

**High Speed, Heavily Loaded and Precision Aircraft
Type Epicyclic Gear System Dynamic Analysis by
Using AGMA Gear Design Guidelines Enhanced by
Exact Definition of Dynamic Loads**

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American Gear Manufacturers Association



TECHNICAL PAPER

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

ABSTRACT:

Dynamic analysis of reliable, lightweight, high speed and high power density epicyclic gears requires special effort to predict their maximum power transmitting capacity. This paper focuses on single-stage epicyclic gears of this category. The true definition of gear system power transmitting capacity requires understanding and proper evaluation of its dynamic capacity, as well as a state-of-the-art elasto-dynamic simulation which responds to input data as a fully instrumented test cell would. This paper presents an overview of key considerations, proper application of AGMA standards, and what makes an epicyclic gear system successful in responding to tomorrow's challenging propulsion needs as a fully reliable component of the aircraft.

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HIGH SPEED, HEAVILY LOADED AND PRECISION AIRCRAFT TYPE EPICYCLIC GEAR SYSTEM DYNAMIC ANALYSIS BY USING AGMA GEAR DESIGN GUIDELINES ENHANCED BY EXACT DEFINITION OF DYNAMIC LOADS

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Introduction

The use of geared devices to optimize speeds of the aircraft and driving engine has a sizeable payoff as the power output of the engine increases. However, this benefit cannot be fully obtained unless we comply with the requirements and design state-of-the-art high power density geared devices. The following are required to define the power transmitting capacity:

- Understanding and proper evaluation of dynamic behavior,
- State-of-the-art and elasto-dynamic system simulation,
- Application of mathematical optimization methods to the design,
- Inclusion of all supporting members into the system simulation.

Goals

Achieving gear system quality-characteristic goals for operability, reliability, and power density will require an accurate design system focused on the right issues. For example, adequate operability margin will require accurate predictions for both system and component natural frequencies and modes as well as for system response to transient events. Achievement of reliability and power density goals will require accurate predictions for steady-state and transient dynamic loads and relative velocities.

The ideal simulation, then, would provide accurate

predictions for dynamic loads and relative velocities, based on specification of certain configuration parameters. The utility of such a system depends on the ability of the simulation to not only make accurate predictions, but to make them with respect to those issues of greatest significance.

In the quest toward accurate simulation, the guideline must be to create the simplest model that retains the essential features of the physical system. After a certain model complexity has been reached, it is likely that improvements in accuracy come in ever smaller increments while solution time increases disproportionately.

Although excitation frequencies are much higher than the lowest natural frequency, they are not so high that the duration of loading is in the order of wave traversal times. The problem, therefore, is a structural dynamics problem rather than a wave propagation problem.

The simulation requirements can be met by developing two distinct capabilities: (1) a natural frequency/mode shape analysis, and (2) a direct integration time history analysis. The first capability provides means of comparing natural frequencies to excitation frequencies to ensure adequate margin, but yields relative displacements. The second capability provides predictions for instantaneous positions, velocities, accelerations, forces and moments acting on all bodies in the system as a result of the application of given forces and moments. The predictions, however, are a function of the mass distribution, stiffnesses, and damping assumed in creating both the analysis and the representation of the physical system being modeled. In addition, the integration process must be stable and accurate. For these reasons, it is extremely important that the simulation model be carefully correlated to verify the validity of the predictions.

Background

The first systematic studies of gear system dynamics were carried out by Peterson (Ref. 1) and Buckingham (Ref. 2). Peterson, a known gear dynamics expert of his time, outlined the basic principles of governing equations and the interrelationship of gear body shape and tooth proportions to resonating frequencies and modes of radial, axial and torsional gear body vibrations.

The basic equations created by Peterson have not been modified since 1924 and are used by engineers as corrective tools to explain the unexplainable by the more conservative, approximate or specially-tailored equations.

The Buckingham study in 1931 was based on simple rigid body analysis with massless springs and lumped masses. This study was the first instrument in understanding load transfer dynamics, dynamic loads, and limitations of rigid body dynamics in gears.

As the mid-20th century approached, a new method of evaluating system dynamics began to emerge with Tuplin. His efforts concentrated on merging the vibratory models of Peterson with the load transfer dynamics of Buckingham. The era started with Tuplin's work which generated several models for considering tooth errors, mesh stiffness, masses with variable rate time and location (distance) dependent springs, and the interval (Refs. 3 and 4).

