

The Influence of the Kinematical Motion Error on the Loaded Transmission Error of Spiral Bevel Gears

by: Claude Gosselin, Louis Cloutier and Q.D. Nguyen
Laval University

American Gear Manufacturers Association



TECHNICAL PAPER

The Influence of the Kinematical Motion Error on the Loaded Transmission Error of Spiral bevel gears

Claude Gosselin, Louis Cloutier and Q. D. Nguyen
Laval University

[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

ABSTRACT:

The optimisation of the kinematical behaviour of spiral bevel gears has been the focus of much attention over the past years. The objectives, aiming to optimize the kinematics of spiral bevel gears under no load, being zero or close to zero motion error coupled to a parabolic motion error curve have been fulfilled by many authors [4, 6, 7]. However, these objectives bypass the more general case of minimizing motion error at a given transmitted load, which is the design goal in aerospace applications where transmissions operate over extended periods at 80 to 90% of available power. Krenzer [6] showed that changing torque significantly modified the shape of the motion error curve, and that minimum motion error under load could be achieved only over short torque ranges.

This paper presents the basis of a Loaded Tooth Contact Analysis (LTCA) program predicting the motion error of spiral bevel gear sets under load and shows some of the influences of the unloaded motion error curve shape and amplitude on the kinematical behaviour under load. The effects of tooth composite deflection (bending and shearing), tooth contact deformation and initial profile separation due to motion error are considered in the development. Due to the complex shape of spiral bevel gear teeth, the tooth bending stiffness is calculated by finite elements. Classical Hertz theory is used to calculate the contact deformation. Numerical examples are presented to illustrate the behaviour of motion error under load as the unloaded motion error is modified. Results show that under general circumstances, low contact ratio gears with parabolic motion error under no load may produce undesirable kinematics under load, while parabolic motion error high contact ratio gears are likely to produce acceptable kinematics under a variety of loads. It is also shown that the governing factors, in loaded motion error, are the contact ratio, thus combined mesh stiffness, and the amplitude of unloaded motion error curve which is linked to load sharing between adjacent tooth pairs.

Copyright © 1992

American Gear Manufacturers Association
1500 King Street, Suite 201
Alexandria, Virginia, 22314

October, 1992

ISBN: 1-55589-590-5

THE INFLUENCE OF THE KINEMATICAL MOTION ERROR ON THE LOADED TRANSMISSION ERROR OF SPIRAL BEVEL GEARS

Claude Gosselin, Professor
Louis Cloutier, Professor
Q.D. Nguyen, Graduate Student

Department of Mechanical Engineering
Laval University, Québec, QC, Canada, G1K-7P4

1. INTRODUCTION

To achieve maximum life for a spiral bevel gear set, low vibration levels and appropriate bearing pattern location and dimensions must be present. The kinematical behaviour of a gear pair (bearing pattern, line of action and motion error) is therefore an important consideration at the design stage. The size of the bearing pattern and the location of the line of action (LOA) both influence the contact deformation and bending displacement of a gear tooth while kinematical motion error determines whether unloaded motion will transfer smoothly between adjacent tooth pairs [5]. However, as teeth bend under load motion error changes and the possibility of excessive motion error and contact entry interference under heavy loads arises, which may generate vibrations and noise [10]. The analysis of motion error of gears under load is therefore a necessary step in the design of a gear train.

One of the recent advances in gear train design is the increase in the number of simultaneous tooth pairs in contact: if two to three tooth pairs, as in high contact ratio gears (HCR), are in simultaneous contact instead of one to two pairs, as in low contact ratio gears (LCR), the overall mesh stiffness can be increased and the load share carried by each tooth pair can be significantly reduced. Although actual load share values can differ substantially, it is possible under certain conditions to increase the load carrying capacity of a gearset without increasing its weight [13]. HCR gears can be obtained in several ways: 1) by decreasing the pressure angle, 2) by increasing the addendum, 3) by using teeth of finer pitch and 4) by increasing bias on the LOA of spiral bevel gears.

Evaluating the load share between meshing teeth in spiral bevel gears is a complex process when more than two tooth pairs are simultaneously in contact and an iterative numerical solution must be used. This paper presents a general approach to calculate how load is shared between meshing teeth in spiral bevel gear pairs, taking into account tooth composite deflection (bending, shearing), tooth contact deformation and initial profile separation due to profile modification.

