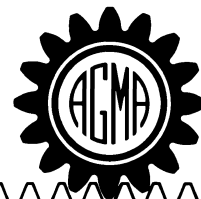


AMERICAN GEAR MANUFACTURERS ASSOCIATION

*Calculated Bending Load Capacity of
Powder Metallurgy (P/M) External Spur
Gears*



AGMA INFORMATION SHEET

(This Information Sheet is NOT an AGMA Standard)

American
Gear
Manufacturers
Association

Calculated Bending Load Capacity of Powder Metallurgy (P/M) External Spur Gears

AGMA 930-A05

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[Tables or other self-supporting sections may be referenced. Citations should read: See AGMA 930-A05, *Calculated Bending Load Capacity of Powder Metallurgy (P/M) External Spur Gears*, published by the American Gear Manufacturers Association, 500 Montgomery Street, Suite 350, Alexandria, Virginia 22314, <http://www.agma.org>.]

Approved January 19, 2005

ABSTRACT

This information sheet describes a procedure for calculating the load capacity of a pair of powder metallurgy (P/M) external spur gears based on tooth bending strength. Two types of loading are considered: 1) repeated loading over many cycles; and 2) occasional peak loading. In a separate annex, it also describes an essentially reverse procedure for establishing an initial design from specified applied loads. As part of the load capacity calculations, there is a detailed analysis of gear teeth geometry. These have been extended to include useful details on other aspects of gear geometry such as the calculations for defining gear tooth profiles, including various fillets.

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Foreword

[The foreword, footnotes and annexes, if any, in this document are provided for informational purposes only and are not to be construed as a part of AGMA Information Sheet 930-A05, *Calculated Bending Load Capacity of Powder Metallurgy (P/M) External Spur Gears*.]

This information sheet was prepared by the AGMA Powder Metallurgy Gearing Committee as an initial response to the need for a design evaluation procedure for powder metallurgy (P/M) gears. The committee anticipates that, after appropriate modification and confirmation based on application experience, this procedure will become part of a standard gear rating method for P/M gears. As such, it will serve the same function for P/M gears as the rating procedure in ANSI/AGMA 2001-C95 for wrought metal gears. Toward this end, the design evaluation procedure described here closely follows ANSI/AGMA 2001-C95, with changes made for the special properties of P/M materials, gear proportions, and types of applications. These design considerations have made it possible to introduce some simplifications in comparison to the above mentioned standard.

The first draft of AGMA 930-A05 was made in June 1996. It was approved by the AGMA Technical Division Executive Committee in January 2005.

Suggestions for improvement of this document will be welcome. They should be sent to the American Gear Manufacturers Association, 500 Montgomery Street, Suite 350, Alexandria, Virginia 22314.

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American Gear Manufacturers Association –

Calculated Bending Load Capacity of Powder Metallurgy (P/M) External Spur Gears

1 Scope

1.1 General

1.1.1 Calculation

This information sheet describes a procedure for calculating the load capacity of a pair of powder metallurgy (P/M) gears based on tooth bending strength. Two types of loading are considered: 1) repeated loading over many cycles; and 2) occasional peak loading. This procedure is to be used on prepared gear designs which meet the customary gear geometry requirements such as adequate backlash, contact ratio greater than 1.0, and adequate top land. An essentially reverse procedure for establishing an initial design from specified applied loads is described in annex D.

1.1.2 Strength properties

Fatigue strength and yield strength properties used in these calculations may be taken from previous test experience, but may also be derived from published data obtained from standard tests of the materials.

1.1.3 Application

This procedure is intended for use as an initial evaluation of a proposed design prior to preparation of test samples. Such test samples might be machined from P/M blanks or made from P/M tooling based on the proposed design after it passes this initial evaluation. Final acceptance of the proposed design should be based on application testing and not on these calculations. If samples made from

tooling fall short in testing, it may be possible to use the same tooling for a design adjusted for greater face width.

1.1.4 Limitations

Gears made from all materials and by all processes, including P/M gears, may fail in a variety of modes other than by tooth bending. This information sheet does not address design features to resist these other modes of failure, such as excessive wear and other forms of tooth surface deterioration.

CAUTION: The calculated load capacity from this procedure is not to be used for comparison with AGMA ratings of wrought metal gears, even though there are many similarities in the two procedures.

1.2 Types of gears

This calculation procedure is applied to external spur gears, the type of gear most commonly produced by the P/M process.

1.3 Dimensional limitations

This procedure applies to gears whose dimensions conform to those commonly produced by the P/M process for load carrying applications:

- Finest pitch: 0.4 mm module;
- Maximum active face width: $15 \times$ module, with a 65 mm maximum;
- Minimum number of teeth: 7;
- Maximum outside diameter: 180 mm;
- Pressure angle: 14.5° to 25° .

1.4 Gear mesh limitations

Some of the calculations apply only to meshing conditions expressed as a contact ratio greater than one and less than two. This translates into the requirement that there is at least one pair of contacting teeth transmitting load and no more than two pairs.

2 Definitions and symbols

2.1 Definitions

The terms used, wherever applicable, conform to ANSI/AGMA 1012-F90.

2.2 Symbols

The symbols and terms used throughout this information sheet are in basic agreement with the symbols and terms given in AGMA 900-G00, *Style Manual for the Preparation of Standards, Information Sheets and Editorial Manuals*, and ANSI/AGMA 1012-F90, *Gear Nomenclature, Definitions of Terms with Symbols*. In all cases, the first time that each

symbol is introduced, it is defined and discussed in detail.

NOTE: The symbols and definitions used in this information sheet may differ from other AGMA documents. The user should not assume that familiar symbols can be used without a careful study of their definitions.

The symbols and terms, along with the clause numbers where they are first discussed, are listed in alphabetical order by symbol in table 1.

Table 1 - Symbols and definitions

Symbol	Terms	Units	Reference
C_A	Operating center distance	mm	Eq 24
d	Gear pitch diameter	mm	Eq 37
d_{AG}	Operating pitch diameter of gear	mm	Eq 25
d_{AP}	Operating pitch diameter of pinion	mm	Eq 24
d_c	Calculation diameter	mm	Eq 1
E	Modulus of elasticity	N/mm ²	Eq 38
F_e	Effective face width	mm	Eq 1
F_o	Overlapping face width	mm	Eq 26
F_x	Each face width extension, not larger than m	mm	Eq 27
F_{xe1}	Effective face width extension at one end	mm	Eq 26
F_{xe2}	Effective face width extension at other end	mm	Eq 26
f_{qm}	Factor relating to axis misalignment adjustment	--	Eq 36
f_{qv}	Factor relating to manufacturing variations adjustment	--	Eq 37
h_t	Whole depth of gear teeth	mm	Eq 32
J	Geometry factor for bending strength	--	Eq 28
J_t	Geometry factor for bending strength under repeated loading	--	Eq 1
J_y	Geometry factor for bending strength under occasional peak loading	--	Eq 2
K_B	Rim thickness factor	--	Eq 31
K_f	Stress concentration factor used in calculating bending geometry factor, J	--	8.2
K_{ft}	Stress correction factor for repeated loading	--	Eq 29
K_{fy}	Stress correction factor for occasional overloads	--	Eq 30
K_L	Life factor	--	Eq 12
K_{LR}	Load reversal factor	--	Eq 12
K_{Ly}	Life factor at 0.5×10^4 cycles	--	Eq 13
K_{mt}	Load distribution factor for repeated loading	--	Eq 31
K_{my}	Load distribution factor for occasional overloads	--	Eq 40
K_{ot}	Overload factor for repeated loads	--	Eq 31
K_{oy}	Overload factor for occasional overloads	--	Eq 40
K_R	Reliability factor	--	Eq 12
K_s	Size factor	--	Eq 12
K_T	Temperature factor	--	Eq 12
K_{ts}	Combined adjustment factor for bending fatigue strength	--	Eq 1
K_{tw}	Combined adjustment factor for repeated tooth loading	--	Eq 1
K_v	Dynamic factor	--	Eq 31
K_y	Yield strength factor	--	Eq 21

(continued)

Table 1 (concluded)

Symbol	Terms	Units	Reference
K_{ys}	Combined adjustment factor for yield strength	--	Eq 2
K_{yw}	Combined adjustment factor for occasional peak loading	--	Eq 2
k_{ut}	Conversion factor for ultimate strength to fatigue limit	--	Eq 5
m	Module	mm	Eq 1
m_B	Backup ratio	--	Eq 32
m_{ct}	Modifying factor due to tooth compliance for repeated loading	--	Eq 35
m_{cy}	Modifying factor due to tooth compliance for occasional overloads	--	Eq 41
m_w	Modifying factor due to tooth surface wear	--	Eq 35
N_G	Number of teeth of gear	--	Eq 24
N_P	Number of teeth of pinion	--	Eq 24
n	Number of tooth load cycles	--	Eq 14
n_u	Number of units for which one failure will be tolerated	--	Eq 20
q_m	Adjustment due to axis misalignment	--	Eq 35
q_v	Adjustment due to manufacturing variations	--	Eq 35
S_b	Bearing span	mm	Eq 36
S_F	Safety factor for bending strength	--	Eq 31
s_t	Design fatigue strength	N/mm ²	Eq 1
s_{tG}	Fatigue limit, full reversal, adjusted for G-1 failure rate	N/mm ²	Eq 3
s_{tT}	G-10 failure rate fatigue limit (published data)	N/mm ²	Eq 3
s_{tTG}	Adjustment in fatigue limit from G-10 to G-1	N/mm ²	Eq 3
s_{uG}	Ultimate tensile strength, adjusted for G-1	N/mm ²	Eq 9
s_{uM}	Minimum ultimate strength listed in MPIF Standard 35	N/mm ²	Eq 10
s_{uT}	Typical ultimate strength (published data)	N/mm ²	Eq 5
s_{uTG}	Reduction in ultimate strength from typical to G-1	N/mm ²	Eq 9
s_y	Design yield strength	N/mm ²	Eq 2
s_{yG}	Yield strength, adjusted for G-1	N/mm ²	Eq 6
s_{yM}	Minimum yield strength listed in MPIF Standard 35	N/mm ²	Eq 7
s_{yT}	Typical yield strength (published data)	N/mm ²	Eq 6
s_{yTG}	Reduction in yield strength from typical to G-1	N/mm ²	Eq 6
T_t	Torque load capacity for tooth bending under repeated loading	Nm	Eq 1
T_y	Torque load capacity under occasional peak loading	Nm	Eq 2
t_R	Rim thickness	mm	Eq 32
V_{qT}	Tooth-to-tooth composite tolerance (or measured variation)	mm	Eq 39
v_t	Pitch line velocity	m/s	Eq 39
Y	Tooth form factor	--	Eq 28

