

Undercutting in Worms and Worm–Gears

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American Gear Manufacturers Association



TECHNICAL PAPER

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UNDERCUTTING IN WORMS AND WORM-GEARS

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Abstract

An equation is developed, which can be used to ensure that there is no undercutting in a worm. For worm-gears, the possibility of undercutting depends on many variables, and no simple criterion has been found. Procedures are therefore described for checking whether there is undercutting, and also whether there are other potential problems, such as interference or non-conjugate contact.

Introduction

In the design of worms and worm-gears, it has been common practice for many years to use larger pressure angles when the lead angle is large, and one of the reasons for this practice is to avoid undercutting in the worm. In this paper, a study is made of the conditions when undercutting may occur, and it is shown that these depend not only on the pressure angle and the lead angle, but also on the number of threads. An equation is derived which can be used to ensure that there is no undercutting.

The paper then examines the possibility of non-conjugate contact between the worm and the gear, and of undercutting in the gear. Previous researchers [1-3] have shown that undercutting occurs if the contact lines on the generating surface form an envelope, and the surface extends beyond the envelope. However, to

use this method the hob surface must be described by analytical equations. In the case of the thread-milled worm, the thread surface shape is generally found numerically. And in nearly all practical cases, the hob profile is modified from that of the worm, so analytical equations are unlikely to exist. It was therefore not found possible to develop any simple equation to predict the occurrence of non-conjugate contact or undercutting, but procedures are described for checking any particular gear-pair design. It is shown that there is a maximum value for the gear face-width, beyond which non-conjugate contact will occur at the exit end of the gear tooth. Additional topics discussed in the paper are the possibility of interference at the worm thread fillet, and of non-conjugate contact at the worm thread tip caused by very small pressure angles. And lastly, the paper describes a method for determining whether there will be undercutting in the teeth of the gear.

A number of examples have been considered, some of conventional designs in which the pitch point coincides with the mean point, and some of recess-action designs. It can be concluded from these examples that non-conjugate contact and undercutting are very much more likely to occur in recess-action designs. In addition, it was found that in some recess-action gears, even though there is no undercutting, the depth of the fillet may be greater than that of the active profile, and the fillet may be cut so deeply that

the tooth is considerably weakened. So, although there are some significant advantages to recess action, it is important that the designs be checked for the possible problems discussed in this paper.

Conjugate Action

The meshing of a worm and its conjugate gear has been fully analysed by Buckingham [4,5]. We regard the worm as a rack, since an axial advance of the worm along its axis is kinematically equivalent to a rotation. The directions of the x_w and y_w axes, shown in Figure 1, therefore remain fixed as the worm translates in the z_w direction. The plane $x_w=0$ in the worm will be called the central axial plane, and any parallel plane at $x_w=\text{constant}$ will be called an offset section. We calculate the thread profile in an offset section, and this profile, regarded as a rack, is used to generate the conjugate tooth profile in the gear transverse section $z_g=-x_w$. At a typical point A of a worm profile, the angle ϕ_r between the profile tangent and the vertical will be called the rack pressure angle, and its value is given [6] by the following expression,

$$\tan \phi_r = \frac{(\tan \phi_t \sin \theta - \cos \theta)}{\tan \psi} \quad (1)$$

where θ , ϕ_t and ψ are the polar coordinate, the transverse pressure angle and the helix angle at point A.

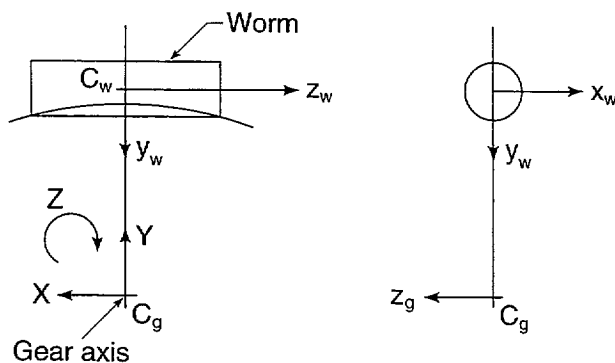


Figure 1. Coordinate axes.

The gear tooth is in contact with the worm profile at point A when the profile normal at A passes through the pitch point P, as shown in Figure 2. The position in space where the contact occurs, i.e. the coordinates of a point on the path of contact, can then be read from Figure 2,

$$X = \frac{(R_{pw} - y_w)}{\tan \phi_r} \quad (2)$$

$$Y = R_{pw} + R_{pg} - y_w \quad (3)$$

$$R_g = \sqrt{X^2 + Y^2} \quad (4)$$

where y_w is the coordinate of worm point A, R_{pw} and R_{pg} are the pitch circle radii, and R_g is the radius of the conjugate point on the gear. When the shape of the worm thread is specified, the axial coordinate z_w of point A is known. The axial translation of the worm and the corresponding rotation of the gear can then be determined, and hence the polar coordinate θ_g of the gear tooth point can be found.

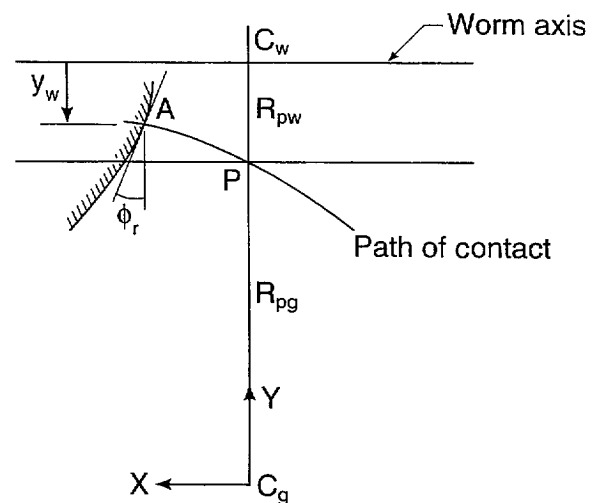


Figure 2. Path of contact in a typical offset section.

Undercutting in the Worm

It is clear from Equation (2) that the coordinate X approaches infinity as ϕ_r tends toward zero. At points where ϕ_r is negative, Equation (2) is no longer valid, and conjugate action is impossible. The locus of points at which ϕ_r is equal to zero is known as the limit of conjugate action. Buckingham has shown [4] that for involute helicoid worms the limit of conjugate action lies along the two horizontal tangents to the base circle, as shown in Figure 3, and that for other types of worm the locus is asymptotic to the same horizontal lines.

Figure 3 is drawn looking in the positive z_w direction (See Figure 1). We are assuming a right-handed thread, and in order to drive the gear clockwise in Figure 1, the worm in Figure 3 must rotate counter-