2. LIST OF SYMBOLS

$\underline{F}_{-1}, \underline{F}_0, \underline{F}_{+1}$	load share vectors for tooth pairs -1, 0 and +1 respectively
$\underline{AS}_{-1}, \underline{AS}_0, \underline{AS}_{+1}$	initial tooth pair profile separation vectors
$\underline{\delta H}_{-1}, \underline{\delta H}_0, \underline{\delta H}_{+1}$	contact deformation vectors (Hertz)
$\underline{R}_{c-1}, \underline{R}_{c0}, \underline{R}_{c+1}$	radius of contact vectors
$\underline{T}_{-1}, \underline{T}_0, \underline{T}_{+1}$	torque vectors
$\underline{T}_{app}$	total applied torque vector
$\underline{K}_{-1}, \underline{K}_{00}, \underline{K}_{+1}$	stiffness vectors at contact points
$\underline{K}_{-1} x, y, z$	stiffness vector x, y and z components at contact points
$\underline{C}_{-1}, \underline{C}_{00}, \underline{C}_{+1}$	compliance vectors
$\underline{\delta}_{pij}, \underline{\delta}_{sij}$	primary and secondary displacement vectors
$C_{00}, C_{10}, C_{01}, C_{11}$	weighting coefficients
$\underline{X}_c, \underline{Y}_c, \underline{Z}_c$	contact point reference frame
$\delta\phi_3$	motion error without load
$\delta\phi_{3L}$	motion error under load
m_g	speed ratio

3. GENERAL ASSUMPTIONS

In order to somewhat simplify the complexity of the problem, the following general assumptions are made:

- there is neither friction in the contact zone, nor presence of lubricant that can affect the distribution of contact pressure and create an oil film of varying thickness between the meshing teeth;
- displacements due to tooth deflection, shearing or contact deformation are sufficiently small that the tooth surface normal at the contact point is not affected;
- a TCA computer program, described in references [1, 2, 3], is used to obtain the kinematical characteristics of the analysed gear sets, such as tooth surface coordinates, contact points along the line of action, bearing pattern, and contact stresses;
- the contact is of hertzian type, and there are no boundary corrections made for edge contact since, in most cases, under the applied torques the motion error curves remain of parabolic shape and the loads at contact entry are minimal;
- at this point, tooth coupling effects are neglected, e.g. the movement of one tooth due to a load applied on an adjacent tooth, although such effects may significantly alter load sharing and are under consideration.

4. PINION BLANK ROTATION COMPATIBILITY CONDITION

Spiral bevel gear teeth are theoretically point contact surfaces. In practice however, because of the low relative longitudinal curvature between the pinion and gear tooth flanks, the instant contact point is close to a line, which we will call the instant line of contact (LOC). Under the action of an applied load, the gear teeth in contact displace by bending and shearing and their surfaces deform at the contact point to form an elongated ellipse.

The load sharing compatibility condition is based on the fact that [6, 11, 12]:

- the pinion and gear blank rotation due to the displacement of any tooth under load must be the same for all simultaneously meshing tooth pairs; this includes tooth bending deflection and shearing displacement, contact deformation and profile separation due to profile modification or motion error (figure 1);
- the sum of the torque contribution of each meshing tooth pair must equal the total applied torque.

Equations (1) and (2) state these conditions for a HCR gear set with three tooth pairs in contact:

$$\left[\frac{F_{-1}}{K_{-11}} + \frac{\Delta S_{-1}}{K_{-11}} + \frac{\delta H_{-1}}{K_{-11}} \right]_{\underline{R}_{c-1}} = \left[\frac{F_0}{K_{00}} + \frac{\Delta S_0}{K_{00}} + \frac{\delta H_0}{K_{00}} \right]_{\underline{R}_{c0}} = \left[\frac{F_{+1}}{K_{+11}} + \frac{\Delta S_{+1}}{K_{+11}} + \frac{\delta H_{+1}}{K_{+11}} \right]_{\underline{R}_{c+1}} \quad (1)$$

$$\underline{T}_{app} = \underline{T}_{-1} + \underline{T}_0 + \underline{T}_{+1} \quad (2)$$

where (subscript t indicates that the tangential direction is considered, e.g. normal to the radius at the contact point - see figure 2):