3 Fundamental formulas for calculated torque capacity

Two types of loading have been identified in 1.1.1. Each has its own formula for calculated torque capacity, reflecting the corresponding critical material properties and other factors. To find the load capacity of a gear under the combined types of loading, calculate the two torque values from the

formulas and use the lower calculated value. To find the overall load capacity of a pair of non-identical gears, or of all the gears in the drive train, the calculated load capacity torque for each gear must be converted to a power value. This is done by multiplying the torque value for each gear by the corresponding gear speed, generally expressed as radians per unit time interval. The lowest of all these power values becomes the calculated power capacity of the complete gear pair or drive train.

3.1 Tooth bending under repeated loading

$$T_t = \frac{s_t K_{ts} d_c F_e J_t m}{2000 K_{tw}} \quad (1)$$

where

- T_t is torque load capacity for tooth bending under repeated loading, Nm;
- s_t is design fatigue strength, N/mm² (see 4.1.2.1);
- K_{ts} is combined adjustment factor for bending fatigue strength (see 5.1);
- d_c is calculation diameter, mm (see clause 6);
- F_e is effective face width, mm (see clause 7);
- J_t is geometry factor for bending strength under repeated loading (see clause 8);
- m is module, mm;
- K_{tw} is combined adjustment factor for repeated tooth loading (see clause 9).

3.2 Tooth bending under occasional peak loading

$$T_y = \frac{s_y K_{ys} d_c F_e J_y m}{2000 K_{yw}} \quad (2)$$

where

- T_y is torque load capacity under occasional peak loading, Nm;
- s_y is design yield strength, N/mm²;
- K_{ys} is combined adjustment factor for yield strength;
- K_{yw} is combined adjustment factor for occasional peak loading;
- J_y is geometry factor for bending strength under occasional peak loading.

4 Design strength values

Design strength values depend not only on the P/M material composition, and any heat treatment, but also on the density achieved during compaction or post-sintering repressing.

4.1 Fatigue strength, s_t

The value for design fatigue strength can be obtained from alternate sources.

4.1.1 Previous test experience

If there has been previous successful experience in the laboratory or field testing of gears from the same material of similar density and processing, it may be possible to perform reverse calculations to arrive at an acceptable design fatigue strength. The value derived from this procedure may be overly conservative unless the test program included a range of load conditions that bracketed the line between successful operation and failure by repeated bending.

4.1.2 Derived from published data

When suitable gear test data is not available, published data based on standard material testing methods can be used, but only after adjustments are made to adapt the fatigue strength values to the design procedures of this information sheet. These procedures are based on values that correspond to the following conditions:

- a) number of test cycles of 10⁷;
- b) test failure rates projected to “less than 1 in a 100”, i.e., 1 percent or “G-1” failure rate;
- c) load cycling of zero-to-maximum load (to reflect typical gear tooth load cycling).

4.1.2.1 Data published as “typical fatigue limit”

Such data for P/M materials generally meet condition (a) of 4.1.2, but not conditions (b) and (c). Values called “typical” generally refer to test results with 50% of the specimens falling below and 50% above the published value. This corresponds to a “G-50” failure rate, also known as mean fatigue life.

Data published by the Metal Powder Industries Federation (MPIF) [1] has been determined as the 90% survival stress fatigue limit, using rotating bending fatigue testing. This fatigue limit data is also known as the “G-10” failure rate fatigue life.

Rotating bending fatigue testing imposes load cycling of full-reversal loads. The critical location on the test specimen is subjected to the maximums of both tensile and compressive stresses.

Adjustments to meet the conditions of 4.1.2(b) and (c) are expressed in the following equations:

$$s_{tG} = s_{tT} - s_{tTG} \quad (3)$$

where

- s_{tG} is fatigue limit, full-reversal, adjusted for G-1 failure rate, N/mm²;
- s_{tT} is G-10 failure rate fatigue limit (published data), N/mm²;
- s_{tTG} is the adjustment in fatigue limit from G-10 to G-1, N/mm².

The adjustment, s_{tTG} , has been estimated for P/M steels as 14 N/mm² from a statistical analysis of recently published data [2].

The design fatigue limit, after adjustments, s_t , is:

$$s_t = \frac{s_{tG}}{0.7} \quad (4)$$

The factor of 0.7 is commonly used to convert from full-reversal to zero-to-maximum load cycling. For those gear applications, such as idler or planet gears, where the gear teeth experience fully reversing loads, this adjustment factor will be corrected through the appropriate choice of load reversal factor, see 5.1.2.

4.1.2.2 Data estimated from “typical ultimate tensile strength”

When fatigue limit data is not directly available, it can be estimated from ultimate tensile strength values. This estimation process is described below.

Convert the typical ultimate tensile strength to the G-10 failure rate fatigue limit by the following expression:

$$s_{tT} = k_{ut} s_{uT} \quad (5)$$

where

s_{uT} is typical ultimate tensile strength value, N/mm²;

k_{ut} is conversion factor for ultimate strength to fatigue limit;

For heat treated steel (martensitic microstructure):

$$k_{ut} = 0.32$$

For as-sintered steel (pearlite and ferrite microstructure):

$$k_{ut} = 0.39$$

For as-sintered steel (ferrite only microstructure):

$$k_{ut} = 0.43$$

Then convert this estimated G-10 failure rate fatigue limit, s_{tT} , to the design fatigue limit for zero-to-maximum loading using equations 3 and 4.

4.2 Yield strength, s_y

The value of design yield strength can be obtained from one of two sources.

4.2.1 Previous test experience

If a gear of the same material and similar density and processing has been tested for the load causing permanent deflection or breakage of the teeth, it may be possible to perform reverse calculations to arrive at a limiting design yield strength.

4.2.2 Derived from published data

When suitable gear test data is not available, published data based on standard material testing methods can be used, but only after an adjustment is made to adapt the yield strength values to the design procedures of this information sheet. These procedures are based on values that correspond to the following condition:

- test failure rates projected to “less than 1 in a 100”, i.e., 1% or “G-1” failure rate.

4.2.2.1 Derived from “typical yield strength”

In as-sintered gears, the published data is generally in the form of a “typical yield strength” based on 0.2% offset. This “typical yield strength”, based on a G-50 failure rate, must be converted to a “design yield strength”, based on a G-1 failure rate. This adjustment may be represented by the following equation:

$$s_{yG} = s_{yT} - s_{yTG} \quad (6)$$

where

s_{yG} is yield strength, adjusted for G-1, N/mm²;

s_{yT} is typical yield strength (published data), N/mm²;

s_{yTG} is reduction in yield strength from typical to G-1, N/mm².

The adjustment, s_{yTG} , is best determined from test observations. An alternative method is to refer to MPIF Standard 35, where this step is accomplished for as-sintered materials by the listing of “minimum” strength values. For these materials:

$$s_{yG} = s_{yM} \quad (7)$$

where

s_{yM} is “minimum” yield strength listed in MPIF Standard 35, N/mm².

The design yield strength is then set equal to this adjusted yield strength:

$$s_y = s_{yG} \quad (8)$$

4.2.2.2 Derived from “typical ultimate strength”

In heat treated materials, typical yield strengths are approximately the same as typical ultimate

strengths. Design yield strength, s_y , may be derived from typical ultimate strength by first converting the typical value for a G-50 failure rate to a design value with a G-1 failure rate, as in 4.2.2.1.

$$s_{uG} = s_{uT} - s_{uTG} \quad (9)$$

where

s_{uG} is typical ultimate strength adjusted to the G-1 failure rate, N/mm².

s_{uT} is typical ultimate strength (published data), N/mm²;

s_{uTG} is reduction in ultimate strength from typical to G-1, N/mm².

The adjustment, s_{uTG} , is best determined from test observations. An alternative method is to refer to MPIF Standard 35, where this step is accomplished for heat treated materials by the listing of "minimum" strength values. For these materials:

$$s_{uG} = s_{uM} \quad (10)$$

where

s_{uM} is "minimum" ultimate strength listed in MPIF Standard 35, N/mm².

