

# A Rayleigh–Ritz Approach to Determine Compliance and Root Stresses in Spiral Bevel Gears Using Shell Theory

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**American Gear Manufacturers Association**



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# **A Rayleigh–Ritz Approach to Determine Compliance and Root Stresses in Spiral Bevel Gears Using Shell Theory**

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

## **ABSTRACT:**

In this paper, a new mathematical model is proposed to predict deflections and root stresses in spiral bevel gears. The tooth shape is modeled as a segment of a shear flexible thick shell of variable rigidity and height corresponding to the spiral bevel gear or pinion tooth dimensions. The shell segment is cantilevered along the circular edge. The shell model is more representative of the spiral bevel tooth geometry compared to a beam or plate model. In addition, the model could be used for both face milled and face hobbed geometries. The compliance computations based on the shell model can be readily integrated into existing computer codes for bevel gear design to determine the load distribution, transmission error, and root stresses. The analysis is performed using the Rayleigh–Ritz approach with algebraic polynomial trial functions. The results are verified with finite element solutions. The results are obtainable on a personal computer and the procedure is computationally much more efficient than the finite element method.

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## A Rayleigh-Ritz approach to determine compliance and root stresses in spiral bevel gears using shell theory

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### INTRODUCTION

Bevel gears are commonly used to transmit motion between angularly disposed shafts. They can be broadly classified into straight bevel gears, spiral bevel gears and hypoids. Straight bevel gears have teeth that radiate from the pitch apex while spiral bevel gears have oblique teeth on which contact begins gradually and continues smoothly from end to end. Spiral bevel and hypoid gears are favored over straight bevel gears in high performance transmissions in automobile, marine and aviation industries because their curved teeth provide smoother and quieter operation along with greater bending resistance. In this paper the ability of the circular cylindrical shell model to represent the salient features of the spiral bevel tooth geometry to determine tooth compliance and root stresses is demonstrated.

The early works on the meshing of bevel gears and the basic relationship of hypoid gears can be found in Wildhaber.<sup>1-2</sup> The work discusses in great detail the method of generation, tooth profile curvature calculations and gear conjugate action. The geometrical characteristics and nomenclature of spiral bevel gears have been documented by AGMA (American Gear Manufacturers Association) and others.<sup>3-5</sup> The Gleason Works Publication provide guidelines for the installation, assembly and inspection of bevel gears.<sup>6-7</sup> The geometry of spiral bevel gears depends on the method of manufacture and various investigators have attempted to characterize the geometric shape of the tooth surface and develop methods to determine the machine settings used to cut the gear.<sup>8-17</sup>

When computers were introduced to the gearing industry in the 1950's, bevel and hypoid gear calculations were among its first applications. Programs were written to calculate machine settings, undercut and to analyze unloaded tooth contact. Since then, computers have played an increasing role in the design, analysis, manufacture and inspection of bevel gears.<sup>18-22</sup> The loaded tooth contact analysis and the finite element stress model<sup>23-25</sup> are important advances in this field

and this paper presents a new model for the bending compliance calculations in the tooth contact analysis programs and outlines its potential for bending strength calculations.

The design of gears includes both the strength determination and a contact analysis which involves the location and movement of the contact zone under load. In spur and helical gears, the earliest gear strength calculations were made using beam theory. Subsequently, plate models have been successfully used in predicting deflections and stresses.<sup>26-29</sup> The use of the annular sector plate to predict deflections and stresses in straight bevel has been recently demonstrated by the authors.<sup>30</sup> In spiral bevel gears however, the longitudinal curvature is significant and it is not possible to use beam or plate theory to adequately represent their flexural behavior. Hence a model based on shell theory is developed to model the geometry and obtain solutions to the deflections and stresses in spiral bevel gears. The spiral bevel tooth geometry can be conveniently represented as a segment of a thick cylindrical shell by taking into account the rigidity variation along the tooth height and facewidth. The effects of shear deformation can become quite significant for small radius to thickness and length to thickness ratios, which are both true for spiral bevel gears. Hence the flexural behavior of spiral bevel gears is modeled employing shear deformation theories<sup>31-32</sup>. The inclusion of variable rigidity and shear deformation effects in the computations poses considerable mathematical difficulties to solve for the deflections and stresses in closed form and numerical solutions are sought.

The application of finite element methods in determining stresses and evaluating compliance in spiral bevel gear design has been demonstrated by Wilcox<sup>33</sup>. A loaded tooth contact analysis based on the actual spiral bevel tooth surface geometry has been demonstrated by Vijayakar et al.<sup>34</sup> The procedure uses a combination of surface integral techniques and finite element methods in the contact analysis. A simulation of the

machine kinematics of the gear generator is carried out based on the precomputed machine settings chosen to cut the gear. A theoretical tooth surface is thus calculated in the mesh generator, which is then used to build the finite element model. While this procedure makes the finite element model very accurate, any small changes in the blank dimensions during preliminary design would require a recomputation of machine settings and recalculation of the surface coordinates. This makes the finite element method in its present form unattractive in an iterative design process. In this paper the proposed model and the method of solution is shown to have considerable advantages. First, the model is based on the tooth and blank dimensions rather than the machine settings. The computations are based on the Rayleigh-Ritz approach which does not require a mesh generator or large computation times and cost. It is thus well suited to iterative computations due to its simplicity and computational efficiency. Finally the use of shell theory to model the flexural behavior of the spiral bevel gear is shown to have comparable accuracy to the finite element solution.