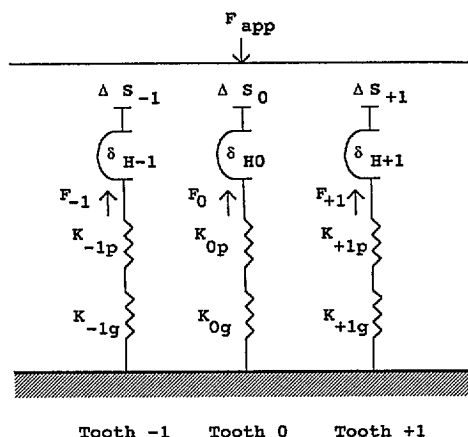


Figure 1: Load Share Components

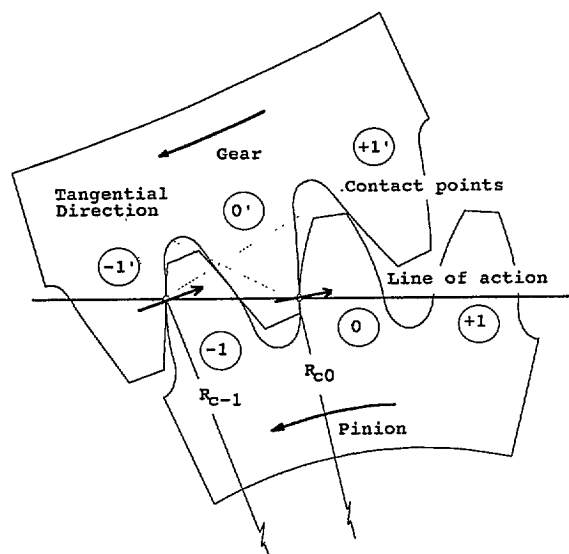


Figure 2: Tooth Numbering Convention

$\underline{K}_{-11}, \underline{K}_{00}, \underline{K}_{+11}$: is the combined stiffness vector, at a contact point, for tooth pairs -1-1', 00' and +1+1' respectively; in figure 2, tooth

pair -1-1' is going out of mesh, tooth pair 00' is the main tooth pair in contact, and tooth pair +1+1' is coming into mesh;

$\underline{\Delta S}_{-1}$, $\underline{\Delta S}_0$, $\underline{\Delta S}_{+1}$: is the initial separation vector between tooth pairs -1-1', 00' and +1+1' respectively, due to profile modification or motion error; the initial separation is obtained from the motion error curve, and is in the tangential direction (figure 2);

$\underline{\delta H}_{-1}$, $\underline{\delta H}_0$, $\underline{\delta H}_{+1}$: is the contact deformation vector of tooth pairs -1-1', 00' and +1+1' respectively; contact is assumed hertzian and is calculated as per references [8, 14];

$\underline{F}_{-1}$, $\underline{F}_0$, $\underline{F}_{+1}$: is the individual load vector for each tooth pair;

$\underline{R}_{c-1}$, $\underline{R}_{c0}$, $\underline{R}_{c+1}$: is the contact radius vector on the pinion tooth;

$\underline{T}_{-1}$, $\underline{T}_0$, $\underline{T}_{+1}$: is the individual vectorial contribution to total applied torque;

$\underline{T}_{app}$: is the total applied torque vector.

The individual torque vectors are defined by the following cross products:

$$\underline{T}_{-1} = \underline{F}_{-1} \times \underline{R}_{c-1} \quad (3)$$

$$\underline{T}_0 = \underline{F}_0 \times \underline{R}_{c0} \quad (4)$$

$$\underline{T}_{+1} = \underline{F}_{+1} \times \underline{R}_{c+1} \quad (5)$$

Since contact deformations do not vary linearly with the applied load, equation (1) cannot be solved directly and a Newton-Raphson based iterative process is used. To speed up convergence and to prevent numerical divergence, close initial load share values can be obtained by modifying equation (1) where only bending deflection and initial separation vectors between meshing tooth pairs are considered:

$$\left[\frac{\underline{F}_{-1} + \underline{\Delta S}_{-1}}{\underline{K}_{-11}} \right]_t = \left[\frac{\underline{F}_0 + \underline{\Delta S}_0}{\underline{K}_{00}} \right]_t = \left[\frac{\underline{F}_{+1} + \underline{\Delta S}_{+1}}{\underline{K}_{+11}} \right]_t \quad (6)$$