The design yield strength is then set equal to this adjusted ultimate strength:

$$s_y = s_{uG} \quad (11)$$

5 Combined adjustment factors for strength

This factor is a combination of factors relating to the strength of the P/M gear material under the operating conditions. Use of such a combined factor helps simplify the fundamental formulas in clause 3. As an added advantage, this combined factor may be used without detailed analysis for subsequent gear designs with similar operating conditions.

5.1 Combined factor for bending fatigue strength, K_{ts}

$$K_{ts} = \frac{K_L K_{LR}}{K_s K_T K_R} \quad (12)$$

where

K_L is life factor;

K_{LR} is load reversal factor;

K_s is size factor;

K_T is temperature factor;

K_R is reliability factor.

5.1.1 Life factor, K_L

The life factor is the ratio of the bending fatigue strength at the required number of tooth load cycles, n , to the strength at 10^7 cycles. It can be estimated from the following equations:

For $0 < n < (0.5 \times 10^4)$,

$$K_L = K_{Ly} = \frac{0.9 s_y}{s_t} \quad (13)$$

For $(0.5 \times 10^4) \leq n \leq (1 \times 10^7)$,

$$K_L = 1 + \left[2.121(K_{Ly} - 1) \right] - \left[0.303(K_{Ly} - 1) \right] \log n \quad (14)$$

For $n > (1 \times 10^7)$,

$$K_L = 1, \text{ for ferrous materials only} \quad (15)$$

(for non-ferrous material, consult test data)

where

n is number of tooth load cycles;

K_{Ly} is life factor at 0.5×10^4 cycles, found from equation 13 with strength values from 4.1.2.1 or 4.1.2.2 and 4.2.2.1 or 4.2.2.2.

5.1.2 Load reversal factor, K_{LR}

In 4.1.2.1, the factor of 0.7 was introduced to adjust the fatigue strength values for the difference in cyclic loading in material testing from the typical cyclic loading of gear teeth. In material testing, the load is fully reversed while in most gear applications the load is zero-to-maximum in one direction only. The K_{LR} factor reverses this adjustment for those less typical gear applications in which the gear tooth loading is bidirectional, as follows:

$$K_{LR} = 1.0 \text{ if load is unidirectional} \quad (16)$$

$$K_{LR} = 0.7 \text{ if load is bidirectional, as in idler or planet gears} \quad (17)$$

5.1.3 Size factor, K_s

In some wrought materials, the stock from which the gear is machined may have non-uniform material properties which are related to size. However, with P/M materials, the properties of the powder mix are independent of the size of the finished gear. The size of the P/M gear may influence processing, which in turn may affect the strength properties at the gear teeth, but only through change to other material characteristics such as density and hardness. In that case, the size effects will be reflected directly in the fatigue strength value, s_b , as described in 4.1. Therefore, for P/M gears, size factor, K_s , is:

$$K_s = 1 \quad (18)$$

5.1.4 Temperature factor, K_T

This factor reflects any loss of strength properties at high operating temperatures. This applies to hardened gears for which a temperature over 177°C may cause some tempering.

For gear blank temperatures below the level at which strength is affected:

$$K_T = 1 \quad (19)$$

For gear blank temperatures above the level at which strength is affected, K_T is increased to reflect the loss in strength. For very low gear blank temperatures in impact prone applications, K_T may be increased to reflect any reduction in impact properties.

5.1.5 Reliability factor, K_R

This factor accounts for the effect of the typical statistical distribution of failures found in fatigue testing of materials. Its value is based on the frequency of failures that can be tolerated in the gear application, expressed as no more than one failure in some number of units, n_u . K_R may be estimated from the following equation:

$$K_R = 0.5 + 0.25 \log n_u \quad (20)$$

where

n_u is number of units for which one failure will be tolerated.

Some values from this equation, along with equivalent “G” values, are given in table 2.

5.2 Combined factor for yield strength, K_{ys}

$$K_{ys} = \frac{K_y}{K_s K_T} \quad (21)$$

where

K_y is yield strength factor;

K_s is size factor (see 5.1.3);

K_T is temperature factor (see 5.1.4).

5.2.1 Yield strength factor, K_y

This factor reflects the difference between the response of hardened versus unhardened materials

to stresses developed during occasional peak loading.

For unhardened materials:

$$K_y = 1.00 \quad (22)$$

For hardened materials:

$$K_y = 0.75 \quad (23)$$

5.2.2 Stress correction factor, K_f

This factor is used in the calculation of J , the geometry factor for bending strength (see clause 8). It reflects the increase in local stresses due to sharp changes in geometry at or near the critical section. These increased stresses directly affect the bending strength under repeated loading. Under occasional loads, however, local yielding may take place and the stress concentration has little or no significant effect on load capacity. In the AGMA gear rating calculation, this difference is treated by re-introducing the stress correction factor as a beneficial adjustment to the yield strength. In the calculation procedures of this document, a different and more direct approach is used, and such an adjustment is not needed and is not included in the above “combined factor for yield strength”. As described in clause 8 and annex C, the J factor for each type of loading is calculated with a stress correction factor which is appropriately modified to reflect the differences.

6 Calculation diameter, d_c

The calculation diameter, as used in equations 1 and 2, must agree with the diameter value used in calculating the Y factor, see annex B. For spur gears, it is the same as the operating pitch diameter of the gear for which the torque capacity is to be calculated. Its value depends on the relative numbers of teeth and the operating center distance and may be, but is not necessarily, equal to the standard pitch diameter, as follows:

Table 2 - Reliability factors, K_R

Requirement of application:	n_u units	Equivalent G-value	K_R
No more than 1 failure in:	10,000	G-0.01	1.50
	1,000	G-0.10	1.25
	100	G-1.00	1.00

For the pinion:

$$d_c = d_{AP} = \frac{2 C_A}{1 + \frac{N_G}{N_P}} \quad (24)$$

where

d_{AP} is operating pitch diameter of pinion, mm;

C_A is operating center distance, mm;

N_P is number of teeth of pinion;

N_G is number of teeth of gear.

For the gear:

$$d_c = d_{AG} = \frac{2 C_A}{1 + \frac{N_P}{N_G}} \quad (25)$$

where

d_{AG} is operating pitch diameter of gear, mm.

7 Effective face width, F_e

The effective face width represents the face width capable of resisting bending loads. If the two mating gears have the same face widths which are fully overlapping, then the effective face width of each is equal to the common face width. If, however, there is a portion of a face width which extends beyond the overlapping width, then this extension may contribute to resisting the bending load.

The extensions may be present at one or both ends of the face width of either of the mating gears.

This may be expressed as equations:

$$F_e = F_o + F_{xe1} + F_{xe2} \quad (26)$$

where

F_e is effective face width, mm;

F_o is overlapping face width, mm;

F_{xe1} is effective face width extension at one end, mm;

F_{xe2} is effective face width extension at other end, mm.

These effective face width extensions may be estimated as follows:

For each extension:

$$F_{xe} = \left(1 - \frac{F_x}{2m}\right) F_x \quad (27)$$

where

F_x is each face width extension (not larger than m), mm;

m is module, mm.

8 Geometry factor for bending strength, J

The geometry factor is a non-dimensional value which relates the shape of the gear tooth, along with some associated geometry conditions, to the tensile bending stress induced by a unit load applied on the tooth flank. For spur gears, there are two elements which go into its calculation:

$$J = \frac{Y}{K_f} \quad (28)$$

where

Y is tooth form factor (see annex B);

K_f is stress correction factor (see annex C).

8.1 Tooth form factor, Y

This factor is dependant only on geometry, with the addition of a coefficient of friction where the tooth sliding friction force may have a significant effect on stresses. As part of making this a non-dimensional factor, the geometry is scaled to a tooth of unit module. The elements of the factor are:

- the location along the tooth flank where the tooth load will have its greatest effect on bending stress;
- the proportions of the tooth shape, especially in the region of the tooth fillet;
- the diameter used to relate applied torque values to a tangential force, by tradition the operating pitch diameter of the gear.

The calculation for determining the Y factor is described in annex B with calculation of some of the required geometry data described in annex A.

8.2 Stress correction factor, K_f

This factor is determined by a combination of tooth geometry, the type of loading, and some property of the material that determines to what extent it is sensitive to stress concentration. The calculation is described in annex C.

Since the type of loading may be a significant factor, there will generally be two values considered for each gear. One, K_{ft} , is for repeated loading and the other, K_{fy} , is for the occasional overload condition. This leads to two possible values for the J factor:

For repeated loading:

$$J_t = \frac{Y}{K_{ft}} \quad (29)$$

where

K_{ft} is stress correction factor for repeated loading.

For occasional overloads:

$$J_y = \frac{Y}{K_{fy}} \quad (30)$$

where

K_{fy} is stress correction factor for occasional overloads.

9 Combined adjustment factors for loading

This is a combination of the remaining load capacity factors, most of which relate to tooth loading under the operating conditions. The use of such a combined factor helps simplify the fundamental formulas in clause 3. As an added advantage, this combined factor may be used without detailed analysis for subsequent gear designs with similar operating conditions.

9.1 Combined adjustment factor for repeated tooth loading, K_{tw}

$$K_{tw} = S_F K_{ot} K_B K_{mt} K_v \quad (31)$$

where

S_F is safety factor for bending strength;
 K_{ot} is overload factor for repeated loads;
 K_B is rim thickness factor;
 K_{mt} is load distribution factor for repeated loading;
 K_v is dynamic factor.