The first detailed documentation for the era of 1960 to 1965 was made by Ozguven and Houser (Ref. 5). They investigated the majority of existing mathematical models which, at that time, exceeded 200 in number. Some of these models (Refs. 6 through 15) were successful in taking the torch one step beyond Tuplin. Successful studies beyond the year 1985 have been published in proceedings of AGMA-FTM, AIAA/ASME-JPC, and ASME-PTG. From 1985 to 1993, a limited number of researchers refined the theories and substantiated their models by experiments (Refs. 16 through 29 and Refs. 35 and 36). The basic systems used by each researcher were extremely similar and were modeled using the ideal of "nothing is rigid and nothing is constant." The systems consisted of a series of springs and responded to input data by time dependent, design oriented, nonlinear and variable spring rates. Thus, the effects of a change in contact ratio and increased tooth deflections (shear, bending, compression, hub, torsion, etc.) independently introduced tooth profile modifications, tooth errors, and fluctuations of the input torque represented a realistic, lifelike simulation via digital computer codes.

Design Considerations of High Power Density Systems

To design a flightworthy, lightweight gear system (for commercial or military use) by using today's tools, force shaft horsepower to weight ratio of 50:1 or greater could only be possible by simulation.

Simulation Base

The fan drive main gear train (Figure 1) equations are based on Newtonian mechanics and kinematics of gears. The motion was expressed with bodies consisting of base diameters and involute forms, rotated so that they are always in contact (Ref. 16).

$$F = ma + k_x = \sum_{i=1}^n F_i \sin(\omega t_i) = F_d \sin(\omega t)$$

$$\text{where } x = \sum_{i=1}^m x_i \quad k = 1, 2$$

- x = systems deformation and deflection (relative distance traveled)
- k = 1, driver k = 2, driven
- i = 1, localized Hertz deformation
- i = 2, deflection due to bending
- i = 3, deflection due to shear
- i = 4, deflection due to normal force
- i = 5, deflection due to deformation of hub
- i = 6, deflection due to torsion of rim and hub
- i = 7, deflection due to other elements
- F_i = individual tooth loads
- F_d = total dynamic load
- t = time

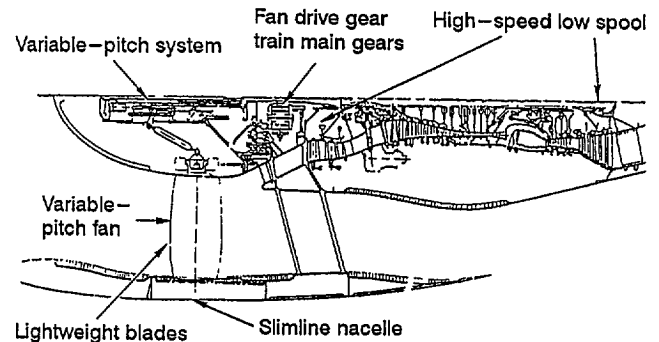


Figure 1 Conceptual Location for Main Gear Train

Profile modifications are expressed as CAM-like deviations from involute profile beginning from the HPSTC (highest point of single tooth contact) or ending at the LPSTC (lowest point of single tooth contact), Figure 2.

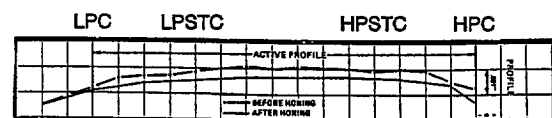


Figure 2 CAM Modified Involute with Tip and Root Relief

Profile errors are expressed as multi-cycle sinusoidal deviations from the line of action or from the tooth surface (Figures 3 and 4).

