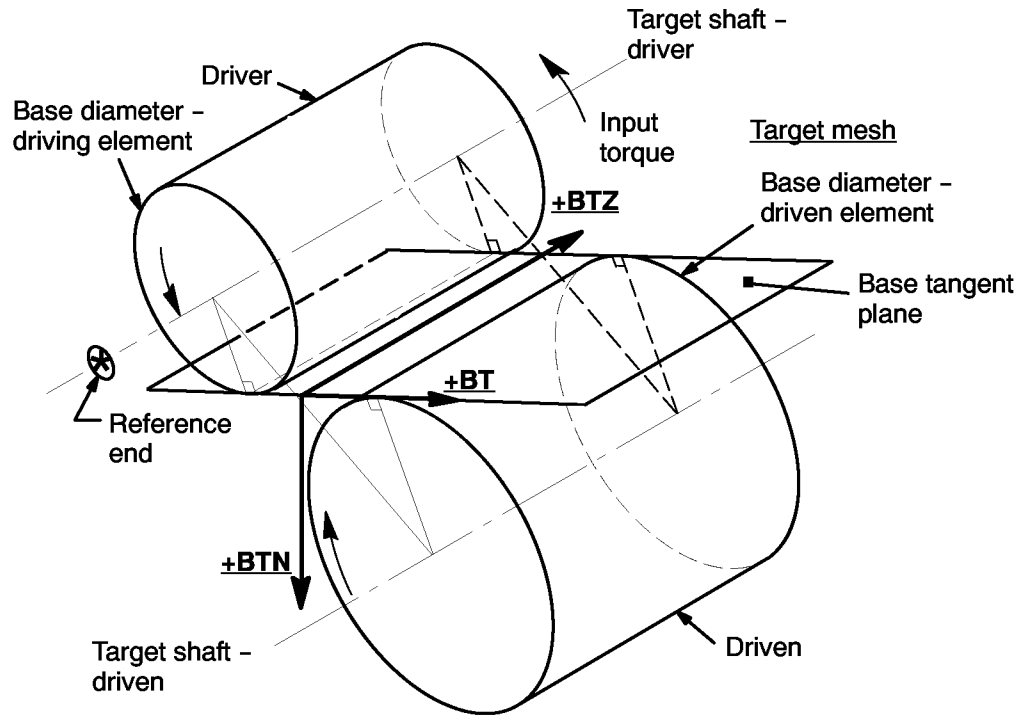


Figure 2 - Base tangent coordinate system for CCW driven rotation from reference end

5.3 Gearing forces and signs

Meshing gear members transmitting torque will cause forces and moments to develop on the shafts that carry these gear members. These forces and moments will cause deflections of the shafts that will tend to affect the alignment and ultimately the distribution of the load across the face width of the mesh. These elastic deflections need to be combined with all other sources of potential misalignment.

The forces on the gear member are given by equations 1 through 3. In these equations, the values of factors H , A , R , and D are obtained using table 2. When properly applied, these factors will ensure that the proper direction of the forces are determined. The directions obtained will be consistent with the BTCS definition presented in 5.2.

The tangential force is calculated as:

$$F_{tG} = \frac{1.91 \times 10^7 P (D R)}{S (D_{pG} - b \sin \gamma_G)} \quad (1)$$

where

- F_{tG} is tangential force, gear member, N;
- P is power transmitted through the mesh, kW;
- D is drive factor (see table 2);

- R is rotation factor (see table 2);
- S is speed of gear shaft, rpm;
- D_{pG} is operating pitch diameter, gear, mm;
- b is helical/bevel gear face width, mm;
- γ_G is bevel pitch angle, gear, degrees.

The separating force is calculated as:

$$F_{sG} = \frac{|F_{tG}| \left[(A D H R \sin \psi \sin \gamma_G + \tan \phi \cos \gamma_G) \right]}{\cos \psi} \quad (2)$$

where

- F_{sG} is separating force, gear member, N;
- A is apex (bevel) factor (see table 2);
- H is hand factor (see table 2);
- ψ is helix angle/spiral angle, degrees;
- ϕ is normal pressure angle, degrees.

The thrust (axial) force is calculated as:

$$F_{aG} = \frac{|F_{tG}| (A) \left[(A D H R \sin \psi \cos \gamma_G - \tan \phi \sin \gamma_G) \right]}{\cos \psi} \quad (3)$$

where

- F_{aG} is axial thrust force, gear member, N.