9.1.1 Safety factor, S_F

A safety factor is commonly introduced into design calculations to provide greater protection against possible failure. This protection may be sought because of concern that some elements of the design process may have overstated the strength of the material or may have understated the level of the loading. Sometimes the added protection against failure is based on concern for some extremely severe result of failure.

In selecting a value for safety factor, it is first necessary to recognize that many of these concerns

have already been addressed elsewhere in the calculations. As for material strength, there have been a whole series of adjustments, such as the selection of the G-1 values from published data, see clause 4, and the various factors defined in clause 5. Similarly for the level of loading, a number of adjustments have been introduced, as described in clause 9. Based on concerns for material strength and loading, unless these adjustments are judged to be inadequate, the suggested value for the safety factor would be one.

This first selection may be increased after consideration of the possible results of failure of the gear under study. If such failure is likely to be followed by severe economic loss, or even more importantly, by injury to those associated with the failed equipment, then the safety factor should reflect the level of the hazards.

Also to be considered is the level of testing that precedes final acceptance of the design. Because the P/M process is used to produce gears for mass production, there is generally the need and opportunity for extensive testing. This, and the recognition that P/M processes are highly consistent, indicates that high safety factors are rarely necessary.

9.1.2 Overload factor for repeated loads, K_{ot}

This factor allows for two types of repeated overloads. One type is the overload that results from operation of the product beyond its nominal rating. If the calculated load capacity is going to be compared to the load associated with the nominal rating, then this factor should be adjusted to reflect this potential overload. The other type is the overload resulting from externally applied dynamic loads. Anything in the drive train that is not steady in its effect on transmitted torque or speed may introduce dynamic torques. For example, non-steady torques are associated with driving members like internal combustion engines or some types of hydraulic motors. They are also associated with varying drive train loads such as reciprocating pumps or intermittent cutting actions.

The selection of the appropriate value of this factor may be based on a thorough dynamic analysis of the drive train with all its inertia, compliance and damping effects. Most often, however, it will be selected in accordance with past experience with similar products and with the application of engineering judgement.

9.1.3 Rim thickness factor, K_B

The calculation of bending strength at the tooth fillet, as in annex B, presupposes that the material in the adjacent areas is adequate to support the stressed regions. If the rim thickness under the root circle is too small to provide this support, or is itself under stress from transmitting torque from the gear web or spokes, then a rim thickness factor is needed to compensate for these rim shortcomings.

The P/M gear is rarely designed with a narrow web and extended rim, as is the common practice in machined or cast wide-face gears. For the typical P/M gear, therefore, the rim thickness factor is set to one. There is a practice of introducing holes into the otherwise solid web of P/M gears to reduce weight and compaction area. If these holes are placed too close to the root circle of the gear teeth, a condition similar to a thin rim results. The rim thickness factor may then be calculated as follows:

Backup ratio, m_B

$$m_B = \frac{t_R}{h_t} \quad (32)$$

where

t_R is rim thickness, mm;

h_t is whole depth of gear teeth, mm.

Rim thickness factor, K_B

For $m_B \geq 1.2$

$$K_B = 1 \quad (33)$$

For $m_B < 1.2$

$$K_B = 1.2916 - 3.682 \log m_B \quad (34)$$

9.1.4 Load distribution factor for repeated loads, K_{mt}

This factor accounts for any lack of complete and uniform contact along the axial length of the mating gear teeth. Such limited contact interferes with a uniform distribution of the transmitted load. The load tends to concentrate where contact is best, which raises the bending stress at the corresponding positions along the base of the tooth. Adjacent portions of the tooth help to support these concentrated loads and, to some extent, limit the rise in local stress.

It is generally impractical to precisely evaluate the exact nature of the non-uniform load distribution, its effect on local bending stress, and the resulting loss in load capacity. Instead, a value for the load

distribution factor is estimated by considering the various items which contribute to, or partially offset, the effect on tooth bending strength.

The common contributing items are:

- misalignment of the gear axes due to manufacturing variations in the geometry of the housing, bearings, shafts, and any other support features;
- manufacturing variations in the geometry of the tooth surfaces, such as axial runout (wobble) or non-uniform tooth thickness across the face width.

The effect of these items on non-uniform load distribution increases with the face width of the mating gears. In the case of gear axis misalignment, the size of the face width in relation to the bearing span is often significant. In the case of tooth surface geometry, the manufacturing variations tend to increase as face width becomes larger in relation to gear diameter.

The common items that tend to improve load distribution are:

- local tooth compliance in the form of bending or twisting of the tooth, combined with contact surface deformations;
- local tooth surface wear, especially in the early cycles of repeated loading.

The load distribution factor for repeated loads can be related to these items by the following equation:

$$K_{mt} = 1 + [(q_m + q_v)(m_{ct} \times m_w)] \quad (35)$$

where

q_m is adjustment due to axis misalignment;

q_v is adjustment due to manufacturing variations;

m_{ct} is modifying factor due to tooth compliance;

m_w is modifying factor due to tooth surface wear.

Procedures for selecting approximate values for these factors are described below. They qualitatively consider many of the elements that can influence the effect of non-uniform load distribution. The quantitative values are only estimates which may be used until more appropriate values are developed by analytical or experimental methods.

9.1.4.1 Axis misalignment adjustment, q_m

This factor recognizes that the extent of axis misalignment will be influenced by the expected

accuracy of the housing, the type of bearings, and the mounting of the gear with respect to bearing locations. It also recognizes that with misalignment determined by these conditions, its contribution to non-uniform load distribution will increase with face width.

$$q_m = f_{qm} \frac{F_o}{S_b} \quad (36)$$

where

F_o is overlapping face width, mm;

S_b is bearing span, mm;

f_{qm} is factor relating to axis misalignment adjustment:

For machined metal housing with rolling element bearings:

$$f_{qm} = 0.1$$

For machined metal housing with straddle mounted sleeve bearings:

$$f_{qm} = 0.2$$

For machined metal housing with overhung mounted sleeve bearings:

$$f_{qm} = 0.5$$

For as-cast or molded housing with straddle mounted sleeve bearings:

$$f_{qm} = 0.6$$

For as-cast or molded housing with overhung mounted sleeve bearings:

$$f_{qm} = 1.0$$

9.1.4.2 Manufacturing variations adjustment, q_v

This factor considers that P/M process variations from ideal gear geometry are influenced by gear proportions. This influence is expressed, for the sake of simplicity, in terms of the ratio of face width to pitch diameter. It also recognizes that gear geometry may be substantially improved by a final finishing process.

$$q_v = f_{qv} \frac{F_o}{d} \quad (37)$$

where

F_o is overlapping face width, mm;

d is gear pitch diameter, mm;

f_{qv} is factor relating to manufacturing variations adjustment (see table 3).

Table 3 - Manufacturing variation adjustment

Typical AGMA accuracy grade ¹⁾	f_{qv}
Q5	1.0
Q6	0.75
Q7	0.6
Q8	0.4
Q9	0.3
Q10	0.2
NOTE: 1) See AGMA 2000-A88.	

9.1.4.3 Tooth compliance modifying factor, m_{ct}

This factor takes into account the compliance of the material, as indicated by its modulus of elasticity, and the degree of loading, as indicated by the design stress.

$$m_{ct} = 1 - 5 \left(\frac{s_t}{E} \right)^{0.5} \quad (38)$$

where

s_t is design fatigue limit, N/mm² (see 4.1.2.1);

E is modulus of elasticity, N/mm².

9.1.4.4 Tooth wear modifying factor, m_w

This factor considers that wear is affected by the hardness of the tooth surfaces, with very slow wear expected from heat treated P/M materials. Also, the kind of wear which best corrects for non-uniform contact conditions takes place when each tooth is contacted by only one tooth on the mating gear. This contact condition is met only when the gear ratio has an integer value.

For one or both gears in as-sintered condition and with an integer value for gear ratio:

$$m_w = 0.6$$

For one or both gears in as-sintered condition and with a non-integer value for gear ratio:

$$m_w = 0.8$$

For both gears in heat treated condition:

$$m_w = 1.0$$

9.1.5 Dynamic factor, K_v

This factor accounts for the added dynamic tooth loads that are developed by the meshing action of the gears. These loads are influenced by:

- imperfections in the geometry of the gear teeth;
- speed of the meshing action;

- size and mass of the gears.

In principle, the appropriate value of this factor may be derived from a thorough dynamic analysis of the drive train with consideration of all these influences. In practice, an approximate value may be calculated from an equation which uses a gear inspection value as the indicator of imperfect geometry and the pitchline velocity as the meshing speed indicator.

The gear inspection most commonly used for P/M gears is the gear rolling check, or double flank test, in which the test gear is rolled with a master gear. See AGMA 2000-A88. One measurement made by this inspection is the tooth-to-tooth composite variation, an approximate indicator of the degree that the gear will contribute to exciting dynamic loads. This value, as expressed by its tolerance, V_{qT} , is part of the specification of gear quality. If measured values are available, they may be used in place of the tolerance. Since meshing conditions are determined by the geometry of both gears, if the tolerances or measurements differ between the two, the value used in the following calculations should be the larger.

$$K_v = \left(1 + 0.0055 V_{qT} \sqrt{v_t}\right)^{0.5} \quad (39)$$

where

V_{qT} is tooth-to-tooth composite tolerance (or measured variation), μm ;

v_t is pitch line velocity, m/s.

9.2 Combined adjustment factor for occasional overloads, K_{yw}

$$K_{yw} = S_F K_{oy} K_B K_{my} K_v \quad (40)$$

where

S_F is safety factor for bending strength;

K_{oy} is overload factor for occasional overloads;

K_B is rim thickness factor;

K_{my} is load distribution factor for occasional overloads;

K_v is dynamic factor.