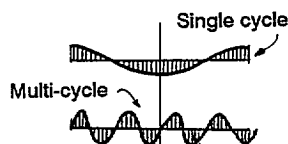


Figure 3

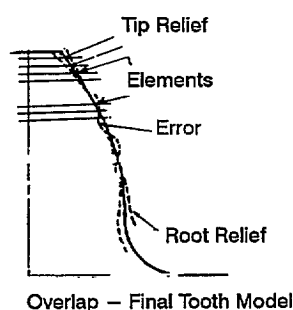


Figure 4

The digitized profile points and tooth deflections are analyzed in a three-dimensional, multi-coordinated rotation system with its speed being equal to the speed of the base diameters. The location of contacting points, number of Hertzian zones, and variation of system stiffness are established by setting iterative computer loops under fully-loaded and deflected conditions. The principle coordinate system was the centerline of gears used to determine the number of contacting pairs. The secondary coordinate system is selected to rotate with the pinion and gear, and the tertiary coordinate system is the local coordinate system of each gear tooth used to determine appropriate deflections of the meshing pair. The fourth coordinate system should be based on the centerline of the planetary system and is used to link the system (gears, bearings, carrier and mount) as one.

In addition to the above, it is emphasized here that all supporting elements and systems must also be included in the model and optimized as one, such as the carrier, bearing, support, mount, lube system, scavenge system and cooling system. All this defines what is expected from the 21st century high power density planetary gearbox for aircraft use.

Design Tools

Methods and tools to design the critical components of a gear system are abundant in number. Several codes exist for gears, gear system dynamics, journal bearings, system simulation, component optimization, structural dynamic and thermal analysis, etc. However, a detailed search showed that none of these codes is sufficient to comply with the special considerations of the aircraft main drive gear systems.

Special efforts concentrated on tooth mesh dynamics and load transfer efficiency. Published studies of NASA, AGMA, ISO and related consortiums were carried steps further. Studies of Buckingham, Cornell, Peterson, Tuplin, Block, Dudley, Alford, AGMA Aerospace Committee, publications of ASME Power Transmission and Gearing Committee, and all known experimental results of United Technologies Corporation in gears, bearings and gear systems were accumulated and used as building blocks for the advancement of design tools and methods.

The following sections discuss the specifics and background of the steps taken to extend the known boundaries of technology in system simulation and gearbox dynamics of high speed and heavily loaded epicyclic gears for aircraft use.

High Involute Overlap Gearing

Industry experience dating back to the early 1920's showed the use and apparent benefits of a high contact ratio (HCR) gear system. The recorded benefits are a reduction in dynamic and static stresses, a reduction in gearbox noise, and system life improvements. These test and production configurations have been documented in the files of the AGMA Gearing Committees.

The configurations are examples of earlier designs of Pratt & Whitney Co. (E. Buckingham), Westinghouse (Peterson and Baud), Wright Aeronautical (Proj. No. XB088), Boeing Vertol (A. J. Lemansky and A. P. Alberti, Navy Contract N00019-69-C-0418), and by R. Drago), Arrow Gear (J. C. Leming), Southwest Research (H. E. Staph), Clark Corporation (R. Green, TW-20/ec 300), Cox Corp. (A. B. Cox), Chrysler Research (K. Buyukataman, NASA/ERDA Contract No. E(11-1)-2749). Most recently, UTC Pratt & Whitney Canada (A. H. Elkholy, 84-DET-65), NASA (TM-103136), AVSCOM (TM 90-C-004), Turbodyne (J. C. Hambric, ASME, 84-DET-157), Dresser Rand (A. Stranford), Lufkin (L. Lloyd), P&I Research Laboratory (Prof. T. Hayashi, ASME, 92-DET-105, ASME, 92-DET-106), UTC-Sikorsky (J. Kish), Boeing (J. Lenski) and NASA (M. Valco) have also contributed extensively during the Advance Rotorcraft Transmission (ART) studies to the leading technology in this field.

Ongoing system studies using advanced codes have shown that benefits could be further improved if the 2.25 contact ratio (CR) in the involute plane is exceeded. The first published studies on a CR greater than 3.00 were made in 1929 by Peterson and Baud (Ref. 30). The test, the purpose of the study, and its effects on greatly improved load carrying capacity are documented in *Mechanical Engineering*, Vol. 51, No. 9. This study also makes extensive use of the results of an earlier study made by Timoshenko and Baud ("Strength of Gear Teeth," *Mechanical Engineering*, Vol. 48, No. 11, 1927).

