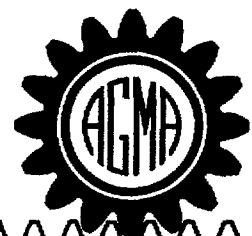


Stress Analysis of Spiral Bevel Gears

A Novel Approach to Tooth Modelling

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American Gear Manufacturers Association



TECHNICAL PAPER

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[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

ABSTRACT:

Spiral bevel gears find extensive application in high performance transmissions such as helicopter drives because their curved teeth provide smoother and quieter operation than straight bevel gear teeth. In this paper, a geometrical approach has been proposed to generate the tooth surface coordinates of spiral bevel gears. The procedure is versatile and can be adapted to any type of spiral bevel gear with appropriate modifications. Various types of spiral bevel gears such as logarithms, circular cut and zerol types are analyzed in this paper.

A new procedure for a theoretical determination of exact tooth load contact line on the surface of the spiral bevel gear tooth, which is vital for estimating the stresses, has been developed. The generalized model developed uses three dimensional finite element method with eight noded isoparametric brick elements.

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STRESS ANALYSIS OF SPIRAL BEVEL GEARS — A NOVEL APPROACH TO TOOTH MODELLING

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1. INTRODUCTION

Spiral bevel gear as a power transmitting element finds extensive applications in engineering industries. For example, they are widely used in automobiles for rear axle drives and in helicopters for rotor transmissions. The suitability of these gears is that their smoothness in operation and large load carrying capacity at high rotational speeds.

The tooth forms of spiral bevel gears are curved in the form of spirals unlike the straight bevel gears. Several spirals are currently used in the design of spiral bevel gears such as logarithmic spiral, the circular cut spiral, and the involute spiral. The logarithmic spiral has the advantage of providing a constant angle between tooth centre line and radial line, at all points along the centre line. The logarithmic spiral bevel gear provides uniform geometrical characteristics for the tooth profile in the transverse plane of the gears. The circular cut spiral, an

approximation of logarithmic spiral, was developed by Gleason works to facilitate relative ease of manufacture.

Huston et al [1-3] have studied the geometrical characteristics and the distortions along the centre line of a circular cut spiral bevel gear. But the study concentrated mainly on crown (flat) gears. Litvin et al [4] have proposed a method for the generation of spiral bevel gears based on parallel motion of a straight line that slides along two mating ellipses. An approach based on differential geometry is explained in the reference [5].

The pitch surface of a bevel gear is a cone and rolling pitch cones have spherical motion. Hence, the exact representation of a bevel gear tooth is possible only on the surface of a sphere and is difficult to visualize. Therefore, the Tredgolds approximation which reduces the problem to one of spur gears is used in which the tooth is represented as a plane surface.

In the present work, spiral bevel gear tooth model is generated using Tredgolds approximation. Since a detailed study is not available about the contact line, which is more vital for the stress analysis using FEM, a geometrical method is developed to generate the same on the surface of the tooth. The stress analysis was conducted on different types of spiral bevel gears such as logarithmic, circular cut and zerol types using three dimensional finite element method.

2. LIST OF SYMBOLS

a	Addendum
d	Dedendum
E	Modulus of elasticity
F	Face width
i	Speed ratio
m	Module
N	Number of teeth
p	Circular pitch
P_b	Base pitch
P_N	Normal base pitch
P_x	Axial pitch
S	Thickness of gear tooth
β	Spiral angle
δ	Pitch angle
ν	Poisson's ratio
ρ	Material density
θ	Angle of rotation

2.1 Attached Words :

B	Referring radii along back cone plane
F	Referring radii along front cone plane

2.2 Suffixes :

i	Refers a section along the face width of the tooth
j	Refers a section in the particular section
o	Along pitch circle
b	Along base circle
t	Transverse section
n	Normal section

(There are other notations which are not mentioned above, but are defined at appropriate places in the text)

3. GEOMETRIC MODELLING OF SPIRAL BEVEL GEAR TOOTH

In this work the geometric modelling of logarithmic spiral is explained first, followed by circular cut and zerol types with suitable modifications.