9.2.1 Safety factor, S_F

This factor is generally the same as the safety factor discussed in 9.1.1 for fatigue loading.

9.2.2 Overload factor for occasional overloads, K_{oy}

This factor should be based on the types of occasional overloads that may be applied to the gears. Some considerations are items such as the inertia and time duration of load in the system under consideration. These may be different from the repeated overloads and will generally require a different factor.

9.2.3 Rim thickness factor, K_B

The same factor discussed in 9.1.3 is used here.

9.2.4 Load distribution factor for occasional overloads, K_{my}

The equation used to estimate this factor is:

$$K_{my} = 1 + [(q_m + q_v)m_{cy}] \quad (41)$$

Note that this equation differs from the equation in 9.1.4 in that the modifying factor due to tooth surface wear has been omitted. Occasional overloads may occur before wear has progressed enough to modify load distribution. The remaining factors are the same except for m_{cy} , the modifying factor due to tooth compliance which is here estimated by:

$$m_{cy} = 1 - 5\left(\frac{s_y}{E}\right)^{0.5} \quad (42)$$

where

s_y is design yield strength, N/mm² (see 4.2).

9.2.5 Dynamic factor, K_v

The same factor discussed in 9.1.5 is used here.

Annex A

(informative)

Calculation of spur gear geometry features

[This annex is provided for informational purposes only and should not be construed as a part of AGMA 930-A05, *Calculated Bending Load Capacity of Power Metallurgy (P/M) External Spur Gears*.]

A.1 Introduction

The calculation of the spur gear form factor in annex B requires data describing a number of gear geometry features. This annex gives the detailed calculations for each of these features as listed below. See A.9 for listing of symbols and terms.

For the individual gear:

- effective outside diameter after tip rounding, see A.3.1;
- tooth thickness at indicated diameter, see A.4.1;
- generated trochoid fillet points, see A.4.5;
- minimum fillet radius, see A.4.6;
- circular-arc fillet points, see A.5.6.

For the gear mesh:

- operating pitch diameters, see A.7.2;
- diameters at highest points of single tooth loading, see A.8.2.

In addition, this annex supplies some detailed calculations for features not required by annex B. These have been included because they are connected to the required calculations and are useful for general reference purposes.

For the individual gear:

- remaining top land after tip rounding, see A.3.2;
- points on the involute profile, see A.4.2;
- bottom land for the circular-arc fillet, see A.5.5.

For the gear mesh:

- profile contact ratio, see A.8.4;
- form limit clearance (test for tip-fillet interference), see annex F.

A.2 Input data

A.2.1 Data common to the mating gears

- module, m ;
- pressure angle, ϕ .

A.2.2 Data for each gear

Member designated by final subscript: P = pinion (driver) and G = gear (driven)

- number of teeth, N ;
- outside diameter, d_O ;
- tip radius, r_r ;
- tooth thickness (at reference diameter), t ;
- root diameter (for circular-arc fillet), d_R ;
- fillet radius (for circular-arc fillet), r_f ;
- basic rack dedendum (for generated trochoid fillet), b_{BR}
- basic rack fillet radius (for generated trochoid fillet), r_{fBR} .

A.2.3 Gear mesh data

- effective operating center distance, C_A .

A.3 Tip radius geometry

See figure A.1.

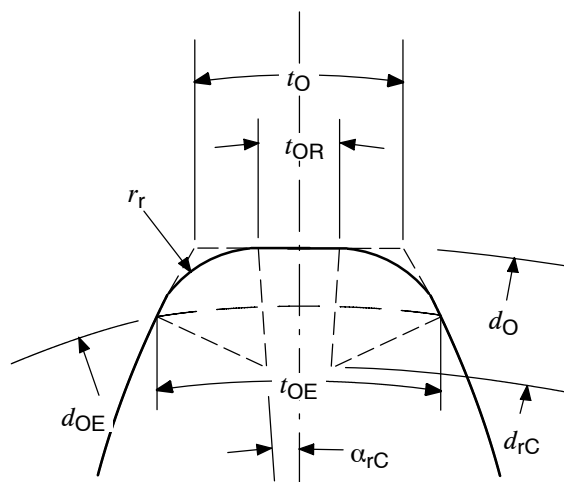


Figure A.1 - Tip round

A.3.1 Effective outside diameter, d_{OE}

This is the diameter at which the involute joins in tangency with the tip round. It is calculated for each gear in the following steps:

Step 1. Diameter at center of tip round, d_{rC} :

$$d_{rC} = d_O - 2r_r \quad (A.1)$$

Step 2. Standard pitch diameter, d :

$$d = N \times m \quad (\text{A.2})$$

Step 3. Base circle diameter, d_B :

$$d_B = d(\cos \phi) \quad (\text{A.3})$$

Step 4. Pressure angle at center of tip round, ϕ_{rC} :

$$\phi_{rC} = \arccos \frac{d_B}{d_{rC}} \quad (\text{A.4})$$

Step 5. Pressure angle at effective outside diameter, ϕ_{OE} :

$$\phi_{OE} = \arctan \left(\tan \phi_{rC} + \frac{2r_r}{d_B} \right) \quad (\text{A.5})$$

Step 6. Effective outside diameter, d_{OE}

$$d_{OE} = \frac{d_B}{\cos \phi_{OE}} \quad (\text{A.6})$$

A.3.2 Remaining top land, t_{OR}

This is the width of the outer tip of the gear that remains after rounding at each corner. The calculation is needed only as a check on the design of the gear. It consists of two steps and uses some of the data found in A.3.1.

Step 1. Tooth thickness half-angle, α :

$$\alpha = \frac{t}{d} \quad (\text{A.7})$$

Step 2. Remaining top land, t_{OR}

$$t_{OR} = d_O \left[\alpha + (\text{inv } \phi) - (\tan \phi_{OE}) + \phi_{rC} \right] \quad (\text{A.8})$$

If the calculated remaining top land is negative, the two tip radii intersect inside of the selected outside diameter. To correct this design flaw, one or more of the following design changes are needed:

- reduce the tip radius;
- reduce the outside diameter;
- increase the tooth thickness.

A.4 Generated trochoid fillet points

The trochoid described below is generated by a rack shaped outline rolling on the standard pitch circle of the gear. This rack shaped outline, universally called a “basic rack”, is often visualized as the outline of an imaginary rack shaped gear generating tool such as

a hob. Although such a tool is not actually used to manufacture a P/M gear, the corresponding basic rack may be used to define the P/M gear trochoid fillet.

If the P/M gear is to replace a gear machined by another type of tool, such as a gear shaper cutter, the trochoid described here will be slightly different from the shape of that machined trochoid. Some gears are machined with a protuberance feature on the tool. The protuberance provides an undercut fillet which can clear the tip of a finishing tool used to modify the involute flank in a secondary operation. This analysis does not cover such a feature, even when it is used on a hob or other rack shaped generating tool. It has been omitted because the addition of an undercut condition is rarely needed in P/M gears.

A.4.1 Basic rack

The calculation uses several data items related to the basic rack. See figure A.2.

A.4.1.1 Specified basic rack proportions

The following data items define the portion of the basic rack that helps determine the trochoid fillet:

- tooth thickness, t_{BR} ;
- dedendum, b_{BR} ;
- fillet radius, r_{fBR} .

These data can be taken from the basic rack specification. It is customary for standards to specify basic rack proportions for unit module. The above items would then be calculated by adjusting the unit pitch data for the actual module of the gear, m .

If a separate basic rack specification is not available, values of the first two of these items can be determined from some of the data in A.2, as follows:

Basic rack tooth thickness, according to common practice:

$$t_{BR} = \frac{\pi m}{2} \quad (\text{A.9})$$

Basic rack dedendum, based on the specified gear root diameter:

$$b_{BR} = 0.5 \left(Nm + \frac{t - t_{BR}}{\tan \phi} - d_R \right) \quad (\text{A.10})$$