The advanced codes in use are now pointing out that 3.00 CR could be exceeded by about 15 percent for the Advanced Ducted Engine (ADE) application. However, a CR equal to 3.00 at the full load condition still appears the best overall design, in the elimination of the majority of dynamic impact energy caused by transmission error related gear load

distribution, system weight reduction and overall system life. A brief schematic of the HCR version of successful test gears confirming the above statement is shown in Figure 5. Figure 6 shows the basic logic behind low and high contact ratio gear impact load, using Tuplin's wedge terminology. Figure 7 shows the variation of bending stresses along the contact surface, as investigated experimentally by Peterson and Baud more than 60 years ago. Their studies were also simulated and confirmed by the Pratt & Whitney GENESIS code used in the analysis.

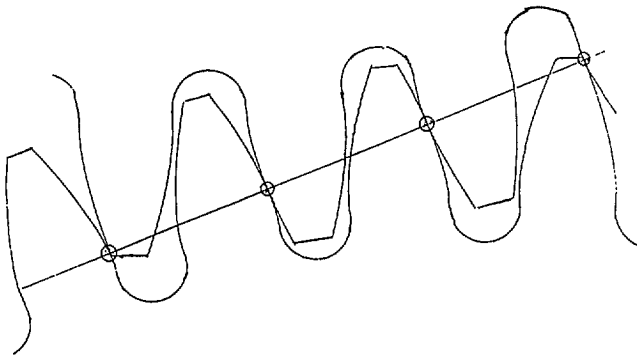
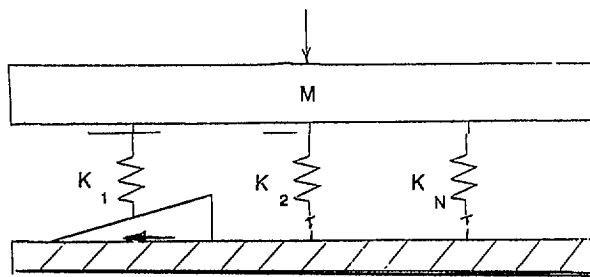


Figure 5 Schematic of High Contact Ratio Version of Successful Test Gears



- (a) $K = k$ = spring rate or stiffness of system
 t = time required to insert wedge (error) or to reach maximum force
 u = displacement of masses during load transfer
 a = acceleration of mass during load transfer
 $u = 1/2 at^2$
 F = applied (tangential load)
 $F_d = F + K(e-u)$ F_d = total dynamic load of motion

Figure 6 Basic Logic of Low and High Contact Ratio Gear Impact Load, Using Tuplin's Wedge Terminology

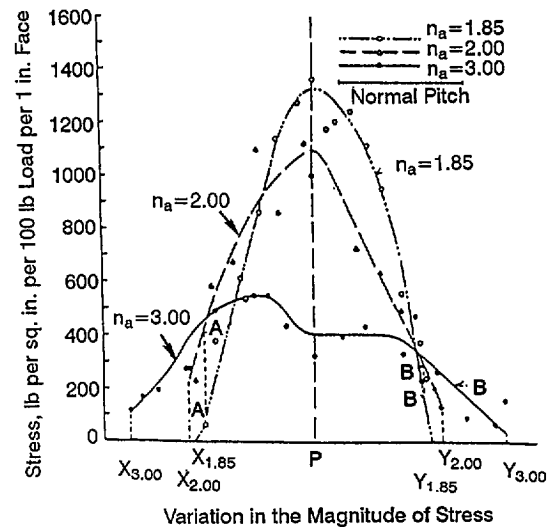


Figure 7 Variation of Bending Stresses Along the Contact Surface, as Investigated by Peterson and Baud

Helical Gear Coordinate System

The cartesian y, z coordinates shown in Figure 8 define the beginning of contact on the faces of a pair of teeth. This is also the deflection of the beginning of the motion, and specified as:

$$S = x - j$$

At the centerline of the contact zone, we have $S = 0$, and thus, $x=j$. This is shown in Figure 9. This figure also defines the center of the y coordinate, $y=0$, which is also called the instantaneous center of rotation. This location coincides with the j th tooth. Thus, at time zero, $S=0, y=0, z=0$, which defines the contact position. The load at time zero by definition is always set equal to zero within the GENESIS code.