## METHOD OF MANUFACTURE

Most spiral bevel and hypoid gears are manufactured either by the face milling process or the face hobbing process. The two methods typically use different blank designs, cutting tools and contact pattern control for producing their respective tooth geometries. Each process has its advantages and disadvantages and the choice of the preferred process is not clear cut. A detailed discussion of the two types of processes and their respective tooth geometries can be found in Krenzer<sup>35</sup>.

The two manufacturing processes produce two very different tooth geometries. The face milling process employs a circular face mill cutter. In the formate process the cutter is set into position relative to the work such that it cuts the correct spiral angle and pressure angle at the calculating point and sweeps out the tooth form of the gear as it rotates about its axis. The lengthwise tooth form is thus a circular arc of curvature equal to the curvature of the cutter. The gear tooth has straight sides in the normal plane. The pinion is generated conjugate to an imaginary gear called the crown gear. The blanks are designed with tapering depth such that the tooth depth is a function of the distance from the pitch apex. The face hobbing process is a conjugate generation method which employs continuous indexing. The lengthwise tooth curve is an extended epicycloid with teeth are of constant depth.

The two processes thus produce different tooth geometries, one with varying depth and circular curvature and the other with constant depth with the lengthwise tooth curve being an epicycloid. The model shown in this paper describes the face milled geometry. The face hobbled gear geometry can be modeled by assuming constant depth and incorporating the radius of curvature change along the face width.

## MODEL DEFINITION AND ASSUMPTIONS

The shape and the nomenclature of a face milled spiral bevel tooth is shown in Fig. 1. It can be seen that the height of the tooth continually increases from the toe to the heel and the tooth is curved along the lengthwise direction. The tooth base thickness increases from the toe to the heel and the thickness decreases along the tooth height at any point on the face width. Hence, any model to determine the compliance should take into

account the rigidity variation along the face width, the lengthwise curvature, as well as the tooth height taper along the face width.

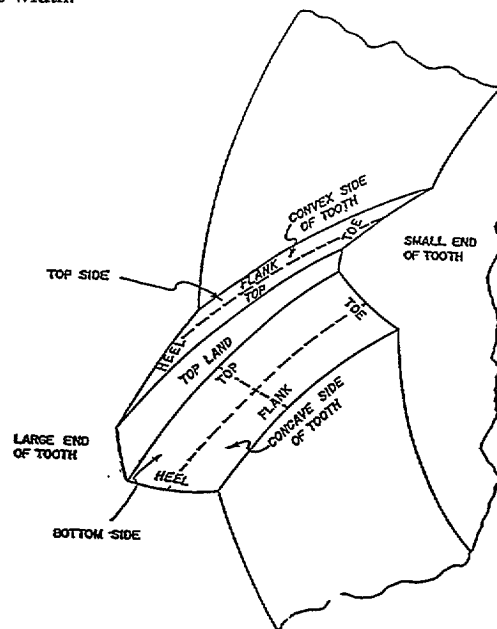


Fig.1 Spiral bevel tooth nomenclature [GWPSD6151B]

Fig. 2 shows the spiral bevel tooth shape modeled as a segment of a thick cylindrical shell, along with the chosen coordinate system. The curved length of the shell segment corresponds to the face width of the gear and the height and thickness of the shell closely approximate the bevel tooth dimensions. The shell segment is clamped along the bottom circular edge. Due to the small radius to thickness and length to thickness ratios, the flexural behavior of the spiral bevel gear is modeled using shear deformation theories. The number of terms to be used in the Ritz series approximation is determined through a convergence study of the free edge deflection curve and the results of the static analysis verified using an ANSYS finite element model. Two types of shear based theories developed by Mindlin and Bhimaraddi, are used to model the flexural behavior of the shell model and the deflections and stresses compared to the finite element model.

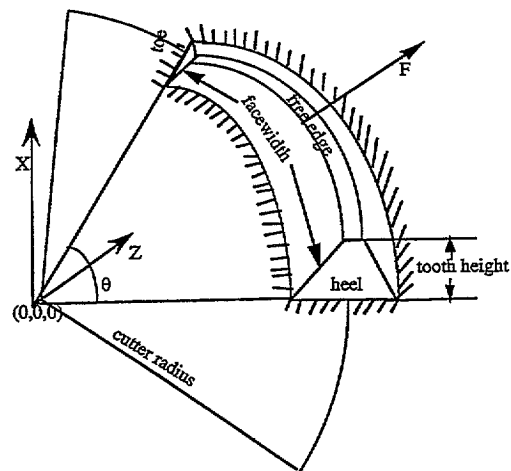


Fig. 2 Shell model of bevel tooth