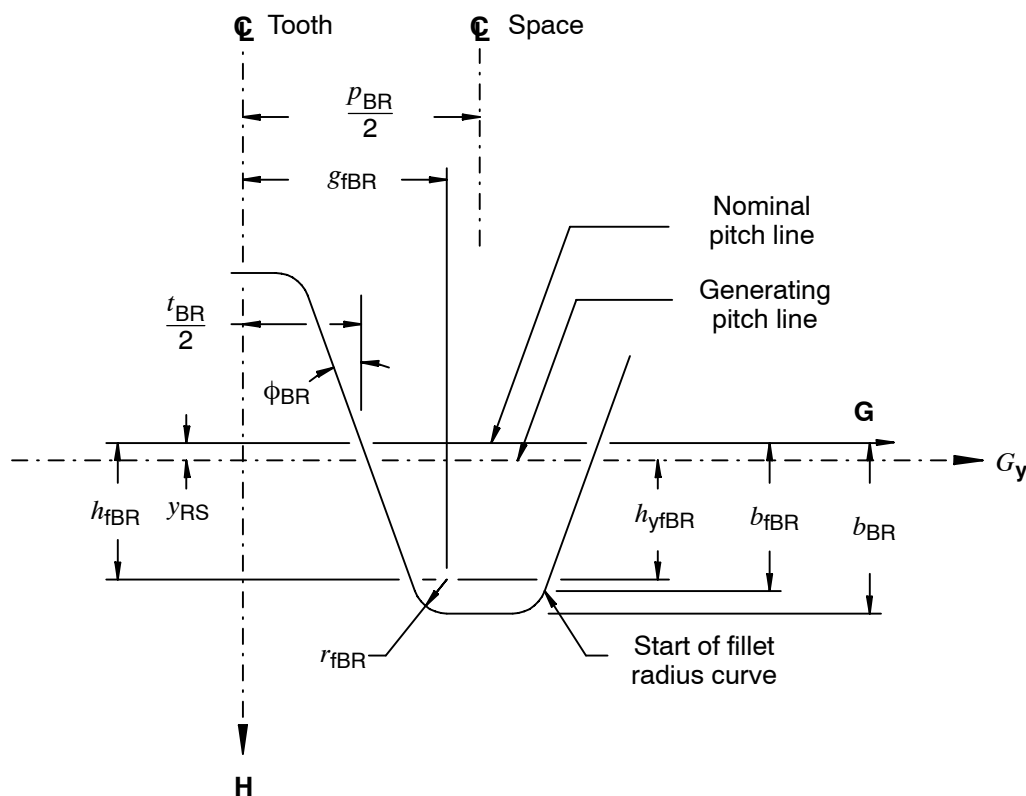


Figure A.2 - Generating basic rack

The third data item, basic rack fillet radius, can not be determined from other data but must be independently specified, as noted in A.2.2. The radius may be zero, indicating a sharp corner, but is almost always a greater value, up to one-fourth of the basic rack dedendum or even larger. However, it may not exceed the size of the full round radius. A full round basic rack fillet will produce a full round gear fillet, leaving no part of a root circle between joined fillets. This maximum basic rack fillet radius is:

$$r_{fBRX} = \frac{\frac{\pi m \cos \phi}{4} - b_{BR}(\sin \phi)}{1 - (\sin \phi)} \quad (\text{A.11})$$

A.4.1.2 Calculated basic rack data

The above data may be used to calculate additional items of basic rack geometry, namely:

- basic rack form dedendum;
- location of the center of the basic rack fillet radius.

The basic rack form dedendum, b_{fBR} , refers to the distance from the basic rack nominal pitch line to the tangent point at the straight line tooth flank and the fillet radius curve. It is calculated as follows:

Basic rack form dedendum:

$$b_{fBR} = b_{BR} - r_{fBR} [1 - (\sin \phi)] \quad (\text{A.12})$$

The center of the fillet radius is located on the basic rack by its coordinates, g_{fBR} and h_{fBR} , relative to the nominal pitch line, as the G-axis, and the tooth centerline, as the H-axis. See figure A.2. These coordinates are calculated as follows:

G-axis coordinate:

$$g_{fBR} = \frac{t_{BR}}{2} + \left[(b_{BR} - r_{fBR})(\tan \phi) \right] + \frac{r_{fBR}}{\cos \phi} \quad (\text{A.13})$$

H axis coordinate (measured from the G-axis located at the nominal pitch line):

$$h_{fBR} = b_{BR} - r_{fBR} \quad (\text{A.14})$$

A.4.2 Rack shift

The generating pitch line on the basic rack, which rolls on the generating pitch circle on the gear, is commonly offset from the nominal pitch line on the basic rack. The rack shift is the offset distance and, as shown in figure A.2, is positive in the direction away from the gear center. This distance is calculated, as follows:

Rack shift:

$$y_{RS} = \frac{t - t_{BR}}{2(\tan \phi)} \quad (A.15)$$

Since the generating action that defines the trochoid is based on the basic rack generating pitch line, the fillet radius center must now be located relative to this line, which is labeled as the Gy-axis. See figure A.2.

Coordinate along the H-axis (measured from the Gy-axis located at the generating pitchline):

$$h_{yfBR} = h_{fBR} - y_{RS} \quad (A.16)$$

The basic rack form dedendum from equation A.12 and the rack shift from equation A.15 are used to test for undercutting as follows:

there is undercutting if:

$$(b_{fBR} - y_{RS}) > \frac{d}{2}(\sin^2 \phi)$$

there is no undercutting if:

$$(b_{fBR} - y_{RS}) \leq \frac{d}{2}(\sin^2 \phi) \quad (A.17)$$

A.4.3 Trochoid generating limits

The trochoid extends from its “start”, point R on the root circle, to its “end”, point F where it connects to the involute profile. This connection is generally a tangency, but becomes an intersection in the case of undercutting.

Figure A.3(a) and (b) show the basic rack positioned to generate the limit points for the first two of these

conditions. At each basic rack position, there is a straight line connecting three points:

- point of contact (pitch point) between the rack generating pitch line and the gear generating pitch circle;
- point at the center of the rack fillet radius;
- point on the generated trochoid (also on the rack fillet radius).

The “pitch-point trochoid line”, makes the “pitch-point polar angle”, θ_f , with the rack pitch line. Each generated point on the trochoid is associated with a value of this angle.

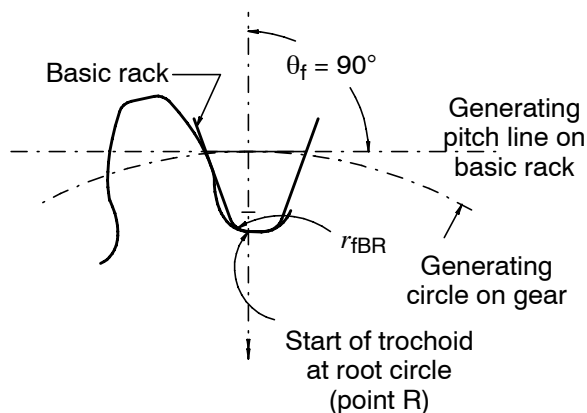
At the start of the trochoid, figure A.3(a), the trochoid point is on the root circle, and the same point is at the root of the rack fillet radius. The pitch-point trochoid line is also a radial line of the gear. The pitch-point polar angle for this trochoid point on the root circle is:

$$\theta_{fR} = 90^\circ \quad (A.18)$$

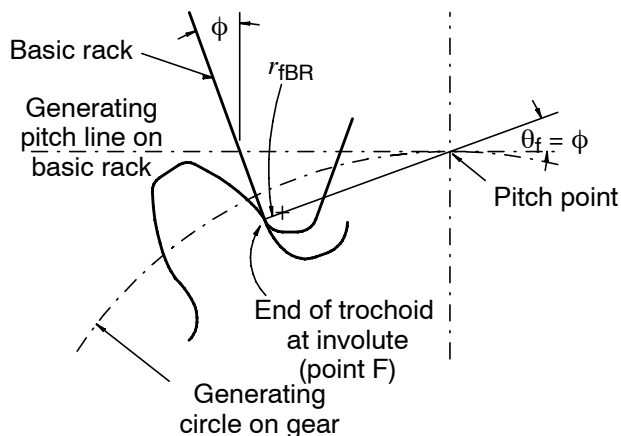
For the typical case of tangency to the involute, the trochoid ends at the point of tangency, or form diameter point, see figure A.4(b). The pitch point polar angle for this trochoid point is:

$$\theta_{fF} = \phi \quad (A.19)$$

In the case of undercut gears, the trochoid ends in an intersection with the involute. The pitch point polar angle corresponding to this intersection point is slightly larger than the value of equation A.19.



(a) Start of trochoid at root circle



(b) End of trochoid at involute

Figure A.3 - Start and end of generated trochoid

The exact value of this angle and the subsequent calculation of the exact values of the coordinates of the intersection point are not essential to the fillet profile data used in annex B. If the exact coordinates are desired for a complete detailed tooth outline, they must be found by an iterative calculation searching for the intersection of the trochoid curve and the connected involute. The numerical steps in such a calculation are beyond the scope of this document. However, this intersection may be found graphically after extending the involute curves. This procedure is supplied in A.6.2.

A.4.4 Fillet point selection

If the trochoid is to be described by a selected number of points, n_f , then the values of equations A.18 and A.19 become the first and n_f -th values of this angle, or:

$$\theta_{f1} = \theta_{fR} = 90^\circ \quad (\text{A.20})$$

$$\theta_{fn} = \theta_{fF} = \phi \quad (\text{A.21})$$

Intermediate points can be found from equally spaced intermediate values of the pitch point polar angle. The following equation gives the value of the “ k -th” point and applies to the intermediate and the start and end points:

$$\theta_f = \frac{\theta_{f1}(n_f - k) + \theta_{fn}(k - 1)}{n_f - 1} \quad (\text{A.22})$$

for $(k = 1 \text{ to } n_f)$

where

n_f is number of points along the fillet.