3.1 Logarithmic Spiral :

The logarithmic spiral has the advantage of providing a constant spiral angle at any point along the spiral curve and is shown in Fig.1. The equation for logarithmic spiral is written as

$$r = r_i e^{\mu(\theta - \theta_{IN})} \quad (1)$$

where r_i = inner radius of the spiral
 r = radius at any point on the spiral
 θ = radial angle at any point on the spiral about the centre line
 θ_{IN} = radial angle made by the inner radius with the center line
 $\mu = \cot \beta$

A logarithmic spiral is developed for the given spiral angle β between the inner radius r_i and the outer radius r_o in a plane sector and then the sector is rolled to form a cone with O as apex and $2\delta_o$ as included angle, where δ_o is the pitch angle of the gear. Each spiral starts from one of the nodal points at the first section of the tooth at the toe side. The inner radius of the spiral is the distance of the point measured from the apex of the front cone and the spiral ends at the back cone on the heel side.

Corresponding to any point P in a bevel gear tooth, two cones can be assumed with apexes O and O' (Fig.2) and are called front cone and back cone respectively. The notations used to represent the various

distances of a point on a bevel gear tooth, based on a front cone, a common central plane and a back cone are also shown in the Fig.2. These terms are used frequently in the description of the generation procedure of spiral bevel gear tooth in the following paragraphs.

Assuming the apex of the front cone as the origin for the coordinate system, the profile of the tooth at the first section of the tooth (section at the toe side) is generated following Tredgold's approximation [6]. The generated coordinates of the first section fall on a plane of the back cone at the toe and are used to generate the entire surface profile along the tooth length.

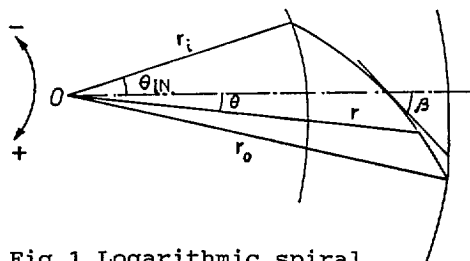


Fig.1 Logarithmic spiral

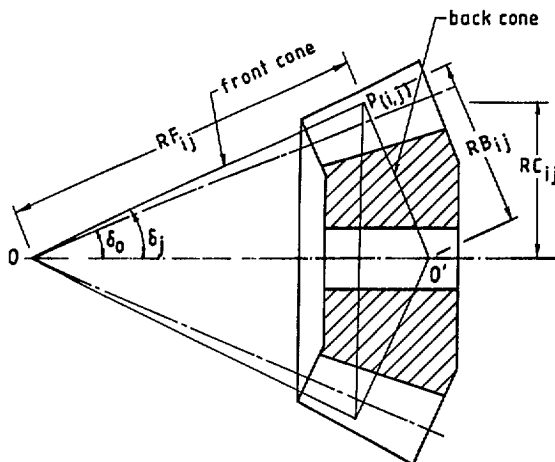


Fig.2 Front and back cones in a bevel gear

Several such sections are considered at equidistant positions along the face width of the tooth and the respective coordinates to describe these subsequent sections are generated from those of the first section.

A right handed logarithmic spiral bevel gear tooth is modelled in the present work.

In a logarithmic spiral, the spiral angle is constant on the plane of a front cone but it is assumed to be varying with the back cone radius at every section. The spiral angle β_j at back cone radius radius RB_j is given as

$$\tan \beta_j = (RB_j / RB_0) \tan \beta_0 \quad (2)$$

where β_0 is the spiral angle measured at the pitch line of the gear.

Through each point in the first section, a spiral is generated on the surface of the respective front cone. The calculated spiral angle β_j at the point where the spiral starts, is assumed to remain the same from the toe to the heel side of the tooth along the generated logarithmic spiral. The initial radius of the spiral is the distance of the point from the apex O. The radius of the spiral at each subsequent section is calculated as follows with reference to the Fig.3.

Making use of the coordinates of the points in the first section, the following values are defined

$$\delta_0 = \tan^{-1}(N_1/N_2) \quad (3)$$

$$\delta_j = \tan^{-1} \left\{ \left[(X_{1j}^2 + Y_{1j}^2) \right]^{1/2} / Z_{1j} \right\} \quad (4)$$

for i=1 ; j=1,n1

$$RF_{1j} = \sqrt{X_{1j}^2 + Y_{1j}^2 + Z_{1j}^2} \quad (5)$$

for j=1,n1

$$RF_{10} = m_1 N_1 / (2 \sin \delta_0) \quad (6)$$

$$RF_{i0} = RF_{10} + (i-1) F / (n-1) \quad (7)$$

for i=1,n

$$RF_{ij} = RF_{1j} RF_{i0} / RF_{10} \quad (8)$$

for i=1,n ; j=1,n1