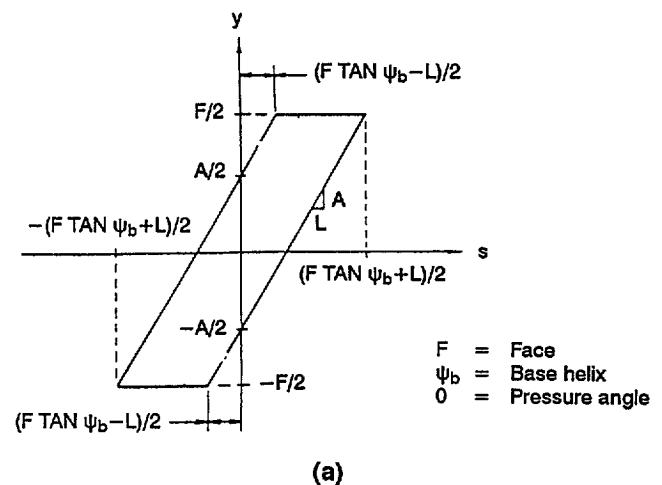
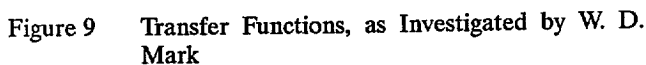
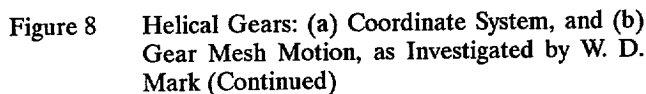


Figure 8 Helical Gears: (a) Coordinate System, and (b) Gear Mesh Motion, as Investigated by W. D. Mark



Critical Issue

carrier and ring gear, as well as some other components) require tailoring of their stiffness and damping characteristics using time dependent equations of motion. This, of course, requires Design Team experience and judgement in system design and simulation. Team efforts to reach the very best are essential in the frontiers of technology advancements.

In this study, the term "stiffness" refers to a function which expresses the rate of application of load and the following changes of TE in attached masses. "Stiffness" is a nonlinear system of differential equations. Similarly, the term "damping" refers to the outcome of the relative motion between load transfer components and/or to the combined effect of the system of masses. "Damping" in this model is also greatly affected by the shear forces of the fluid films which exist within the gear meshes and bearing surfaces.

System Model

The dynamic model of the gear system is set to satisfy design considerations. A detailed description of the model is beyond the scope of this paper. However, a partial schematic illustrating the system model is shown in Figure 10. The number of planets in this model can be any number greater than or equal to one. Figure 10 shows a two-dimensional cross section of one planet and its immediate surroundings. Each component in the system is represented by rotational and translational masses, M_r and M_t .

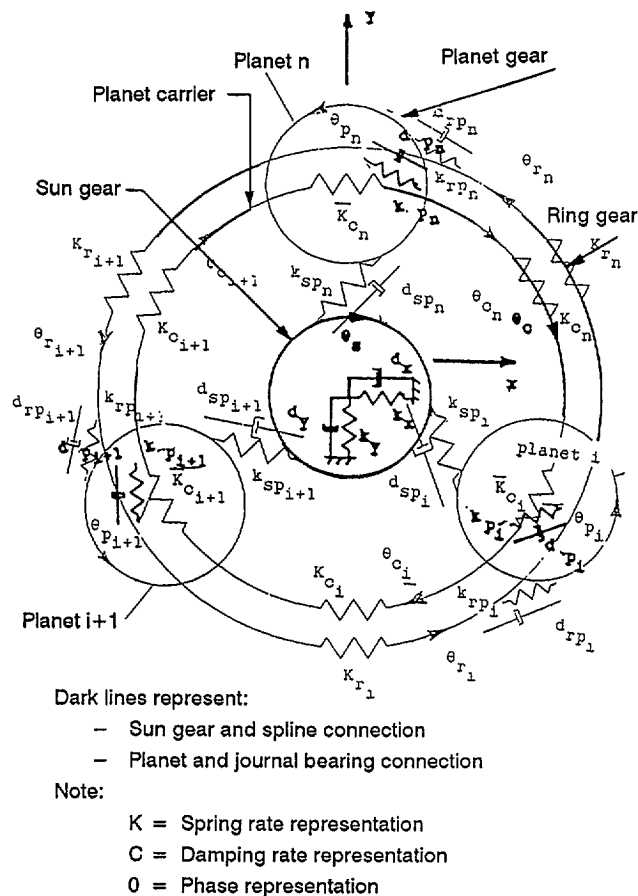


Figure 10 Partial Schematic of System Model