A.4.5 Fillet point coordinates

These coordinates can be calculated as follows, see figure A.4(a), (b) and (c):

Step 1. Pitch point polar radius:

$$\rho_f = \frac{h_{yfBR}}{\sin \theta_f} + r_{fBR} \quad (\text{A.23})$$

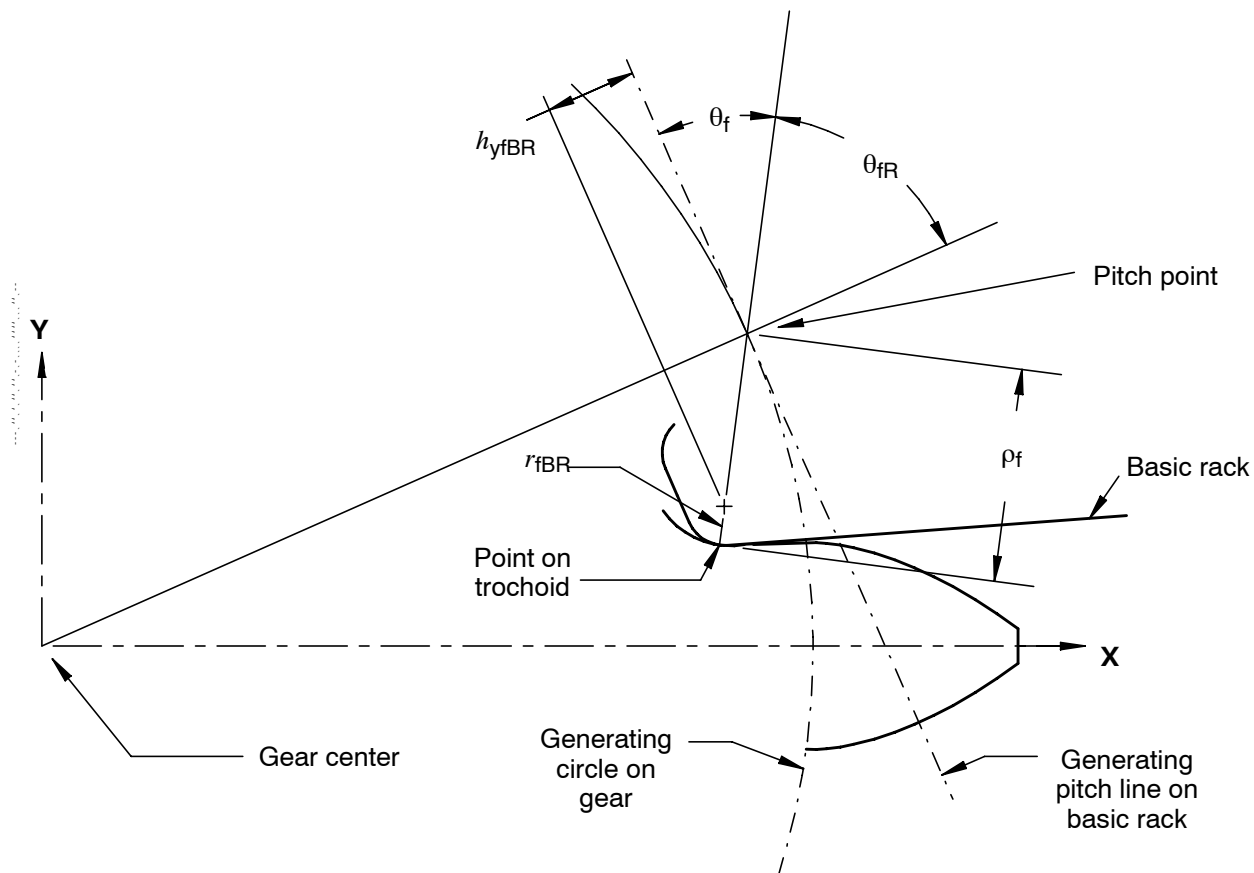


Figure A.4(a) - Generation of fillet point of spur gear tooth

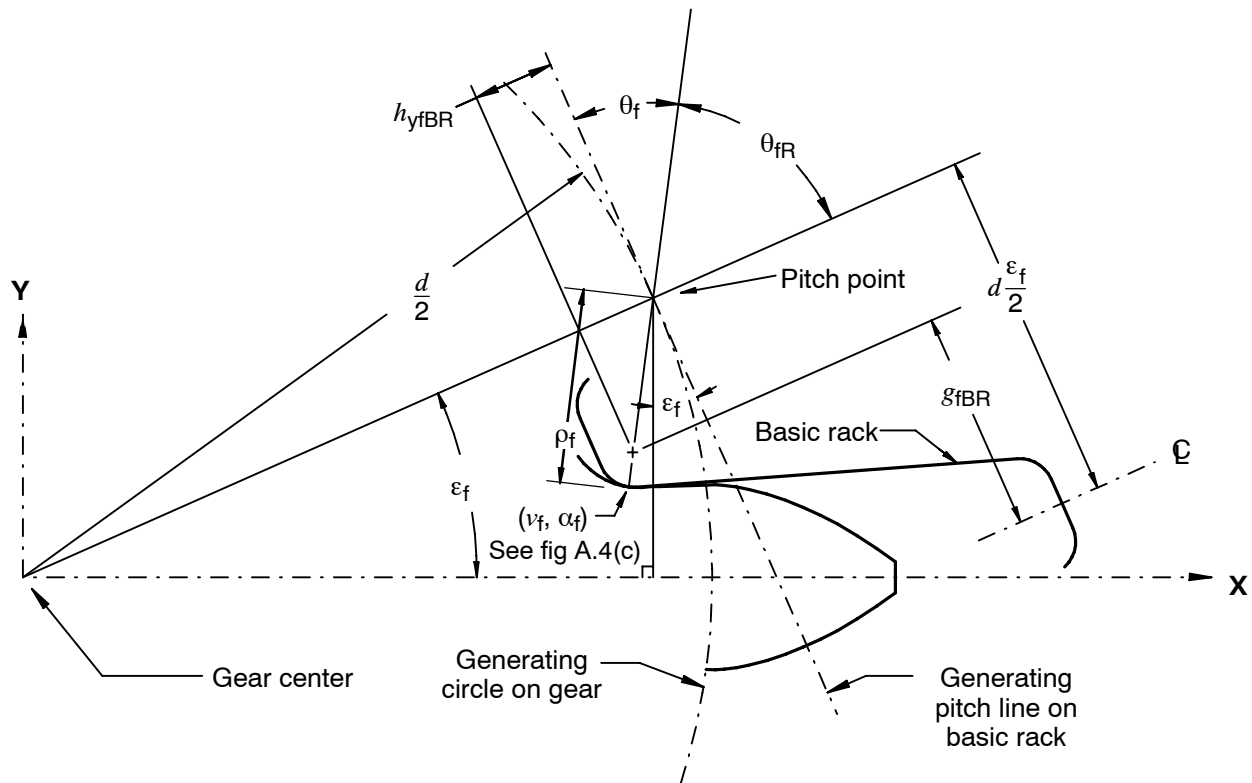


Figure A.4(b) - Generation of fillet point of spur gear tooth

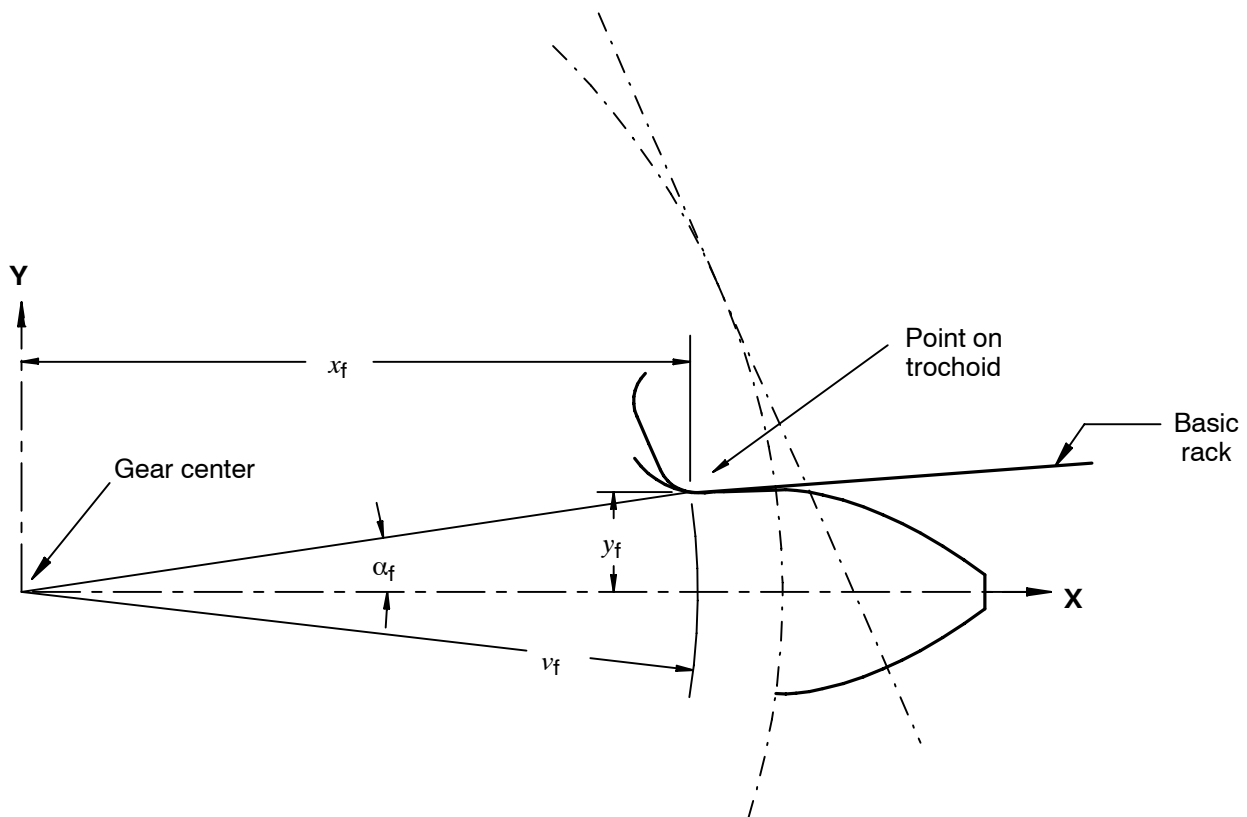


Figure A.4(c) - Generation of fillet point of spur gear tooth

Step 2. Generating roll angle from a pitch point at tooth centerline to a pitch point at which k -th trochoid point is generated:

$$\varepsilon_f = \frac{2 \left(s_{fBR} + \frac{h_{yfBR} \cos \theta_f}{\sin \theta_f} \right)}{d} \text{ radians} \quad (\text{A.24})$$

NOTE: $\frac{\cos \theta_f}{\sin \theta_f}$ is used in place of $\frac{1}{\tan \theta_f}$ to permit evaluation for $\theta_f = 90^\circ$.