Assuming that the modules of contact radii vectors $\underline{R}_{c-1}$, $\underline{R}_{c0}$ and $\underline{R}_{c+1}$ are close, equations (6) and (2) can be further simplified to:

$$\left[\frac{\underline{F}_{-1} + \underline{\Delta S}_{-1}}{\underline{K}_{-11}} \right]_t = \left[\frac{\underline{F}_0 + \underline{\Delta S}_0}{\underline{K}_{00}} \right]_t = \left[\frac{\underline{F}_{+1} + \underline{\Delta S}_{+1}}{\underline{K}_{+11}} \right]_t \quad (7)$$

and

$$\underline{F}_{app} = \underline{F}_{-1} + \underline{F}_0 + \underline{F}_{+1} \quad (8)$$

where the $\underline{F}_i$ are the applied normal loads on the teeth. To further simplify the solution, equation (7) is split in two parts and rewritten in scalar form as:

$$F_{-1} C_{-1} + \Delta S_{-1} = F_0 C_0 + \Delta S_0 \quad (9)$$

$$F_{+1} C_{+1} + \Delta S_{+1} = F_0 C_0 + \Delta S_0 \quad (10)$$

where compliance C_i is the inverse of stiffness K_i . Equations (9) and (10) are rearranged as:

$$F_{-1} = \frac{(\Delta S_0 - \Delta S_{-1}) + F_0 C_0}{C_{-1}} \quad (11)$$

$$F_{+1} = \frac{(\Delta S_0 - \Delta S_{+1}) + F_0 C_0}{C_{+1}} \quad (12)$$

Combined with equation (8), equations (11) and (12) yield:

$$F_{-1} = \frac{(\Delta S_0 - \Delta S_{-1}) + (F_{app} - F_{-1} - F_{+1}) C_0}{C_{-1}} \quad (13)$$

$$F_{+1} = \frac{(\Delta S_0 - \Delta S_{+1}) + (F_{app} - F_{-1} - F_{+1}) C_0}{C_{+1}} \quad (14)$$

Extracting F_{-1} from equations (13) and (14) and subtracting gives:

$$F_{+1} = \frac{C_{-1} (\Delta S_0 - \Delta S_{-1}) + C_0 (\Delta S_{-1} - \Delta S_{+1}) + F_{app} C_0 C_{-1}}{C_{+1} C_{-1} + C_{+1} C_0 + C_0 C_{-1}} \quad (15)$$

$$F_{-1} = \frac{(\Delta S_0 - \Delta S_{-1}) + (F_{app} - F_{+1}) C_0}{C_{-1} + C_0} \quad (16)$$

$$F_0 = F_{app} - F_{+1} - F_{-1} \quad (17)$$

Equations (15) to (17) can now be used to closely estimate the initial load shares of each tooth pair, and help speeding up convergence and prevent numerical divergence.

5. BENDING AND SHEARING STIFFNESS VECTORS

Combined stiffness vectors $\underline{K}_{-11}$, $\underline{K}_{00}$ and $\underline{K}_{+11}$ are calculated from the individual pinion and gear tooth stiffness components which are combined as springs in series (figure 1):

$$\frac{1}{K_{-11 \ x,y,z}} = \frac{1}{K_{1p \ x,y,z}} + \frac{1}{K_{1g \ x,y,z}} \quad (18)$$

$$\frac{1}{K_{00 \ x,y,z}} = \frac{1}{K_{0p \ x,y,z}} + \frac{1}{K_{0g \ x,y,z}} \quad (19)$$

$$\frac{1}{K_{+11 \ x,y,z}} = \frac{1}{K_{+1p \ x,y,z}} + \frac{1}{K_{+1g \ x,y,z}} \quad (20)$$

where $K_{ip \ x,y,z}$ and $K_{ig \ x,y,z}$ are the individual pinion and gear tooth bending stiffness components, given in the pinion fixed reference frame $X_1 X_2 X_3$ (figure 3), defined by:

$$K_{i \ x,y,z} = \frac{F_{i \ x,y,z}}{\delta_{i \ x,y,z}} \quad (21)$$

and where $F_{i \ x,y,z}$ are the load components and $\delta_{i \ x,y,z}$ are the bending deflection components in the X_1 , X_2 and X_3 directions respectively (figure 3).