Step 3. Polar coordinates of trochoid point relative to tooth centerline, gear center polar radius and gear center polar angle:

$$v_f = \sqrt{\left(\frac{d}{2}\right)^2 + (\rho_f)^2 - d(\rho_f)(\sin \theta_f)} \quad (\text{A.25})$$

$$\alpha_f = \varepsilon_f - \arcsin \frac{\rho_f (\cos \theta_f)}{v_f} \text{ radians} \quad (\text{A.26})$$

Step 4. Rectangular coordinates of trochoid point, relative to gear tooth centerline as the X-axis with the origin at the gear center:

$$x_f = v_f (\cos \alpha_f) \quad (\text{A.27})$$

$$y_f = v_f (\sin \alpha_f) \quad (\text{A.28})$$

A.4.6 Minimum radius along trochoid curve

The shape of the trochoid is such that the radius of curvature varies from point to point. The value of this radius at any point is determined by the generating action of the pitch point polar radius. The minimum value is used in the stress concentration calculations of annex C. This minimum value, R_{fN} , corresponds to this radius at the start of the trochoid, where the trochoid is tangent to the root circle and the pitch point polar angle, θ_f , is equal to 90° . See figure A.3(a).

$$R_{fN} = \frac{h_{yfBR}^2}{0.5 d + h_{yfBR}} + r_{fBR} \quad (\text{A.29})$$

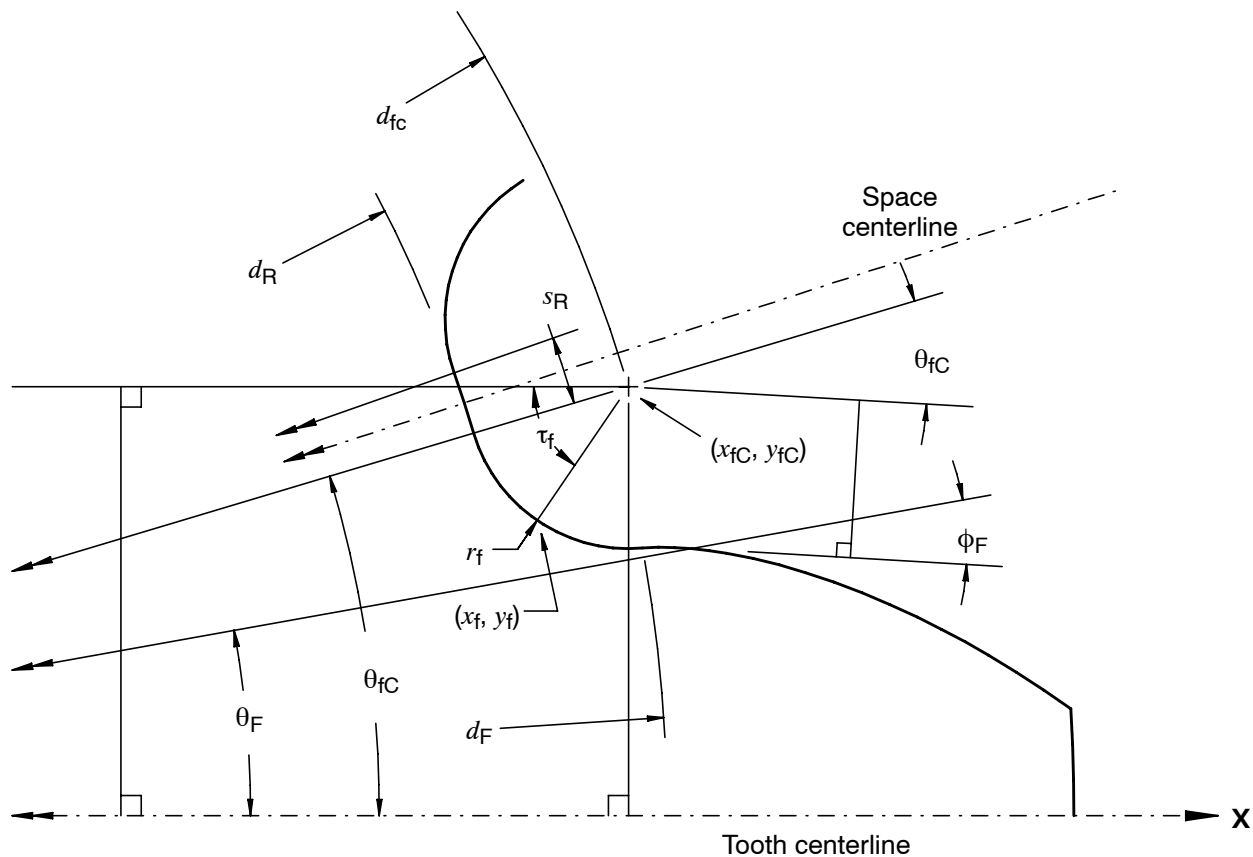


Figure A.5 - Circular arc fillet

A.5 Circular-arc in place of trochoid

See figure A.5. It is a common practice in P/M gear design to introduce a fillet in the form of a single circular arc. In this practice, the arc will start at a tangent point on the root circle and generally end at a tangent point on the involute profile at each side of the tooth space. A fillet of this form simplifies the manufacture of the compacting tool. The selection of the fillet type should consider the following (see figure A.6):

- A small radius may increase stress concentration and reduce tooth bending strength;
- A large radius may introduce interference with the tip of the mating gear;
- A large radius may lead to fillet arcs intersecting outside of the root circle;
- For root diameters smaller than the base circle diameter, a small radius may not give tangent points at both the root circle and the involute profile;
- For profiles that must be undercut to avoid interference with the tip of the mating tooth, there cannot be tangency to the involute. A more complex fillet form is preferred if interference, on one hand, or excessive undercutting, on the other, are to be avoided.

- Circular-arc fillet (shown shallow for clarity)
- Full-fillet radius
- - - - - Trochoid fillet without undercutting
- Trochoid fillet with undercutting

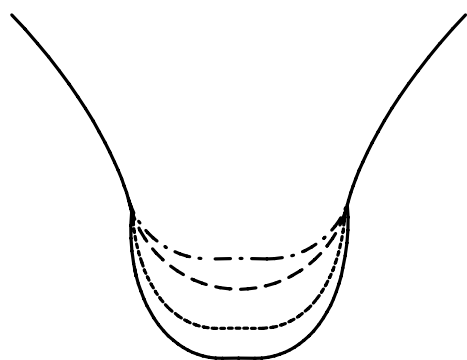


Figure A.6 - Fillets

The fillet radius may be selected so that the two fillets on adjacent teeth form a single continuous arc, constituting a full-fillet radius fillet. This feature will dispose of above items a), c) and in some cases d).

Reduction of the root diameter may help in avoiding item b).

Calculations for determining the size of this full-fillet radius for a specified root diameter are given in A.5.2. If the root diameter is smaller than the base circle diameter, it is not always possible to fit such a fillet to the specified conditions. The calculations indicate if this limiting condition has been reached.

A.5.1 Test for minimum fillet radius

This test is required only if the root diameter is smaller than the base circle diameter. If the root diameter is larger, fillet radii approaching zero will meet the geometry condition of tangency to both the involute tooth flanks and the root circle.

Minimum fillet radius

$$r_{fN} = \frac{d_B^2 - d_R^2}{4d_R}; \text{ but greater than zero} \quad (\text{A.30})$$

A.5.2 Full-fillet radius

Calculation of the full-fillet radius also serves as a test for maximum fillet radius. If the originally specified fillet radius falls between the minimum fillet radius of A.5.1 and the maximum fillet radius calculated below, the calculation of fillet features may proceed. If the original fillet is smaller than the minimum, it must be increased to that value subject to the test in A.8.4. If it is larger than the full-fillet radius fillet, the fillet radius must be reduced to that maximum.

Step 1. Test for the fit of a full-fillet radius fillet:

$$[BT_{ff}] = \left[\frac{\pi}{N} + \frac{d_R}{d_B} - \alpha - (\text{inv } \phi) \right] \quad (\text{A.31})$$

If BT_{ff} is less than 1, the root diameter is smaller than the base circle diameter and a full-fillet radius fillet will not fit the specified gear data.

Step 2. Pressure angle along imaginary involute at the center of the full-fillet radius fillet, ϕ_{bC} :

$$\phi_{bC} = \text{arc sev} [BT_{ff}] \quad (\text{A.32})$$

NOTE: This equation introduces a new trigonometric function, the sevolute function, defined as follows:

$$\text{sev } \phi = \text{sevolute } \phi = \frac{1}{\cos \phi} - \text{inv } \phi \quad (\text{A.33})$$

The “arc sev” or inverse of this function may be found from tables of the function [9] or by the calculation procedure in annex E.